Response to Anonymous Referee #1

Comments from Referee #1

In this paper, the authors mainly developed a partial wavelet coherency method, for identifying the relationship between variables. It is an important issue but also a difficult problem for geo-data analysis, and the method developed would be helpful for the data analysis in geosciences. The following comments are suggested to be considered for further improving the paper:

Comment #1:

(1) In lines 108-110: the "sufficient number" should be clarified, as it has a big influence on the uncertainty estimation, that is, what number is sufficient? Furthermore, the reason of using first-order autocorrelation coefficient for MC simulation should be explained and discussed.

Response #1:

Many thanks for your review and positive general comment.

To address the "sufficient number" issue, different combinations of r1 (first-order autocorrelation coefficient) values (i.e., 0.0, 0.5, and 0.9) were used to generate 10 to 10 000 AR(1) series with three, four and five variables. Our results indicate that the noise combination has little impact on the PWC values at the 95% confidence level as also found by Grinsted et al. (2004) for the BWC case (data not shown). The relative difference of PWC at the 95% confidence level compared to that calculated from 10 000 AR(1) series decreases with increase in number of AR(1) series. When the number of AR(1) is above 300, very low maximum relative difference (e.g., <2%) is observed (Fig. RC1 which will be put in the Supplement as Fig. S1 of Sect. S3). Therefore, a replication of 300 seems to be efficient for significance test. If calculation time is not a barrier, however, greater replication number such as ⩾1000 is recommended. This will be added into the revision.

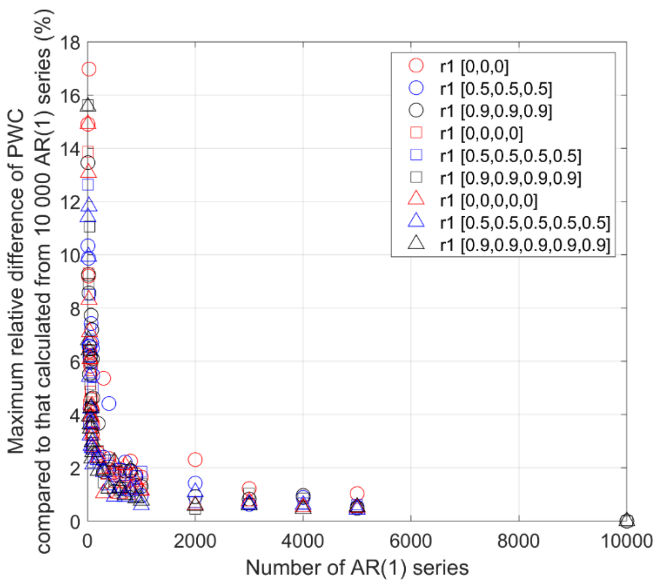

Figure RC1. Relationship between maximum relative difference (%) of PWC compared to that calculated from 10 000 AR(1) series (surrogate dataset) versus the number of AR(1) series during the significance test using the Monte Carlo test. Number of scales per octave is 12.

The first-order autocorrelation coefficients (r1) in brackets refer to those for the response variable (first), predictor variable (second), and excluding variables (third and onwards).

"The first-order autoregressive model (AR(1)) is chosen because it can be used to simulate most geoscience data very well (Grinsted et al., 2004; Si and Farrell, 2004; Wendroth et al., 1992)"

Comment #2:

(2) Lines 121-122, some theoretical lines can be provided to show the difference between Eq. (9) and Eq. (14).

Response #2:

The difference between Eq. (9) and Eq. (14) will be explained by derivation of PWC in the case of one excluding variable from Eq. (1).

So, when only one variable (e.g., $Z1$) is excluded, Eq.(9) ($\rho^2_{y,x\cdot Z} = \frac{\left|1-R^2_{y,x\cdot Z}(s,\tau)\right|^2 R^2_{y,x}(s,\tau)}{\left(1-R^2_{y,Z}(s,\tau)\right)\left(1-R^2_{x,Z}(s,\tau)\right)}$)

can be written as

$$\rho^2_{y,x\cdot Z1} = \frac{\left|1-R^2_{y,x\cdot Z1}(s,\tau)\right|^2 R^2_{y,x}(s,\tau)}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)} \qquad \text{(RC1)}$$

Based on equations (2) in our paper,

$$\rho^2_{y,x\cdot Z1} = \frac{\left|1 - \frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\,\overleftrightarrow{W}^{Z1,Z1}(s,\tau)^{-1}\,\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\overleftrightarrow{W}^{y,x}(s,\tau)}\right|^2 \frac{\left|\overleftrightarrow{W}^{y,x}(s,\tau)\right|^2}{\overleftrightarrow{W}^{y,y}(s,\tau)\,\overleftrightarrow{W}^{x,x}(s,\tau)}}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left|\overleftrightarrow{W}^{y,x}(s,\tau) - \frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}\right|^2}{\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\frac{1}{\sqrt{\left(\overleftrightarrow{W}^{y,y}(s,\tau)\overleftrightarrow{W}^{x,x}(s,\tau)\right)^2}}\left|\overleftrightarrow{W}^{y,x}(s,\tau) - \frac{\overleftrightarrow{W}^{y,Z1}(s,\tau)\overline{\overleftrightarrow{W}^{x,Z1}(s,\tau)}}{\left(\sqrt{\overleftrightarrow{W}^{Z1,Z1}(s,\tau)}\right)^2}\right|^2}{\left(1-R^2_{y,Z1}(s,\tau)\right)\left(1-R^2_{x,Z1}(s,\tau)\right)}$$

$$= \frac{\left| \frac{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}} - \frac{\underset{W}{\leftrightarrow}^{y,Z1}(s,\tau)\ \overline{\underset{W}{\leftrightarrow}^{x,Z1}(s,\tau)}}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{Z1,Z1}(s,\tau)}\ \sqrt{\underset{W}{\leftrightarrow}^{Z1,Z1}(s,\tau)}} \right|^2}{\left(1-R_{y,Z1}^2(s,\tau)\right)\left(1-R_{x,Z1}^2(s,\tau)\right)}$$

$$= \frac{\left| \frac{\underset{W}{\leftrightarrow}^{y,x}(s,\tau)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}} - \frac{\underset{W}{\leftrightarrow}^{y,Z1}(s,\tau)}{\sqrt{\underset{W}{\leftrightarrow}^{y,y}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{Z1,Z1}(s,\tau)}} \cdot \frac{\overline{\underset{W}{\leftrightarrow}^{x,Z1}(s,\tau)}}{\sqrt{\underset{W}{\leftrightarrow}^{x,x}(s,\tau)}\sqrt{\underset{W}{\leftrightarrow}^{Z1,Z1}(s,\tau)}} \right|^2}{\left(1-R_{y,Z1}^2(s,\tau)\right)\left(1-R_{x,Z1}^2(s,\tau)\right)}$$

$$= \frac{\left|\gamma_{y,x}(s,\tau)-\gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}\right|^2}{\left(1-R_{y,Z1}^2(s,\tau)\right)\left(1-R_{x,Z1}^2(s,\tau)\right)} \qquad \text{(RC2)}$$

Namely, when only one variable (e.g., $Z1$) is excluded, Eq.(9) can be written as

$$\rho_{y,x\cdot Z1}^2 = \frac{\left|\gamma_{y,x}(s,\tau)-\gamma_{y,Z1}(s,\tau)\overline{\gamma_{x,Z1}(s,\tau)}\right|^2}{\left(1-R_{y,Z1}^2(s,\tau)\right)\left(1-R_{x,Z1}^2(s,\tau)\right)} \qquad \text{(RC3)}$$

Eq. (RC3) will be added to revision as Eq. (14), and the derivation process for this equation will be added to the supplementary.

In the case of one excluding variable ($Z = \{Z_1\}$), Mihanović et al. (2009) suggested that the PWC can be calculated by an equation analogous to the traditional partial correlation squared (Kenney and Keeping, 1939). Their equation is the same to Eq. (14 or RC3). Unfortunately, Ng and Chan (2012a) might have misinterpreted the equation of Mihanović et al. (2009) and developed Matlab code for calculating PWC using the equation expressed as

$$\rho_{y,x\cdot Z1}^2 = \frac{\left|R_{y,x}(s,\tau)-R_{y,Z1}(s,\tau)\ R_{x,Z1}(s,\tau)\right|^2}{\left(1-R_{y,Z1}^2(s,\tau)\right)\left(1-R_{x,Z1}^2(s,\tau)\right)} \qquad \text{(RC4) (or 15 in the revision)}$$

where $R_{y,x}(s,\tau)$, $R_{y,Z1}(s,\tau)$, and $R_{x,Z1}(s,\tau)$ are the square root of $R_{y,x}^2(s,\tau)$, $R_{y,Z1}^2(s,\tau)$, $R_{x,Z1}^2(s,\tau)$, respectively. $R_{y,Z1}^2(s,\tau)$ and $R_{x,Z1}^2(s,\tau)$ can be calculated from Eq. (10) by replacing y and x with their corresponding variables. Eq. (15) has been widely used to calculate PWC in case of one excluding variable (Aloui et al., 2018; Altarturi et al., 2018; Jia et al., 2018; Li et al., 2018; Mutascu and Sokic, 2020; Ng and Chan, 2012b; Rathinasamy et al., 2017; Wu et al., 2020).

Note that complex coherence and real coherence are involved in the numerators of Eqs. (14) and (15), respectively, while the denominators are exactly the same. Further comparison indicates that Eq. (RC4) underestimates PWC value relative to Eq. (14) unless $\gamma_{y,x}(s,\tau)$

and $\gamma_{y,Z1}(s,\tau)\ \overline{\gamma_{x,Z1}(s,\tau)}$ in Eq. (14) are collinear (i.e., their arguments are identical) under which the two equations produce the same PWC values. Differences between Eqs. (14) and (15) will be discussed further using both artificial data and real dataset. For the comparison purpose, we refer to Eqs. (14) and (15) as new method and classical method, respectively.

Therefore, the differences in PWC values calculated from the two methods are context-specific. As the Referee #2 mentioned, although the difference between the Mihanovic et al. (2009) model (Eq.15) and the proposed model (Eq.14) are small, i.e., the difference of PWC values is only 0.03 for the artificial data, but Eq.14 produces PWC closer to 1.

In addition, the comparison of these two methods using real data indicated that the difference between the two methods can be large. As an example, Figure RC2 and Figure RC3a shows big differences of PWC values between these two methods at scales of around 12 months (1 year). Mean PWC values by the new method were consistently higher than the classical method, and the differences ranged from 0.4 to 0.6 around the scale of 1 year (Figure RC3b).This highlights that the new method produces more accurate results than the classical method.

In the revision, we will incorporate these discussions either in the main body of the paper or in the supplementary.

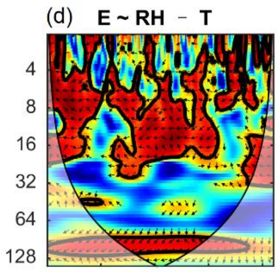

Figure RC2. Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) calculated by the new method. (subplot of Figure 3d in the revision)

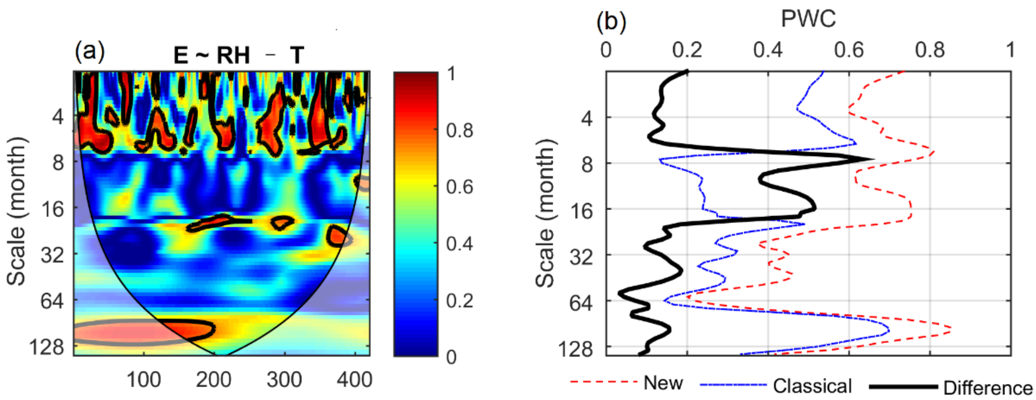

Figure RC3. Partial wavelet coherency (PWC) between evaporation (E) and relative humidity (RH) after excluding the effect of mean temperature (T) using the classical method (a) and differences in PWC between the new method and classical method as a function of scale (b).

Comment #3:

(3) Regarding the structure, is it more suitable to reorganize the Section 3 and 4, that is, the artificial data and their results are analyzed and discussed in Section 3, while those of real data are analyzed and discussed in Section 4?

Response #3:

Thanks for the good suggestion on paper structure. In the revision, we will follow the order of data description, data analysis, results and discussion for each of artificial dataset and real data. To reduce the length of this paper, we will take the suggestion from Referee #2 to remove the real data related to soil water content by adding more about the introduction of the wavelet methods and in-depth discussion of the advantages and weaknesses of the new method.

Thanks again for your constructive comment.

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