

1 Response to reviewer comments

2 We have already responded at length to the reviews by Han and Tian of the first draft of our  
3 comment (that earlier response starts on the following page of this document). We are not going  
4 to address all of the feedback provided in that review, but we do want to highlight some changes  
5 we made in the manuscript in light of their review.

- 6 1. In the introduction, in response to the review by Han and Tian of the draft of this  
7 comment, we removed the wording about different definitions of  $\alpha$  and focused on the  
8 incorporation of advective effects in the sigmoid function.
- 9 2. In section 2, we addressed the role of empirical versus physically-based models as well as  
10 calibration. This topic was raised in the review by Han and Tian of the draft of this  
11 comment, and we felt it was appropriate to address it here.
- 12 3. In section 3, we discussed the argument made in the review by Han and Tian of the draft  
13 of this comment, namely that the flat part of the sigmoid curve only appears very near  
14  $y_H=1$ .
- 15 4. In section 5, we changed the wording regarding how “normal” it is for wet advection to  
16 occur near  $y_H=1$ . At the end of the section we added two notes. First, an expression of the  
17 desirability of handling advection in a CR formulation, and then a note that advection  
18 plays an important role even for  $y_H<1$ .
- 19 5. In Section 6 we re-worded the summary of the argument in item 4 above.
- 20 6. We made multiple revisions to the reference citations.

21

22 Response to “Review of HESS-2020-310 ‘Comment on: A review of the complementary  
23 principle of evaporation: From the original linear relationship to generalized nonlinear functions  
24 by S. Han and F. Tian” (Reviews written by S. Han and F. Tian)

25

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#### 34 Introduction

35 We thank S. Han and F. Tian for their thoughtful review (hereafter, “HT2020b”) of our comment  
36 (hereafter, “CSQ2020”) on Han and Tian (2020; hereafter “HT2020”) and appreciate this  
37 continued discussion of the complementary principle (CP). In CSQ2020, we agreed that the  
38 Sigmoid Generalized Complementary (SGC) formulation is a serious development in CP  
39 research that deserves careful consideration and analysis. However, we concluded that it was not  
40 superior to other recent developments in the CP (e.g., Brutsaert, 2015; Crago et al. 2016; Crago  
41 and Qualls, 2018; Szilagyi et al., 2017, Ma and Szilagyi, 2019). HT2020b was structured around  
42 four claims, which we will discuss in order.

#### 43 HT2020b Claim 1

44 HT2020b argue that two different approaches are both common and valuable in hydrology  
45 research. The first consists primarily of “calibrating parameters for the fitting of observed points  
46 and proposing a method to determine the parameters *in priori*.” The second consists primarily of  
47 developing “approaches...carefully conducted on a physical basis.” We agree--methods that  
48 consistently and accurately reproduce measurements are the most valuable. However, we find the  
49 second type of models to be more likely to generalize well and to apply well outside the  
50 validation range. We also acknowledge the reviewers’ efforts as much as possible to ground their  
51 own research on a physical basis. We agree both methodologies should be explored, but would  
52 much prefer to proceed with physically-based approaches when possible.

#### 53 HT2020b claim 2

54 Second, HT2020b address interpretation of the CP in conditions where large-scale advection or  
55 entrainment of free-atmosphere air partially disconnect the atmospheric boundary layer (ABL)  
56 from the condition of the surface. CSQ2020 argued that the CP is no longer valid under these  
57 conditions. That is, the logic of the CP requires that the ground and ABL are connected, so that  
58 the condition (temperature, humidity, wind speed, etc.) of the atmosphere is adjusted to the

59 condition of the surface, particularly the availability of moisture at the surface. We agree that it is  
60 possible, in principle, to extend a method originally formulated as a CP equation so that it applies  
61 under conditions dominated by these large-scale conditions. Han and Tian (2018; hereafter  
62 “HT2018”) attempt to do this by arguing that over wet surfaces actual regional evaporation  $E$   
63 and Penman evaporation  $E_{\text{pen}}$  are nearly identical so that if  $E_{\text{pen}}$  is increased by dry advection,  $E$   
64 would increase at essentially the same rate. We agreed in our comment that this is possible, but  
65 that it implies that the CP is invalid because the conditions in the ABL are disconnected from  
66 those at the surface. This brings the argument back to claim 1, because if the SGC works under  
67 these conditions, it is not because it captures the physical processes, but because it successfully  
68 matches the data.

69

#### 70 HT2020b claim 3

71 In HT2020 and HT2018, experimental data from around the world are presented to demonstrate  
72 the existence of the three-stage pattern they advocate. CSQ2020 noted that other formulations,  
73 such as that of Brutsaert (2015) could also be said to have three comparable phases, and that the  
74 claim to have a horizontal upper (wet-surface) limit to the third stage is not supported by these  
75 data. HT2020b responded that the flat portion (derivative of zero) only strictly applies at a single  
76 point on the curve, so that graphs of data points would not necessarily reveal the flatness of the  
77 curve. This is a perfectly logical argument, but it means that the primary evidence for a proposed  
78 flat third-stage is not empirical but theoretical.

#### 79 HT claim 4

80 The most powerful theoretical defense of the flat third stage of the SGP is found in HT2018, in  
81 which they derive slopes for the SGP curve at  $x_{\text{min}}$  and  $x_{\text{max}}$ , the dry and wet limits, respectively.  
82 HT2020b wrote that the SGC equation can be expressed  $E/E_{\text{pen}}=f(E_{\text{rad}}/E_{\text{pen}}, m, n, x_{\text{min}}, x_{\text{max}})$ ,  
83 where  $E_{\text{rad}}$  is the first term of  $E_{\text{pen}}$ . But HT2020b stated that, in HT2018,  $E_{\text{rad}}/E_{\text{pen}}$  was treated as  
84 the only independent variable, with the others as parameters. HT2018 and HT2020b were not  
85 obligated to include  $x_{\text{min}}$  as an important variable that can be calculated independently for each  
86 data point as proposed in our papers (Crago et al. 2016; Crago and Qualls, 2018; Szilagyi et al.,  
87 2017, Ma and Szilagyi, 2019). However, CSQ2020 noted that the assumption that  $E_{\text{rad}}/E_{\text{pen}}$  was  
88 the only variable in  $f$  ruled out any version of our “rescaled” CP formulation. Incorporation of  
89 this variable  $x_{\text{min}}$  into the CP actually changes the functional form of the CP, which presumably  
90 could change the slope, particularly at the lower limit.

91 The first step in the derivation by HT2018 (after defining  $E/E_{\text{pen}}$  as a function of  $E_{\text{rad}}/E_{\text{pen}}$  only)  
92 was to take partial derivatives of  $E$  with respect to  $E_{\text{rad}}$  and  $E_{\text{aero}}$  (i.e., the second term of  $E_{\text{pen}}$ ),  
93 resulting in equation (17) of HT2018. CSQ2020 found this problematic because the process did  
94 not consider  $x_{\text{max}}$  (or  $x_{\text{min}}$ , but we will focus on  $x_{\text{max}}$  in this paragraph) to be a variable in this  
95 process. The partial derivatives would have involved more terms, such as  $(\partial E/\partial x_{\text{max}})(\partial x_{\text{max}}/\partial E_{\text{rad}})$   
96 which would not be easy to analyze. Treating  $x_{\text{max}}$  as only a parameter resulted in (17). But later  
97 in the derivation, HT2018 claimed that  $\partial x_{\text{max}}/\partial E_{\text{rad}}$  is not zero; this claim led directly to the flat  
98 third stage of the SGC curve. But CSQ2020 noted that, if  $x_{\text{max}}$  is a constant or parameter, this

99 derivative must be zero. HT2020b responded that  $x_{\max}$  was in fact treated as a parameter, not a  
100 variable, but also that “ $x_{\max}$  is thought to vary with the environment,” and “ $x_{\max}$  is not  
101 independent of  $E_{\text{rad}}$ .” These quotes seem to support the critique of CSQ2020 that  $x_{\max}$  is treated  
102 as both a constant and as a variable in the same derivation. If  $\partial x_{\max} / \partial E_{\text{rad}}$  is not zero, then  $x_{\max}$   
103 must be treated as a variable when the partial derivatives are taken in the first step of the  
104 derivation.

105 To their credit, HT2020b do acknowledge that the limits to the CP are not well understood. Their  
106 surmise that this is due to the relative roles of advection and surface wetness at  $x_{\max}$  seems  
107 plausible.

#### 108 Summary

109 The CP is a fascinating concept. The principle can be stated in one or two sentences and in  
110 equations with only a few variables, but the application of the principle and interpretation of the  
111 variables is surprisingly complicated and some of the concepts are elusive. We have learned a  
112 great deal in thinking through the issues raised by these authors. We find at the end of this  
113 process that there are significant areas of agreement between us and HS2020b, and decreasing  
114 areas of disagreement. Specifically, we agree that both largely empirical and process-based  
115 approaches are valuable, and that large-scale advection must have an impact on the CP. But,  
116 while we appreciate the contributions of S. Han and F. Tian to this research, we still do not find  
117 arguments for the SGC formulation of the CP to be convincing.

118

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143 Comment on: “A review of the complementary principle of evaporation: From the original linear  
144 relationship to generalized nonlinear functions” by S. Han and F. Tian

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154

155 **Abstract**

156 The paper by Han and Tian reviews the history of developments in the complementary  
157 relationship (CR) between actual and potential evaporation and introduces the generalized  
158 complementary principle (GCP) developed by the authors. This comment assesses whether the  
159 GCP: 1) Can give reasonable results from a wide range of surfaces worldwide; 2) is supported by  
160 experimental data that verify the three-stages of evaporation implicit in the GCP, particularly in  
161 the wet-surface limit; 3) has been proven to be correct by the authors in a previous paper; and 4)  
162 is supported by model studies showing that wet surfaces occur predominantly during periods of

163 large-scale moisture convergence. The assessment finds that arguments in favor of the GCP  
164 deserve to be taken seriously, but ultimately remain unconvincing.

165

## 166 **1. Introduction**

167 Han and Tian (2020) (hereafter HT20) provide important insights into the growing body of  
168 literature regarding the Complementary Relationship (CR) of evaporation, and serves well as an  
169 accessible review of the literature. The sigmoid formulation (their equation 13), a key feature of  
170 their Generalized Complementary Principle (GCP) (Han and Tian, 2018; hereafter HT18) is  
171 presented and defended in their paper.

172 Two of the present authors (Szilagyi and Crago, 2019, hereafter SC19) wrote an earlier comment  
173 critiquing the sigmoid function for violating established physical principles (see also the reply by  
174 Han and Tian, 2019a). After further consideration, the present authors recognize that the sigmoid  
175 curve proposed by HT18 and HT20 is intended to incorporate the effects of both the CR and of  
176 large-scale advection under wet-surface conditions. While we do not find the sigmoid function to  
177 have a strong theoretical or empirical basis, we agree with HT18 and HT20, at least in principle,  
178 that this need not violate any laws of nature. (Note that, unless otherwise indicated, all notation  
179 herein follows that of HT20.) the Priestley and Taylor (1972) line at  $x_H = E_{rad}/E_{pen} = 1/\alpha = 1/1.26$   
180 that appears in HT20 (their Figure 3), could be intended by HT18 and HT20 to mark a reference  
181 point on the graph, rather than to establish a limiting value that cannot be crossed. Unless  
182 otherwise noted, all notation herein follows that of HT20—see also Tables I and II for notation  
183 and variable names. Also, the role of a related (but different) adjustable parameter (also named

184  ~~$\alpha$  seems to be used in the formulation primarily to adjust the shape of the sigmoid curve, rather~~  
185 ~~than to set a limit on wet surface evaporation.~~

186



187 Table I Variables used

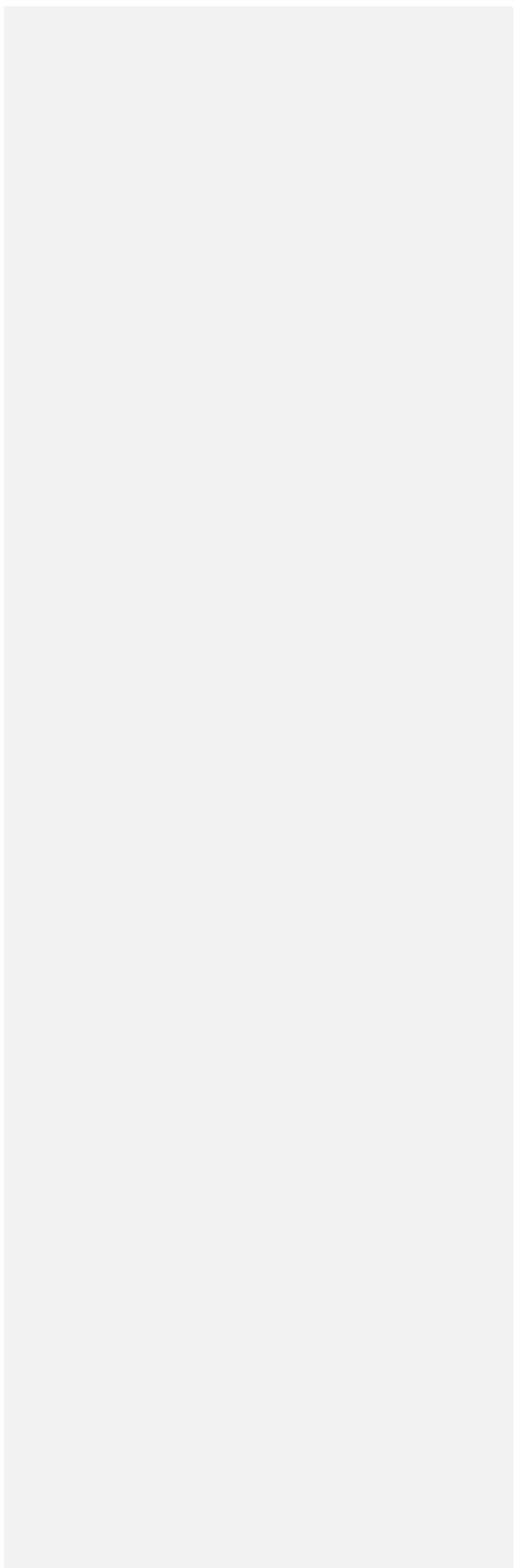
$b$	A GCP model parameter that adjusts the shape of the sigmoid function
$E$	Actual regional evaporation rate
$E_{aero}$	The second term of Penman's (1948) equation, related to the drying power of the air.
$E_{MT}^{max}$	Hypothetical maximum value of $E$ that would occur from a wet patch in an otherwise completely desiccated region
$E_{Pen}$	Evaporation rate from Penman's (1948) equation
$E_{PT}$	$\alpha E_{rad}$ proposed by Priestley and Taylor (1972) for a wet regional surface with minimal advection
$E_{rad}$	The first term of Penman's (1948) equation, <u>with the slope of the saturation vapor pressure typically taken at the measured air temperature (HT18, c.f., Slatyer and McIlroy equivalent to the equilibrium evaporation rate of Slatyer and McIlroy (1961))</u>
$E_{PT}^{T_{ws}}$	Value of $E_{PT}$ found if the slope of the saturation vapor pressure curve is estimated at the wet surface temperature, $T_{ws}$ (see Szilagyi et al., 2016)
$f(E_{rad}/E_{Pen})$	A hypothesized function of $E_{rad}/E_{Pen}$
$x_H$	$E_{rad} / E_{Pen}$
$x_m$	$E_{PT}^{T_{ws}} / E_{MT}^{max}$ the value of $E_{PT}^{T_{ws}} / E_{Pen}$ at which $E$ goes to zero in the rescaled CR (Crago et al, 2016)
$x_{max}$	Parameter that sets the maximum value $x_H$ can reach
$x_{min}$	Parameter that sets the value of $x_H$ at which $y_H \rightarrow 0$
$y_H$	$E / E_{Pen}$

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$\alpha$	The Priestley & Taylor (1972) parameter
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189 Table II. Abbreviations

BC4	Boundary condition 4: $d(E/E_{pen})/d(E_{rad}/E_{pen}) = dy_H/dx_H \rightarrow 0$ as $y_H \rightarrow 1$
CR	Complementary Relationship (between actual and potential evaporation) proposed by Bouchet (1963)
GCP	Han and Tian's (2020) Generalized Complementary Principle
HT18	Han and Tian (2018)
HT20	Han and Tian (2020)
SC19	Szilagyi and Crago (2019)

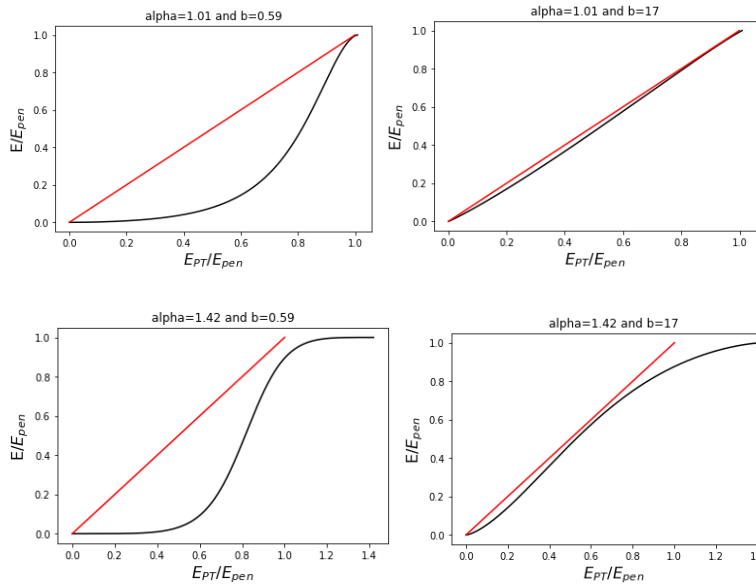
190  
 191 The most controversial feature of the sigmoid function is the slope of the curve at the wet-surface  
 192 limit. Namely, it requires that  $d(E/E_{pen})/d(E_{rad}/E_{pen}) = dy_H/dx_H \rightarrow 0$  as  $y_H \rightarrow 1$  (hereafter, this  
 193 boundary condition will be denoted "BC4"). That is, rather than a complementary relationship,  
 194 BC4 requires that  $E$  and  $E_{Pen}$  are equal and that  $E$  exactly follows any variability by  $E_{Pen}$  in the  
 195 wet surface limit.

196 BC4 deserves careful attention. A major purpose of this comment is to show that there are some  
 197 indications such behavior can occur, but when it does it is a consequence of large-scale processes  
 198 that disconnect the regional land surface from the overlying atmosphere, thus violating the basic  
 199 assumptions behind the CR (namely, that atmospheric and surface conditions are tightly linked  
 200 through surface fluxes). In light of this, corrections to the CR attempting to account for these  
 201 ~~exceptional~~ cases will ~~inevitable~~ly result in a formulation that does not accurately represent  
 202 ~~ordinary~~ (minimally-advective) conditions.

203 This comment will consider the evidence for the following four claims made by HT18 and HT20  
204 in support of the sigmoid function and BC4: First, that the function works reasonably well to  
205 model evaporation from sites around the world; second, that data from these sites support a  
206 three-stage evaporation process and BC4, both of which are required by the sigmoid function;  
207 third, that HT2018 have provided a rigorous proof of the boundary conditions underlying the  
208 formulation; and fourth, that a partial explanation of BC4 has been provided by the study of  
209 Lintner et al. (2015).

## 210 **2. Claim regarding modeling results**

211 First, it is clear that the sigmoid function has been used successfully to model evaporation from  
212 flux stations around the world (see HT18). It is quite a flexible formulation that can match a wide  
213 range of data patterns on an  $(x_H, y_H)$  graph. Calibrated values of  $a$  and  $b$  published in HT18 (their  
214 Table 5) range from about 1.01 to 1.49 and from 0.59 to 17, respectively. Figure 1 shows the  
215 sigmoid function for the four combinations of these extreme parameter values (with  $x_{\min}=0$  and  
216  $x_{\max}=1$ ). These show the wide range of possible curve shapes; allowing  $x_{\min}$  and  $x_{\max}$  to take other  
217 fixed values further increases the flexibility. Such an equation is likely to fit many datasets well,  
218 if tuning is permitted. ~~Of course, any~~ While we believe the ultimate goal of CR research should  
219 be a physically-based formulation that can work well without requiring local  
220 calibration of parameters, there is, nevertheless, value in formulations that can reliably match  
221 datasets with local calibration (including several of our respective publications).



222

223

224 Figure 1. The sigmoid function (black curves) and the Priestley-Taylor line ( $\alpha=1.26$ , straight line  
 225 in red) for the most extreme parameter values documented in HT18. The scales of the horizontal  
 226 axes differ.

227

228 **3. Claim regarding empirical support for three evaporation stages and for BC4**

229 Second, there does seem to be some empirical support for different slopes at different positions  
 230 on  $(x_H, y_H)$  graphs (HT18, their Table 3). However, the curve proposed by Brutsaert (2015) also  
 231 proposes a shallow slope for small  $y_H$ , (stage 1) a steep slope in the middle (stage 3), and a less  
 232 steep slope near  $y_H=1$  (stage 3). Similar behavior is also possible with the rescaled models of the  
 233 present authors. The stage 3 slopes at large  $y_H$  values (HT18, Table 3) would be near zero  
 234 according to BC4, but are generally near 1 instead. HT18 directly address BC4 with data in their  
 235 Figure 6, which plots empirical data along with red curves resulting from the sigmoid function

236 relating  $E/E_{PT}$  to  $E/E_{pen}$ . The sigmoid function curves show  $E/E_{PT}$  increasing as  $E/E_{pen}$  increases,  
 237 until  $E/E_{PT}$  reaches a peak and then begins to decrease with further increases in  $E/E_{pen}$ .  
 238 Correlational evidence for this downturn is given by HT18, but the actual data plotted do not  
 239 visibly follow the downturn in  $E/E_{PT}$  in either panel of Figure 6; the dramatic downturn in the red  
 240 curve Figure 6(a) (the left panel) certainly is not matched by the data. While the limiting  
 241 behavior would only be expected very near  $y_H=1$ , this very fact makes it difficult to argue that  
 242 this behavior exists when nearly all data points on the graph fall below  $y_H=1$ . Similarly, some  
 243 values of parameters for the sigmoid function make the flattening of the third stage nearly  
 244 indistinguishable and therefore inconsequential (i.e., the top two panels of Figure 1).

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#### 245 4. Claim regarding the derivation by HT18

246 Third, the derivation by HT18 is inconclusive. The derivation begins [HT18, their Eq. (8)]:

$$247 E = (E_{pen}) * f(E_{rad} / E_{pen}), \text{ where } E_{pen} = E_{rad} + E_{aero} \quad (1)$$

248  
 249 where  $f$  is a function of  $(E_{rad}/E_{pen})$ . Partial derivatives of  $E$  were taken from Eq. (1) with respect  
 250 to  $E_{rad}$  and  $E_{aero}$ . Further manipulations of these derivatives resulted in the four boundary  
 251 conditions corresponding to the sigmoid curve (HT18). The function  $f(E_{rad}/E_{pen})$  in Eq. (1) could  
 252 include constants or parameters (for instance  $\alpha$ ,  $x_{min}$ , and  $x_{max}$ ), whose “correct” values can be  
 253 found by calibration, after which they must be treated as constants. This means that, once the  
 254 parameters are determined, the shape of  $f(E_{rad}/E_{pen})$  is also determined.

255  
 256 Unfortunately, this leads to two problems. First, the present authors' work with the "rescaled" CR  
 257 (Crago et al., [20172016](#), Szilagyi et al., 2017, Crago and Qualls, 2018) gives evidence that the  
 258 variable  $x_m = E_{PT}^{Tws} / E_{MT}^{max}$ , ( $x_m$  is our own notation) related to the value of  $E_{PT}^{Tws} / E_{pen}$  at

259 which  $E$  goes to zero, is in fact a variable, not a constant. It must be calculated for each  
260 individual data point, and it results in a significant re-arrangement of the data. It could have been  
261 included in Eq. (1) by writing Eq. (1) as:  $y_H = f(x_H, x_m)$ . By taking derivatives without including  
262 the impact that a variable  $x_m$  might have, HT18 assumed from the beginning that  $E/E_{pen}$  does not  
263 vary with  $x_m$ , so a variable  $x_m$  boundary condition could not possibly arise from this derivation.  
264 On the other hand, if  $x_m$  is in fact a significant variable (as the papers cited above suggest), it  
265 could impact the entire derivation, but particularly the two dry-limit boundary conditions.

266  
267 The parameter  $x_{max}$  is the maximum value  $x_H$  can reach, and is usually taken by HT18 and HT20  
268 to be  $1.26^{-1}$ , where 1.26 is the commonly-accepted value for the Priestley and Taylor parameter  
269  $\alpha$ . To prove that  $dy_H/dx_H \rightarrow 0$  as  $y_H \rightarrow 1$  (the most controversial finding of the derivation), HT18  
270 had to show that  $\partial x_{max}/\partial E_{rad}$  evaluated at  $y=1$  cannot be 0 (see the paragraph starting at the  
271 bottom of page 5054 and ending at the top of page 5055 of HT18). But if Eq. (1) is true,  $x_{max}$  has  
272 to be treated as a constant, so the partial derivative must be 0. It is impossible for  $x_{max}$  to be a  
273 constant for the purpose of taking derivatives of Eq. (1), but a variable when evaluating  
274  $\partial x_{max}/\partial E_{rad}$ . Thus, there is a logical inconsistency hidden in this derivation. SC19 showed that, if  
275 the Priestley-Taylor  $\alpha$  (equivalent here to  $1/x_{max}$ ) is actually a constant, HT18's derivation does  
276 not result in a specific required value for  $dy_H/dx_H$  at  $y=1$ . Thus, the boundary condition  
277  $dy_H/dx_H \rightarrow 0$  as  $y_H \rightarrow 1$  does not follow from (1).

278  
279 To sum up consideration of the derivation, three of the four boundary conditions (slope and  
280 intercept at the point where  $y_H \rightarrow 0$ , and slope as  $y_H \rightarrow 1$ ) are doubtful due to the assumptions made  
281 when (1) was used as the definition of  $E$ .

282

283 **5. Claim regarding support from the modeling study of Lintner (2015)**

284 HT18 cite the modeling results of Lintner et al. (2015) in support of BC4. This study used a  
285 steady-state model that captured the key physical processes affecting evaporation. Model results  
286 show decreases in both  $E_{Pen}$  and  $E$  as soil moisture approaches saturation, similar to the behavior  
287 required by BC4. According to Lintner [et al. \(2015\)](#); see also HT18), large-scale horizontal  
288 moisture convergence decreases  $E_{Pen}$  by increasing atmospheric humidity, and at the same time it  
289 increases precipitation and thus soil moisture content. Near the wet limit, water availability  
290 matters less than  $E_{Pen}$  in determining  $E$ , so  $E$  and  $E_{Pen}$  decrease at the same rate. Thus, at the point  
291 of saturation,  $E=E_{Pen}$ , and  $d(E/E_{Pen})/d(E_{PT}/E_{Pen}) = 0$ , apparently satisfying BC4.

292 ~~But note that the normal (i.e., minimal moisture convergence, divergence, or advection) behavior~~  
293 ~~for a wet surface is~~ CR researchers have long held that  $E=E_{Pen}=E_{PT}$  for a wet regional surface  
294 (e.g., Brutsaert, [1982](#), [2005](#), 2015). The only way to get BC4-type behavior is to impose a large-  
295 scale process that causes  $E_{Pen}$  to differ from this value. That is, BC4 is not describing the drying  
296 process and the CR at all; rather, it is describing what happens when large-scale processes cause  
297 the CR simply does not apply to break down. The scenario described by Lintner et al. (2015)  
298 requires a clear disconnect between the land surface processes and the overlying atmospheric  
299 conditions, violating the central assumption of the CR (e.g., Brutsaert, 1982, 2005).

300 It need not be the case that nearly-saturated surfaces coincide with moisture convergence in the  
301 real world outside of steady-state models. Nearly-saturated surface conditions can exist under a  
302 range of large-scale patterns, including positive, negative or negligible moisture convergence or  
303 advection. This is the case because soil moisture content varies at larger time scales than most



304 other components of the surface water and energy budgets (e.g., Sellers et al, 1992), so nearly-  
305 saturated surface conditions can persist after a period of moisture convergence has ended.  
306 Furthermore, saturated surfaces can occur from other processes, such as thunderstorms driven by  
307 surface heating.

308 A formulation that can account for varying advection would be desirable, and such methods have  
309 been previously proposed (e.g. Parlange and Katul, 1992). As already discussed, evidence that  
310 the sigmoid curve does this successfully is lacking. Furthermore, it seems to address advective  
311 effects only for wet surfaces, while advection clearly affects drying surfaces as well.

## 312 **6. Conclusions**

313 HT18 and HT20 have marshaled several empirical and theoretical arguments in support of their  
314 proposed sigmoid formulation of the CR. The range of arguments and data sources used is  
315 impressive, and the present authors only recently recognized the specific nature and the impact  
316 of this challenge to other CR formulations. There is little doubt that some aspects of their  
317 argument are true, including the ability of their formulation to match numerous experimental  
318 datasets. Nevertheless, the specific boundary conditions leading to the sigmoid function are not  
319 well-supported by empirical data; the derivation of the boundary conditions by HT18 was  
320 inconsistent regarding which model values are constants and which are variables; and the  
321 argument that large-scale processes require adoption of BC4 fails because it implies that  
322 essentially makes a disconnect between the land surface and the near-surface atmospheric  
323 conditions is the norm under near-wet-surface conditions, thus changing the shape of the CR  
324 with no solid theoretical or empirical arguments that it is in fact the norm, the exception (large-  
325 scale processes dominating land surface processes in determining near-surface atmospheric  
326 conditions) into the rule, and in doing so, it violates the assumptions of the CR. The CR should

327 ~~ideally only be used under circumstances where advection is minimal.~~ Attempts to adjust for  
328 other conditions (e.g., Parlange and Katul, 1992) are possible, but should not over-ride  
329 consideration of the basic CR concept. This may require developing specific conditions for  
330 screening data.

331 There does not seem to be consensus in the research community on any of the boundary  
332 conditions of the CR except for  $x_H=1$  when  $y_H=1$ . The current authors find the evidence for a  
333 variable  $x_m$  to be strong. This value can be calculated separately for each data point and it leads  
334 to a rescaling of the  $x_H$ -axis, and a resulting reduction in the scatter of the data points (Crago and  
335 Qualls, 2018).

336 While the sigmoid formulation is clearly the result of a serious and substantial research program,  
337 the difficulties with it described here are serious enough that we cannot see it as an improvement  
338 over other recent CR formulations.

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