



1 **The era of Infiltration**

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8
9 **Abstract**

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11 Inspired by a quotation from Howard Cook in 1946, this paper traces the evolution of the
12 infiltration theory of runoff from the work of Robert Horton and LeRoy Sherman in the
13 1930s to the early digital computer models of the 1970s and 1980s. The reasons for the
14 popularity of the infiltration theory are considered, as well as its impact on the way in which
15 hydrological responses were perceived by several generations of hydrologists.
16 Reconsideration of the perceptual model for many catchments, partly as a result of the
17 greater appreciation of the contribution of subsurface flows to the hydrograph indicated by
18 tracer studies, suggests a more precise utilisation of hydrological terms and, in particular,
19 that the use of runoff and surface runoff should be avoided.

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24 *Some future historian of the development of scientific hydrology will probably be*
25 *tempted to call the present period the "era of infiltration." At any rate, the*
26 *preoccupation of contemporary hydrologists with "the infiltration theory of runoff,"*
27 *and the vast amount of energy they have expended in an effort to turn this concept to*
28 *practical account, will certainly be put down as a distinctive feature of our times.*

29 Howard L. Cook, 1946 (p.726)

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31 **The Background to the Era of Infiltration**

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33 This quotation from Howard L. Cook has stimulated this paper which has the aim of trying to
34 understand why the "infiltration theory of runoff" came to have such an impact on
35 hydrological understanding and analysis from the 1930s onwards, particularly in the work of
36 American hydrologists such as Robert Elmer Horton¹, LeRoy Kempton Sherman², Waldo
37 Smith³, Cook himself and many others. In particular to consider the question of why, when
38 in many parts of the United States overland flow is just not observed that often, the
39 infiltration theory of runoff achieved such a widespread acceptance both in the US and
40 elsewhere. The literature in relation to infiltration and surface runoff is, however, vast and
41 a complete review is not possible. I hope to have brought out the most important points and
42 references relevant to this question, particularly from some of the earlier publications.

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¹ (1875-1945), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Horton,_Robert_Elmer

² (1869-1954), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Sherman,_LeRoy_K.

³ (1900-1994), Executive Director of AGU from 1944-1970, see <https://honors.agu.org/waldo-e-smith-1900-1994/>



44 The context of the Cook quotation was the report of the AGU Committee for Infiltration for
45 1946, chaired by G. W. Musgrave who worked in the Soil Conservation Service at that time.
46 This committee had a number of sub-committees: on Infiltration and the Physics of Soil
47 Moisture and of the Infiltration Process; on Infiltration in Relation to Ground Water; on
48 Infiltration in Relation to Snow and Its Physical Properties; on Infiltration in Relation to Surface
49 Runoff; on Infiltration in Relation to Irrigation; and on Infiltration in Relation to Evapo-
50 transpiration and the Consumptive Use of Water. Infiltration was therefore considered to be
51 both central and fundamental to hydrological understanding⁴. The preface to the Cook
52 article provided by Musgrave is pertinent to our question:

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54 *“In the early phases of the development of a new concept, it is common to find*
55 *considerable diversity of thought among the workers in that field. Subsequently,*
56 *through the exchange of ideas, and particularly through the development of factual*
57 *evidence, abstract ideas are crystallized into specific entities. Progress in the*
58 *development of the field is increased, and practical application of ideas that originally*
59 *were abstract now proceeds with greater and greater success.*

60

61 *The concept of infiltration as a factor modifying runoff phenomena is still relatively*
62 *new. Discussions quite diverse in their conclusions abound in the literature. Is it not*
63 *true that at least some of the diversity of thought is due to diverse interpretations of*
64 *terms and definitions? Indeed, it would seem that there is need for re-examination of*
65 *some of the very fundamentals of the problem.*

66

67 *Many have realized during the past several years that there is great need for*
68 *clarification of thought in this relatively new phase of hydrology. Many have realized*
69 *that whatever may be done to promote thinking and expression in terms that are*
70 *specific and are understood by all other workers is certain to result in improved*
71 *research and improved application of research findings.*

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73 *This paper should do much in the way of promoting unanimity in use of terms, of*
74 *opinion as to their significance, and of clarity of concept.” (Musgrave, 1946a, p726)*
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76 The Sub-committee on Infiltration in Relation to Surface Runoff was chaired by Howard Cook,
77 the other members being W. W. Horner, R. A. Hertzler, G. A. Hathaway, and Walter B.
78 Langbein⁵. Cook had been one of the principal assistants of Robert Horton at the Horton
79 Hydrologic Laboratory in Voorheesville, New York⁶.

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The popularity of the infiltration theory

⁴ It is worth noting that well before the development of the infiltration theory in the US, there had been experimental studies of infiltration, particularly in relation to irrigation practices (e.g. Muntz et al., 1905) and at the plot scale (e.g. Houk, 1921) as well as the model of infiltration of Green and Ampt (1911).

⁵ (1907-1982), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Langbein,_W

⁶ Howard L. Cook graduated in Civil Engineering from the State University of Iowa in 1929 then worked at the Horton Hydrological Laboratory as assistant to Robert Horton from 1929 before moving to the Soil Conservation Service in 1934 where he was in charge of hydraulic research. He later worked as an engineer for the Department of the Army. I have not been able to find a full obituary of his life and career.



84

85 Following the quotation at the head of this paper, Cook starts his outline of the subject by
86 considering why the infiltration concept had become so popular:

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88 *“There have, of course, been logical reasons for this remarkable interest in the subject.*
89 *As in all sciences, many have been attracted to it simply because of its newness.*
90 *Another class—and the one that has participated most eagerly—is composed of those*
91 *intrepid practicing engineers who are obliged to make the runoff estimates upon which*
92 *depend the failure or success of costly flood control, water supply, and similar works.*
93 *Still another group has been intrigued by a purely scientific interest, sharpened by the*
94 *fact that the calculation of runoff is the central problem of the science of hydrology*
95 *and involves all phases of the hydrologic cycle. Among these are scientists in the fields*
96 *of soils, plants, and meteorology. As a result of these various motivations, vast*
97 *amounts of labor have been expended - much of it misdirected - and many*
98 *exaggerated claims have been made, to be countered, naturally, by the disparaging*
99 *murmurs of the “old guard,” and other important lines of investigation have been*
100 *temporarily slighted. But real progress has been made. Better estimates of runoff are*
101 *now possible than could be made previously. Problems that would not yield at all to*
102 *earlier methods are now soluble, albeit the solutions are sometimes only rough*
103 *approximations. The inescapable conclusion is that a tool of considerable practical*
104 *value has been added to the equipment of the hydrologist.” (Cook, 1946, p727)*

105

106 This quotation already reveals some quite modern elements of the sociology of an inexact
107 science. The infiltration concept provided a new paradigm for thinking about runoff. It did
108 so in a *rational* way *“simply by providing a physically correct concept of the runoff process”*
109 (p.730), but which also provided the engineer with a tool that could be usefully applied to
110 provide better estimates of runoff for design purposes (even if sometimes only *rough*
111 *approximations*). I do wonder if any of that old guard were murmuring ... but should you not
112 be able to see the surface runoff occurring during storms to apply this type of analysis
113 properly?

114

115 **Surface and subsurface runoff**

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117 Cook, in fact, almost immediately recognises the difficulty of applying the concept in
118 practice in a section on surface and subsurface runoff. He notes that:

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120 *“The runoff from an area is the water flowing from it over the surface of the Earth,*
121 *either in streams or as overland flow. Part of this water has never been below the*
122 *surface. This is called surface runoff. Another part has previously passed into the*
123 *Earth and subsequently returned to the surface. This is called subsurface runoff...”*
124 (op. cit. p.728).

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126 He continues:

127 *“(1) Only surface runoff can be directly determined from Infiltration data. (2) When*
128 *runoff contains subsurface flow, the gaged discharge cannot be used to derive*
129 *infiltration data for the area unless the surface runoff can be separated from the total.*



130 (3) *In general, there is no way of separating surface and subsurface runoff when only*
131 *records of the rates of flow are available.*" (op. cit. p.728)

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133 There is also an interesting comment that:

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135 *"A normal stream. carries both surface and subsurface flow in proportions varying*
136 *widely from time to time. During floods most of the water discharged from deep-soiled*
137 *drainage basins is ordinarily made up of surface runoff. However, in areas of low*
138 *storage capacity (such as thin-soiled basins) a large proportion of the flood water may*
139 *consist of subsurface runoff.*" (op. cit., p.728)

140

141 The reasoning behind this statement is not clear. It implies an expectation that catchments
142 with thin soils and small storage capacities would be associated with higher infiltration
143 capacities and higher downslope transmissivities such that there could be a greater
144 contribution of subsurface stormflow. However, the reasoning might have run more along
145 the lines that high storage capacity will mean a longer mean residence time so that any
146 infiltrated water would simply not be able to contribute within the time scale of the
147 hydrograph. Cook also notes later that in deeper soils when water tables are low in summer,
148 infiltrating water may not actually reach the saturated zone.

149

150 In fact, the role of subsurface runoff production was being promoted more generally at this
151 time. Charles R. (Chuck) Hursh⁷, Director of the Coweeta watershed experiments in North
152 Carolina, had long been promoting the idea that in places where overland flow was only
153 rarely seen, such as in the forests of the Appalachians, the hydrograph was necessarily
154 dominated by direct channel precipitation and subsurface flows, with only slow responses
155 observed in boreholes (Hursh, 1936, 1944; Hursh and Brater, 1941). It is also not as if
156 hydrologists did not realise that in different parts of the US there was less expectation of
157 overland flow. In a national review of flood runoff published during the era of infiltration
158 Hoyt and Langbein (1939) noted, with some surprise, that: *"To those who are acquainted*
159 *with the flood-producing possibilities of isolated storms of from 10 to 12 inches [250-*
160 *300mm] in humid areas, the absence of flood-runoff under single storm-experiences of the*
161 *same magnitude on steep mountain slopes of parts of the southern coast range [in*
162 *California] is amazing"* (p.172). They continue:

163

164 *"Although the small plots may indicate the absence of direct run-off and the*
165 *differences between rainfall and runoff an absorption of between 15 and 20 inches,*
166 *there is a rapid passage of a part of the infiltrated water into stream channels, either*
167 *through the relatively shallow earth-mantel or through the upper parts of the*
168 *shattered bedrock. To the extent that the observations and deductions are correct,*
169 *the flood-hydrograph in these areas is composed largely of ground-water which has*
170 *concentrated very quickly as to time superimposed on which is a small amount of direct*
171 *runoff with irregularities closely following irregularities in the maximum rates of*
172 *precipitation. This condition may also apply on other parts of the country where floods*
173 *occur although studies on small areas indicate very high infiltration capacities."* (Hoyt
174 and Langbein, 1939, p.174)

⁷ (1895-1988), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Hursh,_Charles_R



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176 That the infiltration concept was used much more widely however was undoubtedly due to a
177 number of factors. The first was that it claimed to be *rational* or physically-based; the second
178 was the simplicity of calculating amounts of runoff given information about rainfalls and
179 infiltration capacities; the third was the strong and rather combative character of Robert
180 Horton.

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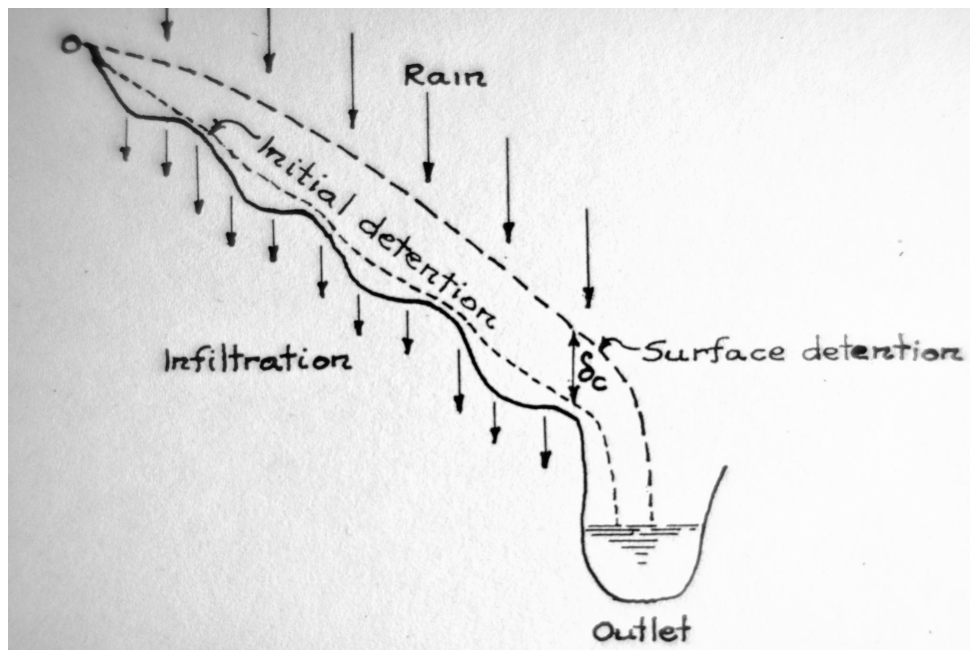
182 Horton's had earlier stated his view of the soil surface acting as a "separating surface" or
183 "sieve" such that:

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185 *"Infiltration divides rainfall into two parts, which thereafter pursue different courses*
186 *through the hydrologic cycle. One part goes via overland flow and stream channels to*
187 *the sea as surface runoff; the other goes initially into the soil and thence through*
188 *ground-water again to the stream or else is returned to the air by evaporative*
189 *processes."* (Horton, 1933 p.446)

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Figure 1. Half-Section of a Small Drainage Basin Illustrating Runoff Phenomena (Vertical Scale Greatly Exaggerated) (from Horton, 1935, with original caption)

It is essentially in this statement that Horton laid the foundation of the infiltration theory concept. He appears to buy in completely to the idea that storm hydrographs are produced by overland flow at this point. This is also clear from his 1935 monograph on Surface Runoff Phenomena, from which Figure 1 is taken. This is perhaps an example of the pragmatics of applications outweighing the information from direct observations (Horton was working as a



202 consultant by the 1930s). This is also evidenced in his paper on Remarks of Hydrologic
203 Terminology later published in the Transactions of the AGU in 1942. He starts by saying that:

204

205 *“When a science is advancing rapidly, as is hydrology today, especially when it is*
206 *changing from an adolescent or qualitative to an adult or quantitative basis, new*
207 *terms are needed in particular for the following two purposes: (1) To give expression*
208 *to new ideas and concepts; (2) to give more definite, specific, quantitative meaning to*
209 *terms and concepts heretofore chiefly qualitative.”* (Horton, 1942, p.479)

210

211 However, in what follows it is clear that Horton’s primary purpose is to favour his own
212 terminology over that of others. There are a number of entries of this type (infiltration rate
213 v. infiltration capacity; recharge v. accretion; plot v. plat⁸), but in the current context the one
214 on subsurface runoff is of most interest. Thus:

215

216 *“Subsurface and concealed-surface runoff. Cases arise where surface-runoff may take*
217 *place in such a manner as not to be visible, as, for example, where it occurs through a*
218 *layer of coarse material, sometimes through a thick matting of grass or mulch-cover;*
219 *through a layer of plant roots close to the soil-surface and under forest-litter; or even,*
220 *in some cases, (through a network of sun-cracks in the soil-surface. This has sometimes*
221 *been called ‘subsurface-runoff’, sometimes ‘ground-water flow’. The term ‘subsurface-*
222 *runoff’ would not be objectionable were it not for the fact that it is likely to be confused*
223 *with true ground-water flow. The term ‘groundwater flow’ applied to this class of flow*
224 *is highly objectionable on several counts; flow occurring close to the surface in the*
225 *manner described has little in common with true ground-water flow. It is mostly*
226 *turbulent flow, while true ground-water flow is mostly laminar. It persists only during*
227 *rainfall-excess or for a short time thereafter, measured in hours or at the most in days,*
228 *whereas ground-water flow persists on perennial streams at all times. Furthermore,*
229 *surface runoff follows the same laws and behaves in the same manner whether it*
230 *actually occurs visibly on the ground surface, or is concealed and invisible, taking place*
231 *just below the soil-surface where it is sustained by temporary detention below the soil-*
232 *surface. Nevertheless, it may be desirable to distinguish between the two cases and, if*
233 *so, flow which is essentially surface-runoff but which is concealed from view in some*
234 *one of the ways described, may appropriately be called ‘concealed-surface runoff.’”*

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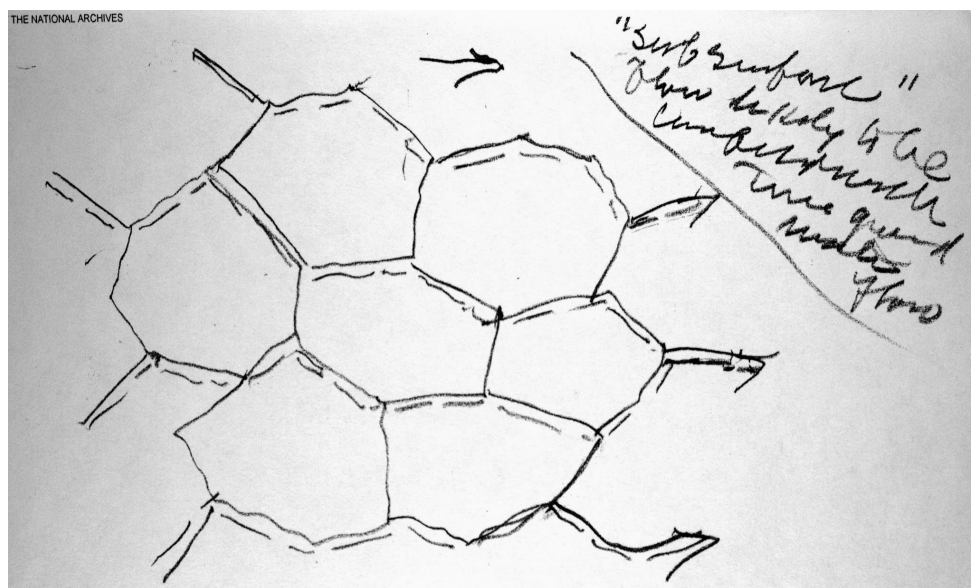
(Horton, 1942, p.481)

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237 Thus, by definition, water contributing to the hydrograph is allowed to be hidden from view
238 and treated as surface runoff *as if* it was in excess of the infiltration capacity of the soil, at
239 least if no longer a laminar flow. An example., taken from the boxes of Horton’s papers in his
240 analysis of downslope flow through sun-cracks (see Figure 2). Again, perhaps underlying this
241 is an interpretation that laminar subsurface flow velocities were far too slow to allow
242 significant contributions to the hydrograph (although, interestingly, observations from the
243 Horton Hydrological Laboratory did show some examples of fast borehole responses, see
244 Beven, 2004c).

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⁸ Horton argued that infiltration capacity, accretion and plat were to be preferred, citing Oxford English Dictionary definitions. In this at least, he has not got entirely his way in the long term.



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Figure 2. Figure explaining lateral “subsurface” flow in sun-cracks as concealed surface runoff (Drawing in Horton’s hand from Box 71 of the Horton Papers in the National Archive)

We should remember that the tracer information that revealed that in many catchments hydrographs are composed largely of pre-event water was not available in the 1930s and 1940s, but Beven (2004a) shows that by comparing rainfall frequency data and Horton’s own infiltration observations it is unlikely that he would have observed widespread overland flow on his own research catchment near Voorheesville more than 1 in 2 to 1 in 5 years (unless of course it was concealed!). Walter et al. (2003) had come to similar conclusions in an analysis of sites in New York State.

The complexity of infiltration processes

Horton’s perceptual model of the response of catchments was, however, much more sophisticated than he is generally given credit for. This was revealed in the 94 boxes of his papers that were classified by Walter Langbein (who had also worked with Horton) and deposited in the US National Archives in 1949 (see the discussions in Beven, 2004a,b,c). Horton argued, for example, that infiltration capacities would be primarily controlled at the soil surface by what he called extinction phenomena, such as compaction of the surface by rainsplash, and blocking of larger pores by displaced fine particles. It was these extinction phenomena that led to the gradual decline in infiltration capacities with time, as described by his well-known infiltration equation that first appears in Horton (1939)⁹. He also recognised that bioturbation and agricultural practices would change the surface between events, resulting in a recovery of infiltration capacities. There could also be marked seasonal changes, something that he observed in his own infiltration observations, and strong

⁹ It is commonly cited to Horton (1933) but does not appear there. It also does not appear in Horton’s Monograph on Surface Runoff Phenomena of 1935.



273 variability in space. He recognised the role of macropores and surface microtopography in
274 concentrating water and allowing the escape of air, which he had shown to be a control on
275 infiltration by experiment (see Beven, 2004b). He also understood that while it was possible
276 to make local predictions of infiltration excess on different land units (effectively producing a
277 distributed model of surface runoff production), it was not possible to calculate the different
278 contributions given only hydrograph contributions.

279
280 Horton was also not alone in recognising the complexity of infiltration processes in this
281 period. In the discussion of a physics-based paper on infiltration by Willard Gardner (1946),
282 G. W. Musgrave commented:

283 *“However, we have before us a type of problem which particularly requires caution in*
284 *extending and applying laboratory-findings to natural field-soils. At least insofar as the*
285 *structure of the laboratory-sample differs from that of the natural soil, caution is*
286 *warranted. Most soils of natural structure contain crevices, channels, and openings*
287 *that transmit free water rather rapidly, though locally, to some depth. It appears from*
288 *many observations in the field that in some cases at least, a very large portion of the*
289 *infiltrating water is thus transmitted. Where a dye is used and the soil-profile is*
290 *dissected following application, the highly irregular nature of the downward moving*
291 *water becomes evident. Dry “islands” are bypassed and left with their air-water*
292 *interfaces intact, at least temporarily. The channels conducting free water act as*
293 *feeders laterally for capillary water, often for a considerable time. The forces of gravity*
294 *and capillarity are not always acting in conjunction. One wonders whether other forces*
295 *such as thermal gradients are involved, and if so, to what extent they are effective.”*

296 G. W. Musgrave (1946b, p.135)

297 298 **Surface runoff and baseflow separation**

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300 This then created a problem for the infiltration theory of runoff because, as noted earlier,
301 Cook points out there was no way of separating surface runoff and subsurface contributions
302 to the hydrograph. But in order to derive the apparent infiltration characteristics from
303 hydrographs and pluviographs it was necessary to do so. The concept of baseflow separation
304 and recession analysis has continued to exercise hydrologists ever since (see Hall, 1968;
305 Tallaksen, 1995; Beven, 1991; Arnold et al., 1995; Chapman, 1999; Eckhardt, 2005), right to
306 the present day (Ladson et al., 2013; Lott and Stewart, 2016; He et al., 2016; Duncan, 2019).
307 Some of these methods allowed for an increase in baseflow during an event, arguing that
308 there would be some accretion to the water table during the time scale of the event (e.g.
309 Horton, 1935; Hursh and Brater, 1941, as based on borehole observations at Coweeta;
310 Hewlett and Hibbert, 1967; or the digital filters of Nathan and McMahon, 1990; Furey and
311 Gupta, 2001; and Aksoy et al., 2009).

312
313 Both Horton (1935) and Cook (1946) suggests the strategy of continuing the past groundwater
314 depletion or recession curve as an indicator of baseflow, with all the flow above that curve
315 being treated as if it was infiltration excess surface runoff but only for *“the special case when*
316 *the subsurface flow is derived entirely from the zone below the permanent groundwater,*
317 *table, and no groundwater accretion occurs, a satisfactory estimate of subsurface flow can be*
318 *made simply by extending the groundwater depletion curve.”* (Cook, op.cit. p728).

319



320 But simply continuing the recession curve results in a problem for the method in calculating
 321 the volume of surface runoff for an event, since the previous recession will always be below
 322 the recession of the current event. Thus, there was also a pragmatic need to allow for a
 323 “baseflow” contribution to rise to meet the falling recession limb of an event. Horton (1935)
 324 had earlier wanted to allow for the accretion of groundwater due to infiltration and specifies
 325 a method for deciding when the field moisture deficit of the soil has been specified, after
 326 which a line can be drawn to where the form of the recession matches the groundwater
 327 depletion curve. He notes that this might occur above or below the point of initial hydrograph
 328 rise (Figure 3). Hewlett and Hibbert (1967) suggested using a standard slope for this rise of
 329 0.05 cfs/mi²/hr (or 0.0567 ls⁻¹/km²/hr), starting from beneath the hydrograph peak, but this
 330 was based only on discharge and borehole data from some small catchments at Coweeta.
 331 Somehow, it became a standard that was used around the world, regardless of soils,
 332 vegetation or geology. Others suggested that the end of surface runoff would be marked by
 333 a break between straight line segments on a semi-logarithmic plot of the recession, indicating
 334 a transition to a process with a slower time constant¹⁰. In essence Cook was correct, there is
 335 no satisfactory way of separating surface from subsurface flow in this way (see also the
 336 discussion in Beven, 1991¹¹).
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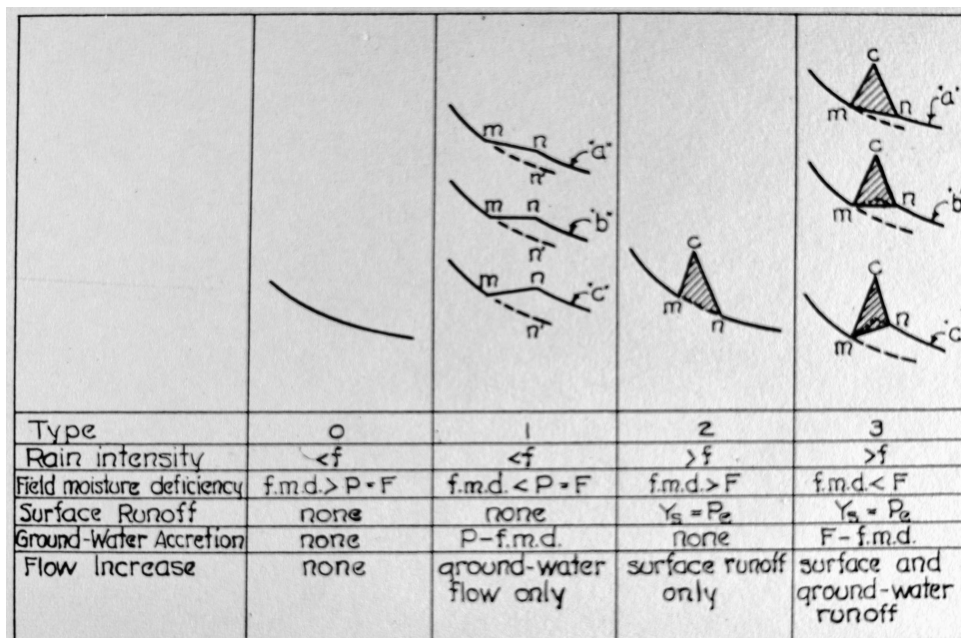


Figure 3. Classification of Stream Rises, with Type 3 showing how to separate ground-water runoff.

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Derivation of infiltration indices from the hydrograph

¹⁰ Barnes (1939, 1944) recognised three such components, overland-flow, ground-water flow, and what he called secondary base-flow, and later storm-seepage or interflow; while Kunkle (1962) distinguished baseflow from the effects of bank storage.

¹¹ Beven (1991) includes a section headed “Choosing a baseflow separation method” that consists only of the one word “Don’t”.



343 Both Horton and Cook recognised that there was a difference between predicting surface
344 runoff locally given information about rainfall and infiltration capacity curves for a soil and
345 deriving apparent infiltration information from rainfalls and an estimate of “surface runoff”.
346 In the first case, the local variability of soils, vegetation and management practices could be
347 taken into account (given the infiltration characteristics of each) on what Cook calls soil-cover
348 *complexes*; the equivalent of modern-day hydrological response units. The second case is
349 more challenging in that it is not possible to obtain more than an index of catchment-wide
350 apparent infiltration. He gives two examples of such indices that can be obtained by
351 matching the observed volume of surface runoff to the observed pattern of rainfall. The first
352 is based on assuming an average declining infiltration capacity to produce an average
353 infiltration rate (the f_{av} or W index) with a special case after significant wetting equivalent to
354 a final constant infiltration rate W_{min} . The second is assuming a constant infiltration rate (the
355 ϕ index). He demonstrates that for this latter index a dependence on rainfall intensity should
356 be expected where there are multiple soil-cover complexes in a catchment “*because the*
357 *higher the intensity the greater the proportion of the area producing runoff throughout the*
358 *rain, not because infiltration capacity increases with intensity of rainfall.*” (Cook, op.cit.
359 p.738). He therefore already recognises the possibility of partial contributing areas of runoff
360 production.

361
362 Further problems arise when there is intermittent rainfall, or where rainfall intensity
363 intermittently falls below the infiltration capacity of the soil and there might be the possibility
364 of some recovery of infiltration capacities between bursts of rainfall. He goes into some detail
365 to explain how different cases might be handled. He does not include, however, the
366 suggestion of using *time condensation* (now more commonly known as the *time compression*
367 *assumption*). This had been introduced 3 years earlier by Leroy Sherman (1943) and then
368 modified by Heggie Nordahl Holtan¹² (1945). Holtan (1961) was also the first person to
369 suggest an infiltration equation that was expressed directly in terms of cumulative infiltrated
370 water, thereby implicitly incorporating a time compression assumption.

371 372 **Infiltration equations** 373

374 Application of the infiltration theory is easiest on a single soil-cover complex given rainfall and
375 information about infiltration capacities of the soil. Quantitative estimation of runoff is
376 easier if the infiltration capacities can be represented as a mathematical function (although
377 in the 1930s and 1940s when the calculations were made by hand, it could actually be faster
378 to read values off of a graph or from a table than to do the calculation, and many papers of
379 the time give examples of hand-worked calculations, e.g. Sherman, 1936, 1943).

380
381 The Green and Ampt (1911) infiltration equation (Table 1), based on a piston-like wetting
382 front approximation to Darcy’s law had been available for some time. Horton (1939, 1940)
383 developed his own form of equation¹³. As noted earlier, he argued that this represented
384 surface controls rather than profile controls on the infiltration capacity. Cook mentions only
385 the Horton equation in his exposition of the infiltration theory but there were other empirical
386 infiltration equations suggested such as the power law form suggested independently by A.

¹² (1909-2006), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Holtan,_H._N.

¹³ Note that Philip (1954) suggests that this equation was first suggested by Gardner and Widtsoe (1921), but Horton (1939, 1940) does not refer back to that earlier paper.



387 N. Kostikov (1932) and Mortimer Reed Lewis (published in 1937 but according to
388 Swarzendruber, 1993, proposed in 1926), the Soil Conservation Service (SCS) curve number
389 method that first appeared in 1954 (SCS, 1954), and later that of Holtan (1961). The idea of
390 solving the Darcy-Richards equation was picked up again in the 1950s, most notably by John
391 Philip¹⁴ (1954) and then in a series of papers for the infiltration problem (Philip, 1957). This
392 was a mathematical challenge for soil physicists and set off a variety of solutions for different
393 types of diffusivity function and boundary conditions, that continued into the 21st Century. A
394 summary of some of these infiltration equations is given in Table 1. Comparison of the
395 behaviours of different equations have been given by, for example, Wilson et al. (1982),
396 Davidoff and Selim (1986), Mishra et al. (2003) and Chahinian et al. (2005).

397
398 The SCS curve number method is of particular interest in terms of its common interpretation
399 as an infiltration equation. Horton frequently clashed with the SCS and seems to have had a
400 low opinion of their engineers (the SCS insisted on interpreting infiltration capacity as a
401 volume rather than as a rate, for example¹⁵). This originally derives from the work of Mockus
402 (1949) who plotted estimates of storm rainfall against the volume of surface runoff, as
403 previously suggested by Sherman (1943). From this analysis Mockus suggests a relationship
404 between them of the form

$$Q = P[1 - (10)^{-bP}]$$

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407
408 with a multiple regression used to estimate the coefficient b based on data for 50 storms
409 collected from catchments “of field size or larger” (p.41). The soil, crop, season, and
410 antecedent precipitation indices used in the regression were derived by an analysis of data
411 from nine USDA research stations. Nowhere does he specify how the amounts of surface
412 runoff were derived. The resulting surface runoff was routed through a dimensionless unit
413 hydrograph to derive hydrograph peaks (Mockus also mentions how a triangular unit
414 hydrograph could be used to approximate the dimensionless unit hydrograph).

415
416 The methods were tested “by estimating total runoff for storms on single- and mixed-cover
417 watersheds”, by which he seems to mean the total volume of surface runoff. The results
418 were better for large storms than small storms and for mixed-cover rather than single cover
419 catchments. Better results were obtained by breaking long duration storms into parts. He
420 notes that rainfall spatial variability and direction of movement might be important in getting
421 better estimates.

422
423 The SCS curve number method took the data of Mockus and also a large number of infiltration
424 capacity measurements on different soil types and land covers in the US, and postulated a
425 proportionality between retention and runoff such that:

$$\frac{F}{S} = \frac{P - Q}{S} = \frac{Q}{P}$$

426
427
428

¹⁴ (1927-1999), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Philip,_John_R

¹⁵ Beven (2004b) reports that in a letter to a Mr. Ramser of the SCS Horton wrote “In reading this discussion I am reminded of the adage that you can lead a horse and some other related animals to water but you can’t make them think.” [Horton papers Box 2: copy of letter dated June 7, 1943]



429 where F is the cumulative infiltration, S is the storage capacity of the soil, Q is the total runoff
430 and P is the total precipitation for an event. According to an interview with Vic Mockus, he
431 had fixed on this functional relationship after dinner one evening, having tried many others,
432 because it best fit the data (Ponce, 1996). An initial abstraction loss, I_a , was also introduced
433 which, on the basis of data from catchments of 10 acres or less, was made proportional to S
434 as $I_a = \lambda S$. While 50% of these observations showed values of λ in the range 0.095 to 0.38,
435 a value of 0.2 was chosen as being at the centre of the data (though Mockus allows that other
436 values might be valid). Combining these equations an expression for Q can be derived as
437

$$438 \quad Q = \frac{(P - 0.2S)^2}{P + 0.8S}$$

439
440 with only the one parameter S . For convenience in engineering applications, this was then
441 scaled to a non-dimensional curve number CN such that (for S in units of inches)
442

$$443 \quad S = \frac{1000}{CN} + 10$$

444
445 where CN has the range of 0 to 100 and is tabulated for different soil classes, land covers and
446 antecedent conditions. The soils information was simplified to only 4 classes for simplicity of
447 use by G. W. Musgrave (Ponce, 1996). It is clear from the literature associated with the curve
448 number methodology that the SCS interpreted the output Q as a volume of surface runoff in
449 excess of the infiltration capacity of the soil (Table 1). Thus, in Module 103 of the SCS-CN
450 Training Manual it is stated that: “Runoff is that part of the precipitation that makes its way
451 towards stream channels, lakes or oceans as surface flow” (p1). The Manual also provides
452 definitions of interflow and baseflow as subsurface contributions to streams but suggests that
453 interflow “is not usually considered in SCS methods of estimating runoff” (p.3).
454

455 There have since been many other interpretations of the SCS Curve Number relationships.
456 Chen (1982) showed how the SCS curve number method could be related to the Holtan
457 infiltration equation, which also allows for a maximum storage capacity, while Mishra and
458 Singh (1999, 2002, 2003) showed how the Mockus relationship could be analytically related
459 to the SCS Curve Number equation and also to the Horton infiltration equation (for the case
460 where the long time infiltration capacity f_c can be assumed negligible). They refer to what is
461 being estimated as *direct surface runoff*. It seems, given the relationship to infiltration
462 equations they derive, they mean by this overland flow to the stream. Steenhuis et al. (1995)
463 suggested that the method could also be interpreted as a saturation excess variable
464 contributing area function, with later verification by Dahlke et al. (2006), while Yu (1998)
465 suggested that it was equivalent to the partial area surface runoff that would be generated
466 on a statistical distribution of soil infiltration characteristics. In all these cases, however, it
467 retains the preconception of representing surface runoff as overland flow. It is important to
468 note, however, that this may not have been the case for the original small catchment
469 observations from which the method was derived (see also the results from Horton’s runoff
470 plat experiment reported in Beven, 2004a, where runoff rate was significantly higher than the
471 observed rainfall intensity). More recently, Ogden et al. (2017) suggest it is really time to
472 move beyond the curve number method suggesting that “sixty five years of use and multiple
473 reinterpretations have not resulted in improved predictability using the method”.



474

475 **Surface detention, channel storage and the unit-graph**

476

477 Horton and others in the era of infiltration recognised that in both analysis and prediction it
478 was not enough to simply calculate the excess of precipitation over the infiltration capacity
479 of the soil. As Horton (1935) put it: “*A striking fact about surface runoff is the manner in which*
480 *a jagged, irregular rain intensity graph is often transformed into a smoothly rounded runoff*
481 *graph.... This is the result of regulation by surface detention and channel storage*” (Horton,
482 1935, p.1). By thinking in terms of a unit strip of hillslope (for which he credits a suggestion
483 of LeRoy Sherman) Horton (1935) analyses the velocities expected for both laminar and
484 turbulent sheet flow, with hydraulic radius assumed equal to the flow depth for a shallow
485 flow, in terms of the Hagan power law equation:

486

$$487 \quad q = K\delta_c^M S^N$$

488

489 where q is the flow per unit width, δ_c is the depth of flow averaged across the width of the
490 slope segment, S is the slope of the surface and K , M , and N are parameters. Horton notes
491 the theoretical values of M and N for laminar and turbulent flows, but also gives analyses of
492 flume provided by Lewis and Neal of the Idaho State Agricultural Experiment Station data that
493 suggest values of M of 0.85 and N of 0.74, suggesting mixed laminar and turbulent flow. He
494 also uses this to derive a profile of overland flow depths under a steady distributed input rate
495 equal to the rainfall rate – a constant infiltration capacity (essentially making the kinematic
496 wave assumption).

497

498 He also recognised the effect of routing through channel storage, both in predicting
499 hydrographs and in the analysis of observed hydrographs to derive infiltration parameters.
500 He suggested a method of routing through a nonlinear (power law) storage based on the
501 storage-discharge curve for the channels, but noting that “*in applying this method for*
502 *correction for channel storage it is important that ground-water flow, if any exists, should be*
503 *eliminated from the hydrograph in advance*” (Horton, 1935, p.41).

504

505 It might be that Horton felt compelled to provide a method of routing runoff because a few
506 years earlier LeRoy Sherman (1932) had already proposed a more general method as an
507 abstraction of the time-area approach that he called the unit-graph method (see the
508 discussion of Beven, 2020). This was then developed into the unit hydrograph theory, with
509 its many variants in terms of mathematical representation, methods of fitting, and
510 parameters related to catchment characteristics. In its classical form the unit hydrograph is
511 used to route estimates of the water contributing to the storm hydrograph after baseflow
512 separation as appropriate (although modern transfer function methods can also be used to
513 predict the complete hydrograph, e.g. Young, 2013). It was thus easy to combine the unit-
514 hydrograph with the infiltration theory *as if* all that water was overland flow in excess of the
515 infiltration capacity of the soil. This provided a convenient engineering procedure that is still
516 in widespread use in many countries.

517

518 **Surface Runoff, Direct Runoff and Stormflow**

519



520 The infiltration theory essentially defines that proportion of the rainfall that will produce
521 surface runoff and contribute to the storm hydrograph. But part of the problem here is what
522 is actually meant by surface runoff. Even going back to the original definitions of Horton and
523 Cook we have seen how surface runoff is what is measured in a stream hydrograph, but that
524 might have reached the stream as either overland flow or subsurface stormflow. We have
525 seen already how Robert Horton suggested that some of this contribution might be *concealed*
526 *surface runoff* and how Howard Cook allowed that effective infiltration rates could not be
527 inferred if there was a significant contribution to the hydrograph from subsurface flows.

528
529 It is also clear that the runoff data analysed by Mockus (1949) and that was used in evolving
530 the SCS-CN model was not necessarily produced entirely by overland flow, despite the
531 common interpretation of the SCS-CN function as an infiltration model. Yet, in setting out the
532 definitions for his analysis, Mockus defines surface runoff as overland flow. He distinguishes
533 between surface runoff, subsurface flow contributing to the hydrograph but which will quickly
534 cease to contribute to streamflow, and groundwater flow which “*may first appear in the*
535 *stream channels during or after the storm, and may continue for a relatively long time*” (p.2).
536 He then defines the term *Direct Runoff* as the sum of surface runoff and subsurface flow
537 “*combined in unknown proportions*”. However, having set out these definitions he proceeds
538 to outline methodologies for estimating surface runoff alone based on nomograms that allow
539 for soil, crop, antecedent conditions, storm duration and seasonal effects. In his use of *Direct*
540 *Runoff*, Mockus was following Franklin F. Snyder a decade earlier who, in a glossary of terms
541 associated with his *Conception of Runoff-Phenomena* defines *surface-runoff* as:

542
543 “*Usually defined as the runoff reaching the surface drainage-channels without*
544 *penetrating the ground-surface. As actually used, surface-runoff usually includes*
545 *considerable subsurface storm-flow and might be better termed direct runoff, since it*
546 *consists of the discharge in excess of a base or ground-water flow which passes a*
547 *gaging station within a rational period of time subsequent to the storm causing the*
548 *rise*”. (Snyder, 1939, p.736).

549
550 Note how this differs from the definition cited earlier in SCS Training Module 103. Later usage
551 was also mixed, and there does not seem to have been a real history of development in the
552 use of the different terms for runoff. To give just a few examples, Leach et al. (1933) use
553 both storm-flow and surface-runoff; Langbein (1940) uses direct runoff, as do Hursh and
554 Brater (1941) who specifically say that storm-runoff as overland-flow has not been observed
555 on the study watershed at Coweeta and give examples of hydrographs dominated by channel
556 precipitation. Hoover and Hursh (1943), however, revert to using storm-runoff. Marston
557 (1952) equates storm runoff to overland flow but Reinhart (1964) includes subsurface
558 stormflow in storm runoff, and in the study of Whipkey (1969) essentially all the storm runoff
559 is subsurface stormflow. Hamon (1963) refers to direct runoff in relation to that predicted
560 by the SCS curve number method, whereas others have continued to use storm runoff as
561 equivalent to overland flow, especially in semi-arid catchments (e.g. Fogel and Duckstein,
562 1970).

563
564 If we turn to the latest issue of the WMO International Glossary of Hydrology (2012) we find
565 *runoff* defined as that part of the precipitation which flows towards a river on the ground
566 surface (*surface runoff*) or within the soil (*subsurface runoff* or *interflow*). *Direct runoff* (or



567 direct flow or storm runoff) is defined simply as water that enters a watercourse without
568 delay (and without any process interpretation). *Infiltration index*, however, is defined as an
569 average rate of infiltration such that precipitation in excess of that value equals the volume
570 of *storm runoff* (implying that the infiltration theory concept still persists, if only in an index
571 form). In the Glossary for *Hillslope Hydrology*, Chorley (1978) also defines *direct runoff* with
572 respect to time, adding that it comprises the sum of channel precipitation, overland flow and
573 subsurface stormflow. His definition of *surface runoff* is limited to flow over the soil surface,
574 and for *quickflow*, *storm runoff*, and *stormflow* he says “see *direct runoff*”.

575
576 There is thus some continuing ambiguity about the use of these terms, particularly surface
577 runoff. This is in part a process issue because, however water flows into a stream by either
578 surface or subsurface flow processes, once in the stream it is measured as a surface runoff
579 (as was the case for the fields and small catchments in the data used by Mockus). The
580 problem is that the word runoff still induces a perception of an overland flow, as in running
581 off over the land surface. This is reinforced by the use of surface runoff even if the ambiguity
582 recognised by Snyder, Cook and Mockus of the unknown mix of surface and subsurface
583 contributions to the hydrograph cannot be easily resolved. This mix, defined by them as
584 *direct runoff* (and now sometimes referred to as *storm runoff* or *stormflow* or *quickflow*) is
585 more commonly what is estimated by the use of hydrograph separation, but it should not
586 then be interpreted as runoff in excess of the infiltration capacity of the soil. That is, perhaps,
587 why the WMO Glossary refers to an infiltration *index* to match the volume of *storm runoff*,
588 even if this perpetuates the perception of runoff as an overland flow. On the other hand, the
589 convenient alliteration of *rainfall-runoff modelling* is generally used to indicate a mix of
590 surface and subsurface processes (except in models that are still limited to predicting only
591 overland flows).

592
593 Given these ambiguities, it might be better to avoid the use of the terms runoff and surface
594 runoff (and concealed surface runoff) altogether and instead refer to *stormflow* or *storm*
595 *discharge* when no process interpretation is inferred, and refer explicitly to overland flow and
596 subsurface stormflow when there is evidence for making a process interpretation. There is
597 also no reason why the general term *hydrological model* should not replace the ambiguity of
598 rainfall-runoff model. This might (just perhaps) lead to a greater appreciation and greater
599 thought about the perceptual model of hydrological processes relevant to particular
600 catchments of interest (Beven, 2001).

601 602 **Persistence of the era of infiltration and perceptual model failures**

603
604 When Cook was writing in 1946, he noted that the infiltration theory of surface runoff was
605 still young and needed to be developed further, such that “*before it can be generally*
606 *employed, many problems must be solved and large quantities of data published*” (op. cit.
607 p.743). He notes in particular, that it would only be valid for cases where subsurface runoff
608 could be neglected, and that infiltration indices derived from hydrograph data would only be
609 satisfactory if there was only one soil-cover complex, otherwise, “*the physical significance is*
610 *obscure*” (p.743). His final statement is to suggest that because of these issues all infiltration
611 data should be accompanied by a statement of how they were derived, so that they would
612 not be misused. It can be said, therefore, that Howard Cook had a rather realistic
613 understanding of the limitations of the infiltration theory.



614

615 It seems that in the years following, however, the pragmatic utility of the methodology to
616 provide estimates of the volume of storm discharge dominated any concerns about the
617 validity of the assumptions. That volume could be combined with the time distribution of
618 the Sherman (1932) unit-graph (and later representations of the unit hydrograph) to allow
619 the prediction of hydrographs, and of hydrograph peaks for design applications. The
620 methodology came to dominate hydrological practice, even well into the computer age, when
621 there were many models essentially based on predicting and routing *effective* or *excess*
622 rainfall based on infiltration equations.

623

624 However, from the late 1960s onwards, the general applicability of the infiltration theory
625 started to be questioned. Cappus (1960) and Moldenhauer et al. (1960) suggested that not
626 all of a catchment would contribute surface runoff, while Betson (1964) concluded that the
627 generally wetter conditions at the base of hillslopes would result in a relatively consistent
628 partial contributing area (see also the consequent partial area model of Betson and Marius,
629 1969). Hewlett and Hibbert (1967) proposed that the contributing area would be dynamic,
630 varying with antecedent conditions and storm rainfalls (see also Dickinson and Whitely, 1970).

631

632 However, particularly after the geochemical hydrograph separation of Pinder and Jones
633 (1969) and the environmental isotope hydrograph separation of Sklash and Farvolden (1979),
634 there was a more general realisation that subsurface processes were necessarily important
635 in runoff generation in many catchments because of the high proportion of pre-event water
636 that appeared to be displaced in the event (something that was later called a double paradox
637 by Kirchner, 2003). Thus, even if there was some overland flow, much of the water in the
638 hydrograph had to be displaced from the soil or deeper layers (Sklash and Farvolden reported
639 that at one site samples of overland flow were indicative of event water in one sampled storm
640 and pre-event water in another). Thus new concepts of runoff generation were required. At
641 Coweeta, where overland flow is rare except in the immediate riparian area (but runoff
642 coefficients can be small), John Hewlett¹⁶ had continued the work of Hursh in trying to
643 understand the role of subsurface flow in hydrograph generation. The idea of runoff and
644 return flow to dynamic saturated areas had appeared in the work of Dunne and Black (1970),
645 a concept later claimed by Hewlett (1974). However, at around the same time, computer
646 models such as the Huggins and Monke (1968) model; the KINEROS model that developed
647 from Smith and Woolhiser (1971); the partial area Quasi-Physically-Based Rainfall-Runoff
648 Model (QPBRM) model of Engman and Rogowski (1974) also included in the study of Loague
649 and Freeze (1985)¹⁷; and the CASC2D model of Downer et al. (2004); were all based on the
650 infiltration theory (and there were many others). Of course, there are still catchments where
651 the infiltration theory might indeed match the perceptual model of overland flow as the
652 dominant process, but it still took time for the perceptual model of how catchments function
653 to recognise the important contribution of subsurface water to stormflow in many
654 catchments.

¹⁶ (1922-2004), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Hewlett,_J_D

¹⁷ In Beven (1989) I criticised the paper of Keith Loague and Al Freeze (1985) because they had applied such the QPBRM model to the Hubbard Brook catchment where surface runoff would be rarely observed. I suggested that was simply poor hydrological practice. Keith Loague replied (pers.comm.), that they had made the choice of applying a model that was widely used in practice, and that these might well be used in practice where the assumptions were not valid.



655

656 A really instructive case in this respect is the history of modelling the R5 catchment at
657 Coshocton by Keith Loague and his colleagues. This is only a small catchment area (0.1 km²)
658 and started out as a study of effect of the variability of infiltration rates in space on runoff
659 generation, making use of the extensive database of infiltration measurements collected by
660 Sharma et al. (1980). It was included in the study of Loague and Freeze (1985) using the
661 QPBRRM computer model. Loague and Gander (1990) added a further 247 infiltration
662 measurements, and Loague and Kyriakidis (1997) used kriging interpolation to produce a fully
663 distributed spatial pattern of infiltration characteristics. Using this information, however,
664 produced less satisfactory hydrograph simulations than the original Loague and Freeze (1985)
665 calibrated model. Various things were tried to improve the results, including allowing for
666 temperature effects in the original infiltration measurements, and taking averages over
667 stochastic fields of parameters consistent with the kriging estimates. It was suggested that
668 there were still limitations of resolution in representing the surface runoff pathways and
669 effects of run-on and reinfiltration. However, improvements in predictions of the peak and
670 time to peak came with a change of model to the finite element based Integrated Hydrological
671 Model (InHM) that included the effects of subsurface flow pathways (VanderKwaak and
672 Loague, 2001; Loague et al., 2005). Following this change of perceptual model from a simple
673 infiltration theory concept, R5 has continued to be used as a case study for the application of
674 integrated models (Heppner et al., 2007; Mirus et al., 2011; Mirus and Loague, 2013).

675

676 Another case is reported in Beven (2002). I was a visiting scientist at the ARS laboratory in
677 Fort Collins, Colorado working with Dave Woolhiser and Roger Smith and helped in an
678 experiment to look at runoff generation on shallow restored soils over mine tailings near
679 Steamboat Springs, Colorado in 1981. The perceptual model in designing the experiment
680 was that the runoff generation would be produced by an infiltration excess mechanism. Thus
681 many dual ring infiltrometer measurements were done, and replicate 25m by 5m sloping plots
682 were watered using a sprinkler system supplied from a large impermeable container of
683 rubberised fabric. Unfortunately, during the experiment the supply started to be limited by
684 movement of the container as it emptied, but some overland flow was generated and
685 collected. It was, however, localised on the surface, and rapidly fell to zero. Meanwhile, in
686 the shallow trench that had been dug to take the collected overland flow from the
687 measurement flume to a small channel, subsurface flow from beneath the collectors
688 continued for some 90 minutes, and at the bankside of the channel there were two outflows
689 from preferential flow pathways through otherwise unsaturated soil. It appeared as if there
690 had been a form of percolation excess process taking place at the boundary between the mine
691 tailings and topsoil, and that the resulting subsurface flow was somehow being channelled
692 within the soil that had been replaced over the mine waste. The volumes of subsurface flow
693 were not measured but were clearly much greater than the surface runoff collected. This
694 was also an instructive case where the perceptual model based on the infiltration theory used
695 in designing the experiment was clearly not correct and needed to be revised.

696

697 There is, therefore, no doubt that the infiltration theory concept led to many misconceptions
698 or perceptual model failures of how the response of particular catchments was dominated by
699 surface flow. There were, of course, many other catchments where subsurface contributions
700 to the hydrograph have been studied in more detail and qualitative perceptual models
701 developed, such as Hursh, Hewlett and others at Coweeta, North Carolina, as mentioned



702 above, and Mosley (1982) and McDonnell (1990) at Maimai, New Zealand, with later additions
703 by Brammer and McDonnell (2003) and McGlynn et al. (2010). These more complete
704 perceptual models, however, tend to be complex and there remains a need to simplify in
705 applying quantitative predictive models. This is perhaps the main explanation of the
706 question posed at the beginning of this paper as to why the infiltration theory of runoff
707 concept has persisted so widely in applications. It still underlies many current hydrological
708 models in one form or another, including the SCS-CN or alternative Green-Ampt methods for
709 estimating direct flow in the Soil Water Assessment Tool (SWAT). In this way, the era of
710 infiltration theory continues, in part because of the convenience of applying the SCS-CN
711 method for practical applications without thinking too much about whether that is
712 appropriate in any particular catchment. In fact, since we do not know too much about the
713 processes in the catchments on which the analysis of Mockus (1949) that led to the SCS-CN
714 method were based, that might be more defensible as a predictor of total direct runoff than
715 the use of point infiltration equations to predict purely overland flow (especially if
716 heterogeneity of soil characteristics and run-on effects are neglected).

717
718 It does seem surprising, however, that more than 70 years after Howard Cook announced the
719 era of infiltration, and 50 years after tracer information showed that hydrographs could be
720 dominated by pre-event water, we should still be so confused about how to describe what is
721 actually being observed and estimated in catchment hydrographs. Cook's observation that
722 it is impossible to separate surface and subsurface contributions to the hydrograph when only
723 records of the rates of flow are available still holds. Learning from tracer separations is not
724 yet standard practice and does not provide unambiguous information about flow pathways.
725 The resulting ambiguity means that there have been no real attempts to define the limits of
726 validity of the infiltration theory, and much confusion about its use. It seems that some of
727 the old guard might still have reason to grumble.

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731 732 **References**

- 733
734 Ahuja, L.R. and Tsuji, G.Y., Use of the Green-Ampt Equation with Variable Conductivity. *Soil Science Society of*
735 *America Journal*, 40(4): 619-622, 1976.
736
737 Aksoy, H., Kurt, I. and Eris, E., Filtered smoothed minima baseflow separation method. *Journal of*
738 *Hydrology*, 372(1-4): 94-101, 2009.
739
740 Arnold, J.G., Allen, P.M., Muttiah, R. and Bernhardt, G., Automated base flow separation and recession analysis
741 techniques. *Groundwater*, 33(6): 1010-1018, 1995.
742
743 Barnes, B. S., The structure of discharge-recession curves, *Trans. Amer. Geophys. Un.* 20: 721-725, 1939
744
745 Barnes, B.S., Subsurface-Flow. *Transactions American Geophysical Union*, 25(5): 746-746, 1944.
746
747 Betson, R. P., What is watershed runoff? *J. Geophys. Res.*, 69(8): 1541-1552, 1964.
748
749 Betson, R. P., and Marius, J. B., Source areas of storm runoff, *Water Resour. Res.* 5(3): 574-582 1969
750
751 Beven, K.J. Infiltration into a class of vertically non-uniform soils, *Hydrological Sciences J.*, 29(4): 425-434, 1984.



- 752
753 Beven, K.J. Changing ideas in hydrology: the case of physically based models. *J. Hydrology*, 105: 157-172,
754 1989.
755
756 Beven, K.J. Hydrograph Separation? Proc.BHS Third National Hydrology Symposium, Institute of Hydrology,
757 Wallingford, 3.1-3.8, 1991.
758
759 Beven, K.J., *Rainfall-Runoff Modelling – the Primer*, 1st edition, John Wiley and Sons: Chichester, 2001.
760
761 Beven, K.J., Runoff production in semi-arid areas, in M J Kirkby and L Bull (Eds.), *Dryland Rivers*, Wiley,
762 Chichester, 57-105, 2002.
763
764 Beven, K.J., Surface runoff at the Horton Hydrologic Laboratory (or not?), *J. Hydrology*, 293, 219-234, 2004a.
765
766 Beven, K.J., Robert Horton's perceptual model of infiltration, *Hydrological Processes*, 18, 3447-3460, 2004b.
767
768 Beven, K.J., Robert Horton and abrupt rises of groundwater, *Hydrological Processes*, 18, 3687-3696, 2004c.
- 769 Beven, K.J., A history of the time of concentration concept, *Hydrology and Earth System Sciences*,
770 <https://doi.org/10.5194/hess-2019-588>, 2020
- 771 Brammer, D.D. and McDonnell, J.J., An evolving perceptual model of hillslope flow at the Maimai
772 catchment. *Advances in hillslope processes*, 1: 35-60, 1996.
773
774 Cappus, P., Bassin versant experimental d'Alrance: études des lois de l'écoulement. Application au calcul et à la
775 prévision des debits, *La Houille Blanche A*, 15: 493-520, 1960
776
777 Chahinian, N., Moussa, R., Andrieux, P. and Voltz, M., Comparison of infiltration models to simulate flood
778 events at the field scale. *Journal of Hydrology*, 306(1-4):191-214, 2005.
779
780 Chapman, T., A comparison of algorithms for stream flow recession and baseflow separation. *Hydrological*
781 *Processes*, 13(5): 701-714, 1999.
782
783 Chen, C.-L.. An evaluation of the mathematics and physical significance of the Soil Conservation Service curve
784 number procedure for estimating runoff volume. *Proc., Int. Symp. on Rainfall-Runoff Modeling*, Water
785 Resources Publications, Littleton, Colo., 387-418, 1982.
786
787 Chorley, R. J., Glossary of Terms, in M. J. Kirkby (Ed.) *Hillslope Hydrology*, John Wiley and Sons: Chichester.
788
789 Collis-George, N., Infiltration equations for simple soil systems. *Water Resources Research*, 13(2): 395-403,
790 1977
791
792 Cook, H.L., The infiltration approach to the calculation of surface runoff. *Trans. Amer. Geophys. Union* 27, 726–
793 743, 1946.
794
795 Dahlke, H. E., Easton, Z. M., Walter, M., and Steenhuis, T.S., Field Test of the Variable Source Area
796 Interpretation of the Curve Number Rainfall-Runoff Equation, *Journal of Irrigation and Drainage Engineering*,
797 138(3): 235-244, 2012.
798
799 Davidoff, B. and Selim, H.M., Goodness of fit for eight water infiltration models. *Soil Science Society of America*
800 *Journal*, 50(3), pp.759-764, 1986.
801
802 Dickinson, W.T. and Whiteley, H. Watershed areas contributing to runoff. *Int. Assoc. Sci. Hydrol. Pubn.* 96: 12-
803 26, 1970.
804



- 805 Downer, C.W., Ogden, F.L., Martin, W.D. and Harmon, R.S., Theory, development, and applicability of the
806 surface water hydrologic model CASC2D. *Hydrological Processes*, 16(2): 255-275, 2002.
807
- 808 Duncan, H.P. Baseflow separation—A practical approach. *Journal of Hydrology*, 575: 308-313, 2019.
809
- 810 Dunne, T., and Black, R. D. Partial area contributions to storm runoff in a small New England watershed, *Water*
811 *Resour. Res.*, 6(5), 1296–1311, 1970.
812
- 813 Eckhardt, K., How to construct recursive digital filters for baseflow separation. *Hydrological Processes: An*
814 *International Journal*, 19(2): 507-515, 2005.
815
- 816 Engman, E. T. and Rogowski, A. S., A partial area model for storm flow synthesis, *Water Resour. Res.*, 10: 464-
817 472, 1974.
818
- 819 Fogel, M.M. and Duckstein, L., Prediction of convective storm runoff in semi-arid regions. *Int. Assoc. Sci.*
820 *Hydrol. Pubn.* 96: 465-478, 1970.
821
- 822 Furey, P.R. and Gupta, V.K., Tests of two physically based filters for base flow separation. *Water Resources*
823 *Research*, 39(10): W1297, 2003.
824
- 825 Gardner, W. and J. A. Widtsoe, The measurement of soil moisture, *Soil Science*, 11: 215-232, 1921.
826
- 827 Gardner, W. Infiltration. *Transactions American Geophysical Union* 27(1): 126-138, 1946.
828
- 829 Green, W. H. and Ampt, G. A., Studies in soil physics. I. The flow of air and water through soils, *J. Agric. Sci.*,
830 4(1): 1-24, 1911.
831
- 832 Hall, F. R., Baseflow recession – a review, *Water Resour. Res.*, 4(5): 973-983, 1968.
833
- 834 Hamon, W. R. Direct Runoff using SCS function. Computation of direct runoff amounts from storm rainfall, *Int.*
835 *Assoc. Sci. Hydrol. Pubn.*, 63: 52-62, 1963
836
- 837 He, S., Li, S., Xie, R. and Lu, J., Baseflow separation based on a meteorology-corrected nonlinear reservoir
838 algorithm in a typical rainy agricultural watershed. *Journal of Hydrology*, 535: 418-428, 2016.
839
- 840 Heppner, C.S., Loague, K. and VanderKwaak, J.E., 2007. Long-term InHM simulations of hydrologic response
841 and sediment transport for the R-5 catchment. *Earth Surface Processes and Landforms*, 32(9), pp.1273-1292.
842
- 843 Hewlett, J. D., Comments on letters relating to the 'Role of subsurface flow in generating surface runoff. 2.
844 Upstream source areas' by R. Allen Freeze, *Water Resour. Res.*, 10: 605-607, 1974
845
- 846 Hewlett, J. D. and Hibbert, A. R., Factors affecting the response of small watersheds to precipitation in humid
847 areas, in: W.E. Sopper and H.W. Lull (Eds.), *International Symposium on Forest Hydrology*, Pergamon: New
848 York, 275-290, 1967
849
- 850 Holtan, H.N., Time-condensation in hydrograph-analysis. *Eos, Transactions American Geophysical Union*, 26(3):
851 407-413, 1945.
852
- 853 Holtan, H. N., A concept for infiltration estimates in watershed engineering. USDA, Agricultural Research
854 Service Publication 41-51, USDA, Washington, DC, 1961.
855
- 856 Hoover, M.D. and Hursh, C.R., Influence of topography and soil-depth on runoff from forest land. *Eos,*
857 *Transactions American Geophysical Union*, 24(2): 693-698, 1943.
858
- 859 Horton, R. E., The role of infiltration in the hydrologic cycle, *Trans. Amer. Geophys. Un.*, 14: 446-460, 1933.
860



- 861 Horton, R. E., Surface Runoff Phenomena. Part I – Analysis of the hydrograph, *Publication 101*, Horton
862 Hydrological Laboratory: Voorheesville, NY, 1935.
863
864 Horton R.E., Analysis of runoff-plat experiments with varying infiltration capacity. *Transactions, American*
865 *Geophysical Union*, 20: 693–711, 1939.
866
867 Horton, R. E., An approach towards a physical interpretation of infiltration rate, *Soil Sci. Soc. Amer. Proc.*, 5:
868 399-417, 1940.
869
870 Horton, R. E., Remarks on hydrologic terminology, *Trans. Amer. Geophys. Un.*, 23: 479-482, 1942
871
872 Houk, I. E., Rainfall and runoff in the Miami Valley. State of Ohio, Miami Conservancy District, Tech. Rept., 8p,
873 1921
874
875 Hoyt, W. G. and W. B. Langbein, Some general observations of physiographic and climatic influences on floods,
876 *Trans. Amer. Geophys. Un.*, 20(2): 166-174, 1939.
877
878 Huggins, L. F. and Monke, E. J., A mathematical model for simulating the hydrologic response of a watershed,
879 *Water Resour. Res.*, 4: 529-539, 1968.
880
881 Hursh, C. R., Storm water and adsorption, *Trans. Amer. Geophys. Un.*, 17: 301-302, 1936
882
883 Hursh C. R., Subsurface-flow. *Transactions, American Geophysical Union* 25: 743–746, 1944.
884
885 Hursh, C. R. and Brater, E. F., Separating storm-hydrographs from small drainage-areas into surface- and
886 subsurface-flow, *Trans. Amer. Geophys. Un.*, 22: 863-870, 1941
887
888 Kirchner, J.W., A double paradox in catchment hydrology and geochemistry. *Hydrological Processes*, 17(4):
889 871-874, 2003.
890
891 Kostiaikov, A. N., On the dynamics of the coefficient of water-percolation in soils and on the necessity of
892 studying it from a dynamic point of view for purposes of amelioration. *Trans. 6th Comm. Intl. Soil Science*
893 *Society Part A*: 17-21, 1932.
894
895 Kunkle, G. R., The baseflow-duration curve: a technique for the study of groundwater discharge from a
896 drainage basin, *J. Geophys. Res.* 67(4): 1543-1554, 1962.
897
898 Ladson, A.R., Brown, R., Neal, B. and Nathan, R., A standard approach to baseflow separation using the Lyne
899 and Hollick filter. *Australasian Journal of Water Resources*, 17(1): 25-34, 2013.
900
901 Langbein, W.B., Channel-storage and unit-hydrograph studies. *Transactions American Geophysical*
902 *Union*, 21(2): 620-627, 1940.
903
904 Leach, H.R., H.L. Cook and R.E. Horton, Storm-flow prediction, *Trans., American Geophysical Union*, 14(1) 435-
905 446, 1933
906
907 Lewis, M. R. The rate of infiltration of water in irrigation practice. *Trans., American Geophysical Union*, 18:
908 361-368, 1937.
909
910 Loague, K. and Freeze, R. A., A comparison of rainfall-runoff modelling techniques on small upland catchments,
911 *Water Resour. Res.*, 21: 229-248, 1985
912
913 Loague, K. and Gander, G. A., R-5 revisited: spatial variability of infiltration on a small rangeland watershed,
914 *Water Resour. Res.*, 26: 957-971, 1990.
915
916 Loague, K. and Kyriakidis, P. C., Spatial and temporal variability in the R-5 infiltration data set: déjà vu and
917 rainfall-runoff simulations, *Water Resour. Res.*, 33: 2883-2896, 1997



- 918
919 Loague, K., Heppner, C.S., Abrams, R.H., Carr, A.E., VanderKwaak, J.E. and Ebel, B.A., Further testing of the
920 Integrated Hydrology Model (InHM): Event-based simulations for a small rangeland catchment located near
921 Chickasha, Oklahoma. *Hydrological Processes*, 19(7): 1373-1398, 2005.
922
923 Lott, D.A. and Stewart, M.T., Base flow separation: A comparison of analytical and mass balance
924 methods. *Journal of Hydrology*, 535: 525-533, 2016.
925
926 Marston, R. B., Ground cover requirements for summer storm runoff control on aspen sites in Northern Utah,
927 J. Forestry 50(4) 303-307, 1952
928
929 McDonnell, J.J., A rationale for old water discharge through macropores in a steep, humid catchment. *Water*
930 *Resources Research*, 26(11): 2821-2832, 1990.
931
932 McGlynn, B.L., McDonnell, J.J. and Brammer, D.D., A review of the evolving perceptual model of hillslope
933 flowpaths at the Maimai catchments, New Zealand. *Journal of Hydrology*, 257(1-4):1-26. 2002.
934
935 Mein, R.G. and Larson, C.L., Modeling infiltration during a steady rain. *Water resources research*, 9(2): 384-394,
936 1973.
937
938 Mezenzev, V.S., Theory of the formation of the surfacerunoff on the slope. *Meteorol. Gidrol.*, 3: 33—40, 1948
939 (in Russian)
940
941 Minshall, N.E., Predicting storm runoff on small experimental watersheds, *J. Hydraul. Div. ASCE* 86(8): 17-38,
942 1960.
943
944 Mirus, B. B. and Loague, K., How runoff begins (and ends): Characterizing hydrologic response at the
945 catchments scale, *Water Resources Research*, 49(5): 2987-3006, 2013
946
947 Mirus, B. B., Ebel, B. A., Heppner, C. S and Loague, K., 2011, Assessing the detail needed to capture rainfall-
948 runoff dynamics with physics-based hydrologic response simulation, *Water Resources Research*, 47(3),
949 W00H10, doi: 10.1029/2010WR009906
950
951 Mishra, S. K. and Singh, V. P., Another look at SCS-CN method, *J. Hydrologic Engineering, ASCE*, 4: 257-264,
952 1999
953
954 Mishra, S.K. and Singh, V.P., SCS-CN method. Part I: derivation of SCS-CN-based models. *Acta Geophysica*
955 *Polonica* 50(3): 457-477, 2002.
956
957 Mishra, S.K. and Singh, V.P., SCS-CN method. Part II: analytical treatment, *Acta Geophysica Polonica*, 51(1):
958 108-123, 2003
959
960 Mishra, S.K., Tyagi, J.V. and Singh, V.P., Comparison of infiltration models. *Hydrological Processes*, 17(13):
961 2629-2652, 2003.
962
963 Mockus, V., Estimation of total and peak rates of surface runoff for individual storms. Exhibit A, Appendix B,
964 Interim Survey Report, Grand (Neosho) River Watershed, US Department of Agriculture: Washington D.D., 1949
965
966 Moldenhauer, W.C., W.C. Barrows, and D. Swartzendruber, Influence of rainstorm characteristics on
967 infiltration measurements, *Trans. Int. Cong. Soil Sci.*, 7, 426-432, 1960.
968
969 Mosley, M.P., Subsurface flow velocities through selected forest soils, South Island, New Zealand. *Journal of*
970 *Hydrology* 55: 65-92, 1982.
971
972 Muntz, M. A., Fauré, M. L., Lain, M. E., Etudes sur la perméabilité des terres faites en vue de l'arrosage, Min. de
973 l'Agr., Dir. Gén. Eaux et Forêts, Ann. Forêts-Hydraulique, 36: 45-96, 1905.
974



- 975 Musgrave, G. W., Report of the Committee on Infiltration 1956-46, *Trans. A,er. Geophys. Un.*, 27(5), 726,
976 1946a.
977
978 Musgrave, G. W., Comment on Infiltration by Willard Gardner, *Transactions American Geophysical Union* 27(1):
979 135, 1946b.
980
981 Nathan, R. J. and MacMahon, A. A. Evaluation of automated techniques for base flow and recession analyses.
982 *Water Resources Research*, 26(7):1465-1473, 1990.
983
984 Ogden, F.L., Hawkins, R.P., Walter, M.T. and Goodrich, D.C., Comment on "Beyond the SCS-CN method: A
985 theoretical framework for spatially lumped rainfall-runoff response" by MS Bartlett et al. *Water Resources*
986 *Research*, 53(7): 6345-6350, 2017.
987
988 Philip, J.R., An infiltration equation with physical significance. *Soil Science*, 77(2): 153-158, 1954.
989
990 Philip, J. R., The theory of infiltration: 1. The infiltration equation and its solution, *Soil Science*, 83: 345-357,
991 1957
992
993 Pinder, G.F. and Jones, J.F., Determination of the ground-water component of peak discharge from the
994 chemistry of total runoff. *Water Resources Research*, 5(2): 438-445, 1969.
995
996 Ponce VM. Notes of my conversation with Vic Mockus, 1996. Available at <http://mockus.sdsu.edu/> (last
997 accessed 8 June 2020).
998
999 Reinhart, K. G., Effect of commercial clearcutting in West Virginia on Overland Flow and Storm Runoff, *J.*
1000 *Forestry* 62(3): 167-171, 1964.
1001
1002 SCS, Hydrology guide for use in watershed planning, Soil Conservation Service, USDA, Washington, D.C., 1954
1003
1004 SCS, Engineering Hydrology Training Series, Module 103: Runoff Concepts, US Department of Agriculture,
1005 Washington DC, 1989.
1006
1007 Sharma, M.L., Gander, G.A. and Hunt, C.G., Spatial variability of infiltration in a watershed. *Journal of*
1008 *Hydrology*, 45(1-2): 101-122, 1980.
1009
1010 Sherman, L. K., Streamflow from rainfall by unit-graph method, *Engineering News Record*, 108: 501-505, 1932
1011
1012 Sherman, L.K., Appendix C—The Horton method for determination of infiltration-rates. *Transactions American*
1013 *Geophysical Union*, 17(2): 312-314, 1936.
1014
1015 Sherman, L. K., Comparison of f curves derived by the methods of Sharp and Holtan and of Sherman and
1016 Mayer, *Transactions American Geophysical Union*, 24: 465-467, 1943.
1017
1018 Singh, V.P. and Yu, F.X., Derivation of infiltration equation using systems approach. *Journal of Irrigation and*
1019 *Drainage Engineering*, 116(6), 837-858, 1990.
1020
1021 Sklash, M.G., Farvolden, R.N., The role of groundwater in storm runoff. *J. Hydrol.* 43: 45-65, 1979.
1022
1023 Smith RE. The infiltration envelope: results from a theoretical infiltrometer. *Journal of Hydrology* 17: 1–21,
1024 1972.
1025
1026 Smith, R. E., and Parlange, J.-Y., A parameter-efficient hydrologic infiltration model, *Water Resour. Res.*, 14(3):
1027 533-538, 1978
1028
1029 Smith, R.E. and Woolhiser, D.A., Overland flow on an infiltrating surface. *Water Resources Research*, 7(4): 899-
1030 913, 1971.
1031



- 1032 Snyder, F. F., A conception of runoff-phenomena, *Trans. Amer. Geophys. Un.*, 20: 725-736, 1939
1033
1034 Steenhuis, T.S., Winchell, M., Rossing, J., Zollweg, J. A. and Walter, M. F., SCS runoff equation revisited for
1035 variable source runoff areas, *J. Irrig. Drain. Div. ASCE*, 121: 234-238, 1995
1036
1037 Swartzendruher, D., Revised attribution of the power form infiltration equation. *Water Resources*
1038 *Research*, 29(7): 2455-2456, 1993.
1039
1040 Tallaksen, L.M., A review of baseflow recession analysis. *Journal of hydrology*, 165(1-4): 349-370, 1995.
1041
1042 Talsma, T. and Parlange, J.-Y., One-dimensional vertical infiltrations, *Aus. J. Soil Res.*, 10: 143-150, 1972
1043
1044 VanderKwaak, J.E. and Loague, K., Hydrologic-response simulations for the R-5 catchment with a
1045 comprehensive physics-based model. *Water resources research*, 37(4): 999-1013, 2001.
1046
1047 Walter, M.T., Mehta, V.K., Marrone, A.M., Boll, J., Gerard-Marchant, P., Steenhuis, T., Walter, M.F., Simple
1048 estimation of prevalence of Hortonian flow in New York City watersheds. *J. Hydrol. Engng, ASCE* 8(4), 214–218,
1049 2003.
1050
1051 Whipkey, R. Z., Storm runoff from forested catchments by subsurface routes, *Int. Assoc. Sci. Hydrol. Pubn.* 85:
1052 773-779, 1969
1053
1054 Wilson B.N., Slack D.C., Young R.A. A comparison of three infiltration models. *Trans. Amer. Soc. Aric. Engrs.* 25:
1055 349–356, 1982
1056
1057 WMO, *International Glossary of Hydrology*, Report WMO-No.385, WMO: Geneva, 2012.
1058
1059 Young, P.C., Hypothetico-inductive data-based mechanistic modeling of hydrological systems. *Water Resources*
1060 *Research*, 49(2): 915-935, 2013.
1061
1062 Yu, B., Theoretical justification of SCS method for runoff estimation, *J. Irrig. Drain. Div., ASCE*, 124: 306-309,
1063 1998
1064
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Table 1. Selected 20th Century infiltration equations with $f(t)$ as infiltration capacity, $F(t)$ as cumulative infiltration, and K_s as saturated hydraulic conductivity (other symbols defined under Comments)

Source	Equation	Comments
Green and Ampt (1911)	$f(t) = K_s \left(\frac{ \psi_f }{z_f} + 1 \right)$	Based on Darcy's law with piston-like wetting front from initial moisture content to saturation. ψ_f is capillary pressure change across wetting front, z_f is current depth to wetting front
Kostiakov (1932), Lewis (1937)	$f(t) = Kt^n$	Empirical, with K and n as parameters
Horton (1939, 1940)	$f(t) = (f_o - f_c)e^{-kt} + f_c$	Empirical. Allows for an initial f_o and final f_c infiltration capacities. Argues that it represents rate equation for extinction phenomena at the soil surface
Mezencev (1948) and later Smith (1972)	$f(t) = Kt^n + f_c$	Extension of the Kostikov-Lewis equation to include a final infiltration capacity
SCS-CN (1954)	$F = P - \frac{(P - 0.2S)^2}{P + 0.8S}$ $S = \frac{1000}{CN} - 10$	Origins lie in estimation of direct flow rather than overland flow, but often interpreted as an infiltration equation. P is event precipitation, S is storage capacity of the soil, CN is the curve number.
Philip (1957)	$f(t) = \frac{St^{-1/2}}{2} + A$	First two terms of series solution to Darcy-Richards equation assuming constant diffusivity. S is the sorptivity of the soil, A is a parameter likely to be somewhat smaller than the saturated conductivity of the soil.
Holtan (1961)	$f(t) = f_o + a(S_o - F)e^n$	Empirical. Makes infiltration capacity dependent on cumulative volume already infiltrated F and initial storage capacity of the soil S_o which also provides an upper limit for infiltration
Talsma and Parlange (1972)	$f(t) = St^{1/2} + \frac{K_s t}{3} + \frac{K_s^2 t^{3/2}}{9S}$	Assumes diffusivity is proportional to rate of change of conductivity with θ
Morel-Seytous and Khanji (1974)	$f(t) = \frac{K_s}{B} \left[\frac{h_o(\theta_s - \theta_i) + C_d}{z_f(\theta_s - \theta_i)} + 1 \right]$	Extension of the Green-Ampt equation with C_d as the capillary drive and h_o as depth of surface ponding.



Ahuja and Tsuji (1976)	$F(t) = K_s t + \frac{K_s - A}{B} [e^{-Bt} - 1] + z_f(\theta_s - \theta_i) \ln[1 + F(t)/z_f(\theta_s - \theta_i)]$	Extension of the Green-Ampt equation to have an exponential time variable hydraulic conductivity function, with parameters A and B , based on comparison with the Philip equation. Claim better fit to observations.
Collis-George (1977)	$F(t) = S(t_c)^{1/2} \left(\tanh \frac{t}{t_c} \right)^{1/2} + f_c t$	Empirical but argues that it provides a better fit to data than Green-Ampt, Horton or Philip equations. S is the sorptivity and t_c a time scale parameter
Smith and Parlange (1978)	$f(t) = K_s \left[\frac{\exp F(t)/C_D}{\exp F(t)/C_D - 1} \right]$	Solution of Darcy-Richards assuming an exponential diffusivity function. Useful when rainfall rates vary, as $f(t)$ is a function of cumulative infiltration $F(t)$.
Beven (1984)	$\frac{dF(t)}{dt} = \frac{K_o \alpha [\psi_f + F/(\theta_s - \theta_i)]}{1 - e^{-\alpha F/(\theta_s - \theta_i)}}$	Extension of Green-Ampt for case of exponential decline of saturated conductivity with depth as $K(z) = K_o e^{-\alpha z}$. Has an implicit solution for F
Singh and Yu (1990)	$f(t) = f_c + \frac{a[S_o - F(t)]^M}{[F(t)]^N}$	Made infiltration dependent on initial storage available and powers of cumulative infiltration and remaining storage. M , N and a are parameters

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