

1 The era of Infiltration

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3 Keith Beven
4 Lancaster Environment Centre
5 Lancaster University
6 Lancaster, UK
7 k.beven@lancaster.ac.uk
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9 Abstract

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11 Inspired by a quotation from Howard Cook in 1946, this paper traces the evolution of the
12 infiltration theory of runoff from the work of Robert Horton and LeRoy Sherman in the
13 1930s to the early digital computer models of the 1970s and 1980s. The reasons for the
14 popularity of the infiltration theory are considered, as well as its impact on the way in which
15 hydrological responses were perceived by several generations of hydrologists.
16 Reconsideration of the perceptual model for many catchments, partly as a result of the
17 greater appreciation of the contribution of subsurface flows to the hydrograph indicated by
18 tracer studies, suggests a more precise utilisation of hydrological terms and, in particular,
19 that the use of runoff and surface runoff should be avoided.
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24 *Some future historian of the development of scientific hydrology will probably be*
25 *tempted to call the present period the "era of infiltration." At any rate, the*
26 *preoccupation of contemporary hydrologists with "the infiltration theory of runoff,"*
27 *and the vast amount of energy they have expended in an effort to turn this concept to*
28 *practical account, will certainly be put down as a distinctive feature of our times.*

29 Howard L. Cook, 1946 (p.726)
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31 The Background to the Era of Infiltration

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33 This quotation from Howard L. Cook has stimulated this paper which has the aim of trying to
34 understand why the "*infiltration theory of runoff*" came to have such an impact on
35 hydrological understanding and analysis from the 1930s onwards, particularly in the work of
36 American hydrologists such as Robert Elmer Horton¹, LeRoy Kempton Sherman², Waldo
37 Smith³, Cook himself and many others. In particular to consider the question of why, when
38 in many parts of the United States overland flow is just not observed that often, the
39 infiltration theory of runoff achieved such a widespread acceptance both in the US and
40 elsewhere. The literature in relation to infiltration and surface runoff is, however, vast and
41 a complete review is not possible. I hope to have brought out the most important points and
42 references relevant to this question, particularly from some of the earlier publications.
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¹ (1875-1945), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Horton,_Robert_Elmer

² (1869-1954), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Sherman,_LeRoy_K.

³ (1900-1994), Executive Director of AGU from 1944-1970, see <https://honors.agu.org/waldo-e-smith-1900-1994/>

44 We will take the start of the era of infiltration as the 1933 paper *On the role of infiltration in*
45 *the hydrological cycle* in the Transactions of the American Geophysical Union by Robert
46 Horton. That was not the start of infiltration studies in the United States. Before that there
47 had been experimental studies of infiltration, particularly in relation to irrigation practices
48 (e.g. Muntz et al., 1905) and at the plot scale (e.g. Houk, 1921) as well as the model of
49 infiltration of Green and Ampt (1911). In the 1933 paper, however, Horton sets out a
50 particular perceptual model of catchment response in an often-cited quotation.

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52 *“Infiltration divides rainfall into two parts, which thereafter pursue different courses*
53 *through the hydrologic cycle. One part goes via overland flow and stream-channels to*
54 *the sea as surface-runoff; the other goes initially into the soil and thence through*
55 *ground-water flow again to the stream or else is returned to the air by evaporative*
56 *processes. The soil therefore acts as a separating surface and the author believes that*
57 *various hydrologic problems are simplified by starting at this surface and pursuing the*
58 *subsequent course of each part of the rainfall as so divided, separately. This has not*
59 *hitherto, in general, been undertaken.”*

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Horton (1933, p.446/447)

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62 This last sentence (not so often cited) suggests that this provides a good starting point. More
63 than a decade later, the context of the Cook quotation was the report of the AGU Committee
64 for Infiltration for 1946, chaired by G. W. Musgrave who worked in the Soil Conservation
65 Service at that time. This committee had a number of sub-committees: on Infiltration and
66 the Physics of Soil Moisture and of the Infiltration Process; on Infiltration in Relation to
67 Ground Water; on Infiltration in Relation to Snow and Its Physical Properties; on Infiltration
68 in Relation to Surface Runoff; on Infiltration in Relation to Irrigation; and on Infiltration in
69 Relation to Evapo-transpiration and the Consumptive Use of Water. Infiltration was
70 therefore considered to be both central and fundamental to hydrological understanding. The
71 preface to the Cook article provided by Musgrave is pertinent to our question:

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73 *“In the early phases of the development of a new concept, it is common to find*
74 *considerable diversity of thought among the workers in that field. Subsequently,*
75 *through the exchange of ideas, and particularly through the development of factual*
76 *evidence, abstract ideas are crystallized into specific entities. Progress in the*
77 *development of the field is increased, and practical application of ideas that originally*
78 *were abstract now proceeds with greater and greater success.*

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80 *The concept of infiltration as a factor modifying runoff phenomena is still relatively*
81 *new. Discussions quite diverse in their conclusions abound in the literature. Is it not*
82 *true that at least some of the diversity of thought is due to diverse interpretations of*
83 *terms and definitions? Indeed, it would seem that there is need for re-examination of*
84 *some of the very fundamentals of the problem.*

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86 *Many have realized during the past several years that there is great need for*
87 *clarification of thought in this relatively new phase of hydrology. Many have realized*
88 *that whatever may be done to promote thinking and expression in terms that are*
89 *specific and are understood by all other workers is certain to result in improved*
90 *research and improved application of research findings.*

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This paper should do much in the way of promoting unanimity in use of terms, of opinion as to their significance, and of clarity of concept.” (Musgrave, 1946a, p726)

The Sub-committee on Infiltration in Relation to Surface Runoff was chaired by Howard Cook, the other members being W. W. Horner, R. A. Hertzler, G. A. Hathaway, and Walter B. Langbein⁴. Cook had been one of the principal assistants of Robert Horton at the Horton Hydrologic Laboratory in Voorheesville, New York⁵.

The popularity of the infiltration theory

Following the quotation at the head of this paper, Cook starts his outline of the subject by considering why the infiltration concept had become so popular:

“There have, of course, been logical reasons for this remarkable interest in the subject. As in all sciences, many have been attracted to it simply because of its newness. Another class—and the one that has participated most eagerly—is composed of those intrepid practicing engineers who are obliged to make the runoff estimates upon which depend the failure or success of costly flood control, water supply, and similar works. Still another group has been intrigued by a purely scientific interest, sharpened by the fact that the calculation of runoff is the central problem of the science of hydrology and involves all phases of the hydrologic cycle. Among these are scientists in the fields of soils, plants, and meteorology. As a result of these various motivations, vast amounts of labor have been expended - much of it misdirected - and many exaggerated claims have been made, to be countered, naturally, by the disparaging murmurs of the "old guard," and other important lines of investigation have been temporarily slighted. But real progress has been made. Better estimates of runoff are now possible than could be made previously. Problems that would not yield at all to earlier methods are now soluble, albeit the solutions are sometimes only rough approximations. The inescapable conclusion is that a tool of considerable practical value has been added to the equipment of the hydrologist.” (Cook, 1946, p727)

This quotation already reveals some quite modern elements of the sociology of an inexact science. The infiltration concept provided a new paradigm for thinking about runoff. It did so in a *rational* way “*simply by providing a physically correct concept of the runoff process*” (p.730), but which also provided the engineer with a tool that could be usefully applied to provide better estimates of runoff for design purposes (even if sometimes only *rough approximations*). I do wonder if any of that old guard were murmuring ... but should you not be able to see the surface runoff occurring during storms to apply this type of analysis properly?

⁴ (1907-1982), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Langbein,_W

⁵ Howard L. Cook graduated in Civil Engineering from the State University of Iowa in 1929 then worked at the Horton Hydrological Laboratory as assistant to Robert Horton from 1929 before moving to the Soil Conservation Service in 1934 where he was in charge of hydraulic research. He later worked as an engineer for the Department of the Army. I have not been able to find a full obituary of his life and career.

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Surface and subsurface runoff

Cook, in fact, almost immediately recognises the difficulty of applying the concept in practice in a section on surface and subsurface runoff. He notes that:

“The runoff from an area is the water flowing from it over the surface of the Earth, either in streams or as overland flow. Part of this water has never been below the surface. This is called surface runoff. Another part has previously passed into the Earth and subsequently returned to the surface. This is called subsurface runoff...”
(op. cit. p.728).

He continues:

“(1) Only surface runoff can be directly determined from Infiltration data. (2) When runoff contains subsurface flow, the gaged discharge cannot be used to derive infiltration data for the area unless the surface runoff can be separated from the total. (3) In general, there is no way of separating surface and subsurface runoff when only records of the rates of flow are available.” (op. cit. p.728)

There is also an interesting comment that:

“A normal stream. carries both surface and subsurface flow in proportions varying widely from time to time. During floods most of the water discharged from deep-soiled drainage basins is ordinarily made up of surface runoff. However, in areas of low storage capacity (such as thin-soiled basins) a large proportion of the flood water may consist of subsurface runoff.” (op. cit., p.728)

The reasoning behind this statement is not totally clear. It implies an expectation that catchments with thin soils and small storage capacities would be associated with higher infiltration capacities and higher downslope transmissivities such that there could be a greater contribution of subsurface stormflow. However, the reasoning might have run more along the lines that high storage capacity will mean a longer mean residence time so that any infiltrated water would simply not be able to contribute within the time scale of the hydrograph. Cook also notes later that in deeper soils when water tables are low in summer, infiltrating water may not actually reach the saturated zone.

In fact, the role of subsurface runoff production was being promoted more generally at this time. Charles R. (Chuck) Hursh⁶, Director of the Coweeta watershed experiments in North Carolina, had long been promoting the idea that in places where overland flow was only rarely seen, such as in the forests of the Appalachians, the hydrograph was necessarily dominated by direct channel precipitation and subsurface flows, with only slow responses observed in boreholes (Hursh, 1936, 1944; Hursh and Brater, 1941). It is also not as if hydrologists did not realise that in different parts of the US there was less expectation of overland flow. In a national review of flood runoff published during the era of infiltration

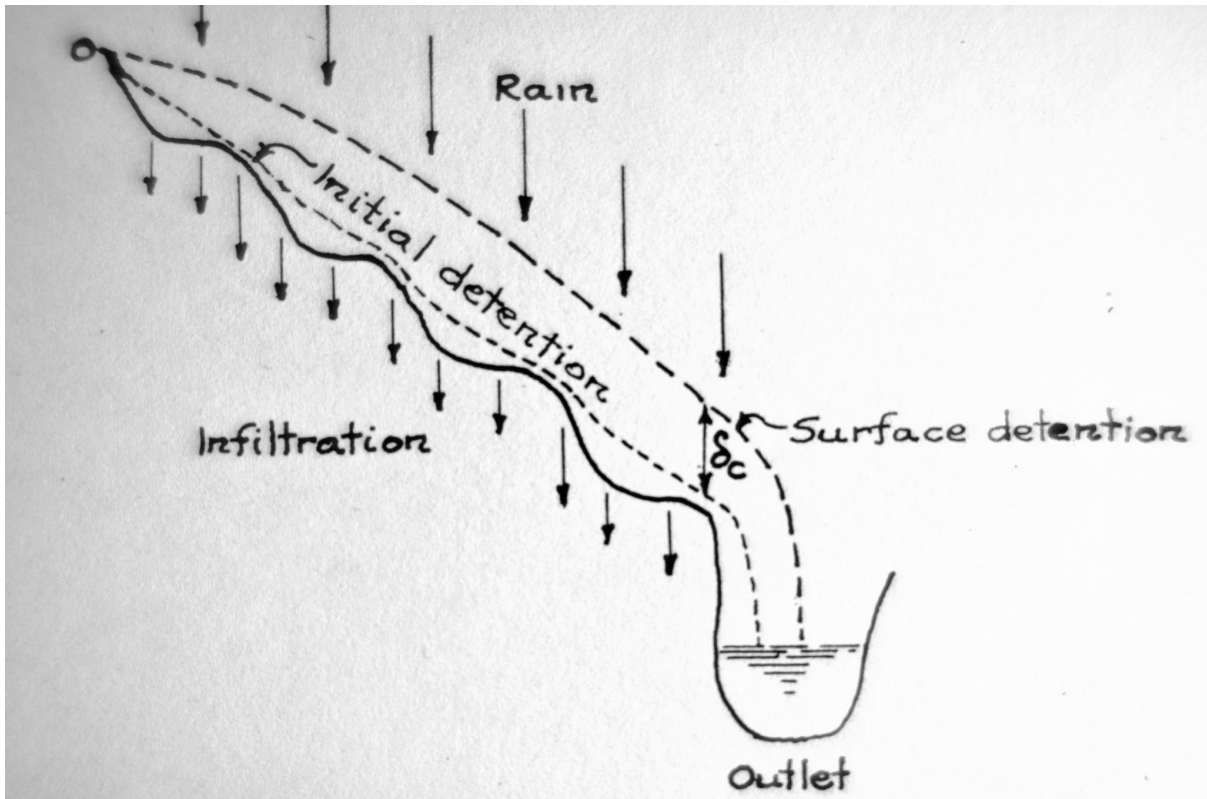
⁶ (1895-1988), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Hursh,_Charles_R

178 Hoyt and Langbein (1939) noted, with some surprise, that: "To those who are acquainted
179 with the flood-producing possibilities of isolated storms of from 10 to 12 inches [250-
180 300mm] in humid areas, the absence of flood-runoff under single storm-experiences of the
181 same magnitude on steep mountain slopes of parts of the southern coast range [in
182 California] is amazing" (p.172). They continue:

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184 "Although the small plots may indicate the absence of direct run-off and the
185 differences between rainfall and runoff an absorption of between 15 and 20 inches,
186 there is a rapid passage of a part of the infiltrated water into stream channels, either
187 through the relatively shallow earth-mantel or through the upper parts of the
188 shattered bedrock. To the extent that the observations and deductions are correct,
189 the flood-hydrograph in these areas is composed largely of ground-water which has
190 concentrated very quickly as to time superimposed on which is a small amount of direct
191 runoff with irregularities closely following irregularities in the maximum rates of
192 precipitation. This condition may also apply on other parts of the country where floods
193 occur although studies on small areas indicate very high infiltration capacities." (Hoyt
194 and Langbein, 1939, p.174)

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196 That the infiltration concept was used much more widely, however, was undoubtedly due to
197 a number of factors. The first was that it claimed to be *rational* or physically-based; the
198 second was the simplicity of calculating amounts of runoff given information about rainfalls
199 and infiltration capacities; the third was the strong and rather combative character of Robert
200 Horton.

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204 Figure 1. Half-Section of a Small Drainage Basin Illustrating Runoff Phenomena (Vertical Scale Greatly
205 Exaggerated) (from Horton, 1935, with original caption)

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In the 1933 paper Horton appears to buy in completely to the idea that storm hydrographs are produced by overland flow. This is also clear from his 1935 monograph on Surface Runoff Phenomena, from which Figure 1 is taken. This is perhaps an example of the pragmatics of applications outweighing the information from direct observations (Horton was working as a consultant by the 1930s). This is also evidenced in his paper on Remarks of Hydrologic Terminology later published in the Transactions of the AGU in 1942. He starts by saying that:

“When a science is advancing rapidly, as is hydrology today, especially when it is changing from an adolescent or qualitative to an adult or quantitative basis, new terms are needed in particular for the following two purposes: (1) To give expression to new ideas and concepts; (2) to give more definite, specific, quantitative meaning to terms and concepts heretofore chiefly qualitative.” (Horton, 1942, p.479)

However, in what follows it is clear that Horton’s primary purpose is to favour his own terminology over that of others. There are a number of entries of this type (infiltration rate v. infiltration capacity; recharge v. accretion; plot v. plat⁷), but in the current context the one on subsurface runoff is of most interest. Thus:

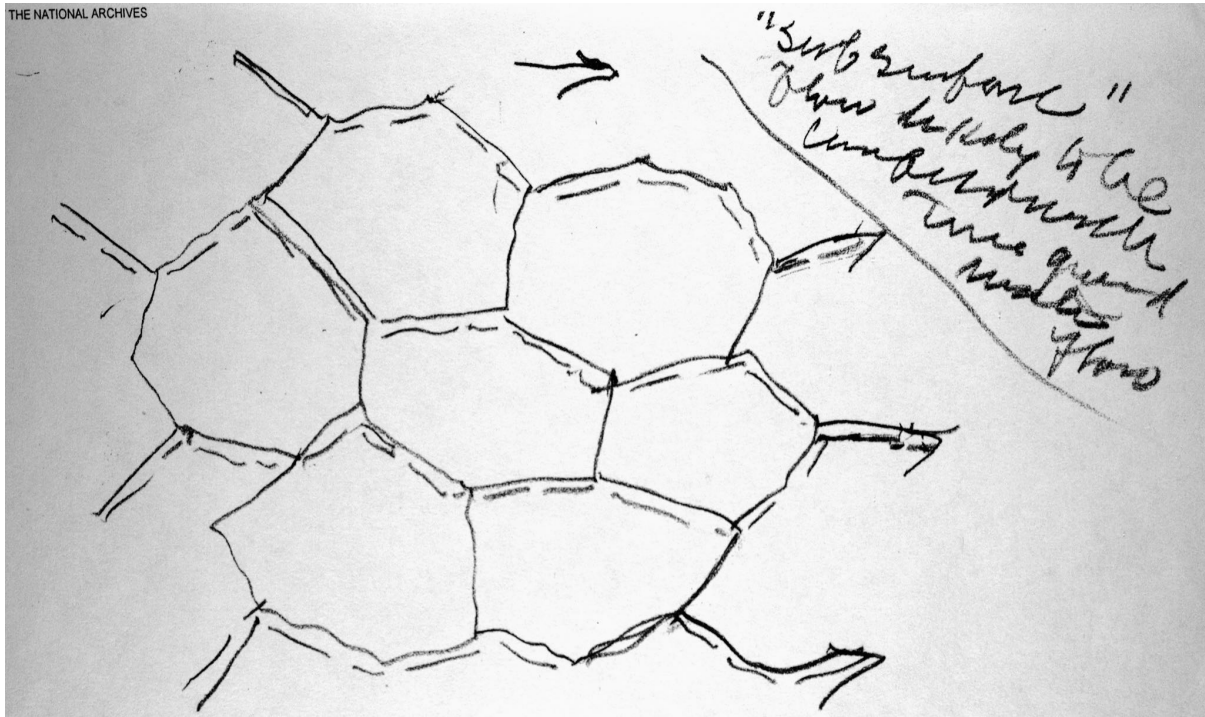
“Subsurface and concealed-surface runoff. Cases arise where surface-runoff may take place in such a manner as not to be visible, as, for example, where it occurs through a layer of coarse material, sometimes through a thick matting of grass or mulch-cover; through a layer of plant roots close to the soil-surface and under forest-litter; or even, in some cases, (through a network of sun-cracks in the soil-surface. This has sometimes been called ‘subsurface-runoff’, sometimes ‘ground-water flow’. The term ‘subsurface-runoff’ would not be objectionable were it not for the fact that it is likely to be confused with true ground-water flow. The term ‘groundwater flow’ applied to this class of flow is highly objectionable on several counts; flow occurring close to the surface in the manner described has little in common with true ground-water flow. It is mostly turbulent flow, while true ground-water flow is mostly laminar. It persists only during rainfall-excess or for a short time thereafter, measured in hours or at the most in days, whereas ground-water flow persists on perennial streams at all times. Furthermore, surface runoff follows the same laws and behaves in the same manner whether it actually occurs visibly on the ground surface, or is concealed and invisible, taking place just below the soil-surface where it is sustained by temporary detention below the soil-surface. Nevertheless, it may be desirable to distinguish between the two cases and, if so, flow which is essentially surface-runoff but which is concealed from view in some one of the ways described, may appropriately be called ‘concealed-surface runoff.’”

(Horton, 1942, p.481)

Thus, by definition, water contributing to the hydrograph is allowed to be hidden from view and treated as surface runoff *as if* it was in excess of the infiltration capacity of the soil, at least if no longer a laminar flow. An example., taken from the boxes of Horton’s papers in his analysis of downslope flow through sun-cracks (see Figure 2). Again, perhaps underlying this

⁷ Horton argued that infiltration capacity, accretion and plat were to be preferred, citing Oxford English Dictionary definitions. In this at least, he has not got entirely his way in the long term.

251 is an interpretation that laminar subsurface flow velocities were far too slow to allow
252 significant contributions to the hydrograph (although, interestingly, observations from the
253 Horton Hydrological Laboratory did show some examples of fast borehole responses, see
254 Beven, 2004c).
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257 **Figure 2. Figure explaining lateral “subsurface” flow in sun-cracks as concealed surface runoff (Drawing in**
258 **Horton’s hand from Box 71 of the Horton Papers in the National Archive)**
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261 We should remember that the tracer information that revealed that in many catchments
262 hydrographs are composed largely of pre-event water was not available in the 1930s and
263 1940s, but Beven (2004a) shows that by comparing rainfall frequency data and Horton’s own
264 infiltration observations it is unlikely that he would have observed widespread overland flow
265 on his own research catchment near Voorheesville more than 1 in 2 to 1 in 5 years (unless of
266 course it was concealed!). Walter et al. (2003) had come to similar conclusions in an analysis
267 of a number of sites in New York State.

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269 **The complexity of infiltration processes**

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271 Horton’s perceptual model of the response of catchments was, however, much more
272 sophisticated than he is generally given credit for. This was revealed in the 94 boxes of his
273 papers that were classified by Walter Langbein (who had also worked with Horton) and
274 deposited in the US National Archives in 1949 (see the discussions in Beven, 2004a,b,c).
275 Horton argued, for example, that infiltration capacities would be primarily controlled at the
276 soil surface by what he called extinction phenomena, such as compaction of the surface by
277 rainsplash, and blocking of larger pores by displaced fine particles. It was these extinction
278 phenomena that led to the gradual decline in infiltration capacities with time, as described by

279 his well-known infiltration equation that first appears in Horton (1939)⁸. He also recognised
280 that bioturbation and agricultural practices would change the surface between events,
281 resulting in a recovery of infiltration capacities. There could also be marked seasonal
282 changes, something that he observed in his own infiltration observations, and strong
283 variability in space. He recognised the role of macropores and surface microtopography in
284 concentrating water and allowing the escape of air, which he had shown to be a control on
285 infiltration by experiment (see Beven, 2004b). He also understood that while it was possible
286 to make local predictions of infiltration excess on different land units (effectively producing a
287 distributed model of surface runoff production), it was not possible to calculate the different
288 contributions given only hydrograph contributions.

289
290 Horton was also not alone in recognising the complexity of infiltration processes in this
291 period. In the discussion of a physics-based paper on infiltration by Willard Gardner (1946),
292 G. W. Musgrave commented:

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294 *“However, we have before us a type of problem which particularly requires caution in*
295 *extending and applying laboratory-findings to natural field-soils. At least insofar as the*
296 *structure of the laboratory-sample differs from that of the natural soil, caution is*
297 *warranted. Most soils of natural structure contain crevices, channels, and openings*
298 *that transmit free water rather rapidly, though locally, to some depth. It appears from*
299 *many observations in the field that in some cases at least, a very large portion of the*
300 *infiltrating water is thus transmitted. Where a dye is used and the soil-profile is*
301 *dissected following application, the highly irregular nature of the downward moving*
302 *water becomes evident. Dry "islands" are bypassed and left with their air-water*
303 *interfaces intact, at least temporarily. The channels conducting free water act as*
304 *feeders laterally for capillary water, often for a considerable time. The forces of gravity*
305 *and capillarity are not always acting in conjunction. One wonders whether other forces*
306 *such as thermal gradients are involved, and if so, to what extent they are effective.”*

307 G. W. Musgrave (1946b, p.135)

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309 **Surface runoff and baseflow separation**

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311 This then created a problem for the infiltration theory of runoff because, as noted earlier,
312 Cook points out there was no way of separating surface runoff and subsurface contributions
313 to the hydrograph. But in order to derive the apparent infiltration characteristics from
314 hydrographs and pluviographs it was necessary to do so. The concept of baseflow separation
315 and recession analysis has continued to exercise hydrologists ever since (see Hall, 1968;
316 Tallaksen, 1995; Beven, 1991; Arnold et al., 1995; Chapman, 1999; Eckhardt, 2005), right to
317 the present day (Ladson et al., 2013; Lott and Stewart, 2016; He et al., 2016; Duncan, 2019).
318 Some of these methods allowed for an increase in baseflow during an event, arguing that
319 there would be some accretion to the water table during the time scale of the event (e.g.
320 Horton, 1935; Hursh and Brater, 1941, as based on borehole observations at Coweeta;
321 Hewlett and Hibbert, 1967; or the digital filters of Nathan and McMahon, 1990; Furey and
322 Gupta, 2001; and Aksoy et al., 2009).

⁸ It is commonly cited to Horton (1933) but does not appear there. It also does not appear in Horton's Monograph on Surface Runoff Phenomena of 1935.

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Both Horton (1935) and Cook (1946) suggests the strategy of continuing the past groundwater depletion or recession curve as an indicator of baseflow, with all the flow above that curve being treated as if it was infiltration excess surface runoff but only for *“the special case when the subsurface flow is derived entirely from the zone below the permanent groundwater, table, and no groundwater accretion occurs, a satisfactory estimate of subsurface flow can be made simply by extending the groundwater depletion curve.”* (Cook, op.cit. p728).

But simply continuing the recession curve results in a problem for the method in calculating the volume of surface runoff for an event, since the previous recession will always be below the recession of the current event. Thus, there was also a pragmatic need to allow for a “baseflow” contribution to rise to meet the falling recession limb of an event. Horton (1935) had earlier wanted to allow for the accretion of groundwater due to infiltration and specifies a method for deciding when the field moisture deficit of the soil has been satisfied, after which a line can be drawn to where the form of the recession matches the groundwater depletion curve. He notes that this might occur above or below the point of initial hydrograph rise (Figure 3). Hewlett and Hibbert (1967) suggested using a standard slope for this rise of 0.05 cfs/mi²/hr (or 0.0567 ls⁻¹/km²/hr), starting from beneath the hydrograph peak, but this was based only on discharge and borehole data from some small catchments at Coweeta. Somehow, it became a standard that was used around the world, regardless of soils, vegetation or geology. Others suggested that the end of surface runoff would be marked by a break between straight line segments on a semi-logarithmic plot of the recession, indicating a transition to a process with a slower time constant⁹. In essence Cook was correct, there is no satisfactory way of separating surface from subsurface flow in this way (see also the discussion in Beven, 1991¹⁰).

⁹ Barnes (1939, 1944) recognised three such components, overland-flow, ground-water flow, and what he called secondary base-flow, and later storm-seepage or interflow; while Kunkle (1962) distinguished baseflow from the effects of bank storage.

¹⁰ Beven (1991) includes a section headed “Choosing a baseflow separation method” that consists only of the one word “Don’t”.

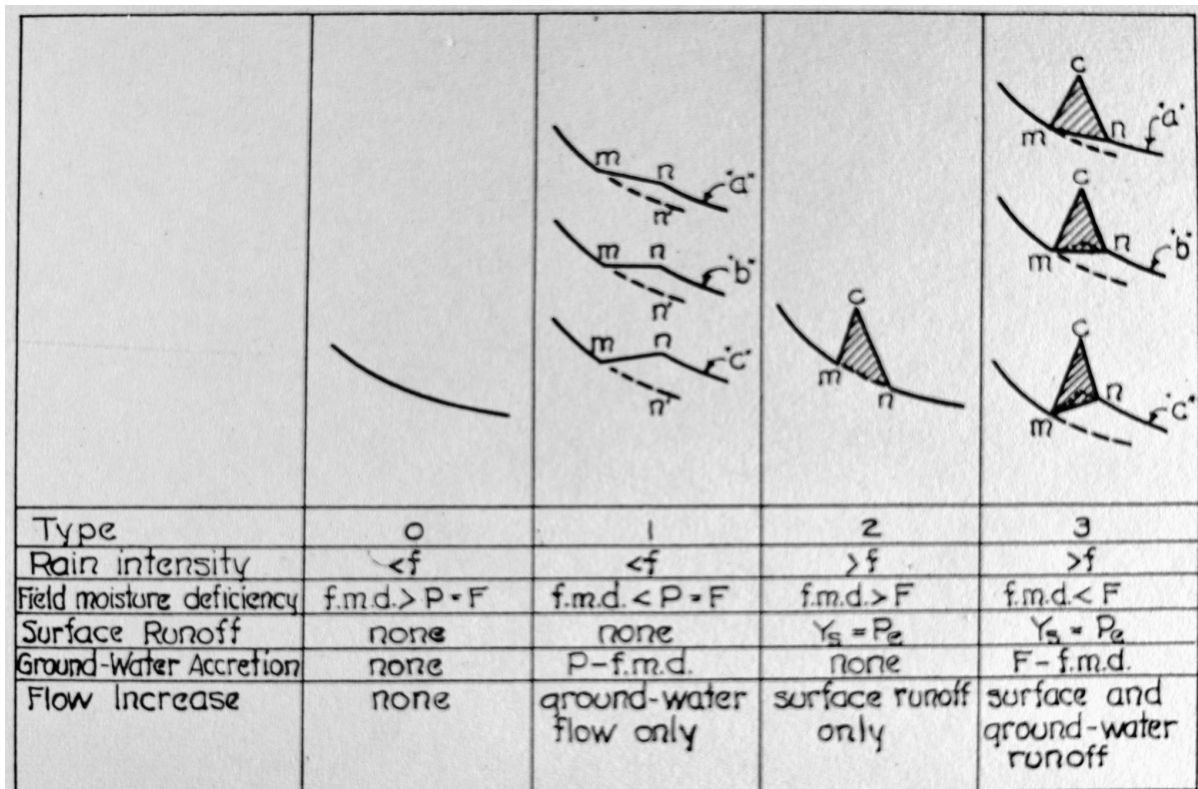


Figure 3. Classification of Stream Rises, with Type 3 showing how to separate ground-water runoff.

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Derivation of infiltration indices from the hydrograph

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354 Both Horton and Cook recognised that there was a difference between predicting surface
355 runoff locally given information about rainfall and infiltration capacity curves for a soil and
356 deriving apparent infiltration information from rainfalls and an estimate of "surface runoff".
357 In the first case, the local variability of soils, vegetation and management practices could be
358 taken into account (given the infiltration characteristics of each) on what Cook calls soil-cover
359 *complexes*; the equivalent of modern-day hydrological response units. Such an approach can,
360 in principle, also allow for the type of time variability of observed infiltration rates described
361 by Horton (1940). However, I have found no real recognition at that time of the difference in
362 scale between the point and plot scale at which observations are possible, and the soil-cover
363 complex scale at which the calculations might be applied.

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365 The second case is more challenging in that it is not possible to obtain more than an index of
366 catchment-wide apparent infiltration. Cook gives two examples of such indices that can be
367 obtained by matching the observed volume of surface runoff to the observed pattern of
368 rainfall, both of which still appear in texts today. The first is based on assuming an average
369 declining infiltration capacity to produce an average infiltration capacity (the f_{av} or W index)
370 with a special case after significant wetting equivalent to a final constant infiltration capacity
371 W_{min} . The second is assuming a constant infiltration capacity (the ϕ index). He demonstrates
372 that for this latter index a dependence on rainfall intensity should be expected where there
373 are multiple soil-cover complexes in a catchment "*because the higher the intensity the greater*
374 *the proportion of the area producing runoff throughout the rain, not because infiltration*
375 *capacity increases with intensity of rainfall.*" (Cook, op.cit. p.738). He therefore already

376 recognises the possibility of partial contributing areas of runoff production (but again, not
377 how scale issues might affect the outcome).

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379 Further problems arise when there is intermittent rainfall, or where rainfall intensity
380 intermittently falls below the infiltration capacity of the soil and there might be the possibility
381 of some recovery of infiltration capacities between bursts of rainfall. He goes into some detail
382 to explain how different cases might be handled. He does not include, however, the
383 suggestion of using *time condensation* (now more commonly known as the *time compression*
384 *assumption*). This had been introduced 3 years earlier by Leroy Sherman (1943) and then
385 modified by Heggie Nordahl Holtan¹¹ (1945). Holtan (1961) was also the first person to
386 suggest an infiltration equation that was expressed directly in terms of cumulative infiltrated
387 water, thereby implicitly incorporating a time compression assumption.

388 389 **Infiltration equations**

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391 Application of the infiltration theory is easiest on a single soil-cover complex given rainfall and
392 information about infiltration capacities of the soil. Quantitative estimation of runoff is
393 easier if the infiltration capacities can be represented as a mathematical function (although
394 in the 1930s and 1940s when the calculations were made by hand, it could actually be faster
395 to read values off of a graph or from a table than to do the calculation, and many papers of
396 the time give examples of hand-worked calculations, e.g. Sherman, 1936, 1943).

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398 The Green and Ampt (1911) infiltration equation (Table 1), based on a piston-like wetting
399 front approximation to Darcy's law had been available for some time. Horton (1939, 1940)
400 developed his own form of equation¹². As noted earlier, he argued that this represented
401 surface controls rather than profile controls on the infiltration capacity. Cook mentions only
402 the Horton equation in his exposition of the infiltration theory but there were other empirical
403 infiltration equations suggested such as the power law form suggested independently by A.
404 N. Kostiaikov (1932) and Mortimer Reed Lewis (published in 1937 but according to
405 Swarzendruber, 1993, proposed in 1926), the Soil Conservation Service (SCS) curve number
406 method that first appeared in 1954 (SCS, 1954), and later that of Holtan (1961). The idea of
407 solving the Darcy-Richards equation was picked up again in the 1950s, most notably by John
408 Philip¹³ (1954) and then in a series of papers for the infiltration problem (Philip, 1957). Given
409 the nonlinearity of the governing equation this was a mathematical challenge for soil
410 physicists and set off a variety of solutions for different types of diffusivity function and
411 boundary conditions, that continued into the 21st Century (e.g. Ogden et al., 2015). A
412 summary of some of these infiltration equations is given in Table 1. Comparison of the
413 behaviours of different equations have been given by, for example, Wilson et al. (1982),
414 Davidoff and Selim (1986), Mishra et al. (2003) and Chahinian et al. (2005).

415
416 The SCS curve number method is of particular interest in terms of its common interpretation
417 as an infiltration equation. Horton frequently clashed with the SCS and seems to have had a
418 low opinion of their engineers (the SCS insisted on interpreting infiltration capacity as a

¹¹ (1909-2006), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Holtan,_H._N.

¹² Note that Philip (1954) suggests that this equation was first suggested by Gardner and Widtsoe (1921), but Horton (1939, 1940) does not refer back to that earlier paper.

¹³ (1927-1999), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Philip,_John_R

419 volume rather than as a rate, for example¹⁴). This originally derives from the work of Mockus
420 (1949) who plotted estimates of storm rainfall against the volume of surface runoff, as
421 previously suggested by Sherman (1943). From this analysis Mockus suggests a relationship
422 between them of the form

$$Q = P[1 - (10)^{-bP}]$$

423
424
425
426 with a multiple regression used to estimate the coefficient b based on data for 50 storms
427 collected from catchments “of field size or larger” (p.41). The soil, crop, season, and
428 antecedent precipitation indices used in the regression were derived by an analysis of data
429 from nine USDA research stations. Nowhere does he specify how the amounts of surface
430 runoff were derived. The resulting surface runoff was routed through a dimensionless unit
431 hydrograph to derive hydrograph peaks (Mockus also mentions how a triangular unit
432 hydrograph could be used to approximate the dimensionless unit hydrograph).

433
434 The methods were tested “by estimating total runoff for storms on single- and mixed-cover
435 watersheds”, by which he seems to mean the total volume of surface runoff. The results
436 were better for large storms than small storms and for mixed-cover rather than single cover
437 catchments. Better results were obtained by breaking long duration storms into parts. He
438 notes that rainfall spatial variability and direction of movement might be important in getting
439 better estimates.

440
441 The SCS curve number method took the data of Mockus and also a large number of infiltration
442 capacity measurements on different soil types and land covers in the US, and postulated a
443 proportionality between retention and runoff such that:

$$\frac{F}{S_{CN}} = \frac{P - Q}{S_{CN}} = \frac{Q}{P}$$

444
445
446
447 where F is the cumulative infiltration, S_{CN} is the storage capacity of the soil (here given a
448 subscript to distinguish from the sorptivity of the soil), Q is the total runoff and P is the total
449 precipitation for an event. According to an interview with Vic Mockus, he had fixed on this
450 functional relationship after dinner one evening, having tried many others, because it best fit
451 the data (Ponce, 1996). An initial abstraction loss, I_a , was also introduced which, on the basis
452 of data from catchments of 10 acres or less, was made proportional to S_{CN} as $I_a = \lambda S$. While
453 50% of these observations showed values of λ in the range 0.095 to 0.38, a value of 0.2 was
454 chosen as being at the centre of the data (though Mockus allows that other values might be
455 valid). Combining these equations an expression for Q can be derived as

$$Q = \frac{(P - 0.2S_{CN})^2}{P + 0.8S_{CN}}$$

456
457
458
459 with only the one parameter S_{CN} . For convenience in engineering applications, this was then
460 scaled to a non-dimensional curve number CN such that (for S_{CN} in units of inches)

¹⁴ Beven (2004b) reports that in a letter to a Mr. Ramser of the SCS Horton wrote “In reading this discussion I am reminded of the adage that you can lead a horse and some other related animals to water but you can’t make them think.” [Horton papers Box 2: copy of letter dated June 7, 1943]

461

462

$$S_{CN} = \frac{1000}{CN} + 10$$

463

464 where CN has the range of 0 to 100 and is tabulated for different soil classes, land covers and
465 antecedent conditions. The soils information was simplified to only 4 classes for simplicity of
466 use by G. W. Musgrave (Ponce, 1996). It is clear from the literature associated with the curve
467 number methodology that the SCS interpreted the output Q as a volume of surface runoff in
468 excess of the infiltration capacity of the soil (Table 1). Thus, in Module 103 of the SCS-CN
469 Training Manual it is stated that: “*Runoff is that part of the precipitation that makes its way
470 towards stream channels, lakes or oceans as surface flow*” (p1). The Manual also provides
471 definitions of interflow and baseflow as subsurface contributions to streams but suggests that
472 interflow “*is not usually considered in SCS methods of estimating runoff*” (p.3).

473

474 There have since been many other interpretations of the SCS Curve Number relationships.
475 Chen (1982) showed how the SCS curve number method could be related to the Holtan
476 infiltration equation, which also allows for a maximum storage capacity, while Mishra and
477 Singh (1999, 2002, 2003) showed how the Mockus relationship could be analytically related
478 to the SCS Curve Number equation and also to the Horton infiltration equation (for the case
479 where the long time infiltration capacity f_c can be assumed negligible). They refer to what is
480 being estimated as *direct surface runoff*. It seems, given the relationship to infiltration
481 equations they derive, they mean by this overland flow to the stream. Steenhuis et al. (1995)
482 suggested that the method could also be interpreted as a saturation excess variable
483 contributing area function, with later verification by Dahlke et al. (2006), while Yu (1998)
484 suggested that it was equivalent to the partial area surface runoff that would be generated
485 on a statistical distribution of soil infiltration characteristics. In all these cases, however, it
486 retains the preconception of representing surface runoff as overland flow. It is important to
487 note, however, that this may not have been the case for the original small catchment
488 observations from which the method was derived (see also the results from Horton’s runoff
489 plot experiment reported in Beven, 2004a, where runoff rate was significantly higher than the
490 observed rainfall intensity). More recently, Ogden et al. (2017) suggest it is really time to
491 move beyond the curve number method suggesting that “*sixty-five years of use and multiple
492 reinterpretations have not resulted in improved predictability using the method*”.

493

494 **Surface detention, channel storage and the unit-graph**

495

496 Horton and others in the era of infiltration recognised that in both analysis and prediction it
497 was not enough to simply calculate the excess of precipitation over the infiltration capacity
498 of the soil. As Horton (1935) put it: “*A striking fact about surface runoff is the manner in which
499 a jagged, irregular rain intensity graph is often transformed into a smoothly rounded runoff
500 graph.... This is the result of regulation by surface detention and channel storage*” (Horton,
501 1935, p.1). By thinking in terms of a unit strip of hillslope (for which he credits a suggestion
502 of LeRoy Sherman) Horton (1935) analyses the velocities expected for both laminar and
503 turbulent sheet flow, with hydraulic radius assumed equal to the flow depth for a shallow
504 flow, in terms of the Hagan power law equation:

505

506

$$q = k_H \delta_c^m s^n$$

507

508 where q is the flow per unit width, δ_c is the depth of flow averaged across the width of the
509 slope segment, s is the slope of the surface and k_H , m , and n are parameters. Horton notes
510 the theoretical values of m and n for laminar and turbulent flows, but also gives analyses of
511 flume data provided by Lewis and Neal of the Idaho State Agricultural Experiment Station that
512 suggest values of m of 0.85 and n of 0.74, suggesting mixed laminar and turbulent flow. He
513 also uses this to derive a profile of overland flow depths under a steady distributed input rate
514 equal to the rainfall rate – a constant infiltration capacity (essentially making the kinematic
515 wave assumption).

516

517 He also recognised the effect of routing through channel storage, both in predicting
518 hydrographs and in the analysis of observed hydrographs to derive infiltration parameters.
519 He suggested a method of routing through a nonlinear (power law) storage based on the
520 storage-discharge curve for the channels, but noting that “*in applying this method for*
521 *correction for channel storage it is important that ground-water flow, if any exists, should be*
522 *eliminated from the hydrograph in advance*” (Horton, 1935, p.41).

523

524 It might be that Horton felt compelled to provide a method of routing runoff because a few
525 years earlier LeRoy Sherman (1932) had already proposed a more general method as an
526 abstraction of the time-area approach that he called the unit-graph method (see the
527 discussion of Beven, 2020). This was then developed into the unit hydrograph theory, with
528 its many variants in terms of mathematical representation, methods of fitting, and
529 parameters related to catchment characteristics. In its classical form the unit hydrograph is
530 used to route estimates of the water contributing to the storm hydrograph after baseflow
531 separation as appropriate (although modern transfer function methods can also be used to
532 predict the complete hydrograph, e.g. Young, 2013). It was thus easy to combine the unit-
533 hydrograph with the infiltration theory *as if* all that water was overland flow in excess of the
534 infiltration capacity of the soil. This provided a convenient engineering procedure that is still
535 in widespread use in many countries.

536

537 **Surface Runoff, Direct Runoff and Stormflow**

538

539 The infiltration theory essentially defines that proportion of the rainfall that will produce
540 surface runoff and contribute to the storm hydrograph. But part of the problem here is what
541 is actually meant by surface runoff. Even going back to the original definitions of Horton and
542 Cook we have seen how surface runoff is what is measured in a stream hydrograph, but that
543 might have reached the stream as either overland flow or subsurface stormflow. We have
544 seen already how Robert Horton suggested that some of this contribution might be *concealed*
545 *surface runoff* and how Howard Cook allowed that effective infiltration rates could not be
546 inferred if there was a significant contribution to the hydrograph from subsurface flows.

547

548 It is also clear that the runoff data analysed by Mockus (1949) and that was used in evolving
549 the SCS-CN model was not necessarily produced entirely by overland flow, despite the
550 common interpretation of the SCS-CN function as an infiltration model. Yet, in setting out the
551 definitions for his analysis, Mockus defines surface runoff as overland flow. He distinguishes
552 between surface runoff, subsurface flow contributing to the hydrograph but which will quickly
553 cease to contribute to streamflow, and groundwater flow which “*may first appear in the*

554 *stream channels during or after the storm, and may continue for a relatively long time”* (p.2).
555 He then defines the term *Direct Runoff* as the sum of surface runoff and subsurface flow
556 “*combined in unknown proportions*”. However, having set out these definitions he proceeds
557 to outline methodologies for estimating surface runoff alone based on nomograms that allow
558 for soil, crop, antecedent conditions, storm duration and seasonal effects. In his use of *Direct*
559 *Runoff*, Mockus was following Franklin F. Snyder a decade earlier who, in a glossary of terms
560 associated with his *Conception of Runoff-Phenomena* defines *surface-runoff* as:

561
562 “*Usually defined as the runoff reaching the surface drainage-channels without*
563 *penetrating the ground-surface. As actually used, surface-runoff usually includes*
564 *considerable subsurface storm-flow and might be better termed direct runoff, since it*
565 *consists of the discharge in excess of a base or ground-water flow which passes a*
566 *gaging station within a rational period of time subsequent to the storm causing the*
567 *rise*”. (Snyder, 1939, p.736).

568
569 Note how this differs from the definition cited earlier in SCS Training Module 103. Later usage
570 was also mixed, and there does not seem to have been a real history of development in the
571 use of the different terms for runoff. To give just a few examples, Leach et al. (1933) use
572 both storm-flow and surface-runoff; Langbein (1940) uses direct runoff, as do Hursh and
573 Brater (1941) who specifically say that storm-runoff as overland-flow has not been observed
574 on the study watershed at Coweeta and give examples of hydrographs dominated by channel
575 precipitation. Hoover and Hursh (1943), however, revert to using storm-runoff. Marston
576 (1952) equates storm runoff to overland flow but Reinhart (1964) includes subsurface
577 stormflow in storm runoff, and in the study of Whipkey (1969) essentially all the storm runoff
578 is subsurface stormflow. Hamon (1963) refers to direct runoff in relation to that predicted
579 by the SCS curve number method, whereas others have continued to use storm runoff as
580 equivalent to overland flow, especially in semi-arid catchments (e.g. Fogel and Duckstein,
581 1970).

582
583 If we turn to the latest issue of the WMO International Glossary of Hydrology (2012) we find
584 *runoff* defined as that part of the precipitation which flows towards a river on the ground
585 surface (*surface runoff*) or within the soil (*subsurface runoff* or *interflow*). *Direct runoff* (or
586 direct flow or storm runoff) is defined simply as water that enters a watercourse without
587 delay (and without any process interpretation). *Infiltration index*, however, is defined as an
588 average rate of infiltration such that precipitation in excess of that value equals the volume
589 of *storm runoff* (implying that the infiltration theory concept still persists, if only in an index
590 form). In his Glossary for *Hillslope Hydrology*, Chorley (1978) also defines *direct runoff* with
591 respect to time, adding that it comprises the sum of channel precipitation, overland flow and
592 subsurface stormflow. His definition of *surface runoff* is limited to flow over the soil surface,
593 and for *quickflow*, *storm runoff*, and *stormflow* he says “*see direct runoff*”.

594
595 There is thus some continuing ambiguity about the use of these terms, particularly surface
596 runoff. This is in part a process issue because, however water flows into a stream by either
597 surface or subsurface flow processes, once in the stream it is measured as a surface runoff
598 (as was the case for the fields and small catchments in the data used by Mockus). The
599 problem is that the word runoff still induces a perception of an overland flow, as in running
600 off over the land surface. This is reinforced by the use of surface runoff even if the ambiguity

601 recognised by Snyder, Cook and Mockus of the unknown mix of surface and subsurface
602 contributions to the hydrograph cannot be easily resolved. This mix, defined by them as
603 *direct runoff* (and now sometimes referred to as *storm runoff* or *stormflow* or *quickflow*) is
604 more commonly what is estimated by the use of hydrograph separation, but it should not
605 then be interpreted as runoff in excess of the infiltration capacity of the soil. That is, perhaps,
606 why the WMO Glossary refers to an infiltration *index* to match the volume of *storm runoff*,
607 even if this perpetuates the perception of runoff as an overland flow. On the other hand, the
608 convenient alliteration of *rainfall-runoff modelling* is generally used to indicate a mix of
609 surface and subsurface processes (except in models that are still limited to predicting only
610 overland flows).

611
612 Given these ambiguities, it might be better to avoid the use of the terms runoff and surface
613 runoff (and concealed surface runoff) altogether and instead refer to *stormflow* or *storm*
614 *discharge* when no process interpretation is inferred, and refer explicitly to overland flow and
615 subsurface stormflow when there is evidence for making a process interpretation¹⁵. There is
616 also no reason why the general term *hydrological model* should not replace the ambiguity of
617 rainfall-runoff model. This might (just perhaps) lead to a greater appreciation and greater
618 thought about the perceptual model of hydrological processes relevant to particular
619 catchments of interest (Beven, 2001; Beven and Chappell, 2020).

620 621 **Persistence of the era of infiltration and perceptual model failures**

622
623 When Cook was writing in 1946, he noted that the infiltration theory of surface runoff was
624 still young and needed to be developed further, such that “*before it can be generally*
625 *employed, many problems must be solved and large quantities of data published*” (op. cit.
626 p.743). He notes in particular, that it would only be valid for cases where subsurface runoff
627 could be neglected, and that infiltration indices derived from hydrograph data would only be
628 satisfactory if there was only one soil-cover complex, otherwise, “*the physical significance is*
629 *obscure*” (p.743). His final statement is to suggest that because of these issues all infiltration
630 data should be accompanied by a statement of how they were derived, so that they would
631 not be misused. It can be said, therefore, that Howard Cook had a rather realistic
632 understanding of the limitations of the infiltration theory.

633
634 It seems that in the years following, however, the pragmatic utility of the methodology to
635 provide estimates of the volume of storm discharge dominated any concerns about the
636 validity of the assumptions. That volume could be combined with the time distribution of
637 the Sherman (1932) unit-graph (and later representations of the unit hydrograph) to allow
638 the prediction of hydrographs, and of hydrograph peaks for design applications. The
639 methodology came to dominate hydrological practice, even well into the computer age, when
640 there were many models essentially based on predicting and routing *effective* or *excess*
641 rainfall based on infiltration equations (see, for example, Beven, 2012).

642
643 However, from the late 1960s onwards, the general applicability of the infiltration theory
644 started to be questioned. Cappus (1960) and Moldenhauer et al. (1960) suggested that not
645 all of a catchment would contribute surface runoff, while Betson (1964) concluded that the

¹⁵ Note that Beven and Young (2013) also suggest some clarifications to the language used in hydrological modelling.

646 generally wetter conditions at the base of hillslopes would result in a relatively consistent
647 partial contributing area (see also the consequent partial area model of Betson and Marius,
648 1969). Hewlett and Hibbert (1967) proposed that the contributing area would be dynamic,
649 varying with antecedent conditions and storm rainfalls (see also Dickinson and Whitely, 1970).
650

651 However, particularly after the geochemical hydrograph separation of Pinder and Jones
652 (1969) and the environmental isotope hydrograph separation of Sklash and Farvolden (1979),
653 there was a more general realisation that subsurface processes were necessarily important
654 in storm flow generation in many catchments because of the high proportion of pre-event
655 water that appeared to be displaced in the event (something that was later called a double
656 paradox by Kirchner, 2003). Thus, even if there was some overland flow, much of the water
657 in the hydrograph had to be displaced from the soil or deeper layers (Sklash and Farvolden
658 reported that at one site samples of overland flow were indicative of event water in one
659 sampled storm and pre-event water in another). Thus new concepts of runoff generation
660 were required. At Coweeta, where overland flow is rare except in the immediate riparian
661 area (but runoff coefficients can be small), John Hewlett¹⁶ had continued the work of Hursh
662 in trying to understand the role of subsurface flow in hydrograph generation. The idea of
663 runoff and return flow to dynamic saturated areas had appeared in the work of Dunne and
664 Black (1970), a concept later claimed by Hewlett (1974). However, at around the same time,
665 computer models such as the Huggins and Monke (1968) model; the KINEROS model that
666 developed from Smith and Woolhiser (1971); the partial area Quasi-Physically-Based Rainfall-
667 Runoff Model (QPBRM) model of Engman and Rogowski (1974) also included in the study of
668 Loague and Freeze (1985)¹⁷; and the CASC2D model of Downer et al. (2004); were all based
669 on the infiltration theory (and there were many others). Of course, there are still catchments
670 where the infiltration theory might indeed match the perceptual model of overland flow as
671 the dominant process, but it still took time for the perceptual model of how catchments
672 function to recognise the important contribution of subsurface water to stormflow in many
673 catchments.

674
675 A really instructive case in this respect is the history of modelling the R5 catchment at
676 Coshocton by Keith Loague and his colleagues. This is only a small catchment area (0.1 km²)
677 and started out as a study of effect of the variability of infiltration rates in space on runoff
678 generation, making use of the extensive database of infiltration measurements collected by
679 Sharma et al. (1980). It was included in the study of Loague and Freeze (1985) using the
680 QPBRM computer model. Loague and Gander (1990) added a further 247 infiltration
681 measurements, and Loague and Kyriakidis (1997) used kriging interpolation to produce a fully
682 distributed spatial pattern of infiltration characteristics. Using this information, however,
683 produced less satisfactory hydrograph simulations than the original Loague and Freeze (1985)
684 calibrated model. Various things were tried to improve the results, including allowing for
685 temperature effects in the original infiltration measurements, and taking averages over
686 stochastic fields of parameters consistent with the kriging estimates. It was suggested that

¹⁶ (1922-2004), see http://www.history-of-hydrology.net/mediawiki/index.php?title=Hewlett,_J_D

¹⁷ In Beven (1989) I criticised the paper of Keith Loague and Al Freeze (1985) because they had applied such the QPBRM model to the Hubbard Brook catchment where surface runoff would be rarely observed. I suggested that was simply poor hydrological practice. Loague (1990) replied that they had made the choice of applying a model that was widely used in practice, and that such models might well be used in practice where the assumptions were not valid.

687 there were still limitations of resolution in representing the surface runoff pathways and
688 effects of run-on and infiltration. However, improvements in predictions of the peak and
689 time to peak came with a change of model to the finite element based Integrated Hydrological
690 Model (InHM) that included the effects of subsurface flow pathways (VanderKwaak and
691 Loague, 2001; Loague et al., 2005). Following this change of perceptual model from a simple
692 infiltration theory concept, R5 has continued to be used as a case study for the application of
693 integrated models (Heppner et al., 2007; Mirus et al., 2011; Mirus and Loague, 2013).

694

695 Another case is reported in Beven (2002). I was a visiting scientist at the ARS laboratory in
696 Fort Collins, Colorado working with Dave Woolhiser and Roger Smith and helped in an
697 experiment to look at runoff generation on shallow restored soils over mine tailings near
698 Steamboat Springs, Colorado in 1981. The perceptual model in designing the experiment
699 was that the runoff generation would be produced by an infiltration excess mechanism. Thus
700 many dual ring infiltrometer measurements were done, and replicate 25m by 5m sloping plots
701 were watered using a sprinkler system supplied from a large impermeable container of
702 rubberised fabric. Unfortunately, during the experiment the supply started to be limited by
703 movement of the container as it emptied, but some overland flow was generated and
704 collected. It was, however, localised on the surface, and rapidly fell to zero. Meanwhile, in
705 the shallow trench that had been dug to take the collected overland flow from the
706 measurement flume to a small channel, subsurface flow from beneath the collectors
707 continued for some 90 minutes, and at the bankside of the channel there were two outflows
708 from preferential flow pathways through otherwise unsaturated soil. It appeared as if there
709 had been a form of percolation excess process taking place at the boundary between the mine
710 tailings and topsoil, and that the resulting subsurface flow was somehow being channelled
711 within the soil that had been replaced over the mine waste. The volumes of subsurface flow
712 were not measured but were clearly much greater than the surface runoff collected. This
713 was also an instructive case where the perceptual model based on the infiltration theory used
714 in designing the experiment was clearly not correct and needed to be revised.

715

716 There is, therefore, no doubt that the infiltration theory concept led to many misconceptions
717 or perceptual model failures of how the response of particular catchments was dominated by
718 surface flow. There were, of course, many other catchments where subsurface contributions
719 to the hydrograph have been studied in more detail and qualitative perceptual models
720 developed, such as Hursh, Hewlett and others at Coweeta, North Carolina, as mentioned
721 above, and Mosley (1982) and McDonnell (1990) at Maimai, New Zealand, with later additions
722 by Brammer and McDonnell (2003) and McGlynn et al. (2010). These more complete
723 perceptual models, however, tend to be complex and subject to limitations of knowledge of
724 hydrological processes in the subsurface. In addition, there remains a need to simplify in
725 applying quantitative predictive models in practice (Beven and Chappell, 2020). In that
726 respect, infiltration theory still provides an approximate engineering solution that is simple
727 to apply, as already recognised in the “*rough approximations*” of Cook (1946).

728

729 This is perhaps the main explanation of the question posed at the beginning of this paper as
730 to why the infiltration theory of runoff concept has persisted so widely in applications. It still
731 underlies many current hydrological models in one form or another, including the SCS-CN or
732 alternative Green-Ampt methods for estimating direct flow in the Soil Water Assessment Tool
733 (SWAT). In this way, the era of infiltration theory continues, in part because of the

734 convenience of applying the SCS-CN method for practical applications without thinking too
735 much about whether that is appropriate in any particular catchment. In fact, since we do not
736 know too much about the processes in the catchments on which the analysis of Mockus
737 (1949) that led to the SCS-CN method were based, that approach be more defensible (if only
738 for the range of conditions for which the data were available) as a predictor of total direct
739 runoff at the scale of interest than the use of point infiltration equations to predict purely
740 overland flow (especially if heterogeneity of soil characteristics, the commensurability issues
741 of scale of infiltration measurements against scale of applications, and run-on effects are
742 neglected).

743

744 It does seem surprising, however, that more than 70 years after Howard Cook announced the
745 era of infiltration, and 50 years after tracer information showed that hydrographs could be
746 dominated by pre-event water, we should still be left with so much ambiguity about how to
747 describe what is actually being observed and estimated in catchment hydrographs. Cook's
748 observation that it is impossible to separate surface and subsurface contributions to the
749 hydrograph when only records of the rates of flow are available still holds. Learning from
750 tracer separations is not yet standard practice and does not provide unambiguous
751 information about flow pathways. Yes, we understand that there are limitations on the
752 knowability of what goes on in the subsurface but such ambiguity means that there have been
753 no real attempts to define the limits of validity of the infiltration theory, and much confusion
754 about its use. It seems that some of the old guard might still have reason to grumble.

755

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758

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Table 1. Selected 20th Century infiltration equations with $f(t)$ as infiltration capacity, $F(t)$ as cumulative infiltration, and K_s as saturated hydraulic conductivity (other symbols defined under Comments)

Source	Equation	Comments
Green and Ampt (1911)	$f(t) = K_s \left(\frac{ \psi_f }{z_f} + 1 \right)$	Based on Darcy's law with piston-like wetting front from initial moisture content to saturation. ψ_f is capillary pressure change across wetting front, z_f is current depth to wetting front
Kostiakov (1932), Lewis (1937)	$f(t) = Kt^N$	Empirical, with K and N as parameters
Horton (1939, 1940)	$f(t) = (f_o - f_c)e^{-kt} + f_c$	Empirical, with k as a time scale parameter. Allows for an initial f_o and final f_c infiltration capacities. Argues that it represents rate equation for extinction phenomena at the soil surface
Mezencev (1948) and later Smith (1972)	$f(t) = Kt^N + f_c$	Extension of the Kostikov-Lewis equation to include a final infiltration capacity
SCS-CN (1954)	$F = P - \frac{(P - 0.2S_{CN})^2}{P + 0.8S}$ $S_{CN} = \frac{1000}{CN} - 10$	Origins lie in estimation of direct flow rather than overland flow, but often interpreted as an infiltration equation. P is event precipitation, S_{CN} is storage capacity of the soil associated with the Curve Number, CN .
Philip (1957)	$f(t) = \frac{St^{-1/2}}{2} + A$	First two terms of series solution to Darcy-Richards equation assuming constant diffusivity. S is the <i>sorptivity</i> of the soil, A is a parameter likely to be somewhat smaller than the saturated conductivity of the soil.
Holtan (1961)	$f(t) = f_o + \alpha(S_o - F)e^\eta$	Empirical with α and η as parameters. Makes infiltration capacity dependent on initial value f_o , cumulative volume already infiltrated F and initial storage capacity of the soil S_o which also provides an upper limit for infiltration
Talsma and Parlange (1972)	$f(t) = St^{1/2} + \frac{K_s t}{3} + \frac{K_s^2 t^{3/2}}{9S}$	Assumes diffusivity is proportional to rate of change of conductivity with θ . K_s is saturated hydraulic conductivity and S sorptivity
Morel-Seytous and Khanji (1974)	$f(t) = \frac{K_s}{B} \left[\frac{h_o(\theta_s - \theta_i) + C_d}{z_f(\theta_s - \theta_i)} + 1 \right]$	Extension of the Green-Ampt equation with C_d as the capillary drive, h_o as depth of

		surface ponding and B as a scaling parameter allowing for lack of complete wetting.
Ahuja and Tsuji (1976)	$F(t) = K_s t + \frac{K_s - a}{b} [e^{-bt} - 1] + z_f(\theta_s - \theta_i) \ln[1 + F(t)/z_f(\theta_s - \theta_i)]$	Extension of the Green-Ampt equation to have an exponential time variable hydraulic conductivity function, with parameters a and b , based on comparison with the Philip equation. Claim better fit to observations.
Collis-George (1977)	$F(t) = S(t_c)^{1/2} \left(\tanh \frac{t}{t_c} \right)^{1/2} + f_c t$	Empirical but argues that it provides a better fit to data than Green-Ampt, Horton or Philip equations. S is the sorptivity, t_c a time scale parameter, and f_c a final infiltration capacity
Smith and Parlange (1978)	$f(t) = K_s \left[\frac{\exp F(t)/C_D}{\exp F(t)/C_D - 1} \right]$	Solution of Darcy-Richards assuming an exponential diffusivity function. Useful when rainfall rates vary, as $f(t)$ is a function of cumulative infiltration $F(t)$.
Beven (1984)	$\frac{dF(t)}{dt} = \frac{K_o \beta [\psi_f + F/(\theta_s - \theta_i)]}{1 - e^{-\beta F/(\theta_s - \theta_i)}}$	Extension of Green-Ampt for case of exponential decline of saturated conductivity with depth as $K_s(z) = K_o e^{-\beta z}$. Has an implicit solution for F
Singh and Yu (1990)	$f(t) = f_c + \frac{a[S_o - F(t)]^{\dot{M}}}{[F(t)]^{\dot{N}}}$	Made infiltration dependent on initial storage available and powers of cumulative infiltration and remaining storage. \dot{M} , \dot{N} and a are parameters

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