Quantifying the Impacts of Compound Extremes on Agriculture

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Abstract. Agricultural production and food prices are affected by hydroclimatic extremes. There has been a large literature measuring the impacts of individual extreme events (heat stress or water stress) on agricultural and human systems. Yet, we lack a comprehensive understanding of the significance and the magnitude of the impacts of compound extremes. Here, we combine a fine-scale weather product with outputs of a hydrological model to construct functional metrics of individual and compound hydroclimatic extremes for agriculture. Then, we estimate a yield response function with individual and compound metrics focusing on corn in the United States during the 1981-2015 period. Supported by statistical evidence, we show that metrics of compound hydroclimatic extremes are better predictors of corn yield variations than metrics of individual extremes. We also confirm that wet heat is more damaging than dry heat for corn. We show that the average yield damage from heat stress has been up to four times more severe when combined with water stress. Keywords. agriculture; climate impacts; water balance model; extreme heat; extreme drought.

1 Introduction

We construct various metrics of individual and compound hydroclimatic extremes appropriate for agricultural studies. In agricultural production, water and heat extremes are key determinants of yield variations. They affect agricultural yields, farm revenues, and crop markets. The relationship between extreme heat and crop yields has been well-documented, particularly across the United States (US) and particularly for corn (Schlenker and Roberts, 2009; Urban et al., 2012; Diffenbaugh et al., 2012; Roberts et al., 2013; Lobell et al., 2013; Urban et al., 2015; Wing et al., 2015; Burke and Emerick, 2016). However, the precipitation-based metrics of water conditions used previously are either mean or cumulative measures calculated over the growing season or stages of crop growth. These cumulative indices, monthly mean, or seasonal average metrics do not capture extreme events during the season (e.g. early-season floods and late-season droughts can cancel out when taking the average). The mean variable can be misleading as the plants respond to day to day variability. Furthermore, the mean water index may not represent hydrological extremes (D’Odorico and Porporato, 2004; Lobell and Burke, 2010; Schaffer et al., 2015; Werner and Cannon, 2016). While the average conditions are important, exposure to extreme water stress can cause permanent unrecoverable damage to the plant (Denmead and Shaw, 1960). In addition, too much water can cause flooding, waterlogging, or may wash out soil nutrients and fertilizers (Kaur et al., 2018; Schmidt et al., 2011; Urban et al., 2015). Therefore, it is necessary to introduce metrics of extreme soil moisture stress. This will be even more important in the future, as climate projections are predicting more extreme drought and precipitation events (Myhre et al., 2019). In other words, mean variables can create biases in future climate impact analysis by ignoring the extreme events. It is important to evaluate new metrics of daily water availability to fully understand the impact of water extremes on crop yields, as this will be important in both fundamental understandings of the crop-water system, and in predicting the impacts of future extreme events.
Further, crops obtain their water directly from soil moisture, yet extreme water metrics based on soil moisture have been only minimally explored (Fishman, 2016). Several studies have highlighted the need for irrigation to compensate for soil moisture deficits (Li et al., 2017; McDonald and Girvetz, 2013; Meng et al., 2016; Williams et al., 2016), further pointing to soil moisture as a potentially more important crop water availability metric than precipitation. However, current statistical studies have had limited success in statistically capturing the yield response to soil moisture metrics (Bradford et al., 2017; Peichl et al., 2018; Siebert et al., 2017). There are several potential reasons for the limited success of previous statistical studies in capturing yield response to soil moisture. Direct measures of soil water availability include complex biophysical and hydrological processes that are difficult to capture in a rather simple statistical model. On the other hand, seasonal mean soil moisture is highly correlated to seasonal precipitation. Thus, including an average metric of soil water content may not add value to a statistical model. Another barrier has been limited availability of daily fine-scale soil moisture data and inconsistency of soil moisture data with heat information. It has become a standard practice either to focus on a limited geographical area (Rizzo et al., 2018; Wang et al., 2017) or to employ a proxy variable like precipitation, evapotranspiration, or vapor pressure deficit estimates (Comas et al., 2019; Roberts et al., 2013). The recent work by Ortiz-Bobea et al. is an exception that highlights the importance of mean soil moisture metrics for estimating crop yields in the US (Ortiz-Bobea et al., 2019).

A key unknown is the extent of the benefits of soil moisture in buffering heat damage to yields. Despite existing theoretical frameworks and controlled experiments, we currently lack a comprehensive understanding of the impact of heat on yields while controlling for water (Bradford et al., 2017; Ortiz-Bobea et al., 2019). The problem is that current studies tend to separate the impact of heat from water stress. These studies estimate the average impact of heat stress on corn yields without distinguishing between a hot-dry day (dry heat) and a hot-wet day (wet heat). There is no robust predictive framework that captures the implications of compound extremes in the determination of national crop yields. Also, the current literature is focused mainly on the impacts of dry-heat and ignoring the impacts of wet-heat stress (Ribeiro et al., 2020). The growth effects of heat and soil moisture are mutually interdependent. Beneficial heat is less beneficial without sufficient soil moisture. On the other hand, soil moisture is not beneficial without sufficient heat for plant growth. Harmful heat can be less harmful when there is enough soil moisture (Hauser et al., 2018). While the amount of daily water requirement depends on the biophysical properties of soil and crop, it changes with temperature, solar radiation, humidity, and wind speed. In this framework, daily weather variability, which is expected to change in the future with climate change, can affect both soil moisture supply and demand by altering the abundance and frequency of precipitation and by increasing the water required to compensate evapotranspiration and evaporation. If the temperature is high and there is not enough soil moisture for a long period (drought conditions), this may cause severe damage to crops (Denmead and Shaw, 1960). Therefore, consideration of the daily compound impacts of soil moisture and heat is necessary to capture the impacts on natural supply and plant demand for soil moisture.

In this paper, we investigate the significance of compound heat and water conditions in predicting crop yields, including dry-heat and wet-heat. We focus on corn as the major field crop in the US. We also compare the metrics of compound extremes versus individual extremes (i.e. only heat stress or only water stress). This study also demonstrates the advantages of using soil moisture metrics over current proxy variables in capturing climate-driven variations in heat and moisture availability.

2 Methods

Technically, we extend the models in Schlenker and Roberts (2009) and Ortiz-Bobea et al. (2019) by assuming the growth effects of heat and water are mutually interdependent. We use detailed soil moisture information available from a physical hydrologic model. Specifically, we estimate 1) the marginal impacts of heat stress (individual extreme) on crop yields; 2) the marginal impact
of daily soil moisture stress (individual extreme) on crop yields, and 3) the marginal impact of heat and soil moisture (compound extremes) on crop yields. Marginal impact and conditional marginal impact are two statistical concepts equivalent to partial derivatives in mathematics. When the partial derivative of one variable does not depend on other variables, we use the term “marginal impact”. When it depends on other variables, we use “conditional marginal impact”. A conditional marginal impact shows the impact of a compound extreme. A non-conditional marginal impact can show the impact of individual extremes.

Here, we introduce two statistical models of crop yield as a function of heat and soil moisture. For each model, we consider different parameterizations of heat and soil moisture to estimate the impacts of water availability on corn yields in the US. Model 1 assumes the impacts of heat and water on corn yields are separable. This model considers metrics of individual extremes (heat stress and water availability). Figure 1 visualizes four soil moisture conditions that are unfavorable for crop yield. Both too much water [i] and intense moisture stress [ii] can cause severe damage to crop yields. Similarly, a long period of mild moisture stress [iii] or a short period of severe moisture stress [iv] can also cause significant yield loss. These measures can help to understand the need for artificial drainage or irrigation as shown in panel (b). Within this framework, we investigate which metric of individual extremes is a better predictor of corn yields. Relaxing the separability assumption, model 2 assumes the yield impacts of heat and water are mutually interdependent. Model 2 considers metrics of compound extremes.

2.1 Data

In estimating the marginal impact of soil moisture on corn yields, we employ information about soil moisture, temperature, precipitation, and corn yields for counties of the United States for the 1981-2015 period as summarized in Table 1. The data on yield is obtained from USDA-NASS (United States Department of Agriculture-National Agricultural Statistics Service) at the county level. The yield is defined as the corn production (in bushels) divided by harvested area (in acres). Precipitation is defined in millimeters as accumulated rainfall during the growing season (Apr-Sep). It is calculated based on PRISM (Parameter-elevation Regressions on Independent Slopes Model) daily information at 2.5 x 2.5 arcmin grid cells over the continental US for 1981-2015. It is aggregated to each county according to cropland area weights. Compound metrics of heat and soil moisture are also calculated daily at the gridded level. Then we aggregate the metrics to the growing season and county level.

3.3.1 Degree days (index for heat)

Following D’Agostino and Schlenker (2015), the daily distribution of temperatures is approximated assuming a cosine function between the daily minimum and maximum temperature. Let \( \bar{t} = \cos \left( \frac{2b - T_{\text{max}} + T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \right) \), then degree days (dday) at each day is defined using

\[
D(b) = \begin{cases} 
\frac{T_{\text{max}} + T_{\text{min}}}{2} - b & \text{if } b \leq T_{\text{min}} \\
\frac{\bar{t}}{\pi} \left[ \frac{1}{2} (T_{\text{max}} + T_{\text{min}}) - b \right] + \frac{\bar{t}}{2\pi} \sin(\bar{t}) & \text{if } T_{\text{min}} < b \leq T_{\text{max}} \\
0 & \text{if } T_{\text{max}} < b 
\end{cases}
\]  

(1)

where \( b \) is the base for calculating degree days and can take the base values as well as critical values. We consider a piecewise-linear function to aggregate the degree days. The major assumption is that plant growth is approximately linear between two bounds. Degree days between two bounds is simply degree days above the smaller bound minus degree days above the larger bound.

We calculate county-level seasonal degree days based on daily weather information. The weather information on daily maximum and minimum temperature are obtained from PRISM at 2.5 x 2.5 arcmin grid cells over the continental US for 1981-2015. Degree days are initially calculated for each day at each 2.5 x 2.5 arcmin grid cell during the growing season (Apr-Sep). Then they are
aggregated for the whole growing season from the first day of April through the last day of September. Finally, they are aggregated to the county level using cropland area weights.

### 3.3.2 Soil moisture (index for water availability)

Daily soil moisture content and soil moisture fraction are obtained from the Water Balance Model (Grogan, 2016; Wisser et al., 2010) based on daily simulations using PRISM data at 6 x 6 arcmin grid cells for the 1981-2015 period over the continental US. Here, we briefly describe WBM’s soil moisture module. However, the model is much more complex and employs a large list of inputs. Full documentation for WBM can be found in Wisser et al. (2010) with updates in Grogan (2016). In WBM, crop-specific soil moisture balance within each grid cell is calculated with an accounting system that tracks a location's water inputs and outputs and is limited by the soil moisture pool’s water holding capacity.

\[
\frac{\partial W_s}{\partial t} = \begin{cases} 
  g(W_s)(I - PET) & \text{if } I < PET \\
  I - PET & \text{if } PET \leq I \text{ and } (I - PET) < (W_{cap} - W_s) \\
  W_{cap} - W_s & \text{if } PET \leq I \text{ and } (W_{cap} - W_s) \leq (I - PET) 
\end{cases}
\]

where \( W_s \) is soil moisture, \( t \) is time, \( I \) is the sum of all water inputs to the soil moisture pool, \( PET \) is potential evapotranspiration, and \( W_{cap} \) is available water capacity. Water inputs to the soil come in the form of precipitation as rain and as snowmelt. Water intercepted by the canopy reduces precipitation reaching the soil. Here, we use the Hamon method for estimating \( PET \) (Federer et al., 1996; Hamon, 1963), and \( g(W_s) \) is 1 for all crops, while it is an exponential function of soil moisture depth for non-crop soil areas. Crop-specific potential evapotranspiration values, \( PET_c \), are calculated following the FAO-recommended crop-modeling methodology (Allen et al., 1998):

\[
PET_c = k_c \cdot PET
\]

where \( k_c \) [-] is a crop-specific, time-varying scalar. Crop scalar values are from Siebert and Döll (2010), and crop maps that identify the area of each rainfed crop type within a grid cell are from the Crop Data Layer (CDL, USDA NASS, 2017). When soil moisture is insufficient for crops to extract water equal to \( PET_c \), actual crop evapotranspiration is limited to available soil water volumes.

Available water capacity, \( W_{cap} \), is a function of vegetation-specific rooting depth, a crop-specific depletion factor, soil field capacity, and soil wilting point:

\[
W_{cap} = D_c R_c (F - W_p)
\]

where \( D_c \) is the depletion factor for crop \( c \), \( R_c \) is the rooting depth of crop \( c \), \( F \) is the soil field capacity, and \( W_p \) is the soil wilting point. Here we use the Harmonized World Soil Database (Fischer et al., 2008) as model input for all soil properties. Corn rooting depth is set to 1 meter; and the depletion factor is 0.5, following Siebert and Döll (2010). Once the soil moisture content reaches field capacity, no further water is added to the soil moisture pool; excess inputs move to the groundwater pool via percolation and the river system via runoff.

The soil moisture metrics used in both statistical models are calculated as the mean of soil moisture content (in mm for the 1000 mm topsoil) or cumulative deviations from normal levels during the growing season (Apr-Sep) for each 2.5 x 2.5 arcmin grid cell. For the compound metrics of soil moisture and heat, we sum up degree days for each temperature interval (5°C) for each soil moisture deviation interval (10 mm) for each 2.5 x 2.5 arcmin grid cells for the 1981-2015 period. We employed Crop Data Layer from the US Department of Agriculture to exclude grid cells with no cropland and to aggregate the grid cell information to the county level (Boryan et al., 2012; USDA-NASS, 2017). We have constructed the soil moisture metric relative to the “normal” levels. In this study, we define normal as the 25-year average soil moisture in the growing season. The water available to plants
depends on volumetric soil moisture as well as soil type. For the same volume of soil moisture, different soil types imply different wilting points and different field capacity which result in different water availability to crops. Figure S1 shows the difference between normal soil moisture content, water available to plants, and unavailable water. This illustrates that sand and sandy-loam soil types have the lowest field capacity (and water availability) while clay and clay-loam have the highest. To operationalize the soil moisture metric, we consider soil moisture deviation from normal. Soil moisture deviation is defined as daily soil moisture minus the normal soil moisture levels. The soil moisture level is considered extreme if it is below/above a threshold. The threshold is obtained by testing the impacts of 5-mm intervals of soil moisture deviation from normal.

We construct our water metrics based on soil moisture conditions shown in Fig. 1 (extreme surplus = A, surplus = B, around normal = C+D, deficit = E, extreme deficit = F). Three types of metrics are constructed for each condition. A simple metric is the number of days during the growing season with each condition. To show the intensity of each condition, the second metric is defined based on cumulative deviation from normal for each condition. Finally, a compound metric is defined as the sum of degree days for each observed soil moisture condition.

2.2 Model (1) individual extremes

Model 1 is a basic model that uses individual extremes, following a similar approach as Schlenker and Roberts (2009). Model 1 assumes that the effects of heat on corn yields are cumulative over the growing season and separable from water. In other words, the end-of-season yield is the integral of daily heat impacts over the growing season. This relationship can be demonstrated via Eq. (5):

\[ y_{it} = \int_{\underline{h}}^{\bar{h}} g(h) \varphi_{it}(h) \, dh + z_{it} \delta + c_i + \epsilon_{it} \]  

(5)

Where \( y_{it} \) is crop yield, \( g(h) \) is a function showing yield as a function of heat, \( \varphi_{it}(h) \) is the time distribution of heat \((h)\) over the growing season in location \(i\) and year \(t\), while the heat ranges between the lower bound \(\underline{h}\) and the upper bound \(\bar{h}\); metrics of water availability (e.g., precipitation or soil moisture) and other control factors are denoted as \(z_{it}\), and \(c_i\) is a time-invariant fixed effect.

All other unobserved variables are in the \(\epsilon_{it}\) term. The fixed effect variable (also termed the unobserved individual effect) allows us to control for other biophysical or economic characteristics of each location which are not varying over time and can potentially explain the yield differences between counties. Note that this form of equation with fixed effects and unobserved variables is a standard econometric method. We evaluate the accuracy of this model, compared to historical data, using first cumulative precipitation, then mean soil moisture as the water availability metric \(z_{it}\).

2.3 Model (2) compound extremes

Here, we introduce a new statistical model to focus on the compound metrics of available water and heat as the major indicators of plant growth to evaluate if including the conditional marginal impact of heat and water on yields provides improved yield estimates. Model 2 is:

\[ y_{it} = \int_{\underline{m}}^{\bar{m}} \int_{\underline{h}}^{\bar{h}} g(h, m) \varphi(h, m) \, dh \, dm + c_i + \epsilon_{it} \]  

(6)

where \( y_{it} \) is the crop yield, \( g(h, m) \) is the yield response function to each combination of soil moisture level, \(m\), and heat, \(h\); \( \varphi(h, m) \) is the distribution of soil moisture and heat; \(\bar{m}\) and \(\underline{m}\) are upper and lower thresholds of soil moisture; \(\bar{h}\) and \(\underline{h}\) are maximum and minimum heat; \(c_i\) is a time-invariant county fixed effect; and \(\epsilon_{it}\) is the residual. Here, we do not
separate the impact of heat from water. In other words, the marginal impact of heat depends on water; and the marginal impact of water depends on heat.

2.4 Estimation strategy

For Model (1), we build on Schlenker and Roberts (2009) by including different representations of water variables. In Model (1-a), \( z_{it} \) includes cumulative precipitation from the first day of April to the last day of September and its square term; this will evaluate the standard way yields have been estimated in previous studies. In Model (1-b), \( z_{it} \) is the seasonal mean soil moisture index and its square term, used to evaluate the use of soil moisture instead of precipitation. Model (1-c) includes the number of days with low soil moisture as well as the number of days with high soil moisture, evaluating the importance of extreme soil moisture events (Fig. 1). In Model (1-d), \( z_{it} \) includes metrics of soil moisture below or above normal levels, evaluating the importance of extreme soil moisture intensity (Fig. 1). For Model (1), we assume a piece-wise linear form for \( g(h) \). We include degree days above 29°C as a metric of extreme heat as well as degree days from 10 to 29°C as a metric of beneficial heat.

Considering the exposure to each temperature interval to capture the marginal impact of heat and water on crop yields, we estimate the following for model (1-a):

\[
y_{it} = \alpha D_{it}^{10-29} + \beta D_{it}^{29} + \delta_1 P_{it} + \delta_2 P_{it}^2 + \lambda_1 t + \lambda_2 t^2 + c_i + \epsilon_{it} \tag{7}
\]

where \( i \) is an index for counties, \( t \) is the index of time, \( s \) is the index for states, \( y_{it} \) is the log corn yields, \( D_{it} \) represents growing degree day variables, \( P \) shows cumulative precipitation over the growing season, \( t \) shows the time trend variable (\( t = \text{year} - 1950 \)), \( c_i \) is a time-invariant county fixed effect, \( \epsilon \) is the residual, and \( \alpha, \beta, \delta, \lambda \) are the regression parameters showing the marginal impacts. The subscript \( a \) is used to show the water coefficients (\( \delta \)) are related to metrics in Model (1-a). To evaluate the importance of soil moisture metrics in Model (1-b), we estimate the following:

\[
y_{it} = \alpha D_{it}^{10-29} + \beta D_{it}^{29} + \delta_b M_{it} + \delta_b^2 M_{it}^2 + \lambda_1 t + \lambda_2 t^2 + c_i + \epsilon_{it} \tag{8}
\]

where the variables are defined as Model (1-a) except for the water availability metric. Here \( M \) shows the seasonal mean soil moisture index calculated as average daily root zone soil moisture from the first day of April to the end of September. The subscript \( b \) is used for \( \delta \) to distinguish the water coefficients in Model (1-b). For Model (1-c) we estimate the following model:

\[
y_{it} = \alpha D_{it}^{10-29} + \beta D_{it}^{29} + \delta_c N_{it}^{\text{def}} + \delta_c^2 N_{it}^{\text{sur}} + \lambda_1 t + \lambda_2 t^2 + c_i + \epsilon_{it} \tag{9}
\]

where we replace seasonal mean or cumulative metrics with two new metrics to control the impacts of water extremes on corn yields. Here, \( N_{it}^{\text{def}} \) is the number of days that soil moisture is under 25 mm below normal levels (deficit); and \( N_{it}^{\text{sur}} \) is the number of days that soil moisture is higher than 25 mm above normal levels. The rest of the variables are defined as Model (1-a). The subscript \( c \) shows \( \delta \) is specific to Model (1-c). Finally, we estimate the following equation for Model (1-d):

\[
y_{it} = \alpha D_{it}^{10-29} + \beta D_{it}^{29} + \delta_d M_{it}^{\text{pos}} + \delta_d^2 M_{it}^{\text{neg}} + \lambda_1 t + \lambda_2 t^2 + c_i + \epsilon_{it} \tag{10}
\]

where \( M_{it}^{\text{pos}} \) is a cumulative measure of positive soil moisture deviations compared to the normal levels (equivalent to A+B+C in Fig. 1). And \( M_{it}^{\text{neg}} \) is the cumulative measure of negative soil moisture deviations compared to the normal levels (equivalent to D+E+F in Fig. 1). The subscript \( d \) distinguished estimated \( \delta \) from previous models.

We assume the errors are serially correlated due to unobservable and systematic measurement errors, and we consider clustering US counties by the state which has been a standard approach in the literature (Blanc and Schlenker, 2017; Hsiang, 2016; Lobell
and Burke, 2010). In this study, the models are estimated using a panel fixed-effect approach. A panel fixed-effect approach is a statistical method for analyzing two-dimensional (e.g. time and location) panel data. This method is helpful for analyzing those data collected for the same locations over time with a relatively short time span (Wooldridge, 2016). As our data set contains information for counties over time, a panel data analysis is appropriate. In addition, a fixed-effect model is appropriate as there are unique biophysical and economic attributes of counties that can explain yield differences across counties and are not changing over time. When we conduct a statistical test (Hausman test), it rejects the random effects model in favor of the fixed effect models we use. The panel consists of 35 years (1981-2015) for all US counties with corn production. For purposes of model comparison, we provide adjusted $R^2$, Akaike’s information criterion (AIC), and Bayesian information criterion (BIC).

For Model (2), we consider the daily interaction of heat and soil moisture as the compound metric. The interaction term is defined when the marginal impact of an explanatory variable depends on the magnitude of yet another explanatory variable (Wooldridge, 2016). Here, the marginal impact of heat on yield depends on water availability; also, the marginal impact of water on yield depends on heat. This is called conditional marginal impact. A key empirical challenge arises when estimating the model with daily interaction of heat and soil moisture. A simple multiplicative interaction of soil moisture variable and heat variables will be problematic (Hainmueller et al., 2019). It implies a linear interaction effect that changes at a constant rate with heat. However, as will be shown below, soil moisture has a non-linear marginal effect. We take two approaches here to calculate the conditional marginal impact of heat on corn yields to address the challenges of aggregating daily soil moisture to seasonal water availability metrics.

First, we construct a binning estimator based on daily interaction on heat and soil moisture in model (2-a). We define several intervals of soil moisture (SM) represented by daily dummy variables and we interact these dummy variables with the daily excess heat index of 29°C. Also, we take 25 mm intervals for soil moisture deviation from normal. In other words, we split the degree days into degree days conditional to soil moisture conditions. This includes dday29°C & SM 75+ mm below normal (extreme deficit), dday29°C & SM 25-75 mm below normal (deficit), dday29°C & SM 0-25 mm around normal (normal), dday29°C & SM 25-75 mm above normal (surplus), and dday29°C & SM 75+ mm above normal (extreme surplus). We estimate a coefficient for each combination of excess heat and soil moisture; ie., we estimate a model with metrics of degree days while controlling for soil moisture. The model provides the conditional marginal impact of excess heat as:

$$y_{it} = \alpha D_{it}^{10-29} + \sum_m \beta_m D_{mit}^{29} + \delta M_{it} + \delta' M_{it}^2 + \lambda_t + \lambda_t' t + c_i + \varepsilon_{it}$$

where $i$ is the county index, $t$ is the time index, $m$ is an index of soil moisture condition (high, low, normal), $s$ is an index for states, $y$ is average corn yields, $D$ represents conditional growing degree day variables, $M$ shows the seasonal mean soil moisture content, $T$ stands for the time trend variable, $c_i$ is a time-invariant county fixed effect. Here, $\beta$ is indexed by $m$; i.e., the marginal impact of heat is conditional to soil moisture conditions. $\alpha, \beta, \delta, \lambda$ are the regression parameters showing the marginal impacts.

Second, we estimate a model with metrics of soil moisture while controlling for temperature in model (2-b). We define an index of soil moisture when the temperature is above the threshold and an index of soil moisture when the temperature is below the threshold. In this model, the soil moisture is separated by a temperature threshold $H^*$. The model provides the conditional marginal impact of heat on corn yields as:

$$y_{it} = \alpha D_{it}^{10-29} + \beta D_{it}^{29} + \sum_m \delta_m M_{mit} \left|_{H<H^*} \right. + \delta'_m M_{mit} \left|_{H>H^*} \right. + \lambda_t + \lambda_t' t + c_i + \varepsilon_{it}$$

where $i$ is the county index, $t$ is the time index, $m$ is an index of soil moisture condition, $s$ is an index for states, $y$ shows average corn yields, $D$ represents growing degree day variables, $M$ shows conditional seasonal mean soil moisture, $T$ stands for the time
trend variable, $H$ is the average daily temperature, $H^*$ is the temperature threshold, and $c_i$ is a time-invariant county fixed effect. Here, we define $\delta$ and $\delta'$ to test whether the marginal impact of soil moisture depends on heat. The soil moisture metrics are calculated from daily gridded data and aggregated to county and growing season. This includes the index of normal soil moisture ($SM_{0-25+}$ mm around normal) when $H > H^*$, the index of normal soil moisture when $H < H^*$, the index of moisture deficit ($SM_{25+}$ mm below normal) when $H > H^*$, index of moisture deficit when $H < H^*$, the index of moisture surplus ($SM_{25+}$ mm above normal) when $H > H^*$, and the index of moisture surplus when $H < H^*$. $\alpha, \beta, \delta, \lambda$ are the regression parameters showing the marginal impacts.

### 2.5 Decomposition method

To show the significance of weather variation for crop yields, we estimate the historical impacts of heat and water. In a general form, we can decompose the impacts by taking the total derivative from the yield function. The general form is:

$$dy = \frac{\partial y}{\partial h} dh + \frac{\partial y}{\partial m} dm$$

where $dy$ shows the deviation of crop yields from the trend, $dh$ is the deviation of heat from the historical mean; and $dm$ is the deviation of soil moisture from normal levels. We apply this to Model (2-a) while the trend is estimated assuming no variation in heat and water availability. We predict the overall variation in yields using the estimated coefficients of Model (2-a):

$$d\hat{y} = \frac{\partial y}{\partial D} dD^{10-29} + \sum_{m} \frac{\partial y}{\partial D_m} dD_m^{29} + \frac{\partial y}{\partial M} dM + \frac{\partial y}{\partial M^2} dM^2$$

(14)

where $d$ shows the differential, $d\hat{y}$ is the predicted variation of crop yields, and partial derivatives are the estimated coefficients. Then, we re-predict the yields using the estimated coefficients of Model (2-a) for normal soil moisture. Thus, the predicted variation in crop yields is driven only by the variation in observed heat.

$$d\hat{y}_{heat} = \frac{\partial y}{\partial D} dD^{10-29} + \frac{\partial y}{\partial D_n} dD_n^{29}$$

(15)

Finally, the difference between (14) and (15) shows the predicted impact of variation in water.

$$d\hat{y}_{water} = d\hat{y} - d\hat{y}_{heat}$$

(16)

Note that the deviations are calculated for each year.

### 3 Results

The overall simulation results from WBM are illustrated in Fig. 2-4, showing gridded historical mean for the cultivated continental US, average annual variations for the cultivated continental US, and bivariate distribution of soil moisture and heat for the corn growing grid cells. To illustrate the spatial heterogeneity, Fig. 2 shows the growing season mean soil moisture content (in mm in 1000 mm topsoil) as calculated based on daily root-zone soil moisture level from Apr-Sep for 1981-2015 at 2.5 x 2.5 arcmin grids excluding non-cultivated area. Average growing season soil moisture is heterogeneous across the Continental US, with distinct regional patterns (see Fig. 2). For the corn belt, the soil moisture level is relatively high compared to other regions. The mean of volumetric soil moisture ranges from below 50 mm in southern California to above 250 mm in the Corn Belt and around Mississippi.
To compare the variation of simulated soil moisture and precipitation, Fig. 2 illustrates the weighted average soil moisture and precipitation over the cultivated US for 1981-2015. In general, variation in soil moisture average is higher than in that of precipitation (Fig. 3), showing how this new water metric is different from previous approaches. One interesting finding is that for some years the mean precipitation and the mean soil moisture move in opposite directions. For example, in 1990 the mean precipitation is declined by around 5% while the mean soil moisture is increased by around 13%.

To show the dynamics of soil moisture and heat, Fig. 4 shows their bivariate distribution by month based on daily information for all the cultivated grid cells in the US Corn Belt for 1981-2015. Heat and soil moisture combinations vary through the growing season (Fig. 4). The data shows significant month-to-month variation, with the second half of the season facing hotter and dryer days. Also, July has the highest variation in soil moisture deviation with a high probability of compound extremes as the distribution moves toward the lower right.

Below, we describe the regression results from each individual model, and compare their performance to identify which metrics are important to include in the statistical estimate of corn yields. The central finding is that metrics of soil moisture extremes are statistically significant, and models including intensity, duration, and severity metrics (as illustrated in Fig. 1) better capture both mean and variation in U.S. corn yields. This point is illustrated in Fig. 5, which compares Model 1a to Model 2a: each model estimates the percentage change in corn yields assuming additional 10 degree-days above 29˚C and no change in mean soil moisture. The figure shows that Model (1) would significantly underestimate the damage for conditions with extreme water surplus or extreme water deficit.

3.1 Model (1): predicting yield responses to individual extremes

The results from Model (1-a) show a strong relationship between corn yields and heat and precipitation (Table 2 column 1-a). The marginal impact of a degree-day within 10-29˚C is significantly positive while that from an additional degree day above 29˚C is strongly negative, confirming the seminal findings of Schlenker and Roberts (2009).

The results from Model (1-b), excluding precipitation, shows the marginal relationship with soil moisture is also significant (Table 2 column 1-b). This confirms the findings of Ortiz-Bobea et al. (2019). It shows that the marginal relationship with soil moisture is increasing up to ~92 mm in 1000 mm topsoil and decreasing for higher values.

In Model (1-c), we consider the number of days that soil moisture is either too high or too low. The model with metrics of soil moisture extremes further improves the fit, revealing a negative marginal relationship associated with the number of days with low/high soil moisture. Regarding Model (1-c), the coefficient on the number of days with low moisture is also significant and negative. Our estimation sample shows 26 days of high soil moisture and 27 days of low soil moisture on average. The implication is that eliminating 25 days of high soil moisture and 25 days of low soil moisture can improve the corn yields by up to 12.6%.

Model (1-d) shows the estimated coefficients when considering surplus and deficit (soil moisture deviation from normal) instead of average seasonal soil moisture. Here, we consider two thresholds for low and high soil moisture. Returning to Fig. 1, we evaluate the area of all blue bars and the area of all red bars. It shows that the marginal impact of the moisture deficit (cumulative negative soil moisture deviation) is significant and positive. This indicates the positive contribution of additional soil moisture when the soil moisture levels are below normal. On the other hand, the marginal impact of additional soil moisture in a wet period – i.e., a positive soil moisture deviation -- is negative. In other words, this measure captures the fact that plants will benefit from reductions in soil moisture when the soil moisture levels are above normal. This is an indicator of the value of sub-surface drainage for agriculture. Note that the Model (1-d) decreases the marginal relationship with extreme heat (dday29˚C). However, this effect is not statistically different from that produced by the first model.
The coefficient of the deficit in Model (1-d) is significant and positive. On the other hand, the coefficient of the extreme deficit is also significant and positive. The estimation sample shows this metric is around 2300 mm on average. It indicates that reducing the deficit by 2300 mm and reducing the surplus by the same amount can improve the corn yield by up to 21.2% on average. Note the mean soil moisture can stay unchanged in this scenario.

3.2 Model (2): predicting yield responses to compound extremes

In Model (2-a) we introduce heat-soil moisture interactions to test whether soil moisture availability changes the marginal impact of heat on yields (estimation results are in Table 3). We find that the average marginal impacts of dday29°Cs (heat stress) are all significant. The coefficient on dday29°C combined with the extreme deficit is -0.0082. The coefficient of ddays29°C (heat stress) combined with extreme water surplus is -0.0140. These figures are significantly different compared to Model (1).

We estimate a model with soil moisture while controlling for temperature (2-b). The results are presented in Table 4. The coefficient of degree days from 10°C to 29°C is significant and positive. This is not significantly different from previous models (1-a, 1-b, 1-c, 1-d, and 2-a). The coefficient on degree days above 29°C is significant and negative. It is close to the estimated values from Model (2-a) but slightly lower than Model (1). This indicates that the average damage from extreme heat index (dday29°C) is around 25% lower than Model (1). The estimated parameters show the yield response to changes in soil water content. Comparing the parameter values can show the difference in yield response to soil moisture in hot weather and moderate weather. The coefficient on normal soil moisture conditional to hot weather is 0.00012. The coefficient on normal soil moisture conditional to moderate weather is 0.00003. This indicates that the yield response to water is up to four times higher in hot weather.

The marginal impact on soil moisture deficit index is 0.00009 in hot weather and is 0.00002 in moderate weather. This also supports the finding that water is up to four times more beneficial to corn yields in hot weather. Also, the results show that the damage from excess water is up to two times larger in hot weather.

3.3 Model comparison

A comparison of model performance metrics is given in Table 5, along with a description of the water metric and extreme metric used in each model. We find that for Model 1b-d and Model 2a-d the coefficients on the soil moisture metrics are significant and with expected signs. Comparing the models’ performance suggests that Model (1-b), with mean soil moisture, performs better than the Model (1-a), with cumulative precipitation. Also, Model (1-d), with the extreme soil moisture metrics, outperforms both previous models (with cumulative precipitation or with mean soil moisture). The best corn yield predictor is from Models (2-a) and (2-b), considering compound extremes through the daily interaction of heat and soil moisture. We find that using a seasonally averaged soil moisture metric is insufficient for capturing yield extremes; i.e., the temporal resolution of the soil moisture metric is important for estimating corn yield variability. Figure 6 illustrates the difference by comparing the modeled impacts of average soil moisture (Model 1-b) on corn yields (Panel a) to the impacts considering the deviation from normal soil moisture (Model 1-d) estimated for a sandy soil type (Panel b) and a clay soil type (Panel c). In other words, when parametrizing the soil moisture as a deviation from normal, we get a specific piece-wise linear yield response to water depending on soil types (and normal levels of soil moisture), the extremes of which are completely missed by the model that only uses mean soil moisture. We find that the average corn yield damage from excess heat is up to four times more severe when combined with water stress. This damage can only be estimated when including soil moisture and metrics of extreme water stress (e.g., Models 2a-d).
3.4 Decomposing the variation in US corn yields

We have decomposed the changes in the US corn yields from 1981 to 2015 to understand the relative roles of soil moisture and heat in interannual corn yield variation. Figure 7 illustrates a decomposition based on our findings while aggregated for the whole US. With no climate variation, the US corn yield is expected to have a smooth positive trend as shown in green color. The deviation from the trend occurs due to changes in water and heat stressors. The blue bars are showing the expected changes in US corn yields due to changes in the water stress while the orange bars are demonstrating the expected yield changes due to changes in heat stress. While there have been years in which the stressors have moved together (e.g. 2011 and 2012), for several years water and heat have offset each other’s benefit or damage. For example, in 1992 the damage from heat is partially offset by benefits from water. Or in 2010, the damage from water stress is partially offset by benefits from heat.

3.5 Robustness checks

The Supplementary Material provides several robustness checks. The goal is to investigate whether different assumptions can improve the predictive power of Model (1) such that it outperforms Model (2). We answer three questions. First, are the estimation results from Model (1) different from those using alternative water metrics from WBM output? Second, are the estimates in Model (1) different from those obtained using a model considering growth stages? And third, do the main findings change if we alter the geographical scope of the study?

For the first robustness question, alternative water metrics, we re-estimate Model (1) using daily evapotranspiration (which is related to the water requirements of plants) and soil moisture fraction. Overall, the findings remain robust to alternative soil moisture metrics from WBM including the mean of soil moisture fraction (soil moisture content divided by field capacity), the seasonal mean of evapotranspiration as well as within season standard deviation of them. We also look at the results using an alternative interpolation of WBM data to PRISM resolution (nearest neighbor versus bilinear interpolations). We reject the null hypothesis that the coefficient on yield response to heat is different between these two metrics. Also, we reject the null hypothesis that the prediction power across these models is higher than Model (2).

To test the second robustness question, time separability, we re-estimate Model (1-b) for two-month intervals (Apr-May, Jun-Jul, Aug-Sep), and the findings remain robust. We find that considering bi-monthly variables does not change the yield response to heat. Although this alternative formulation does improve the predictive power of Model (1-b) a little bit, the performance is not better than the original Models (2-a) and (2-b) with compound extremes.

To test the sensitivity of our findings to geographical area, we re-estimate the models for Eastern US and Western US. We find that the estimated coefficients of Models (1-a) and (1-b) are not robust to the geographical choice, while those of Model (2) remain robust.

4 Discussion

In this paper, we have identified new water availability metrics that improve the predictive power of statistical corn yield models. While predictive power is an important outcome of this analysis, the insights gained from incrementally adding higher temporal-resolution metrics of water extremes to the models are also valuable for understanding the drivers of corn yield variability, and for revealing the resolution of water availability data required to capture future extremes under climate change scenarios. Statistical crop models have been used to both elucidate drivers of crop yield trends and variability, and to evaluate potential climate change impacts on crop production in the future (e.g., Lobell and Burke, 2010; Diffenbaugh et al. 2012). However, these models typically use seasonally averaged water availability metrics (e.g., total growing season precipitation), and utilize precipitation more often
than soil moisture. Generally, if the location of the study does not expect a significant change in the within-season distribution of the soil moisture, a mean soil moisture index will work. However, if there is an expected change in this distribution, using the mean variable will create biased yield projections. Because climate models project significant changes in the frequency and intensity of both extreme precipitation and temperature (Bevacqua et al., 2019; Manning et al., 2019; Myhre et al., 2019; Poschlod et al., 2020; Potopová et al., 2020; Wehner, 2019; Zscheischler et al., 2018), the results presented here show that the mean metrics of water availability – especially mean precipitation - are not sufficient to capture the impacts on yields. It is necessary to consider the metrics of extreme events as illustrated in Fig. 1. As we find that the coefficient on extreme heat is significantly different when considering soil moisture, it is possible that previous climate impact studies have over- or under-estimated the yield impacts. Further, farm management practices can alter soil moisture – and therefore yields – independent of precipitation. Supplemental irrigation, as well as no-till farming, cover cropping, and soil conservation, can increase soil moisture. These adaptations may occur in places predicted to face higher mean precipitation coupled with more extreme water events. The results of these management practices cannot be captured by statistical models looking at precipitation metrics alone. Such precipitation-based studies could potentially lead to over-estimation yield damages under future climate extremes by not accounting for human adaptations designed to conserve soil moisture.

Applying this framework to climate impact studies will face a key challenge – namely projecting the future compound extremes with the high temporal resolution of Model 2. It requires collaboration between hydrologists, climate scientists, and statisticians (Zscheischler et al., 2020). For future yield projections, we need reliable future projections of daily temperature (maximum and minimum) and soil moisture. Unfortunately, to the best of our knowledge, available data sets including predictions of future soil moisture have a relatively coarse spatial and temporal resolution, and rely on climate model projections with known difficulties representing daily temporal resolution events (Hempel et al., 2013). Further research is required to improve the ability of climate models and impact models in projecting the bivariate distribution of heat-moisture (Sarhadi et al., 2018).

5 Conclusions

This study serves to bridge the gap between statistical studies of climate impacts on crops and their biophysical counterparts by recognizing the central role of soil moisture – which is not a simple linear transformation of precipitation – in understanding crop yields. We employ a fine-scale, high temporal resolution dataset to investigate the conditional marginal value of soil moisture and heat in US corn yields for the 1981-2015 period employing a statistical framework. The major contribution of this study is showing that the coefficient on extreme heat (dday29˚C) is significantly different while considering daily interactions with soil moisture, emphasizing the importance of compound hydroclimatic conditions.

Our first key finding is that seasonal mean soil moisture performs better than average precipitation in statistically predicting corn yield. While the majority of current empirical studies employ precipitation as a proxy of water availability for crops, we show that the precipitation coefficient may not be always an appropriate measure of water availability. This study suggests that soil moisture content should be used in estimating crop yields instead of cumulative rainfall for locations with high runoff, drainage, or irrigation (e.g. Western and Central US).

Also, the metrics of soil moisture extremes can explain a portion of the damages to corn yield. On average, farmers can improve corn yields by up to 24% only by avoiding extreme water stress. We also find that the coefficient of excess soil moisture is negative.

This is in line with the current agronomic literature (Torbert et al., 1993; Urban et al., 2015) which points out that high soil moisture content can result in nutrient loss through excess water flows. In addition, at high humidity, the plants may have difficulty remaining cool at high temperatures. There is also a risk of waterlogging soils. With a few notable exceptions (e.g., rice), most
crops do not grow well in inundated conditions as the plant roots need oxygen, so the direct impact of excess water stress is because of the anoxic conditions.

Finally, the marginal impact of heat index on crop yields depends on the soil moisture level. We show the average yield damage from heat stress is up to four times more severe when combined with water stress; and therefore the value of water in maintaining crop yield is up to four times larger on hot days.

Code availability. The codes are available at DOI:10.4231/Q07D-J369.

Data availability. The historical weather data (PRISM) is available at http://www.prism.oregonstate.edu. The input data for estimations are available at DOI:10.4231/0M14-EY38.

Author contribution. All authors contributed to conceptualization, methodology, formal analysis, and writing- review & editing. IH and DSG collected model input, performed the simulations, and contributed to the investigation, resources, software, and validation. IH contributed to writing the original draft and visualization. TWH contributed to supervision and funding acquisition.

Competing interests. The authors declare that they have no conflicts of interest.

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McDonald, R. I. and Girvetz, E. H.: Two Challenges for U.S. Irrigation Due to Climate Change: Increasing Irrigated Area in Wet States and Increasing Irrigation Rates in Dry States, PLOS ONE, 8(6), e65589, doi:10.1371/journal.pone.0065589, 2013.


Table 1. Yield, heat, and water metrics for 1981-2015 (Apr-Sep)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn yield (bushels / acre)</td>
<td>109.8</td>
<td>37.8</td>
<td>4.5</td>
<td>246.0</td>
</tr>
<tr>
<td>Cumulative precipitation (mm)</td>
<td>564</td>
<td>183</td>
<td>1</td>
<td>1469</td>
</tr>
<tr>
<td>Mean daily soil moisture content (mm) in Apr-May (mm)</td>
<td>47</td>
<td>39</td>
<td>0.1</td>
<td>262</td>
</tr>
<tr>
<td>Mean daily soil moisture content (mm) in Jun-Jul (mm)</td>
<td>48</td>
<td>45</td>
<td>0</td>
<td>270</td>
</tr>
<tr>
<td>Mean daily soil moisture content (mm) in Aug-Sep (mm)</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td>264</td>
</tr>
<tr>
<td>Degree days from 10°C to 29°C when soil moisture is low</td>
<td>1848</td>
<td>434</td>
<td>693</td>
<td>3083</td>
</tr>
<tr>
<td>Degree days from 10°C to 29°C when soil moisture is normal</td>
<td>397</td>
<td>430</td>
<td>0</td>
<td>2629</td>
</tr>
<tr>
<td>Degree days from 10°C to 29°C when soil moisture is high</td>
<td>1112</td>
<td>572</td>
<td>0</td>
<td>3044</td>
</tr>
<tr>
<td>Degree days above 29°C when soil moisture is low</td>
<td>61</td>
<td>61</td>
<td>0</td>
<td>723</td>
</tr>
<tr>
<td>Degree days above 29°C when soil moisture is normal</td>
<td>18</td>
<td>31</td>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>Degree days above 29°C when soil moisture is high</td>
<td>37</td>
<td>48</td>
<td>0</td>
<td>680</td>
</tr>
<tr>
<td>Index of soil moisture above normal levels (mm)</td>
<td>330</td>
<td>346</td>
<td>0</td>
<td>2665</td>
</tr>
<tr>
<td>Index of soil moisture below normal levels (mm)</td>
<td>2370</td>
<td>2135</td>
<td>0</td>
<td>20319</td>
</tr>
<tr>
<td>Number of days with moisture deficit &gt; 25 mm</td>
<td>-2384</td>
<td>2147</td>
<td>-23978</td>
<td>0</td>
</tr>
<tr>
<td>Number of days with moisture surplus &gt; 25 mm</td>
<td>27</td>
<td>30</td>
<td>0</td>
<td>182</td>
</tr>
<tr>
<td>Mean daily soil moisture fraction</td>
<td>0.71</td>
<td>0.18</td>
<td>0.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Mean daily evapotranspiration (mm)</td>
<td>0.55</td>
<td>0.58</td>
<td>0.00</td>
<td>2.95</td>
</tr>
<tr>
<td>Number of observations</td>
<td>69923</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table reports descriptive statistics for major variables in this study. The mean and standard deviations are calculated over US counties for the 1981-2015 period. All the weather data are calculated for each 2.5 x 2.5 arcmin grids, averaged over the time interval, and then averaged to counties using cropland area weights. Soil moisture seasonal normal is defined as the average of 1981-2015 daily soil moisture level from the first day of April to the last day of September.
Table 2. Corn yield estimation without the interaction of heat and soil moisture in Model 1 (a-d).

<table>
<thead>
<tr>
<th>Model 1</th>
<th>(1-a) Log CornYield</th>
<th>(1-b) Log CornYield</th>
<th>(1-c) Log CornYield</th>
<th>(1-d) Log CornYield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree Days 10-29°C Apr-Sep</td>
<td>.000336*** (.000087)</td>
<td>.000343*** (.000080)</td>
<td>.0003486*** (.0000725)</td>
<td>.0003083*** (.0000683)</td>
</tr>
<tr>
<td>Degree Days above 29°C Apr-Sep</td>
<td>-.005307*** (.000673)</td>
<td>-.005114*** (.000691)</td>
<td>-.005277*** (.0006678)</td>
<td>-.005041*** (.0005999)</td>
</tr>
<tr>
<td>Precipitation Apr-Sep</td>
<td>.000658** (.000254)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precipitation Apr-Sep Square</td>
<td>-5.16e-07** (-9.35e-07)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seasonal Mean Soil Moisture Content</td>
<td></td>
<td>.003593*** (.000664)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seasonal Mean Soil Moisture Content Square</td>
<td></td>
<td>-.000017*** (3.000e-06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of days with SM 25+ mm above normal</td>
<td></td>
<td></td>
<td>-.001838*** (.0003816)</td>
<td></td>
</tr>
<tr>
<td>Number of days with SM 25+ mm below normal</td>
<td></td>
<td></td>
<td>-.002089*** (.0002817)</td>
<td></td>
</tr>
<tr>
<td>Index of Soil Moisture above Normal (mm)</td>
<td></td>
<td></td>
<td></td>
<td>-.000040*** (2.800e-06)</td>
</tr>
<tr>
<td>Index of Soil Moisture below Normal (mm)</td>
<td></td>
<td></td>
<td></td>
<td>.000044*** (7.100e-06)</td>
</tr>
<tr>
<td>Obs.</td>
<td>69923</td>
<td>69923</td>
<td>69923</td>
<td>69923</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.4686</td>
<td>0.4714</td>
<td>0.4795</td>
<td>0.4914</td>
</tr>
<tr>
<td>AIC (Akaike’s information criterion)</td>
<td>-21238.1</td>
<td>-21612.3</td>
<td>-22696.8</td>
<td>-24303.4</td>
</tr>
<tr>
<td>BIC (Bayesian information criterion)</td>
<td>-21201.4</td>
<td>-21575.7</td>
<td>-22660.2</td>
<td>-24266.8</td>
</tr>
</tbody>
</table>

Standard errors are in parenthesis & adjusted for state clusters
*** p<0.01, ** p<0.05, * p<0.1

Notes: Table lists regression coefficients and shows standard errors in brackets. The constant term and coefficients on the interaction of each state and time trends are not reported.
Table 3. Corn yield estimation while splitting heat stress index in Model 2a

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree days from 10°C to 29°C</td>
<td>.0003083***</td>
<td>(.0000685)</td>
<td></td>
</tr>
<tr>
<td>dday29°C &amp; SM 75+ mm below normal (extreme deficit)</td>
<td>-.0082398***</td>
<td>(.0014372)</td>
<td></td>
</tr>
<tr>
<td>dday29°C &amp; SM 25-75 mm below normal (deficit)</td>
<td>-.0062069***</td>
<td>(.0009793)</td>
<td></td>
</tr>
<tr>
<td>dday29°C &amp; SM 0-25 mm around normal (normal)</td>
<td>-.0037559***</td>
<td>(.0004045)</td>
<td></td>
</tr>
<tr>
<td>dday29°C &amp; SM 25-75 mm above normal (surplus)</td>
<td>-.0055709***</td>
<td>(.0012041)</td>
<td></td>
</tr>
<tr>
<td>dday29°C &amp; SM 75+ mm above normal (extreme surplus)</td>
<td>-.0140295***</td>
<td>(.0019083)</td>
<td></td>
</tr>
<tr>
<td>Mean daily soil moisture content (mm)</td>
<td>.0026635***</td>
<td>(.0008153)</td>
<td></td>
</tr>
<tr>
<td>Square of mean daily soil moisture content</td>
<td>-.0000161***</td>
<td>(2.600e-06)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>69923</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>.4921</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Akaike's Crit</td>
<td>-24401.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bayesian Crit</td>
<td>-24328.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table lists regression coefficients and shows standard errors in brackets. The constant term and coefficients on the interaction of each state and time trends are not reported.

Standard errors are in parenthesis & adjusted for state clusters

*** p<0.01, ** p<0.05, * p<0.1
Table 4. Estimation of corn yields while splitting the soil moisture metrics in Model 2b

<table>
<thead>
<tr>
<th>Metric</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree days from 10°C to 29°C</td>
<td>.0003154***</td>
<td>(.0000689)</td>
</tr>
<tr>
<td>Degree days above 29°C</td>
<td>-.004044***</td>
<td>(.00005384)</td>
</tr>
<tr>
<td>Index of normal soil moisture when T &gt; T*</td>
<td>.0001199***</td>
<td>(.0000342)</td>
</tr>
<tr>
<td>Index of extreme moisture surplus when T &gt; T*</td>
<td>-.0000628***</td>
<td>(.0000151)</td>
</tr>
<tr>
<td>Index of extreme moisture deficit when T &gt; T*</td>
<td>.000092***</td>
<td>(.0000234)</td>
</tr>
<tr>
<td>Index of extreme moisture deficit when T &lt; T*</td>
<td>.0000209***</td>
<td>(7.100e-06)</td>
</tr>
<tr>
<td>Index of extreme moisture surplus when T &lt; T*</td>
<td>-.0000326***</td>
<td>(3.200e-06)</td>
</tr>
<tr>
<td>Index of normal soil moisture when T &lt; T*</td>
<td>.000028**</td>
<td>(.0000105)</td>
</tr>
</tbody>
</table>

Observations: 69923
R-squared: .5006
Akaike's Crit: -25582.4
Bayesian Crit: -25509.2

Standard errors are in parenthesis & adjusted for state clusters

*** p<0.01, ** p<0.05, * p<0.1

Notes: Table lists regression coefficients and shows standard errors in brackets. The constant term and coefficients on the interaction of each state and time trends are not reported.
Table 5: Performance metrics for Models 1(a-d) and 2(a-b).

<table>
<thead>
<tr>
<th>Model</th>
<th>Water metric</th>
<th>Extreme metric</th>
<th>R-squared</th>
<th>AIC (Akaike’s information criterion)</th>
<th>BIC (Bayesian information criterion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-a</td>
<td>Avg. precipitation</td>
<td>Precipitation sqr</td>
<td>0.469</td>
<td>-21,238</td>
<td>-21,201</td>
</tr>
<tr>
<td>1-b</td>
<td>Avg. soil moisture</td>
<td>Soil moisture sqr</td>
<td>0.471</td>
<td>-21,612</td>
<td>-21,576</td>
</tr>
<tr>
<td>1-c</td>
<td>Avg. soil moisture</td>
<td>Number of days with low/high soil moisture</td>
<td>0.480</td>
<td>-22,697</td>
<td>-22,660</td>
</tr>
<tr>
<td>1-d</td>
<td>Avg. soil moisture</td>
<td>Avg soil moisture deficit/surplus</td>
<td>0.491</td>
<td>-24,303</td>
<td>-24,267</td>
</tr>
<tr>
<td>2-a</td>
<td>Avg. soil moisture</td>
<td>T binned by extreme deficit/surplus</td>
<td>0.492</td>
<td>-24,402</td>
<td>-24,328</td>
</tr>
<tr>
<td>2-b</td>
<td>normal soil moisture x T</td>
<td>extreme deficit/surplus x T</td>
<td>0.501</td>
<td>-25,582</td>
<td>-25,509</td>
</tr>
</tbody>
</table>
Figure 1. Soil moisture dynamics within a typical growing season. Some soil moisture conditions can be harmful to crops including excess wetness [i], moisture stress intensity [ii], duration of moisture stress [iii], and severity of soil moisture stress [iv].
Figure 2. Growing season mean soil moisture content (in mm in 1000 mm topsoil) as calculated based on daily root-zone soil moisture level from Apr-Sep for 1981-2015 at 2.5 x 2.5 arcmin grids excluding non-cultivated area. The soil moisture level is obtained from the Water Balance Model (WBM) and non-cultivated area information is from USDA National Cultivated Layer. This map illustrates the heterogeneity of simulated soil moisture over the Continental US and even within states.
Figure 3. Variations of average precipitation versus average soil moisture over corn areas in the United States. The precipitation is aggregated from PRISM and soil moisture is aggregated from WBM from 2.5 arcmin grid cells weighted by cropland area.
Figure 4. The bivariate density of heat and soil moisture for 1981-2015 for all the grid cells in the US Corn Belt. The precipitation is aggregated from PRISM and soil moisture is aggregated from WBM based on 2.5 arcmin grid cells.
Figure 5. Estimated damage to corn yield from an additional 10 degree-days above 29°C and no change in seasonal mean soil moisture.
Figure 6. Estimated impact of soil moisture on log corn yields. Including soil moisture in the regression and its square term, as in model 1-b, will give us a quadratic relationship between soil moisture and yields as in panel (a). A piece-wise linear parametrization, as in model 1-d, can provide location-specific piece-wise linear relationship based on soil moisture deviation from normal as in panels (b) and (c). This will cause the maximum of the response curve to be in lower soil moisture levels for sand and in higher soil moisture levels for clay soil texture.
Figure 7. The bars show the “contribution of water” and “contribution of heat” in variation of US corn yields (left axis). The lines illustrate actual yields and trend (right axis).