

## ***Interactive comment on “Possibilistic response surfaces combining fuzzy targets and hydro-climatic uncertainty in flood vulnerability assessment” by Thibaut Lachaut and Amaury Tilmant***

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**We would like to thank the referee for this thorough review.**

*“Lachaut and Tilmant introduce the concept of “possibilistic surfaces” to describe conditions under which success or failure of a water resources system is possible, where regions of “possibility” are defined in three different ways: 1) using logistic regression and defining success regions as conditions under which the logistic regression predicts*

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*success in meeting a threshold of satisfaction with at least some probability  $p$ , 2) using fuzzy performance thresholds in which a hard success/failure threshold does not need to be defined for a logistic regression model, rather a fuzzy membership function is used to assign continuous performance values to fuzzy sets, and 3) using convex hulls to define regions of success based on the outer bound of scenarios in which performance was found to be acceptable. The authors also discuss benefits of employing non-gridded sampling of conditions under which to evaluate water system performance to generate these surfaces.*

*Of the 3 possibilistic surfaces introduced by the authors, I believe only the second is new to the literature. As noted by the authors, Kim et al. (2019) use logistic regression to define success and failure regions. “*

**We would like to stress that the main goal of the paper is not to propose new methods to account for remaining uncertainty within response surfaces, but rather to consider fuzzy thresholds when partitioning a response surface. The research question needs to be made clearer in the paper: how to divide a response function in success and failure regions, when the threshold that defines success is ambiguous?**

**This was exemplified for three different methods used to generate regions of success and failure when the response surface itself is uncertain.**

**What we proposed was how to modify each of these partitioning methods to accommodate fuzzy thresholds. They corresponded to different assumptions about how to approach the remaining uncertainty of the response surface:**

**1. When assuming a continuous and probabilistic response surface: logistic regression. It normally requires a binary definition of success or failure, to produce a probability map. We decompose it for a sample of alpha-cuts to consider a non-binary definition.**

**2. When assuming a continuous response, but do not probabilities: analytical approx-**

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imation. We modify the membership function with an error interval and apply it to the fitted estimates.

3. When avoiding both a continuous approximation and reliance on probabilities: convex hulls. Here there is no membership function, hulls are enlarged to consider a looser definition of success and failure.

In the revised version we will refine the scope of the paper. We realize that the paper, even with these clarifications, attempts too many things at the same time. Testing several partitioning methods leads to confusion about the scope of the paper and detracts from the main contribution on fuzzy thresholds. We thus suggest an important departure from the first version: removing methods 2 and 3. The focus is then on introducing fuzzy thresholds to the logistic regression-based response surface.

The title, abstract, introduction and methods chapters will be revised accordingly, insisting on the following points:

1) Response surfaces as a common tool in DMDU (decision-making under deep uncertainty) normally rely on a binary definition of success and failure. In practice however, the difference between success and failure is not always clearly defined.

2) The research question is: how to use response surfaces in such cases? We propose a method to incorporate fuzzy thresholds to the response surface.

3) Doing so on a response surface that is itself well-defined is straightforward. Instead of a frontier, a clear transition area separates the regions of “full” success and “full” failure. However, we face the challenge of applying a fuzzy definition of success and failure to an uncertain surface. The notion of possibility attempts to synthesize the different nature of the uncertainties that we try to associate in a single figure for decision support: uncertainty of inputs - and thus performance - on the response side, uncertainty of the appreciation from the decision-maker or ambiguity, on the valuation side.

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4) We propose a mathematical justification for an approximated logistic regression that would consider a sample of values between 0 and 1, instead of binary only.

*“However, the authors do not discuss Quinn et al. (2018), who used logistic regression as described in this paper to define success/failure regions that account for stakeholders’ different levels of risk aversion by choosing different probabilities of success from the logistic regression to define the boundary. The authors also state that logistic regression cannot capture nonlinear relationships in the mapping of climate conditions to success/failure, but this is not true. One can easily incorporate interaction or nonlinear predictors in a logistic regression just as in a linear regression. See Hadjimichael et al. (2020) for an example. Other studies which use logistic regression for scenario discovery that were not cited by the authors include Lamontagne et al. (2019) and Marcos-Garcia et al. (2020).”*

Thank you for the suggested references, they will be included in the revised version. It will be interesting to discuss how stakeholders’ risk aversion defines success/failure regions in Quinn et al. (2018), how fuzzy thresholds rather relate to stakeholders’ ambiguity (or sometimes loss aversion), and how possibilistic surfaces attempt to combine both.

The statement concerning the lack of non-linearity between variables was indeed not correct, the revised manuscript will be changed accordingly.

*“With respect to the convex hull representation of possibilistic surfaces, this sounds like info-gap decision theory (Ben-Haim, 2006), which the authors do not discuss in the paper. It is not clear what their method contributes beyond this approach.”*

We propose removing the convex hull approach from the paper. Still we here discuss the difference with the info-gap method, that the original paper should indeed have mentioned.

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The info-gap method starts from a first guess coordinate in the exposure space and increases the uncertainty horizon from this estimate through a nested set of convex hulls. Their increasing size is controlled by a parameter. In this study, we assume no initial estimate, we consider the uncertainty space as a whole and draw the hulls containing all successes or failures, and we thus do not quantify the uncertainty horizon. There was also a difference in scope. We did not propose a complete framework like info-gap. We integrate fuzzy targets to different tools that are commonly employed in DMDU, which can then be used within larger frameworks like info-gap, eco-engineering decision scaling, etc.

*“It is also worth noting potential problems with this approach. One, which is briefly described by the authors, is that if the failure boundary is not convex, it could be too conservative. For example, a failure region like the red region in the attached figure could be estimated by logistic or linear regression with an interaction term between the two factors on each axis to capture the non-convexity. The convex hull, however, would include everything to the right of the black line, which includes a substantial region of successes in blue. But a convex hull might not always be more conservative like the authors imply. This is because it is defined by the realized values from their model simulations. As discussed by the authors with respect to their logistic regression model, none of the GCMs met their failure definition, but the probability of success in those worlds did not always meet their threshold of acceptability. They might not fall within the convex hull of failures, though, making the convex hull a less conservative definition in that case.”*

This should indeed have been specified as a limitation to convex hulls. Risk aversion is indeed not always an advantage for the convex hull. Its main strength is rather as an alternative when no good extrapolating tool (logistic regression or direct approximation) can be used. E.g. if the stressors are poor descriptors and/or the response is too noisy, or if the decision maker just wants to rely on actual simulations only to define the

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regions. The border of the hulls represents a variation in the exposure space, without having to make any statement about the distribution of the observed phenomena within or outside the hull.

As previously mentioned, the 3 methods explored how to incorporate fuzzy thresholds for different ways to handle the remaining uncertainty of the response. In the case of convex hulls, the fuzzy definitions of success and failure simply change the samples that they delineate.

*“Where I think the authors have introduced a new approach to the literature is in combining logistic or linear regression with fuzzy set membership. The question is, what value does this method add to the alternative approaches? I think this should be the focus of the paper, and it is not currently clear what that value is. Personally, I find the first and third approaches more intuitive. It is easy to understand what a probability of success represents, so defining success regions based on probability contours from a logistic regression makes sense to me. Similarly, it is easy to understand a failure region defined by lines connecting the farthest scenarios in which failures have occurred. I find fuzzy sets much harder to interpret, and more subjective to define. But I think it could provide value in that no hard success/failure threshold has to be assumed if using it with linear regression, whereas this is not true for the other two approaches. It would be helpful to expound more on this benefit, and the differences that come out of using this approach as opposed to Method 1. It is likely no method dominates all others, but why is this new method on the Pareto front of options? This needs to be better emphasized by comparing and contrasting the regions that come out of the alternative approaches.”*

This is an important element, also asked by the first referee.

As previously mentioned, we were not comparing a fuzzy set approach to two other approaches. The three methods are different ways to divide an uncertain response in

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success and failure regions. Fuzzy sets were meant to extend these different methods for cases where the threshold defining success and failure is ambiguous. Removing methods 2 and 3 should clarify this.

The revised manuscript will focus on the incorporation of fuzzy thresholds using only one partitioning method. What we propose is an extension of bottom-up vulnerability assessment studies for specific situations where the distinction between failure and success states are not clearly defined. This is not an alternative to traditional bottom-up approaches. The ultimate goal of the proposed extension is to be able to address a particular situation whereby crisp thresholds do not exist due to a variety of reasons including the lack of consensus amongst stakeholders, the ambiguous definition of the associated objective, etc.

This comment also asks to emphasize the differences that might come out of this approach, we agree it is worth discussing. However, the result section will remind that when comparing crisp and fuzzy thresholds, the crisp threshold is only counterfactual, not an alternative method to be compared to. It helps to visualize how fuzzy thresholds affect the partition of the exposure space in regions (which can be further emphasized at figure 11) but we assume that the crisp threshold is not available in the first place.

Similarly, it is also worth considering if fuzzy thresholds can lead to a different decision, compared to a counterfactual crisp threshold. In the attached figures (to be included in section 2 of the revised manuscript), we see that it is theoretically the case if the response functions of two options have different slopes. Other case studies show this change in slope, e.g. Quinn et al. (2017) where an improvement for expected flooding can make extreme floods worse, thus changing the slope of performance as function of stressors. However, in the present paper, the difference in slopes between the alternative options is small and does not lead to a different decision. Still, we will make clear that the results specific to this case-study are an illustration of how our proposed extension works, rather than a justification of its comparative value.

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*“Finally, the authors discuss shortcomings of using gridded scenarios to build models of success/failure regions, but they never compare their non-gridded sampling to a gridded sample to illustrate its claimed superiority. I suggest the authors remove this argument entirely as it is a secondary argument anyway, and is never actually illustrated. Please see the annotated manuscript for additional, more minor comments.”*

We will indeed follow the suggestion and remove the argument.

We here respond to some of the additional comments in the manuscript. We again really appreciate the time and dedication to this review, including the English corrections.

Line 5, abstract: we could say there is both noise and error, but those are hard to distinguish in the response surface approach. We will replace the sentence by: “furthermore, response surfaces have their own irreducible uncertainty, from the limited number of descriptors and the stochasticity hydro-climatic conditions”.

Line 58: the proposed definition is indeed more adequate, modified.

Line 90, modified: “the choice of a longer modelling time scale. . .”

Line 95: although the logistic regression is indeed used in scenario discovery, we here focus only on how it probabilistic regions instead of binary regions in the exposure space (scenario discovery starting with this step and focusing on the transition boundary). The sentence is reworked: “Quin et al. (2018), Kim et al. (2019), Lamontagne et al. (2019), Hadjimichael et al. (2020), and Marcos-Garcia et al. (2020) use a logistic regression to divide the exposure space based on probability of success (often as a step in a scenario-discovery approach).”

As requested at line 209, and based on comments lines 128, 156, 181, 211, we need to clarify some definitions on performance, probability of success, and thresholds.

- As noted in the later comments, here  $p$  is not probability of success but performance (in this case study, reliability). Replaced by  $r$  in the manuscript for clarity (in turn, error

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R replaced by  $\delta$ ).

- The performance is the reliability, i.e. the frequency of non-flooding days in a single realization, a time series, that is a point on the response surface.

- System success and failure are defined over a time series, by the performance satisfying a threshold (we now replace “performance target” by “acceptability threshold” in all instances for clarity). They can be confused with single-time step “failures” or “successes” (flooding, or lack of flooding), so we make sure to insist in the revised manuscript. In turn when defining a state of flooding, we specify “flooding threshold”.

- The logistic regression provides a probability of success, while success itself is a frequency (of local successes, i.e. non-flooding days).

Line 116, added: “overall system success or failure are measured over a time series, as opposed to local successes or failures that happen at a given time step. . .”

Line 218, modified: “Removing the noise through aggregation. . .”

Line 181: this uncertainty is not considered in the calculation, only in the discussion. We will also remind it in this section.

Line 206, added: “a sigmoid error function”

Line 215: this part will be deleted, but indeed, intuitively the actual error distribution would make more sense.

Section 3.2, lines 272-302: paragraphs restructured as recommended.

Line 306, coefficient of variation added with literature examples (Nazemi 2020).

Line 324: removed as unnecessary.

Lines 352 and 357: the un-gridded argument is removed, as suggested in the main comment.

Line 364: the McFadden pseudo R square is computed in the revised manuscript.

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Line 400: sentence removed.

Line 512: sentence removed.

#### References:

Quinn, J. D., Reed, P. M., Giuliani, M., and Castelletti, A. (2017). Rival framings: A framework for discovering how problem formulation uncertainties shape risk management trade-offs in water resources systems, *Water Resources Research*, 53, 7208–7233.

Additional references to be included in the paper on membership functions:

R. E. Haber, R. Haber, A. Alique, and S. Ros (2002). Application of knowledge-based systems for supervision and control of machining processes. *Handbook of software engineering and knowledge engineering*, vol. 2, pp. 673-710.

J. M. Garibaldi and R. I. John (2003). Choosing membership functions of linguistic terms. *The 12th IEEE International Conference on Fuzzy Systems, 2003. FUZZ '03.*, St Louis, MO, USA, 2003, pp. 578-583 vol.1, doi: 10.1109/FUZZ.2003.1209428

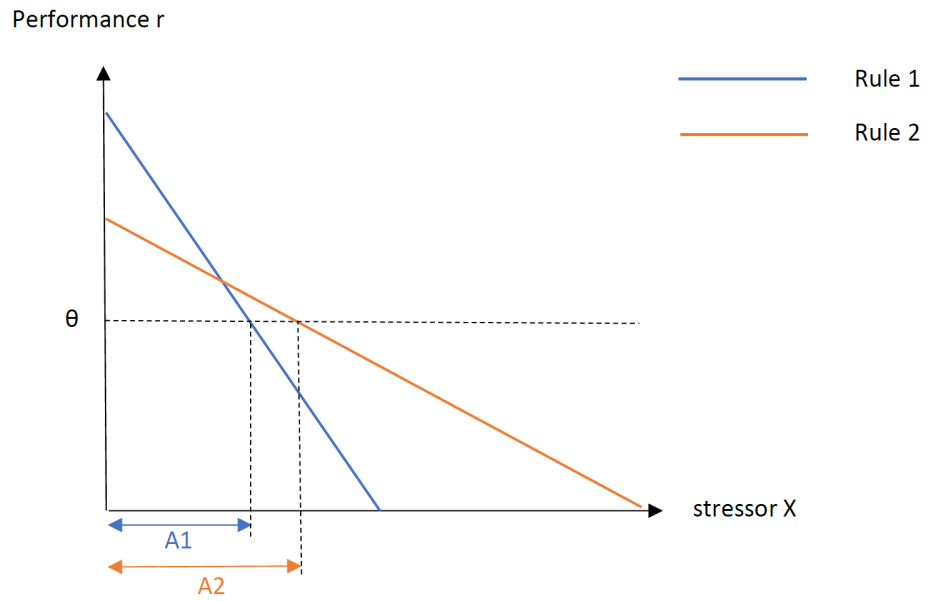
B. Bouchon-Meunier, M. Dotoli, B. Maione and D. Bari. (1996). On The Choice Of Membership Functions In A Mamdani-Type Fuzzy Controller.

Wu D. (2012). Twelve considerations in choosing between Gaussian and trapezoidal membership functions in interval type-2 fuzzy logic controllers. *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*; Brisbane, QLD, Australia

Sadollah, A. (2018). Introductory Chapter: Which Membership Function is Appropriate in Fuzzy System? [10.5772/intechopen.79552](https://doi.org/10.5772/intechopen.79552).

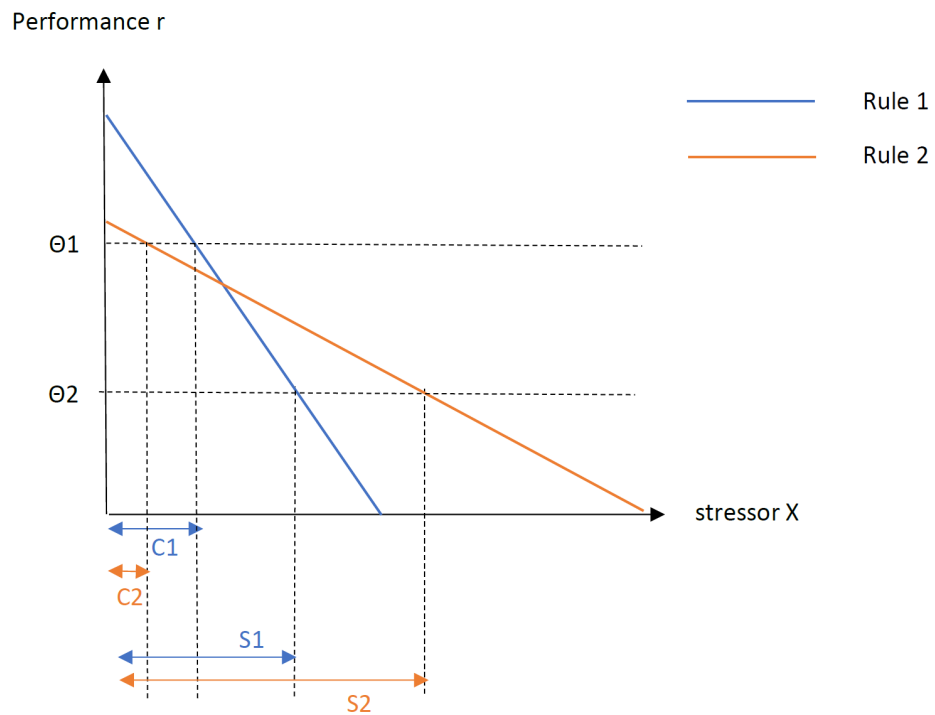
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Interactive comment on *Hydrol. Earth Syst. Sci. Discuss.*, <https://doi.org/10.5194/hess-2020-214>, 2020.



**Fig. 1.** With a crisp threshold  $\theta$ , rule 2 has a larger success region  $A2$ .

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**Fig. 2.** With a fuzzy threshold  $(\theta1, \theta2)$ , Rule 2 has a larger “at least partial” success domain  $S2$ , but a smaller “full” success domain  $C2$ , than Rule 1.

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