Responses to Anonymous Referee #1

The authors present a new flood routing model, "HDGNM", which is a further development of the "DGNM" model, which was developed by the primary author. The authors expand on the DGNM model, a Nash-cascade model, by incorporating a heteregeneous S-curve. The motivation of the authors is to improve flow routing in rivers that exhibit changes in the slope and geometry along its reach. They apply their model for flow routing in a 105 km stretch of the Hanjiang River, and demonstrate that the HDGNM model provides smaller error statistics.
I want to preface this discussion by stating that I am not an expert on the mathematical development of Nash-cascade models, and I recommend to the editor to rely on a different referee to judge the novelty or necessity of this development within that branch of study.

Reply: We very much appreciate the careful reading of our manuscript and valuable suggestions of the reviewer. According to the reviewer’s helpful suggestion, we have rewritten the introduction and added the purpose and main contribution.

The goal of this paper is to take into consideration spatial heterogeneity in DGNM, which is a river flood routing model that accounts for the separate contributions of old water and new water. The spatial heterogeneity of underlying surface is one main source of the nonlinearity of the hydrological processes and a key factor restricting the development of the hydrological model, but it is also an important breakthrough to improve the forecast accuracy. The topographic heterogeneity of rivers has great effects on river flood routing. Most hydrologic routing models have not considered such spatial heterogeneity, which restricts the application of this type models, especially in river reaches where the river slopes and cross-sections change greatly. It is beneficial to take into consideration such spatial heterogeneity. DGNM based on the linear cascade concept has the potential to address heterogeneity. Fortunately, the explicit expression of DGNM addressing the spatial heterogeneity is obtained by strictly mathematical derivation and conceptual interpretation of the routing system.

First, I want to complement the authors for presenting their study in such a concise format. Although the authors should expand on a few sections to ease understanding, the manuscript has a very respectable size. However, I would strongly recommend for the authors to let their manuscript be proofread by a native speaker.

Reply: We have adjusted the layout of the manuscript, as well, the manuscript was carefully reread to check for language issues. We have replaced the initial mistakes and edited the sentences carefully.

According to the two reviewers’ suggestions, the layout is adjusted as follows:
Section 1. Introduction

Section 2. Heterogeneous Discrete Generalized Nash Model
This new section includes the parts “Conceptual interpretation of the DGNM”, “Heterogeneous S curve”, and “Derivation of the heterogeneous DGNM” as subsections.

Section 3. Application
This section includes: Case study, and Model calibration.

Section 4. Results and discussion

Section 5. Conclusions

I think my main comment on the manuscript is that, in its current state, I fail to see the benefits of the proposed approach. The quantified improvement over the DGNM model is well described in the case study (although it does seem a marginal improvement at best), but I’m not convinced that the entire approach is conceptually ill-conceived. This may in part be due to a lack of context or well defined objective in the introduction, but points to some deeper concern as well.

Reply: Most hydrologic routing models are lumped, and hence fail to describe the spatial heterogeneity of the river reach. Developed from the linear cascade concept, the DGNM has the potential to take into account such spatial heterogeneity. The heterogeneous linear cascade concept is introduced in the DGNM. Based on the water balance equation for each reservoir and the hydraulic relation between cascade reservoirs as well as the conceptual interpretation of the DGNM, the heterogeneous DGNM (HDGNM) is fortunately deduced in an explicit expression of heterogeneous S curve and inputs and outputs. The HDGNM is strictly deduced in mathematics and has a rigorous conceptual interpretation of the routing process. That is the main contribution of the manuscript. To make it much clear, we have rewritten this part.

First a small note on the literature review. The authors discuss a wide arrange of literature (starting from Nash’s original paper) from L23-L54 on Rainfall Runoff modelling, even though the manuscript focuses on flow routing. Only from L55 onward the authors turn toward the relevant literature. Perhaps a restructuring to better lead up to the main objective would be advisable. Regarding relevant literature, I feel the authors focus to much on the Nash-cascade types of models and developments thereof, at the expense of other state-of-the-art literature on distributed hydrological modelling (e.g. see Imhoff et al., 2020 and references therein. DOI:"10.1029/2019WR026807").

Reply: Thanks for the reviewer’s suggestions. We have rewritten the introduction and added the developments of some distributed hydrological modeling in deal with the spatial heterogeneity.
In this part, with respect to the spatial heterogeneity exhibited in hydrological processes, we mainly focus on the way to deal with the spatial heterogeneity in conceptual models.

Discretization of watershed is common practice to transform the lumped conceptual models to semi-distributed and distributed models. Physically based discretization and conceptual discretization are summarized as two discretization approaches, and this manuscript is based on the second approach.

The added part of introduction is as follows:

"*The hydrological processes usually exhibit substantial spatial heterogeneity. That might be due to the spatial heterogeneity of rainfall and underlying surface. Spatial heterogeneity of a river basin increases the predicting uncertainty of streamflow using hydrological models (Adhikary et al., 2019), it is the key factor restricting but also promoting the development of hydrologic models. In the early hydrological modelling, a watershed is considered as a single unit for computations where the watershed parameters and variables are averaged over this unit (Dwarakish and Ganasri, 2015). These lumped models fail to describe the spatial heterogeneity of inputs, parameters, and processes. To better describe such spatial heterogeneity, the hydrological models have been developed from lumped to semi-distributed and distributed. It is common practice to transform the lumped conceptual models to semi-distributed and distributed models by discretizing the watershed into sub-watersheds (Arnold et al., 1990), grid cells (Liang et al., 1994; Vertessy and Elsenbeer, 1999; Yao et al., 2009), representative elementary watershed (Reggiani et al., 1998; Reggiani and Rientjes, 2005), hydrological response units (Arnold et al. 1998a, b), and so on. Such transition enables the model to take into account the spatial heterogeneity. However, not like the physically based models, e. g. SHE (Abbott et al., 1986a, b), whose most parameters can be measured in the field (observable), without some kind of calibration, much parameters of the conceptual models need to be calibrated and hence may cause the overparameterization and equifinality problems (Imhoff et al, 2020). To overcome such problems, Samaniego et al. (2010) introduced a multiscale parameter regionalization (MPR) to obtain seamless parameters across scales, where upscaling operators are used to aggregate catchment characteristics at a very detailed resolution to the modeling resolution and then a transfer function is applied to these catchment characteristics to calculate hydrological model parameters. It is an effective technique to integrate the spatial heterogeneity of physiographic characteristics. As a result, the number of parameters to be calibrated can be greatly reduced. To further lower the number of calibrated parameters in the MPR methodology, Imhoff et al. (2020) investigate the applicability of (pedo) transfer functions (PTFs) in combination with suitable upscaling operators for deriving seamless hydrologic model parameters. These PTFs are derived from laboratory experiments with point-scale samples in a bottom-up approach (see Van Looy et al., 2017), and are not constrained to the model but to actual field measurements, thus requiring no further model calibration. Contrasted to such bottom-up approach, Tran et al. (2018) proposed an alternative top-down approach by disaggregating parameters to higher resolutions using catchment properties and their spatial heterogeneity. This disaggregation approach is promising and can serve as an alternative to regionalization techniques. Either in a bottom-up or* *top-down approach, the spatial heterogeneity of catchment characteristics is represented by relationships between the spatial distribution of parameter values and them."*

Second, if I had to distil an objective from this manuscript it would be (Paraphrasing from L16) "To adapt the DGNM for flow routing to better deal with river reaches with varying geometry". This objective overlooks other, perhaps better suited, methods to deal with flow routing in river reaches of varying geometry. Conceptually, I would expect models derived from the shallow water equations to provide strong competition indeed. A literature review discussing alternatives outside from Nash models, would help to persuade the reader that the proposed alternative is worthwhile.

Reply: Similar to the distributed hydrologic models, physically based hydraulic flood routing approach can significantly account for the spatial heterogeneity and should be an appropriate method to deal with flow routing in river reaches of varying geometry. However, the detailed channel geometry information required in these hydraulic models are difficult to obtain in some rivers. As a result, the simplified hydrologic routing methods such as Muskingum method and IUH method are usually used as an alternative in these rivers. In fact, the comparison to other methods, including the widely used Muskingum method and dynamic wave model (DWM), has been made in Yan et al. (2019). The results show that the DGNM can provide comparable (to DWM) or even better results (to Muskingum). The HDGNM is a modification of DGNM, and we don't make such repeated comparison any more.

Third, building on the previous section, I'm having trouble seeing the inherent conceptual benefit for the broader scientific community. Applying Nash models to flow routing in rivers like the Hanjiang is really stretching the conceptual interpretation of the model to (in my opinion) untenable limits. The authors state that introducing heterogeneity would theoretically improve the model, but this is not supported by a rigorous analysis of the physics of river flow that their modification tries to alleviate. An interesting addition could perhaps be found in discussing how, from a physical point of view, changes in slope and cross-sections are expected to influence travel times and distortion of the flood wave, highlighting the flaws in the DGNM and hypothesizing how the HDGNM addresses these flaws. In its current form, I lean toward seeing the HDGNM model as an (overly) complex data-based model, more akin to machine-learning models than to process-based models - which have their applications as well, but if seen as such, require proper introduction and review of relevant literature.

Reply: In the introduction part, we have reviewed the modifications of some linear cascade based conceptual models in addressing the spatial heterogeneity. No matter the watershed is conceptually discretized into a cascade or parallel of unequal reservoirs or channels, the spatial heterogeneity of watershed or rainfall can be partially reflected, and hence improving the forecast accuracy in different degrees. The HDGNM is also a linear cascade based model addressing the spatial heterogeneity in a similar way, and an improvement should be expected similarly. From a physical point of view, the storage coefficient K is a reservoir detention characteristic and has a physical meaning of travel time. Physically, the changes in slope and cross-sections are expected to influence travel times and distortion of the flood wave. This influence can then be reflected by the storage coefficient K. In the DGNM, all linear reservoirs have a same K, such abrupt changes of the slope and cross-sections can not be addressed. In contrast, with different K, the HDGNM is more adaptive to these changes. We have added this analysis in the revised discussion part.

As we have emphasized that the HDGNM is a conceptual model, whose parameter has a specific physical meaning. Not like the machine-learning models, it is potential to estimate these parameters by the relationships with other physical characteristics, such as the bed slope and length. We will study this possibility in the future.

The case study itself is interesting and well defined, although some expansion on the case study (see specific comments) is required. The application of the HDGNM model is clear and results are well described, although somewhat marginal compared to DGNM. I would encourage the authors to publish the source code of their model and test data along-side the manuscript as well.

Reply: Thanks for the reviewer's positive comment for the application part. We would open the source code and data in the future.

In summary, I think the manuscript needs extensive revision before publication in HESS would be advised - mainly to better place it in light of the state-of-the-art and highlight the academic advancement made. Although to be fair, I fear the inherent academic progress made by this manuscript, even if thoroughly revised as advised above, may remain too little to be considered for publication in HESS, and that a different journal may be better suited. I include some specific comments below, in the hope they will be useful to the authors.

Reply: We have tried our best to revise the manuscript. The layout has been re-organized, especially the derivation of the model has been rewritten to make the contribution and novelty much clearer. The introduction has also been rewritten focusing on the spatial heterogeneity in conceptual hydrological models to reflect a wider scope of the study, which will be benefit for the broader scientific community. River flow routing is one of the key components of the hydrological modeling, and hydrologic routing is still an important way. Most of the recent hydrologic routing models are still lumped that cannot reflect the spatial heterogeneity of the river reaches. The DGNM based on the linear cascade concept has the potential to account for such spatial heterogeneity by replacing the equal linear reservoirs into unequal ones. However, it seems impossible to obtain the solution of high order differential equation of the Nash cascade model with unequal storage parameters directly. In this manuscript, the strict mathematical derivation is combined with the deeper conceptual interpretation of IUH, S-curve and linear cascade as well as the DGNM to obtain the HDGNM. This integration is a perfect solution of the above difficult problem. The application results also proves the improvement of the proposed approach. We hope that the revised manuscript can be acceptable.

Specific comments

L10: "The heterogeneous... the DGNM". This is very vague wording: I did not understand what the authors meant by 'conceptual interpretation of the DGNM' until much later on. Consider rephrasing this.

Reply: In the revised abstract, this vague description has been replaced by "The discrete generalized Nash model (DGNM) based on the Nash cascade model has the potential to address spatial heterogeneity by replacing the equal linear reservoirs into unequal ones."

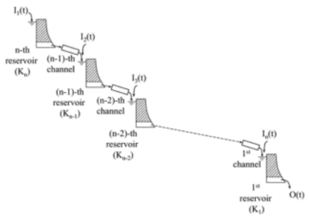
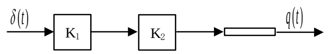
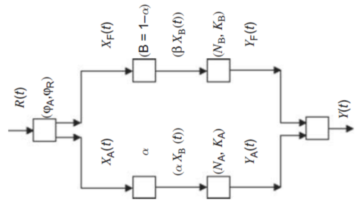
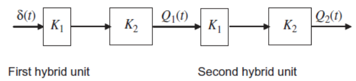
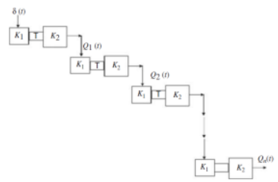
L16: "The HDGNM ... change greatly". Be more specific (here, but certainly later in the manuscript) what is meant by 'greatly'

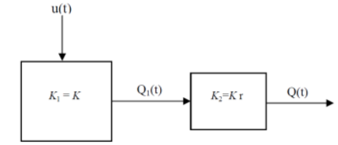
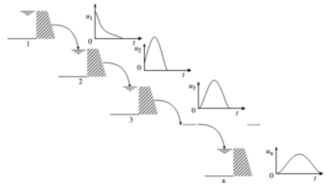
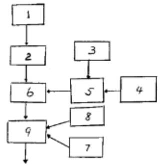
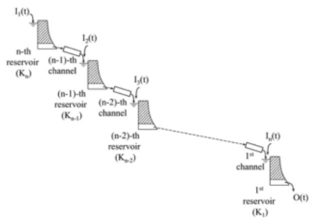
Reply: The main purpose of this study is addressing the spatial heterogeneity in the DGNM. In the river flow routing system, the spatial heterogeneity is reflected in the greatly changed river slopes and cross-sections. Greatly changed is relative to the homogeneity, no specific scopes.

L53: "All of these ... Runoff modelling". Be more specific which improvements are relevant for the objective.

Reply: In the revised introduction, we have summarized a table (Table 1) to illustrate the contributions of each literature. The improvements of each model has been added too.

Table 1 Summary of the IUH based models in considering the spatial heterogeneity

Authors	**Models**	**Formulas**	**Visual representation**
Dooge (1959)	Doogle IUH model	$q(t)=\frac{S}{T}\int_0^{t'\leq T}\left(\frac{\delta(t-\tau)}{\prod_{i=1}^{i(\tau)}(1+\mathrm{K}_i\mathrm{D})}\right)\omega(\frac{\tau}{T})\,\mathrm{d}\,\tau$	
Singh (1964)	Nonlinear IUH model	$q(t)=\frac{1}{(\mathrm{K}_2-\mathrm{K}_1)}\int_0^{t'\leq T}\left[\exp(\frac{-(\mathrm{t}-\tau)}{K_2})-\exp(\frac{-(\mathrm{t}-\tau)}{K_1})\right]\omega(\tau)\,\mathrm{d}\,\tau$	
Diskin et al. (1978)	Urban Parallel Cascade (UPC) IUH Model	$q(t)=\frac{\alpha_A}{K_A\Gamma(\mathrm{n}_A)}\left(\frac{t}{K_A}\right)^{n_A-1}\exp\left(\frac{-t}{K_A}\right)+\frac{\alpha_B}{K_B(\mathrm{n}_B-1)}\left(\frac{t}{K_B}\right)^{n_B-1}\exp\left(\frac{-t}{K_B}\right)$	
Bhunya et al. (2005)	Hybrid Model (HM)	$Q_2(\mathrm{t})=\frac{1}{\left(K_1-K_2\right)^2}\left[\left(t\exp\left(\frac{t}{K_1}\right)+t\exp\left(\frac{t}{K_2}\right)\right)-\frac{2K_1K_2}{\left(K_1-K_2\right)}\left(\exp\left(\frac{t}{K_1}\right)-\exp\left(\frac{t}{K_2}\right)\right)\right]$	
Singh et al. (2007)	Extended hybrid model (EHM)	$Q_2(\mathrm{t})=\frac{1}{\left(K_1-K_2\right)^2}\left\|\exp\left(\frac{t-2T}{K_1}\right)\left[t-2\left(T+\frac{K_1K_2}{\left(K_1-K_2\right)}\right)\right]+\exp\left(\frac{\left(t-2T\right)}{K_2}\right)\left[t-2\left(T-\frac{K_1K_2}{\left(K_1-K_2\right)}\right)\right]\right\|$	

Bhunya et al. (2008)	Two-reservoir variable storage coefficient (2VSC) model	$Q_2(t)=ER_1\left\{\frac{1}{1-r}\left(\exp\left[-\frac{t-\Delta t}{K}\right]-\exp\left(-\frac{t}{K}\right)\right)+\frac{r}{1-r}\left[\exp\left(-\frac{t}{K_r}\right)-\exp\left(-\frac{t-\Delta t}{K_r}\right)\right]\right\}$	
Li et al. (2008)	Heterogeneous IUH (HIUH)	$q(t)=\sum_{j=1}^{n}\frac{K_j^{n-2}}{\prod_{i=1,i\neq j}^{n}\left(K_j-K_i\right)}e^{-\frac{t}{K_j}}$	
Wang and Chen (1996)	Spatially distributed IUH	$Q_k(t)=\sum_{L=0}^{m}\sum_{j=0}^{n}A(\mathrm{L},\mathrm{n}-\mathrm{j}+1)\mathrm{u}(\mathrm{t}-\mathrm{L}\,\Delta\,\mathrm{t})\left[1-\sum_{i=0}^{j-1}(i!)^{-1}\left(\frac{\mathrm{t}-\mathrm{L}\,\Delta\,\mathrm{t}}{k}\right)^{i}\exp\left(-\frac{\mathrm{t}-\mathrm{L}\,\Delta\,\mathrm{t}}{k}\right)\right]$	
Wan et al. (2016)	Generalized concentration curve (GCC)	$Q_k(t)=\sum_{i=1}^{n}\sum_{t=1}^{tu_i}\left[I_i(k-t+1)\omega_i\sum_{j=1}^{i}\frac{K_j^{i-2}e^{-{t-(i-1)\tau}/{K_j}}}{\prod_{m=1,\,m\neq j}^{i}\left(K_j-K_m\right)}\right]$	

L73: "The DGNM ... topograpy too". Check language

Reply: We have rewritten the introduction part.

L83: what is a combination formula?

Reply: Here we have not explained accurately enough, it represents the calculation formula of the combinatorial number.

L106: "another way to deduce the HDGNM": what is the first? Why is another way required?

Reply: This expression is not accurate. It will be corrected in the revision text.

L111: "But for the basins with large topographical changes": Some form of conceptual sketch of what the authors mean by 'large topographical change' would be appreciated.

Reply: Large topographical change is relative to the homogeneity, no specific scopes. The schematic of the heterogeneous river flow routing system is illustrated in Fig. 1.

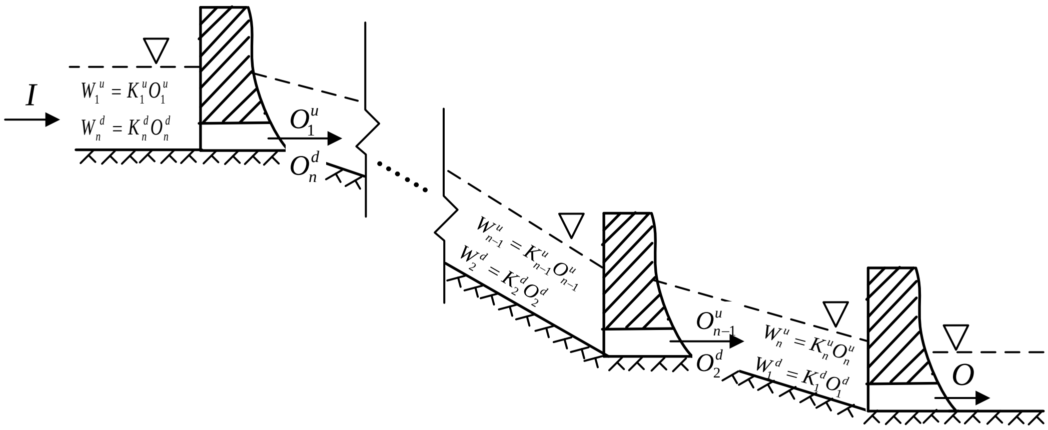


Fig. 1 Schematic of the heterogeneous river flow routing system

L125: it is unclear why this formula is introduced, nor how it follows from (4).

Reply: This formula is used in Appendix. We have removed it to the Appendix.

L133: what is a sub-river?

Reply: Sorry, the expression is not adequate. It should be "sub-reach".

L150: "but it seems impossible..." I'm not sure I follow why it is supposed to seem impossible.

Reply: Sorry, the expression is not adequate. Not like the outflow O^{new}, K can be replaced by K_i directly. In the expression of O^{old}, K is not explicit in the equation, but implicit in the calculation of coefficients of S_{n-j}. Hence, O^{old} cannot be obtained by directly replacing K to K_i.

L211: please specify on what basis the river is subdivided into these reaches.

Reply: In the cited reference, the subdivision is based on the location of the cities.

L211: please use scientific notation for the slopes (1.76*10**-4)

Reply: Will be corrected in the revised text.

L212: It is indicated by whom?

Reply: It is indicated by the cited reference.

L213: please make clear what sub-reaches 1 and two are (Huangjiagan-Guanghua and Guanghua-Taipingdian?)

Reply: Yes, sub-reaches 1 and 2 are the reaches Huangjiagan-Guanghua and Guanghua-Taipingdian, respectively. We will clarify this in the revision.

L219: The selection criteria of floods should be better described. Are these all the flood waves that fulfil the stated criteria? What do the authors mean by (delta t = 3)?

Reply: The expression is not adequate. It should be "the performance".

L221: I don't understand what the authors mean by 'the simulation effect'

Reply: The expression is not adequate. It should be "the performance".

L221: The forecast capability of HDGNM cannot be tested by comparing to DGNM. Improvement over DGNM can be tested, but any forecasting prowess should be based on evidence (measurements)

Reply: Yes, the expression is not adequate. Will be corrected in the revised text.

L223: please specify how 'flood data' was obtained and what it consists of.

Reply: Will be clarified in the revised text.

L226: please specify which parameters were optimized.

Reply: Will be clarified in the revised text.

L229: It would be very helpful if the authors could expand on the outcome of their optimisation exercise. Specifically, assuming that n=3 is an optimised value, is this an expected value? The authors state the the HDGNM is better suited to deal with topographical change, and this case study indeed shows four subreaches, of which the first one has a shallower slope than the final three. So, based on this information, would n=2 not be a more expected value? Or perhaps n=4, based on the number of subreaches the authors divide the river into.

Reply: Will be added in the revised text.

L267: "The heterogeneous ... the DGNM". I think the way this sentence is phrased does not help the author’s case. Would ’The HDGNM was derived by implementing a heteregeneous S curve into the DGNM model’ not be more to the point?

Reply: A good suggestion! Will be corrected in the revised text.

L295: What would constitute a reasonable request?

Reply: A reasonable request is usually about the purpose of the usage. It is not supported if the data was used as a commercial purpose.

Technical corrections

Figure 2: The size of the labels is a bit small and difficult to read.

Reply: Thank you for your suggestion. In the revised version we will take care to improve the readability of all tables and figures.

Figure 2: Please indicate which of these flood are the calibration events and which are the validation events

Reply: Will be corrected in the revised version.