Dear Editor and Referees!

The authors want to thank you for your valuable comments and suggestions. We are sure they led to an improvement of the manuscript.

Referee comments RC1 to RC3 as well as short comment SC1 were answered in detail during the discussion process already (see AC1-4). Now we provide the revised paper as a fluently readable document and as a version where all changes are marked and commented (see below). All specific referee comments were considered according to AC1-4. In addition, all general criticism is addressed and the paper was improved accordingly:

- The paper was restructured and sharpened. Its aim introducing and evaluating a new method to model *SWE* was emphasized.
- The placement of the model within the snow modeling spectrum was specified, not least by providing Table 1.
- The entangled descriptions of model code, modules and snow physics were clarified, not least by including Table 2.
- Language and writing were improved; repetitions eliminated and parentheses avoided.
- Requested citations were analyzed and included, whenever possible.
- More details on the data used for calibration and validation were given. A respective table was added in the appendix (Table A1).
- The application example was moved to the appendix.
- Discussion-like parts from the initially submitted manuscript's Results section were moved to the Discussion section.
- Main text was shortened by ca. 3000 words (>20%).

We updated the results of our error analysis, initiated by RC2 but without providing relative numbers (see AC2). We now provide RMSEs for all *SWE* and *SWE*_{pk}, as well as gradings depending on absolute *SWE* (cf. Fig. 2, Table 4 etc.), instead of statistics of RMSEs from different stations. This changed some key values concerning model accuracy, but none of the overall statements. Revisiting data sources and providing Table A1 revealed some minor inconsistencies concerning the number of observations and years, which could be solved easily.

We hope the revised manuscript fulfills HESS journal requirements for a final revised paper, and we are looking forward to your answer.

With best regards, Michael Winkler, Harald Schellander, Stephanie Gruber

Snow Water Equivalents exclusively from Snow Depths and their temporal Changes: The Δ SNOW.MODEL

Michael Winkler^{1,*}, Harald Schellander^{1,2,*}, and Stefanie Gruber¹

²Department of Atmospheric and Cryospheric Sciences, University of Innsbruck, Austria

*These authors contributed equally to this work.

Correspondence: Michael Winkler (michael.winkler@zamg.ac.at) and Harald Schellander (harald.schellander@zamg.ac.at)

Abstract. ¹Snow depths<u>Snow heights</u>² have been manually observed for many years, sometimes decades, at various places around the globe. These records are often of good quality. In addition, more and more data from automatic stations and remote sensing are available. On the other hand, records of snow water equivalent (SWE)-synonymous for snow load or mass – are sparse, although it might be the most important snowpack feature in fields-like hydrology, climatology, agriculture, natural

5 hazards research, etc. *SWE* very often has to be modeled, and respectivethose models either depend on meteorological forcing or are not intended to simulate individual *SWE* values, like the substantial seasonal "peak *SWE*".

The Δ SNOW.MODEL is presented as a new method to simulate local-scale *SWE*. It solely needs a regular time series of snow depths as input.snow depths as input, though a gapless record thereof. Temporal resolution of the data series is no restriction per se. The Δ SNOW.MODEL is a semi-empirical multi-layer model and freely available as R-package. Snow compaction is

10 modeled following the rules of Newtonian viscosity. The model considers measurement errors, treats overburden loads due to newfresh³ snow as additional unsteady compaction, and melted mass is stepwise distributed top-down in the snowpack.

Seven model parameters are subject to calibration. Snow observations of 67 winters from 14 stations, well-distributed over different altitudes and climatic regions of the Alps, are used to find an optimal parameter setting. Data from another 71 independent winters from 15 stations is used for validation., which was performed using 71 winters from 14 stations, well-distributed

- 15 over different altitudes and climatic regions of the Alps. Another 73 rather independent winters act as validation data. Results are very promising: Median bias and root mean squaresquared⁴ error for SWE are only -3.0 kg m⁻² and 30.8 kg m⁻², and +0.3 kg m⁻² and 36.3 kg m⁻² -4.0 kg m⁻² and 23.9 kg m⁻², and +2.3 kg m⁻² and 23.1 kg m⁻² for peak SWE, respectively. This is a major advance compared to snow models relying on empirical regressions and even sophisticated thermodynamic snow models do, but also much more sophisticated thermodynamic snow models not necessarily perform better.
- 20 Not least, this study outlines the need for comprehensive comparison studies on *SWE* measurement and modeling at the point and local scale.

¹ZAMG – Zentralanstalt für Meteorologie und Geodynamik, Innsbruck, Austria

¹Changes based on specific and general reviewer comments are marked in blue and orange, respectively.

²Snow "height" was changed to snow "depth" throughout the paper and also in the title. The respective symbol H was changed to HS, and h was changed to hs.

³"Fresh" snow was changed to "new" snow throughout the paper.

⁴Root mean "squared" error was changed to root mean "square" error throughout the paper.

1 Introduction

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Depth (*HS*) Total *H* and bulk density (ρ_b) are fundamental characteristics of a seasonal snowpack (e.g., Goodison et al., 1981; Fierz et al., 2009). Equation (1) links them to the areal density [kg m⁻²] of the snowpack, which – in hydrological applications – is usually referred to as snow water equivalent (*SWE*), as it resembles "the depth of water that would result if the mass of snow melted completely" (Fierz et al., 2009).

 $SWE = HS \cdot \rho_{\rm b}$ $[1 \, \text{kg m}^{-2} \equiv 1 \, \text{mm water equivalent (w.e.)}]$ (1)

1.1 Measurements of *HS* and *SWE*

Measuring HS is relatively easy (e.g., Sturm and Holmgren, 1998): Manual measurements at a certain point only require a rod
or ruler (e.g., Kinar and Pomeroy, 2015), and decades-long series of daily HS measurements exist in many regions – in lowlands as well as in alpine areas (e.g., Haberkorn, 2019). In modern times more and more HS data from automated measurements (mostly sonic or laser distance ranging) become available, typically in sub-hourly resolution (McCreight and Small, 2014). In addition, remote sensing techniques currently increase the number of HS data significantly, having the advantage of an areal picture instead of point information but at the cost of accuracy and in most cases also– in most cases – temporal resolution

35 and regularity (Cf., e.g., Dietz et al. (2012) for a general review of methods, and Deems et al. (2013) for a review on lidar measurements. Painter et al. (2016) provide a thorough overview. Garvelmann et al. (2013) and Parajka et al. (2012), e.g., illustrate the potential of timelapse photography.)

In contrast, measurements of SWE (or ρ_b) are more difficult (e.g., Sturm et al., 2010): Manual measurements require some basic equipment like snow tubes or snow sampling cylinders, a bit of dexterity, and are time consuming. In case snow depth

- 40 exceeds the sampling tool's size a pit has to be dug to consider the layered structure of the snowpackare time consuming (especially if a snow pit is dug), require some basic equipment like snow tubes or snow sampling cylinders and not least some dexterity (e.g., Kinar and Pomeroy, 2015). As a consequence, SWE measurements are carried out at much fewer locations than HS measurements (e.g., Mizukami and Perica, 2008; Sturm et al., 2010), their accuracy is lower, and series are shorter. Only in very rare cases consecutive, decades-long measurement series are available (e.g., in Switzerland; cf. Jonas
- 45 et al., 2009). Often they are only carried out at irregular time intervals ("snow courses") and even if regularly measured, temporal resolution is hardly ever higher than two weeks. ABesides these restrictions concerning manual measurements, also automatic measurements of *SWE* are not at all comparable in quality and quantity with automated *HS* measurements. They are quite expensive, often inaccurate, still at a developmental stage, and/or suffer from significant problems if not intensively maintained throughout the snowy season. Methods involve weighing techniques (snow scales; e.g., Smith et al., 2017; Johnson
- 50 et al., 2015), pressure measurements (snow pillows; e.g., Goodison et al., 1981), upward-looking ground penetrating radar (GPR; e.g., Heilig et al., 2009), passive gamma radiation (e.g., Smith et al., 2017), cosmic ray neutron sensing (CRNS; e.g., Schattan et al., 2019), L-band Global Navigation Satellite Signals (GNSS; e.g., Koch et al., 2019), etc. Presumably, the biggest

and best serviced network of automated *SWE* measurements is SNOTEL with about 800 sites in Western North America (Avanzi et al., 2015).

- SWE data from remote sensing are not operationally available for the local and point scaleFurthermore, there is no way to directly monitor SWE by remote sensing techniques (Schaffhauser et al.,2008), and deriving this snow property from satellite products at sub-kilometer resolution the local scale (< 1 km) is still not possible (Smyth et al., 2019). On top of that, there is the issue of longterm availability: automated measurements and (at least rough) remote sensing of SWE have not been available for more than some twenty years at their best (e.g., SNOTEL, operated since the late 1990s), a fairly short timespan compared
- 60 to decades-long daily *HS* data (e.g., Kinar and Pomeroy, 2015).

Regardless of these problematic circumstances accompanying SWE measurements, many hydrological, agricultural, and otherhydrological and agricultural applications depend on good estimates of SWE (e.g., Goodison et al., 1981; Sturm and Holmgren, 1998). Ultimately, Very often, in the end the mass of water stored in the snowpacks matters very often and, therefore, , and that's why the majority of those fields is especially interested in seasonal SWE maxima, i.e. "peak SWE" (SWE_{pk}).

65 SWE_{pk} are also the main focus of different kinds of extreme value and climatic analyses, both of which additionally very much rely on longterm or even "historical" data. Not least, snow load standards (e.g., International Organization for Standard-ization, 2013) rely on extreme value analyses of longterm snow load records (or SWE records, as snow load is defined as the product of SWE and the gravitational acceleration., respectively; snow load SL = SWE · g, with gravitational acceleration g = 9.8 m s⁻²). These points reveal the great discrepancy between the good data situation in terms of HS on the one hand, and the insufficient availability of SWE data on the other.

1.2 Modeling SWE

Modern snow models like Crocus (e.g., Vionnet et al., 2012), SNOWPACK (e.g., Lehning et al., 2002), SNTHERM (Jordan, 1991), or the dual-layer model SNOBAL (Marks et al., 1998) resolve mass and energy exchanges within the ground-snowatmosphere regime in a detailed way by depicting the layered structure of seasonal snowpacks. (Echoing Langlois et al. (2009),

- 75 these models will be termed "thermodynamic snow models" in the following.) All of them need atmospheric variables as input, primarily precipitation, temperature, humidity, wind speed, and radiative fluxes. Also relatively simple thermodynamic models at least require temperature and/or precipitation (e.g., De Michele et al., 2013) or climatological means thereof (Hill et al., 2019). Avanzi et al. (2015) provide a good review. Estimations of absolute model errors for *SWE* are rather searce. However, it might be at the order of 10 to 15 kg m⁻² (Langlois et al., 2009), but probably sometimes significantly more (Vionnet et al.)
- 80 al., 2012). See Sect. 3.1 and Table 4 for more details.⁵ All thermodynamic snow models need meteorological input data like temperature or radiation. Unfortunately, many valuable longterm *HS* series which are so valuable for a variety of applications (see above) do not involvecome along with these data, and parameterizing or downscaling them from other sources in turn is susceptible to errorsisn't straightforward either. Even markedly simpler models at least require temperature and precipitation measurements as inputs (e.g., De Michele et al. (2013) or in a very recent work by Hill et al. (2019) climatological means
- 85 thereof. Thermodynamic snow models are not Consequently, no thermodynamic snow model is applicable to derive SWE

⁵Moved to Discussion and Outlook section (Sect. 4.) NICHT VERGESSEN

exclusively from *HS*. (Avanzi et al. (2015) provide a good review and also introduce models including "statistical descriptions" which, in turn, need *SWE* measurements as input and, therefore, are not further addressed here. In this study these are counted as "thermodynamic snow models" as well.)

On the other side of the SWE modeling spectrum there are those models which – aside HS – only depend on date d (Pis-

- 90 tocchi, 2016), d and altitude z (Gruber, 2014, see statistical approach therein), d and regional parameters (e.g., Mizukami and Perica, 2008; Guyennon et al., 2019) or d, z and regional parameters (e.g., Jonas et al., 2009)(e.g., Jonas et al. (2009); and applications thereof e.g., by Achleitner and Schöber (2017), who fitted the parameters to Austrian data). Again, Avanzi et al. (2015) provide a thorough listing of those models, which will be termed "empirical regression models" (ERMs)abbreviated as ERMs (for "empirical regression models") in the following. ERMs very much rely on the strong, near-linear dependence
- 95 between *HS* and *SWE* (cf., e.g., Jonas et al., 2009). According to Gruber (2014) and Valt et al. (2018) *HS* describes 81% and 85% of *SWE* variance, respectivelymore than 80% of *SWE* variance (81% and 85%, respectively). This behavior bases on the narrow range within which the majority of bulk snow densities is found, and it leads to the well-known characteristic of $HS-SWE-\rho_b$ datasets: log-normally distributed *HS* and *SWE* as well as normally distributed ρ_b (e.g., Sturm et al., 2010). Unfortunately, ERMs cannot adequately model (unchanged) *SWE* during periods with snow densification only due to meta-
- 100 morphism and deformationdeformation strain⁶ (Jordan et al., 2010) but without (significant) mass loss. Sturm and Holmgren (1998) already state that HS and "load" (or SWE, respectively) play a more limited role in determining the compaction behavior in seasonal snow than "grain and bond characteristics" and temperature.

Interestingly, in most ERMs absolute, single-day *HS* observations are the only snow characteristics used. Depending on calibration focus they either adequately model mean-*SWE* or *SWE*_{pk}, mid winter or spring, etc. This is an inherent fact due to their model architecture. Those calibrated for good estimates of mean-*SWE*(necessarily) fail to model *SWE*_{pk} sufficiently well, those designed for *SWE*_{pk} often give bad *SWE* results during phases with shallow snowpacks. Typically, they simulate unrealistic mass losses during phases with compaction only by metamorphism and deformation, and the timing of *SWE*_{pk} as

well as the duration of high snow loads cannot be modeled well. As it is honestly stated by Jonas et al. (2009) – the authors of one of the most influential ERMs – those models cannot be used to "convert time series of *HS* into *SWE* at daily resolution
or higher" because they may "feature an incorrect fine structure in the temporal course of *SWE*". Therefore, ERMs are not

suitable to calculate SWE for individual days. (Still, well made for means.)

McCreight and Small (2014) go an interesting step further and not only use single-day HS values for their regression model, but also the "evolution" of daily HS. They make use of the negative correlation of HS and ρ_b at short timescales (10 days) and their positive/negative correlation at longer timescales (3 months) during accumulation/ablation phases. This promising

115 step of development is limited by the fact that the model parameters can only be estimated through regressions relying on at least three training datasets of *HS* and ρ_b (or *SWE*) from nearby stations. (MeCreight and Small (2014) used ultrasonic *HS* measurements in conjunction with *SWE* pillow measurements.) Unfortunately, this disqualifies the model of McCreight and Small (2014) for assigning *SWE* to longterm and historical *HS* series as consecutive *SWE* measurements are not available for those.

⁶"Deformation strain", literally termed as such by Jordan et al. (2010), is called "deformation" throughout the revised manuscript.

- 120 An alternative approach that links *HS* and *SWE* throughout a snowy season without the need of further meteorological input is provided by Martinec (1977) and revisited by Martinec and Rango (1991). In some respect their semi-empirical model Searching for alternative approaches that link *HS* and *SWE* throughout a snowy season without the need of meteorological input like temperature one might find Martinee and Rango(1991)'s work which provides a semi-empirical model that in some respect bridges the gap between thermodynamic models (needing lots of meteorological input) and ERMs (being
- 125 "overregulated" by snow depth). They use a method already developed by Martinec (1956) "to compute the water equivalent from daily total depths of the seasonal snow cover". Snow compaction is expressed as a time-dependent power function, and they end up with the following equation for each. Each layer's snow density ρ_n after *n* days is given by $\rho_n = \rho_0 \cdot (n+1)^{0.3,7}$ where ρ_0 is the initial density of the snow layer. A fixed exponent of 0.3 is used, without going into detail. Martinec and Rango (1991) set ρ_0 to 100 kg m⁻³, Martinec (1977) varied it from 80 to 120 kg m⁻³. This computation is meant to give good results
- 130 for the seasonal maximum snow water equivalent (SWE_{pk}) . It is shown, the older the snow the less important is the correct choice of the crucial parameter ρ_0 (Martinec, 1977; Martinec and Rango, 1991). Martinec and Rango (1991) use a constant ρ_0 of 100 kg m^{-3} and a fixed exponent of 0.3, without going into detail how these values were found. They show that the error made by a bad choice of ρ_0 is rapidly decreasing with *n* and therefore the power function gives robust results at least for old(er) snow layers. They claim (without further explanation) that snow "luckily" does not settle according to an exponential
- 135 curve, and show that in that case the error of ρ_0 would be independent from *n* and would not diminish while the snow layers are aging. Their model interprets "each increase of total snow depth [...] as snow fall" and if ""[if]— the total snow depth remains higher than the settling by [the power function][Eq. (2) of the article in hand], this is also interpreted as new snow. If the snow depth drops lower than the value of the superimposed settling curve of the respective snow layers, it is interpreted as snowmelt, and a corresponding water equivalent is subtracted. In this way the water equivalent of the snow cover can be
- 140 continuously simulated [...]" (Martinec and Rango, 1991). Rohrer and Braun (1994) improved this model particularly for the ablation season by further increasing density whenever melt conditions are modelled and by introducing a maximum possible snow density of 450 kg m⁻³. Martinee and Rango (1991) get promising results using this simple but robust snow compaction law. They only need daily snow depths as input and end up with a modeled SWE record at daily resolution.

Table 1 summarizes the classification of SWE models with respect to their essential input.⁸

145 1.3 Motivation for a New Approach

The question evolves, whether thosesuch a semi-empirical, layer-resolving snow models like the rather old one of Martinec and Rango (1991) can be improved and modernized, in order to provide an up-to-date snow model standard somewhere between sophisticated, thermodynamic models and modest, purely-statistical $HS-SWE-\rho_b$ models, like ERMs. Looking at the ease of Martinec (1977)'s and Rohrer and Braun (1994)'s approaches requiring only regular HS as input (see Table 1)Martinec and

150 Rango (1991)'s approach, thinking about modern computational possibilities, and given the introductorily described strong need for an implementable method"something handy", it seems interesting that there are no recent publications on this topic.

⁷Equation 2 of the submitted manuscript is not numbered in the revised manuscript.

⁸Table 1 was added during the revision. It was not part of the primarily submitted manuscript.

This leads to a possible null hypothesis of the paper-in-hand: "It is not possible to better model snow water equivalents than empirical regression models do, by exclusively using snow depths and their temporal changes as input." This statement can be rejected after the following presentation of a new way to assign *SWE* values to the huge number of longterm, historical, and

high-quality HS records of daily resolution. It follows Martinee and Rango (1991)'s key feature of considering daily change

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of snow depth as a proxy for the various processes altering bulk snow density ρ_b and snow water equivalent *SWE*, but further In the following, an advancement of semi-empirical *SWE* models is presented, which maintains their key feature of considering daily *change* of snow depth as a proxy for the various processes altering bulk snow density and snow water equivalent, but further

- 160 bases its (dry) snow densification function on Newtonian viscosity,
 - provides a way to deal with small discrepancies between model and observation (in the order of HS measurement errors),
 - takes into account unsteady compaction of underlying, older snow layers due to overburden snow loads, and
 - densifies snow layers from top to bottom during melting phases without automatically modeling mass loss due to runoff. $\frac{9}{2}$
- 165 The ideas for the latter three features were already developed by Gruber (2014), but not suitably realized. The This new modeling approach is named Δ SNOW.MODEL and an easy-to-use R-package is available through https://cran.r-project.org/ package=nixmass. The package is called *nixmass*, and it not only involves the Δ SNOW.MODEL, but also other models that use snow depth (*nix*...Latin for "snow") to simulate *SWE* (i.e., snow *mass*).

The way how physical processes are coded in the Δ SNOW.MODEL is thoroughly described in the The Δ SNOW.MODEL neither gives any new crucial insights in snow physics nor involves substantially new approaches. Still, the Δ SNOW.MODEL "rearranges" existing components in a physically consistent way and – as a whole – represents a new *method*. That is why it is described in the following Method section of this publication (Sect. 2). The calibration is outlined in Sect. 2 as well. Results, like best parameter choices and validation of the model output when compared to measurements, are given in Sect. 3. Section 4 provides an application of the Δ SNOW.MODEL for spatially modeling extreme snow loads in Austria. Sections 5 and 6 discuss possible

175 future developments and provide concluding remarks.¹⁰In Sect. 4 model sensitivity, open questions, and possible future developments are discussed and Sect. 5 provides concluding remarks.

2 Method

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As a successor of Martinee and Rango(1991)'s model the Δ SNOW.MODEL also builds on a semi-empirical approach and, therefore, can be regarded as standing between thermodynamic and empirical regression models. Its basic version was already presented by (Gruber, 2014, chapter 4) and a revision was presented by Gruber et al. (2018), but the Δ SNOW.MODEL described

⁹The last point, addressing the possible implementation of rain-on-snow, was omitted during the revision.

¹⁰Section 4 was moved to the appendix. In the revised paper the Discussion and Outlook section is numbered 4, Sect. 5 is the Conclusions section.

in the present article experienced a significant updating and recoding. It is designed for seasonal snowpacks and not intended for (multiannual) firn. The Δ SNOW.MODEL does not need any further input but a gapless snow depth record. It uses changes in observed snow depth to simulate a record of *SWE*. A scheme of the Δ SNOW.MODEL's principle is shown in Fig. 1.

Snow compacts over time due to various processes. Jordan et al. (2010) categorize them in snow drift, dry and wet meta-

- 185 morphism, and deformation. The Δ SNOW.MODEL cannot deal with snow drift, however, it differentiates between the latter processes: two processes. Dry metamorphism is mainly processed in the *Dry Compaction module*¹¹ of the Δ SNOW.MODEL, in case of a significant increase in snow depth also in the *Overburden submodule* of the *New Snow module*. Wet metamorphism is treated in the *Drenching module*.and the Metamorphism and deformation are processed in the three modules *Dry Metamorphism* (2.1), *New Snow* (2.2.1), and *Drenching* (2.2.3). The fourth module of the Δ SNOW.MODEL, the *Scaling*
- 190 *module*, *Scaling* (2.2.2), accounts for small discrepancies between model and observations. Table 2^{12} correlates Jordan et al. (2010)'s compaction processes with the Δ SNOW.MODEL modules and outlines the processes that are ignored. The specific modules are described in Sects. 2.1 and 2.2, a scheme of the model principle is shown in Fig. 1.

2.0.1 Preliminary: The First Snow Layer

For non-zero snow depth observations ($HS_{obs} > 0$) after a snow-free period the Δ SNOW.MODEL assigns the following features 195 to the model snowpack: There is one snow layer (layer counter ly = 1) and the age of this layer is set to age = 1. Thickness Snow height¹³ of this model layer (hs) and total model snow depth (HS) are equal, and set to observed snow depth: $hs = HS := HS_{obs}$. Analogously, the layer's snow water equivalent equals total snow water equivalent: $swe = SWE := \rho_0 \cdot HS_{obs}$, with new snow density ρ_0 being an important parameter of the Δ SNOW.MODEL (cf. Sect. 3). The treatment of the first snow event is illustrated at t = 2 in Fig. 1; it is processed within the New Snow Module (Sect. 2.2.1).

200 2.1 Dry Compaction moduleMetamorphism

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As it was mentioned in the Introduction, Martinec and Rango (1991) used a power function (Eq. (2)) to describe densification of aging snow, because this way errors in initial density ρ_0 get less relevant over time. For the Δ SNOW.MODEL this kind of high error tolerance of ρ_0 is a rather feeble argument to use a power law, since it only holds for old snow and deep snowpacks, but with the Δ SNOW.MODEL also *SWE* of ephemeral snowpacks (e.g., at low elevation sites) should be modeled as good as possible. Furthermore, as the Δ SNOW.MODEL considers overburden load in a particular way (Sect. 2.2.1), it is not expedient to have a direct dependence between density and age of a layer. Aside from that drawbacks of aMartinee and Rango (1991)'s power law compaction (and in contrast to Martinee and Rango (1991)'stheir unproven claim "snow would [not] settle [...] according to an exponential curve"), most modern snow models very well simulate snow compaction by way of Newtonian viscosity with associated exponential densification over time (e.g., Jordan et al., 2010). In the Δ SNOW.MODEL's *Dry*

¹¹The Dry Metamorphism module was renamed to Dry Compaction module during the revision.

¹²Table 2 was added during the revision. It was not part of the primarily submitted manuscript.

¹³Changed throughout the paper according to Fierz et al. (2009) as a consequence of changing snow "height" to snow "depth" (see above). The respective symbol h was changed to hs.

210 Compaction module the densifying effects of dry metamorphism and deformation are combinedThe △SNOW.MODEL's Dry Metamorphism Module combines the effects of dry metamorphism and deformation, by applying the following adaption of Sturm and Holmgren (1998)'s relation, with the help of De Michele et al. (2013). (For wet metamorphism – as defined by Jordan et al. (2010) – see Drenching module in Sect. 2.2.3.)

$$\frac{hs(i,t-1)}{hs(i,t)} = 1 + \Delta t \cdot \frac{\widehat{\sigma}(i,t)}{\eta(i,t)}$$
with $\widehat{\sigma}(i,t) = g \cdot \sum_{\hat{i}=i}^{ly(t)} swe(\hat{i},t)$
and $\eta(i,t) = \eta_0 \cdot e^{k \cdot \rho(i,t)}$.
(2)

215 Model timestep Δt in general is arbitrary, but usually it is one day. If so, t can be explained as "today" and t-1 as "yesterday" here. Accordingly, hs(i,t) is today's modeled thickness of the *i*-th snow layer. Snow layers are counted from bottom to top; layer i = 1 is the lowest and oldest layer. Today's depth of the total snowpack is $HS(t) = \sum_{i} hs(i,t)$.

The individual snow water equivalents of the layers are given by swe(i,t), and their sum represents total mass of the snowpack $SWE(t) = \sum_{i} swe(i,t)$. The vertical stress at the bottom of layer *i* is given by $\hat{\sigma}(i,t)$ (De Michele et al., 2013). It is constituted by the sum of loads overlying layer *i* (including layer *i*'s own load), with ly(t) being today's total number of snow layers or – in other words – ly(t) is the index *i* of today's uppermost (i.e., "surface") layer.

The Newtonian viscosity of snow η is made density-dependent in the framework of the Δ SNOW.MODEL (following Kojima, 1967), but dependencies on temperature, grain characteristics etc. are very consciously ignored – due to the lack of information on it when dealing with pure snow depth data. Today's density of layer *i* is $\rho(i,t)$; it equals $\frac{swe(i,t)}{hs(i,t)}$. *k* and η_0 are tuning parameters of the *Dry Compaction module* Dry Metamorphism Module (see Sect. 3).

To avoid excessive compaction a crucial parameter is introduced in the Δ SNOW.MODEL, as it was already done by Rohrer and Braun (1994): ρ_{max} . It defines the maximal possible density of a snow layer and, -(consequently,) also the maximum bulk snow density. Rohrer and Braun (1994) set ρ_{max} to 450 kg m^{-3} ; finding its optimal value for the Δ SNOW.MODEL Finding its optimal value is subject to calibration (Sect. 2.3). ρ_{max} figures the density a snow layer (or the whole snowpack) can reach at most, unless it looses mass by melting. ρ_{max} , of course, is a model parameter and cannot be observed in real snowpacks. In case the *Dry Compaction module*Dry Metamorphism Module increases the density of one or more layers beyond ρ_{max} , $\rho(i,t)$ of the respective layer(s) is set equal to ρ_{max} .

According to Eq. (2) the rate of densification of a certain snow layer is linearly depending on the overlying snow load $\hat{\sigma}(i,t)$ and exponentially depending on the layer's density $\rho(i,t)$. Sturm and Holmgren (1998) conclude that this difference is one

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reason why "snow load plays a more limited role in determining the compaction behavior than grain and bond characteristics and temperature". Nonetheless, higher overloads result in stronger compaction. Denser and older layers, respectively, compact less than newer layers with lower densities. This links the densification rate to the layer age, but indirectly by the use of density, and not directly. Therefore, <u>ASNOW.MODEL's compaction is not directly depending on layer age</u> as it was the case withif using

Martinec and Rango (1991)'s power law approach. The usage of an exponential function for compaction is one major difference

240 between the ASNOW.MODEL and Gruber (2014), who uses a power law approach similar to Martinec and Rango (1991).

The *Dry Compaction module* Dry Metamorphism Module of the Δ SNOW.MODEL is illustrated by the light blue arrows in Fig. 1. This module is applied at every point in time (except if there is no snow; see t = 1 in Fig. 1). The *Dry Compaction module* Dry Metamorphism Module is the core "highest-ranking" module because based on its result the Δ SNOW.MODEL decides between three different processes, realized by the other three modules:

245 2.2 Process Decisions

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At every point in time, after the Dry Compaction module was run, The Δ SNOW.MODEL simulates the layers' thicknesses for the next point in time hs(i,t) using Eq. (2) (Dry Metamorphism Module). As one time step typically equals one day, figuratively spoken the Dry Metamorphism Module acts "over night", from "yesterday" to "today". "Today" observed $HS_{obs}(t)$ and modeled HS(t) are compared. The Δ SNOW.MODEL's process decision algorithm now takes the result of the difference $\Delta HS(t) = HS_{obs}(t) - HS(t)$ and confronts it with τ [m]. τ is another tuning parameter of the Δ SNOW.MODEL (see Sect. 2.3). Its value is in the order of a few centimeters (see Sect. 3) since τ could also be regarded as a measure for observational error. Technically, τ is a threshold deviation and defines a limit of $\Delta HS(t)$ whose overshooting, adherence, and undershooting heads

for one out of the modules described in the following Sects. 2.2.1 to 2.2.3. Table 2 links them to snow physics.three branches, which – together with the Dry Metamorphism Module – build the four modules of the Δ SNOW.MODEL:

- 255 New Snow (2.2.1): In case observed snow depth is significantly higher than modeled snow depth $(\Delta HS(t) > +\tau)$, a snowfall event is assumed. This means mass gain as well as enhanced compaction of underlying layers due to the overburden load.
 - Scaling (2.2.2): In case there is no significant difference between observed and modeled snow depth $(-\tau \le \Delta HS(t) \le +\tau)$, neither mass gain nor loss is modeled. However, modeled snowpack is "scaled", i.e. compressed or stretched, to fulfill the condition $HS(t) = HS_{obs}(t)$.
 - Drenching (2.2.3): In case observed snow depth is significantly lower than modeled snow depth (ΔHS(t) < -τ), it is interpreted as wet snow metamorphism. In the snowpack this "drenching" happens from top to bottom, resulting in the associated (strong) decline of snow depth. The drenching can either be caused by melt (mass loss) or rain (mass gain), whereas treating the latter is optional, not finalized in the current version of the ΔSNOW.MODEL, and not further detailed here.

2.2.1 New Snow module

In case $\Delta HS(t) > +\tau$, meaning observed snow depth is significantly higher than modeled snow depth, a new snow event is supposed and a new top snow layer is modeled by the Δ SNOW.MODEL (see at t = 2 and t = 7 in Fig. 1 for a schematic illustration). This is a consequential step and nothing innovative at all. Other models have implemented this mechanism as

- 270 well (e.g., Martinec and Rango, 1991; Lehning et al., 1999; Sturm et al., 2010)(e.g., Martinec and Rango, 1991; Sturm et al., 2010. However, the Δ SNOW.MODEL goes beyond and introduces another feature: It explicitly models the peculiar effect of overburden load on underlying layers, defined as their enhanced densification due to (sudden)stress, which is put on by the weight of new snow (or rain-on-snow). Grain bonds get broken, grains slide, partially melt, and warp (Jordan et al., 2010), and the layers densify comparatively rapidly and strongly. The Δ SNOW.MODEL interprets overburden load as an "unsteady and
- 275 discontinuous" stress on the snowpack, under which snow presumably does not react as a viscous Newtonian fluid. (As long as Δt —the time between two consecutive observations Δt —is in the order of at least some hours, discontinuity is an intrinsic feature of the process. Mostly daily snow depths are available, a new snow event is always an unsteady case then.) The part of Jordan et al. (2010)'s "deformation strain", that is created by the individual layer loads, is interpreted as a "continuous" effect and is processed by the Dry Metamorphism Module; see $\hat{\sigma}$ in Eq. (2).
- 280 The *New Snow module* realizes the effect of overburden load through the *Overburden submodule* by reducing each layer's thickness hs(i,t) with the help of the dimensionless "overburden strain" $\epsilon(i,t)$, defined as

$$\epsilon(i,t) = c_{\rm ov} \cdot \sigma_0 \cdot e^{-k_{\rm ov} \frac{\rho(i,t)}{\rho_{\rm max} - \rho(i,t)}} \tag{3}$$

with $\sigma_0 = \Delta HS(t) \cdot \rho_0 \cdot g$.

 c_{ov} [Pa⁻¹] is another tuning parameter of the model (see Sect. 2.3) and. It controls the importance of the unsteady sudden, enhanced compaction due to overburden load. According to Sturm and Holmgren (1998) and in consistency with Eq. (2) snow 285 load has a linear effect on the bulk density. Therefore, $\epsilon(i, t)$ is made linearly depending on the load, which the overlying new snow is putting on the underlying layers. This load (or stress or pressure) is well approximated by σ_0 [Pa]; the bigger the overburden load, the stronger the compaction. (The overburden load does not fully equal σ_0 , since $\Delta HS(t)$ is not the depth of the new snow, but the difference between modeled depth -- "before" knowing about the new snow event -- and observed depth "after" the new snow event. An iterative calculation would be more precise, however, Eq. (3) proved to be an adequate compromise between simplicity and accuracy.) In order to avoid $\epsilon(i,t) > 1$, c_{ov} is restricted (at least) to the range of values 290 between 0 and the minimum value of the data record for $\frac{1}{\sigma_0}$. As σ_0 hardly ever exceeds 1000 Pa, $\frac{1}{\sigma_0}$ normally is larger than 1×10^{-3} Pa⁻¹. This value, thus, marks a good upper bound for $c_{\rm ov}$ (Sect. 2.3). Dimensionless $k_{\rm ov}$ controls the role of a certain snow layer's density (i.e., age, respectively) on $\epsilon(i,t)$, and has to be specified by calibration (see Sect. 2.3). The densitydependence of $\epsilon(i,t)$ was chosen to be exponential, and using ρ_{max} in the denominator of Eq. (3)'s exponent secures that 295 overburden loads cannot make snow layers denser than ρ_{max} . The closer a snow layer's density is to the maximum density ρ_{max} , the less it will be compacted by additional load. Relatively new and, therefore, - therefore - not very dense layers, are exposed to greater densification, which is exactly what is observed in reality. As it will be shown in sections 2.2.2 and 2.2.3 ρ_{max} also governs mass loss and melt in the model. Not least, ρ_{max} illustrates the possible maximum density of a wet seasonal snowpack in the \triangle SNOW.MODEL--world and <u>— as it can be seen in Sect. 3 —</u> it is possible to assign a reasonable value to it (cf. Sect. 3).

300 (Sturm et al. (2010), who revisited Sturm and Holmgred (1998), already introduced a maximum density for seasonal snow.

They used it very prominently in their formula for modeling bulk density and defined five (snow) climate classes with different values of ρ_{max} ranging from 217 to 598 kg m⁻³.)

The "overburden strain" $\epsilon(i,t)$ theoretically lies between 0 and 1 and compresses all (old) snow layers of the model in case of a new snow event. Practically, $\epsilon(i,t)$ is often close to zero (in this study 90% of all computed ϵ are smaller than 0.09) and 305 extremely rarely higher than 0.3 (in this study only 9 out of 10000).

The following intermediate (asterisked) variables are defined due to the overburden load. The compressed layer's masses, swe(i,t), remain unaffected during this process.

$$\epsilon(i,t) = \frac{hs(i,t) - hs^*(i,t)}{hs(i,t)} \quad \text{leading to} \quad hs^*(i,t) = (1 - \epsilon(i,t)) \cdot hs(i,t)$$

$$HS^*(t) = \sum_i hs^*(i,t)$$

$$\rho^*(i,t) = \frac{swe(i,t)}{hs^*(i,t)} \quad (4)$$

A new snow event, identified by the condition ΔHS(t) > +τ, of course not only impacts the older snow and compacts it
310 more strongly, but it also adds a new snow layer and mass to the snowpack (pink arrow at t = 2 and t = 7 in Fig. 1). The number of layers is increased by one and the following attributes are given to the new layer:

$$age(ly,t) = 1$$

$$hs(ly,t) = HS_{obs}(t) - HS^{*}(t)$$

$$swe(ly,t) = hs(ly,t) \cdot \rho_{0}$$
(5)

The total snow water equivalent is risen: SWE(i,t) = SWE(i,t-1) + swe(ly,t), and the intermediate variables of Eq. (4) overwrite their originals: $hs(i,t) = hs^*(i,t)$, $HS(t) = HS^*(t) + hs(ly,t)$, and $\rho(i,t) = \rho^*(i,t)$. The model--snowpack with 315 this new properties now again compacts according to Eq. (2), time t is risen by one increment, and at the next point in time "tomorrow" the process again starts with the decision described in Sect. 2.2. The *Overburden submodule* treatment of overburden snow loads as triggers for enhanced compaction of the underlying snow is illustrated with a purple arrow at t = 7 in Fig. 1. Although the overburden strain ϵ in most cases hardly deviates from zero (see above), the value of this feature for the performance of the Δ SNOW.MODEL is supposed to be rather high, at least it should be worth the effort (cf. Sect. 4).¹⁴

320 2.2.2 Scaling module

EOf course, equations (2) and (3) are highly simplified representations of the complex viscoelastic behavior of snow-and make no claims of being particularly precise. Still, also snow depth observations typically only show an accuracy of a few centimeters. The Δ SNOW.MODEL accepts these inherent inaccuracies and apparent discrepancies between model and measurements and

¹⁴The discussion of this issue is provided in the Discussion and Outlook section of the revised manuscript. NICHT VERGESSEN!

copes with them by not applying too strict criteria in the process decisions described in Sect. 2.2. The threshold deviation

uncertainty measure τ (introduced above) acts as a buffer to avoid too frequent gain or loss of mass in the model world: In 325 case $|\Delta HS| \leq |\tau|$ neither the snowpack looses mass nor gains mass, but mass is kept constant. In order to benefit from having a new measurement at every point in time, HS(t) is intentionally set to $HS_{obs}(t)$ by the Scaling module.

The Scaling module forces a partial revaluation reversal of the previous compaction, which was modeled by the Dry Com*paction module* Dry Metamorphism Module between t-1 and t. The best-fitted parameter setting for η_0 is temporarily rejected and substituted by η_0^* . It would be straight forward to use one adjusted $\eta_0^*(t)$ for all layers. However, this leads to a ratio-330 nal function with multiple solutions for $\eta_0^{\circ}(t)$, <u>Consequently</u>, this approach shows a clear non-physical behavior making it necessary to calculate different $\eta_0^*(i,t)$ for each layer i. See Appendix B for details on that.

 $\eta_0^*(i,t)$ is then used instead of η_0 in Eq. (2) to recalculate the compaction of individual layers. HS(t) now equals $HS_{obs}(t)$. In most cases all layers get "slightly more" or "slightly less" compacted by the Scaling module than by the Dry Compaction *module*Dry Metamorphism Module. Only at rare occasions the scaling does not compact, but a small "stretching" of the 335 snowpack is necessary. This only happens if there was a small increase in observed snow depth and very little modeled dry metamorphic compaction; the condition $HS(t) + \tau > HS_{obs}(t) > HS_{obs}(t-1)$ has to be fulfilled. Of course, such "stretching" does not occur in reality, but also in the Δ SNOW.MODEL it is an infrequent case that only acts at a small scale. In: in any case the "stretching" is smaller than τ . The issue is accepted as a model artifact, not least, because the "stretching" enables the very

valuable adjustment to HS_{obs} at every point in time – without forcing mass gains for any-insignificant HS raises that are within the measurement accuracy.

In case the density of an individual layer exceeds ρ_{max} by the scaling process, the excess mass is distributed layerwise from top to bottom. SWE remains constant during scaling, unless it would be necessary to compact all layers beyond ρ_{max} . In this case the appropriate excess mass is taken from the model- snowpack and interpreted as runoff, SWE is reduced and all

layer thicknesses are cut accordingly (see *Runoff submodule* in Sect. 2.2.3 for details). As τ turns out to be – reasonably and 345 preferably – chosen in the order of a few centimeters (Sect. 3), the resulting reduction of SWE within the Scaling module is always quite small: (e.g., with $\tau = 2 \text{ cm}$ and maximum density chosen 450 kg m⁻³ (like Rohrer and Braun, 1994) the mass loss due to runoff, i.e. runoff, is only 9 kg m^{-2} .

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The *Scaling module* is illustrated as black arrows in Fig. 1. Note, again that the scaling is nothing "physical", but also nothing "substantial" in terms of SWE, yet it is a smart way to utilize the advantage of having a measured snow depth at every point in time.

Drenching module 2.2.3

The Drenching module, finally, defines compaction due to liquid water percolating from top to bottom through the snowpack, loosening grain bonds and leading to densification (wet snow metamorphism). In case observed snow depth at a certain

355 point in time is significantly lower than modeled snow depth ($\Delta HS(t) < -\tau$), the Drenching module is activated. Drenching compaction is the Δ SNOW.MODEL's synonym of wet snow metamorphism.

The drenching can either be caused by melt or rain and the Δ SNOW.MODEL can principally deal with both processes, which are (often) contradictory in terms of mass change (melt: mass loss or invariant; rain: mass gain or - only if combined with runoff - invariant or mass loss). However, distinguishing between them is indeed extremely difficult (if not impossible) if only

- 360 snow depths are available. For the time being T the Δ SNOW.MODEL ignores rain on snow since it concentrates on modeling SWE for pure snow depth records without having any further information on e.g.e.g precipitation, temperature, snowfall level, etc.... Possibilities how rain could be addressed inat future developments are outlined in Sect. 4 MACHEN, VIELLEICHT MITHILFE DER GESTRICHENEN SÄTZE. This drawback seems disappointing, however, given the relative success of the ASNOW.MODEL "without rain" (see Sect. 3) one should not expect too much improvement when incorporating rain in one or another - potentially elaborate - way. 365

To cope with the model-observation-model/observation discrepancy $\Delta HS(t) < -\tau$ the Drenching module densifies the model layers until ρ_{max} is reached, – starting from the uppermost one. Figuratively spoken, it wettens or drenches a certain layer gets drenched until saturation and meltwater is further distributed until it is "saturated" and further distributes the melt water to the underlying layer. This process is repeated until (transient, therefore asterisked) HS^* equals $HS_{obs}(t)$. One or more

370

(upper) layers might reach ρ_{max} . In case $\Delta HS(t)$ is so negative that all model snow layers (from top to bottom) are compacted and densified to ρ_{max} , but still $HS^* > HS_{\text{obs}}(t)$ the *Runoff submodule* is activated and runoff R(t) is defined as, the product of the remaining depth difference and the maximum density constitutes mass loss, i.e. runoff R(t).

$$R(t) = (HS^* - HS_{obs}(t)) \cdot \rho_{max}.$$
(6)

All layer thicknesses are "cut" by a respective portion: $(HS^* - HS_{obs}) \cdot \frac{hs_i^*}{HS^*}$. This mechanism does not reduce total number of layers, but layers potentially get very thin. During the melt season, where most of the runoff is produced, the *Runoff* 375 submoduleDrenching module is more or less continuously active until it is snow-free $HS_{obs}(t) = 0$ and all the snow has been converted to runoff. For aone distinct snowpack from the first snow fall (t_1) until getting snow-free again (t_2) one has $\sum_{t=1}^{t_2} R(t) = SWE_{\rm pk}.$

In Fig. 1 the *Drenching module* is shown by the brown-arrow (as long as there is no mass loss) and its *Runoff submodule* by the green arrowsevan arrow (in case runoff is modeled). 380

2.3 Calibration

The Δ SNOW.MODEL has seven parameters that can be used for calibration: ρ_0 , ρ_{max} , η_0 , k, τ , c_{ov} , and k_{ov} (cf. Tab. 3). For the first four parameters one finds suggestions and ranges in the literature:

Sturm and Holmgren (1998) do not address the criticality for the choice of new snow density, however, they use constant $\rho_0 = 75 \text{ kg m}^{-3}$. It is a well known characteristic of newfreshly fallen snow to show large variations in densities. Helfricht 385 et al. (2018) reviewed many studies and give a general range of $10-350 \,\mathrm{kg \, m^{-3}}$, narrowing it down to "mean values" between $70 - 110 \text{ kg m}^{-3}$. Note, that this is daily densities. Sub-daily means of new snow densities are lower. Helfricht et al.

(2018), for example, come up with an average of 68 kg m^{-3} for hourly time intervals. During the calibration process for the Δ SNOW.MODEL ρ_0 was varied from 50 to 200 kg m^{-3} .

- The second density-related calibration parameter is ρ_{max} , the maximum possible density within the model framework. As mentioned, Rohrer and Braun (1994)Sturm et al. (2010) already setdefined such a maximum at 450 kg m⁻³. Also Sturm et al. (2010) defined it for five different climate classes, ranging. They range from 217 to 598 kg m⁻³. Glaciologists set the "critical density" before snow turns into firn (which is wetted snow that has survived one summer) to 400 to 800 kg m⁻³ (e.g., Paterson, 1998). Still, manual density measurements of seasonal snow used in previous studies hardly ever exceeded $\rho_b = 500$ kg m⁻³
- 395 (e.g., Jonas et al., 2009; Guyennon et al., 2019). Armstrong and Brun (2010) limit it to approximately 400 to 500 kg m^{-3} too. In order to find the fittest value for ρ_{max} used in the Δ SNOW.MODEL, it was varied from 300 to 600 kg m^{-3} .

Equation (2) needs η_0 , the "viscosity at [which] ρ equals zero" (Sturm and Holmgren, 1998). It is found to be in the order of 8.5×10^6 Pas (Sturm and Holmgren, 1998), 6×10^6 Pas (Jordan et al., 2010), and 7.62237×10^6 Pas (Vionnet et al., 2012). During the calibration process for the Δ SNOW.MODEL η_0 was varied from 1 to 20×10^6 Pas. Parameter k, the second

- 400 necessary parameter in Eq. (2), was varied from 0.011 to $0.08 \text{ m}^3 \text{ kg}^{-1}$ by Sturm and Holmgren (1998) depending on climate region and respective different types of snow. However, they cite Keeler (1969) in their Table 2 with values for k for "Alpinenew" snow of up to $0.185 \text{ m}^3 \text{ kg}^{-1}$. In more complex snow models k is set to $0.023 \text{ m}^3 \text{ kg}^{-1}$ (see Crocus: b_{η} in Vionnet et al. (2012)'s Equation (7); and also in Equation (2.11) of Jordan et al., 2010) or $0.021 \text{ m}^3 \text{ kg}^{-1}$ (see SNTHERM: Equation (29) in Jordan, 1991). Its range for the Δ SNOW.MODEL calibration was set from 0.01 to $0.2 \text{ m}^3 \text{ kg}^{-1}$, which is quite generous.
- 405 There are no references for the latter three parameters. Threshold deviation τ, as mentioned, might be interpreted as a measure of observation error, is regarded to be in the order of a few centimeters, and was modified from 1 cm to 20 cm for calibration. The last two parameters, c_{ov} and k_{ov}, determine the role of overburden strain and are newly introducted in kind of a specialty of the ΔSNOW.MODEL: c_{ov} and k_{ov}. At least the limits of c_{ov} could be defined (Sect. 2.2.1) as c_{ov} ∈ [0, min(¹/_{σ0})]. k_{ov} is only known to be a dimensionless,(dimensionless) real, positive number. For calibrating the ΔSNOW.MODEL c_{ov} and k_{ov}
 410 were restrained by [0, 10⁻³ Pa⁻¹] and [0.01, 10], respectively.

As mentioned, timestep Δt principally can be chosen arbitrarily. Mostly it might be one day, because many (longterm) snow depth measurements are on a daily basis. The calibration performed in this study is based on $\Delta t = 1$ day. Still, longer Δt (e.g., three days) as well as shorter Δt (e.g., one hour) are conceivable and could be handled by the Δ SNOW.MODEL. Note, however, at least some(at least some) calibration parameters will change significantly when changing Δt . This gets obvious

415 when thinking about new snow density ρ_0 , which of course is different if defined for one hour or for a three day timestep. The usage of this publication's calibration parameters can, therefore, only be suggested for daily snow depth records.

2.3.1 Calibration Data and Method

The calibration process needs *SWE* data, but *SWE* measurements are quite rare (see Sect. 1)data – either from observations or from a much more sophisticated snow model, whose simulated *SWEs* are sufficiently reliable. As it was outlined in the

420 Introduction, *SWE* measurements are quite rare. FurthermoreHowever, for calibration not only *SWE* observations are needed, but also regular snow depthgapless snow depths records from the same places, at least at daily resolution. Gruber (2014) col-

lected 1415 years of weekly *SWE* data from six stations in the Eastern Alps, measured by the observers of the Hydrographic Service of Tyrol (Austria) between winters 1998/99 and 2011/122012/13. The measurements of snow depth and water equivalent were made manually in snow pits with rulers and snow sampling cylinders (500 cm^3) , respectively. The sites range from

- 425 590 m to 1650 m altitude and are situated in relatively dry, inneralpine regions as well as in the Northern and Southern Alps, which are more humid due to orographic enhancement of precipitation (see Gruber, 2014, for details). The sites in the Southern Alps even show a moderate maritime influence due to their vicinity to the Mediterranean Sea, the most important source of moisture for this region (e.g., Seibert et al., 2007). These $6 \times 14 = 846 \times 15 = 90$ winter seasons cover 1166 measured *HS*-*SWE* pairs. Besides these *SWE* measurements manual *HS* measurements are available for every day at the respective stations.
- 430 Figure A1 and Table A1 provide a map and a list, respectively.¹⁵

The second source for *SWE* measurements used for calibration is Marty (2017). The Swiss SLF freely provides biweekly *SWE* and daily *HS* data from 11 stations in Switzerland (mostly in the Northern Alps, some inneralpine) spanning an altitude range from 1200m to 2540m. The biweekly *HS* measurements, accompanying (corresponding to the biweekly *SWE* measurements,) were compared with the contemporary value of the daily *HS* records. Only those sites and years were used for

- 435 calibration where the respective values of the daily HS record match the values of the biweekly measurements. If this condition is fulfilled, it is supposed that SWE and HS measurements fit together sufficiently well, although they unfortunately cannot always be taken exactly at the same place, which introduces uncertainty (e.g., López-Moreno et al., 2020). Consequently, 9 stations were used, most of them in the Northern Alps, some inneralpine, spanning an altitude range from 1200m to 1780m This was the case for 9 stations, with all in all 56 winters and 388363 pairs of HS and SWE measurements. Details are
- 440 given in Fig. A1 and Table A1. (Other stations and years suffer from discrepancies caused by too far distances between the measurements etc.)

In order to ensure an unperturbed validation, the observation data sets from Austria and Switzerland (15541529 SWE - HS pairs) were split in two almost equally big parts, one for model calibration (SWE_{cal}) and one for validation (SWE_{val}).into even years for model calibration (SWE_{cal}) and odd years for validation (SWE_{val}).¹⁶ The two data sources (Gruber, 2014; Marty,

- 445 2017) do not address the accuracy of the manual SWE observations. Mostly, SWE measurements made with snow sampling cylinders are used as references in comparison studies, without addressing *their* accuracy (e.g., Sturm et al., 2010; Dixon and Boon, 2012; Kinar and Pomeroy, 2015; Leppänen et al., 2018). López-Moreno et al. (2020) provide a reported range of 3-13%, and condense the results of their own, very thorough and valuable experiments to an error range of 10-15% for bulk snow density. The majority of SWE_{cal} and SWE_{val} comes from the Hydrographic Service of Tyrol, Austria, where snow sampling
- 450 cylinders (500 cm^3) are used (Sect. 2.3.1). The repeatability of this kind of measurement is estimated at $\pm 4\%$ for glacier mass balance studies (R. Prinz, Univ. of Innsbruck, Austria; pers. comm.). Roughly interpreting these density measurement "variabilities" as relative observation errors for *SWE*, the results for absolute accuracy would typically spread across the wide range of about $2 \text{ to } 50 \text{ kg m}^{-2}$.

¹⁵Figure A1 and Table A1 were added during the revision. They were not part of the primarily submitted manuscript.

¹⁶The following remarks on *SWE* observation accuracy were placed at the beginning of the Validation and Illustration section of the initially submitted manuscript and moved to this place during revision.

Model calibration was performed with the statistical software *R* (R Core Team, 2019) and the *R* package *optimx* (Nash, 2014). Results were obtained with optimization methods *L-BFGS-B* (Byrd et al., 1995) followed by *bobyqa* (Powell, 2009), which both are able to handle lower and upper bounds constraints. The function to be minimized was the root mean square error (RMSE) of *SWEs* from the Δ SNOW.MODEL and observed *SWEs*, using the calibration data set *SWE*_{cal}.

3 Results

The following evaluates the ability of the Δ SNOW.MODEL to calculate snow water equivalents exclusively from snow depths, and its practicability.¹⁷ See sections 3.2 and 4 for more. Before that, however, the model parameters have to be optimized during the calibration process (which was described in Sect. 2.3). The results thereof – the best-fitted parameters – are important model results per se, since they show insights how well model world and real world fit together.

3.1 Optimized Parameters and Sensitivities

Table 3 gives an overview of all parameters and summarizes the optimal setting for the Δ SNOW.MODEL. A discussion of the best-fitted values and of the model sensitivity to parameter changes can be found in Sect. 4.

The minimal RMSE between all *SWE* observations used for calibration (*SWE*_{cal}) and the respective modeled values is 30.1 kg m^{-2} . It is reached for new snow density $\rho_0 = 81 \text{ kg m}^{-3}$, maximum density $\rho_{\text{max}} = 401 \text{ kg m}^{-3}$, "viscosity parameters" $\eta_0 = 8.5 \times 10^6 \text{ Pa s}$ and $k = 0.030 \text{ m}^3 \text{ kg}^{-1}$, threshold deviation $\tau = 2.4 \text{ cm}$, and "overburden parameters" $c_{\text{ov}} = 5.1 \times 10^{-4} \text{ Pa}^{-1}$ and $k_{\text{ov}} = 0.38$.¹⁸ A graphical analysis of the model sensitivity to parameter changes is shown in Fig. 2. *SWE*_{pk} was chosen to indicate sensitivity, since it is probably the most important quantity for snow-hydrology and other fields

470 SWE_{pk} was chosen to indicate sensitivity, since it is probably the most important quantity for snow-hydrology and other fields where snow mass is a key variable (see Introduction).

3.1.1 New Snow Density ρ_0

Being aware of both – the huge possible variations of new snow density ρ_0 depending on meteorological conditions during snowfalls and the possible cruciality of this parameter for *SWE* simulation by the Δ SNOW.MODEL – ρ_0 was chosen to be a

- 475 constant in the framework of the model. For $\rho_0 = 81 \text{ kg m}^{-3}$ the minimal root mean square differences/errors (RMSE) between all *SWE* observations used for calibration (*SWE*_{cal}) and the respective modeled values was reached. This value clearly lies within the broader frame of possible new snow densities and quite closely to Sturm and Holmgren (1998)'s 75 kg m⁻³ (The Δ SNOW.MODEL could be seen as an extended combination of the Sturm and Holmgren (1998) and Martinec and Rango (1991) approaches.), but it is found in the lower part for "typical" new snow densities (e.g., Helfricht, 2018). A possible explanation
- 480 could be that the SWE measurement records used for the calibration tend to underrepresent late winter and spring conditions. Regular (weekly, biweekly) observations capture the short melt seasons worse than the (much) longer accumulation phases.

 $^{^{17}}$ Most content of subsections 3.1.1 to 3.1.5 of the submitted manuscript was transferred to the Discussion section during revision. The segmentation 3.1.1-3.1.5 was omitted.

¹⁸All the optimized values (except ρ_{max}) where rounded to two significant figures for the revised manuscript.

Therefore, *SWE* records might be biased towards early and mid winter new snow densities, which are lower (e.g., Jonas et al., 2009). Still, there are also some indications that using e.g., 100 kg m^{-3} as constant new snow density when modeling *SWE* results in an overestimation of precipitation (up to 30% according to Mair et al., 2016). The calibrated value for ρ_0 can be regarded as a reasonable result, even more when only considering it as a model parameter but not as a physical constant.

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The sensitivity analysis illustrated in Fig. 2 confirms the importance of a good choice of ρ_0 . Increasing ρ_0 quite fast leads to a decrease of the relative bias of seasonal *SWE* maxima (*SWE*_{pk}). (Note the definition of the relative bias in Fig. 2's caption.) In absolute values: too small ρ_0 cause too small *SWE*_{pk}, using higher values leads to an overestimation of *SWE*_{pk}. This behavior supports above-mentioned tendency to overestimate precipitation when choosing 100 kg m^{-3} as new snow

- 490 density. As expected, the new snow density is the most crucial parameter of the Δ SNOW.MODEL (cf. Tab 3). The median relative bias of SWE_{pk} changes by -0.46% per +1 kg m⁻³, if the whole calibration range of ρ_0 is considered to calculate the sensitivity (50 - 200 kg m⁻³). This means a median change in SWE_{pk} of +0.37 kg m⁻² when ρ_0 is risen by +1 kg m⁻³. If the limits are chosen tighter around the optimal value, the gradient is even steeper: -0.62% and +0.50 kg m⁻² per +1 kg m⁻³, respectively, when the gradient is approximated for the range 70 - 90 kg m⁻³. Widely-used $\rho_0 = 100 \text{ kg m}^{-3}$, consequently,
- 495 causes a median overestimation of SWE_{pk} of about 12% in the Δ SNOW.MODEL (same for daily SWE). Users should be aware of this. The suggestion clearly is to either use the best-fitted parameters of this study or recalibrate *all* parameters (with appropriate SWE data), but not adjusting only single parameters. As the calibration data of this study are spread across various climates and altitudes, users can be quite confident to get good results if using $\rho_0 = 81 \text{ kg m}^{-3}$. This value seems to be a good compromise – at least at alpine areas. However, for (very) maritime, very dry, polar or tundra regions the optimized ρ_0 should 500 be used with caution; if possible, recalibration is recommended.

3.1.2 Maximum Density ρ_{max}

Of course, the maximum bulk snow density of a snowpack changes from year to year and site to site. For the ΔSNOW.MODEL simplicity and independence from meteorological variables outweigh precision. Even more so, when there are good arguments for the existence of a "typical" maximum bulk density ρ_{max}. Put simply, not too old seasonal and also ephemeral snowpacks melt
away when they get water saturated. Before that, there is limited time for dry densification; dry winter snow's bulk density is widely described as staying below about 350 kg m⁻³ (e.g., Paterson, 1998; Sandells et al., 2012). Accounting for the fact that volumetric liquid water content of about 10% marks the funicular mode of liquid distribution in old, coarse-grained snow (Denoth, 1982; Mitterer et al., 2011), this leads to the rough estimate of a typical maximum bulk density of about ⁹/₁₀ · 350 + ¹/₁₀ · 1000 = 415 kg m⁻³. Convincingly, the fittest value for ρ_{max} in the ΔSNOW.MODEL turns out to be 401 kg m⁻³, which is close to that value and well situated within the range given in the literature (Table 3). Moreover this is virtually the

510 which is close to that value and well situated within the range given in the literature (Table 3). Moreover this is virtually the same value like the median maximum seasonal density of the SWE_{val} data records (400 kg m^{-3} , see box plot in Fig. 3) – another indication why ρ_{max} could be regarded as a typical seasonal maximum of ρ_{b} .

Figure 2 illustrates the similarity between ρ_0 and ρ_{max} regarding their influence on *SWE* simulations. Keeping the other six Δ SNOW.MODEL parameters constant but increasing ρ_{max} leads to increased *SWE*_{pk} and vice versa. This is not surprising, 515 however reasonable. The Δ SNOW.MODEL is not as sensitive to changes in ρ_{max} than to changes in ρ_0 : Raising ρ_{max} by $+1 \text{ kg m}^{-3}$ leads to a mean decrease of the relative bias of SWE_{pk} of -0.06%, which corresponds to an increase in absolute SWE_{nk} of +0.24 kg m⁻² per +1 kg m⁻³. The same argumentation like for ρ_0 in Sect. 3.1.1 lets users of the Δ SNOW.MODEL be quite sure when taking $\rho_{\text{max}} = 401 \text{ kg m}^{-3}$, the best-fitted value according to this study's calibration. Be aware that solely changing parameter ρ_{max} for an application of the Δ SNOW.MODEL elsewhere, without proper recalibration of the other parameters,

might lead to significant changes in the results for SWE. For the data set of this study, e.g., using $\rho_{\text{max}} = 500 \text{ kg m}^{-3}$ (instead 520 of 401 kg m⁻³) would lead to a median overestimation of SWE_{pk} of about 24 kg m⁻².

3.1.3 Viscosity Parameters no and k

Equation (2) represents the settlement and densification function of the Δ SNOW.MODEL. Viscosity of layer *i* at time *t* is defined as $\eta(i,t) = \eta_0 \cdot e^{k \cdot \rho(i,t)}$. Two parameters η_0 and k act as adjustment screws and have to be calibrated. Literature places η_0 in the order of some 10^6 Pa s, k is supposed to be in the range of 0.01 to $0.2 \text{ m}^3 \text{ kg}^{-1}$ (Table 3). The latter was varied over the

- 525 mentioned range during the calibration process and – satisfactorily – it's optimized value is $k = 0.030 \text{ m}^3 \text{ kg}^{-1}$, which is very elose to the ones used in widely accepted, sophisticated, thermodynamic models (Jordan et al. (2010), Vionnet et al. (2012); see also Sect. 2.3). The range over which η_0 was varied during the calibration was 1 to 20×10^6 Pa s. Best-fitted value is 8.5×10^6 Pa s, pretty close to other studies' values (Table 3).
- 530 As far as the Δ SNOW.MODEL's sensitivity to changes in the "viscosity parameters" η_0 and k is concerned. Fig. 2 shows that an isolated rise of the model snow viscosity – either by enhancing η_0 or k – increases the relative bias of SWE_{pk}, which means a decrease in absolute values of SWE_{DK} . This behavior is consistent, since higher viscosity reduces the densification rate and the model snowpack tendentially stays deeper. Consequently, increases in observed snow depth tend to bring less new snow in the Δ SNOW.MODEL (New Snow Module). Finally, simulated SWE_{pk} is reduced when η_0 or k are increased and vice versa.

535 3.1.4 Discrepancy Parameter τ

The Δ SNOW.MODEL's parameter to cope with uncertainties in snow depth is τ . It is supposed to be in the order of a few centimeters (at maximum). In particular, it should avoid excessive production of snow mass in the model through too frequent simulation of new snow events (see Sect. 2.2). τ is kind of a peculiarity of the Δ SNOW.MODEL and therefore no bounds can be found in literature. It was generously accepted to range between 1 and 20 cm for calibration and turned out to be optimal

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at $\tau = 2.4$ cm (Table 3). Given the wide range of possible values, this is very close to what it would be expected to be as a measure for HS obs accuracy.

Model sensitivity to changes in τ turns out to be quite low for values in the order of a few centimeters, but the influence on simulated SWE_{pk} is strongly increasing if τ is chosen greater than about 5 cm (Fig. 2). This result makes a lot of sense, if τ is seen as a measure of observation accuracy, because this is very likely to be better than 5 cm. Like changes in η_0 and k.

ehanges in τ are indirect proportional to changes in SWE_{pk} – for a closely related reason: The bigger τ the more often (small) 545 new snow events are not counted as such because the Scaling module is more frequently activated at the cost of the New Snow Module (see Sect. 2.2). Mass gains are tendentially modeled less frequently and, as a consequence, snow water equivalents stay smaller.

3.1.5 Overburden Parameters cov and kov

550 Aside τ , there are two more parameters that are peculiar to the Δ SNOW.MODEL. They are needed to simulate unsteady compaction by overburden load of new snow (Eq. (3)). Because of their (presumed) uniqueness in the snow model spectrum there is no information available on how to choose them (see Sect. 2.3 for more). However, the calibration produces $c_{ov} = 5.10 \times 10^{-4} \text{ Pa}^{-1}$ and $k_{ov} = 0.379$ as fittest values (Table 3).

As outlined in Sect. 2.2.1, the implementation of overburden strain in the ΔSNOW.MODEL is supposed to be an important
aspect of the model. Still, the sensitivity of modeled SWE_{pk} to changes in either c_{ov} or k_{ov} – without changing the respective other – are quite minor. (See Fig. 2) for c_{ov}. k_{ov} is not shown, because it is comparable, but with opposite sign.) The reason for this relative insensitivity of the model to changes in c_{ov} and k_{ov} could be the contradicting effects of these two "overburden parameters": Higher c_{ov} push overburden strain ε towards 1.0 (Eq. (3)), which increases the role of overburden snow. hs* and HS* of Eq. (4 are reduced and, consequently, the new layer thickness (and mass) is increased (Eq. (5)). Higher c_{ov}, therefore, lead to higher SWE and SWE_{pk}. For k_{ov} it is the opposite, higher values of k_{ov} cause lower SWE.

3.1 Validation and Comparison to other modelsIllustration

¹⁹As pointed out, the observations of snow water equivalent where divided in two sets, one for calibration (SWE_{cal}) and one for validation (SWE_{val}) . The question arises, how accurate the manual SWE observations are. The two data sources do not address this point (Gruber, 2014; Marty, 2017). In general, this is not easy to be answered since SWE measurements made with

- 565 snow sampling cylinders mostly are used as references in comparison studies, without addressing *their* accuracy (e.g., Sturm et al., 2010; Dixon and Boon, 2012; Kinar and Pomeroy, 2015; Leppänen et al., 2018a). The majority of SWE_{cal} and SWE_{val} comes from the Hydrographic Service of Tyrol, Austria, where snow sampling cylinders (500 cm^3) are used (Sect. 2.3.1). The repeatability of this kind of measurement is estimated at $\pm 4\%$ for glacier mass balance studies, with filling height having the largest influence on accuracy (R. Prinz, Univ. of Innsbruck, Austria; pers. comm.). According to Leppaenen et al. (2018b), who
- 570 compared various density samplers, the "variability among the replications of each sampler and variability among the samplers [is] both less than 10% [...] However, the uncertainty introduced by using different samplers was higher than the uncertainty caused by observer error". Roughly interpreting these density(!) measurement "variabilities" as relative observation errors for SWE, the results for absolute accuracy would typically spread across the wide range of about $2 \text{ to } 50 \text{ kg m}^{-2}$.
- ²⁰Table 4 provides an overview of model uncertainties for *SWE*. Vionnet et al. (2012) find a root mean square error and bias
 of 39.7 kg m⁻² and -17.3 kg m⁻², respectively, comparing 1722 manual samplings at Col de Porte (Chartreuse Mountains, France) and Crocus. These seem to be quite pessimistic values, since root mean square differences found by Langlois et al. (2009) are significantly lower. (However, they base on much fewer data.) Roughly summarized, *SWE* observations as well as "first-class" snow models' *SWE* simulations are associated with comparable uncertainties; RMSEs might be favorably approximated in the order of 10 to 20 kg m⁻².

 $^{^{19}}$ The remarks on *SWE* observation accuracy were moved from the beginning of the Validation and Illustration section of the initially submitted manuscript to the Methods section of the revised manuscript. Content-related changes of this paragraph are minor.

 $^{^{20}}$ The paragraph about SWE accuracy of thermodynamic snow models was moved to Sect. 4.7 during the revision.

- 580 In this study no quantitative comparison with thermodynamic snow models was performed, since they need further meteorological data and the focus was on data records constrained to snow depths. However, the Δ SNOW.MODEL was thoroughly evaluated against ERMs. Figure 2 and Table 4 show the results. Even though ERMs do not need meteorological data, it is not straight forward to calibrate them for new sites and applications. From the vast number of ERMs (cf. Avanzi et al., 2015) the ones of Pistocchi (2016) and Guyennon et al. (2019) were chosen to be fitted to SWE_{cal} (standard: Pi16, Gu19; calibrated:
- Pil6cal, Gul9cal). These models are (quite) new and easy to calibrate. Additionally, an approach simply using a constant bulk 585 snow density at every point in time was calibrated to fit this study's data. Interestingly, 278 kg m^{-3} turned out to be the optimal value minimizing root mean square errors of all $SWE_{cal}SWE$ values (ρ_{278}). Moreover, Jonas et al. (2009) and Sturm et al. (2010) were used for comparison (Jo09, St10). Unfortunately, calibration of these powerful models would have needed much more data than the 780 SWE-HS-pairs of the SWE cal data set, because this would have needed much more data these powerful
- models could not be calibrated with SWE cal, because this would have needed much more SWE-HS-pairs than those about 590 1500 having been processed in this study. Therefore, Jonas et al. (2009) and Sturm et al. (2010) were used with their standard parameters, but for Jonas et al. (2009) it was distinguished between regions (see Fig. 2's caption): Region 6 (Jo09R6) having the highest and Region 7 (Jo09R7) having the lowest "region-specific offset", respectively. Other contemporary approaches had to be ignored, mostly because of the problematic transferability of regional parameters (e.g., McCreight and Small, 2014,

595 or Mizukami and Perica, 2008).

> Figure 4's upper left panel indicates the decent performance of the ERMs at the mean when applied to the validation-half of the data set. The bias of modeled SWE (lower left panel in Fig. 2) is quite low and tendentially positive, meaning SWE is often slightly overestimated by the ERMs. (Distinct values are given in Table 4.) The Δ SNOW.MODEL — on the contrary - slightly underestimates SWE on average, the median bias is -3.0 kg m^{-2} . The overall good results for the

- 600 ERMs is not particularly surprising, since they are dedicated to perform well on average. The specially calibrated versions of Pistocchi (2016) and Guyennon et al. (2019) show a significantly smaller bias than their originals. The model of Jonas et al. (2009) has the smallest (actually a negative) bias for their "Region 7", encompassing the dry, inneralpine Engadin as well as parts of the Southern Alps and the very East of Switzerland (Samnaun), which is partly influenced by orographic precipitation from Northwesterly flows. In terms of heterogeneity in precipitation climate "Region 7" is comparable to the region where
- 605 the SWE data of this study comes from. Sturm et al. (2010) assess the bias for their model (with their "alpine" data set) at $+29 \text{ kg m}^{-2}$ with a standard deviation of 57 kg m^{-2} , and they outline that "in a test against extensive Canadian data, 90% of the computed SWE values fell within $\pm 80 \text{ kg m}^{-2}$ of measured values". This is a much more conservative estimation than the results for this study would suggest.

The other three indicators illustrated in Fig. 2 and summarized in Table 4-bias of seasonal snow mass maximum SWE_{pk} (upper right panel) and the root mean square errors of individual SWE (lower left panel) as well as of SWE_{pk} (lower right 610 panel) – signify the better performance of the Δ SNOW.MODEL compared to ERMs:²¹ The latterERMs are intrinsically tied to snow depth (see Sect. 1.2) and are systematically forced to overestimate SWE_{pk} . Developers of ERMs are well aware of this, nevertheless, this is a pity since SWE_{pk} is probably the most-wanted snowpack feature in hydrology, climatology, and

²¹The line break of the discussion paper was omitted in the revised manuscript.

extreme value analysis. The Δ SNOW.MODEL proofs to perform much better here. Note, the maximal SWE of a winter season

- not necessarily equals the highest measured SWE, because measurements are only taken weekly or biweekly. In the vast 615 majority of the SWE records used for this study, the highest seasonal observation is followed by at least one lower SWE reading. Sometimes real SWE might be higher after the highest measurement of a winter season was taken, but a thorough data check revealed, this is of minor importance here. It is sufficiently precise to assume that measured seasonal maximum SWE equals SWE_{pk}. The Δ SNOW.MODEL's bias of SWE_{pk} is very minor, only $+0.3 \text{ kg m}^{-2} + 2.3 \text{ kg m}^{-2}$ at median. ERMs
- typically suffer from (far) too high SWE_{pk} simulations. The reasoning was given in the Introduction: "ERMs calibrated for 620 good estimates of mean-SWE (necessarily) fail to model SWE_{DK} sufficiently well", since they are "overregulated" by the snow depth. Moreover, the Δ SNOW.MODEL works better for the timing of SWE_{pk} (not shown in Fig. 2 and Table 4)does not only work well for the bias in SWE and SWE_{pk}, but also for the bias in the timing of SWE_{pk} (not shown in Fig. 2, but in the last column of Table 4). ERMs tend to model SWE_{pk} some days too early, because – the date of modeled SWE_{pk} is shifted pulled towards the date of highest HS (cf. Fig. 4). The Δ SNOW.MODEL virtually reduces this bias to zero. 625
- Another satisfactory validation result for the Δ SNOW.MODEL is shown in Fig. 2's upper-lower panels. RMSEs for all SWE values are constantly lower than if modeled with ERMs: a RMSEa median error of $30.8 \text{ kg m}^{-2} \frac{23.9 \text{ kg m}^{-2}}{23.9 \text{ kg m}^{-2}}$ (Δ SNOW.MODEL) contrasts RMSEsfaces median errors between 39.1 and 50.9 kg m^{-2} (ERMs). Calibrating the models of Pistocchi (2016) and Guyennon et al. (2019) results in some improvement, at least they perform much better than the "constant density approach" p_{278} approach after the calibration. The model of Jonas et al. (2009) does a decent job also without recalibration, which is 630 remarkable. Sturm et al. (2010)'s method isprobably suffers from the handicap of being calibrated with data from the Rocky Mountains. For this comparison the "alpine" parameters of Sturm et al. (2010) were taken, however, conditions might differ (too) much from the European Alps. Absolute errors in SWE increase with increasing SWE. For snowpacks lighter than $75 \text{ kg m}^{-2} \Delta$ SNOW.MODEL RMSE is 17 kg m^{-2} , between 75 kg m^{-2} and 150 kg m^{-2} it is 26 kg m^{-2} , and for snowpacks heavier than 150 kg m^{-2} it increases to 43 kg m^{-2} . 635

Not least as a consequence of the mentioned drawbacks of the ERMs when dealing with distinct values, the advantages of the Δ SNOW.MODEL get most obvious when modeling $SWE_{\rm pk}$ (Fig. 4, lower right; Table 2, last three columns). The Δ SNOW.MODEL also hasmanages to have a small RMSErange of RMSEs, with a median of 36.3 kg m^{-2} 23.1 kg m⁻² when modeling $SWE_{\rm pk}$ (Fig. 2, upper right; Table 4, last column) and 75% of the errors staying below $34.4 \,\mathrm{kg \, m^{-2}}$. Also the SWE_{pk} -RMSEs for the different SWE classes These values for SWE_{pk} are very close to those for (daily) SWE, which em-640 phasizes the Δ SNOW.MODEL's ability to model all individual SWEs comparably equally well. The evaluated ERMs have much higher, mostly at least doubled doubled to tripled errors in simulated SWE_{pk} and rather big spreads. Remarkably, the simple ρ_{278} approach is performing relatively well. In case the Jonas et al. (2009) model is suitably adjusted to regional specialties, it performs better than the other ERMs, but still significantly worse than the Δ SNOW.MODEL. The Jonas et al. (2009) model – if suitable adjusted to regional specialties – performs best, still, well beaten by the Δ SNOW.MODEL.

The Δ SNOW.MODEL root mean square errors – slightly lower than 25 kg m^{-2} , also for certain values like SWE_{nk} – are higher than the errors of the thermodynamic snow models (roughly estimated at 10 to 20 kg m^{-2}), which is not surprising given the latter's demands on input data and computational power. However, the ASNOW.MODEL outperforms empirical regression

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models. This can be argued on base of this studythe study in hand (especially Fig. 2), but even more when looking at the ERM studies themselves: Jonas et al. (2009) provide RMSEs between $50.9 \text{ and } 53.2 \text{ kg m}^{-2}$ for their standard model, which are quite high values compared to the findings of the study in hand (39.4 kg m^{-2} for their Region 7ea. 30 kg m^{-2} for SWE, see Table 4). One explanation could be that Jonas et al. (2009), as well as other ERM studies, rely on a huge amount of, but still diverse measurements in terms of record length, observations per season etc.very diverse measurements (Still, lots of them!). The Δ SNOW.MODEL-study Δ SNOW.MODEL study only consists of data from selected stations with long and regular

655 SWE readings, where also ERMs seem to work better. Guyennon et al. (2019) summarize their and other studies' validation results using MAE, the mean absolute error, and Sturm et al. (2019) publish values for SWE-BIAS. Sturm et al. (2010) assess the bias for their "alpine" model at +29 kg m⁻² with a standard deviation of 57 kg m⁻², and they outline that "in a test against extensive Canadian data, 90% of the computed SWE values fell within ±80 kg m⁻² of measured values". Table 4 provides an overview and showsthe overview, and again it gets obvious, that ERMs generally perform better with this study's data, than 660 with their originalGuyennon et al. (2019)'s data.

3.2 Illustration

²²Figure 1 schematically shows the functioning of the Δ SNOW.MODEL. A practical example is now-provided in Fig. 3, based on the optimal calibration parameters found during this study and using the same colors as in Fig. 1. Kössen, the station shown, is situated in the Northern Alps at 590m above sea level (cf. Fig. A1). Although it is a low-lying place it is known to be snowy,

- 665 which is, firstly, due to intense orographic enhancement of precipitation associated with Northwesterly to Northeasterly flows in the respective region (Wastl, 2008) and, secondly, comparably frequent inflow of cold continental air masses from Northeast. Showing Kössen—rather than a high altitude station— should emphasize the versatile usability of the Δ SNOW.MODEL:. It is not only designed for high areas with deep, long-lasting snowpacks, but also for, e.g., valleys with shallow, ephemeral snowpacks. Winter 2008/09 was chosen because the Δ SNOW.MODEL shows a rather typical performance in terms of RMSE and BIAS
- 670 **RMSEs etc.** in Kössen then (see Table 4, values in brackets) and because some important, model-intrinsic features can be addressed and discussed:

Late November 2008 brought the first, however transient snowpack of the season (Fig. 3). The Δ SNOW.MODEL identifies two days with snow fall (purple markings) and models two respective snow layers, which can be distinguished by the thin black line in Fig. 3. After about a week the snowpack starts to melt, the snow layers reach ρ_{max} very fast (the blue shading gets dark), and

- 675 finally all the snow was converted to runoff (green markings). In the second half of December there were three days with new snow, followed by a strong decline in snow depth. In the frame of the Δ SNOW.MODEL this *HS* decrease is only possible, if the layers "get wet" and —the *Drenching module* is activated (marked in brown). The layers get denser, starting at the top. However, the decrease was "manageable" by only increasing the two uppermost layer densities to ρ_{max} and making the third layer just a bit denser. Not all layers got to ρ_{max} ("saturated") and no runoff was modeled. The Δ SNOW.MODEL conserves the two dense
- 680 layers until the end of the winter, which can clearly be seen in Fig. 3. (One could interpret the layers as consisting of melt forms or a refrozen crust. However, interpretations like that require caution, because modeling such detailed layer features is not the

²²The Illustration subsection was newly introduced during the revision.

intention of the Δ SNOW.MODEL!) During January Fig. 3 shows a phase where mnothing special happens. Modeled values and observations agree to a high extent and only the *Scaling module* 8^{id}=general]is activated formakes small adjustments (white markings). Small "stretching events" can be recognized, e.g. on January 2nd and 3rd, where model snow layers are set less dense

- 685 in order to avoid too frequent mass gains. (This model behavior was thoroughly described in Sect. 2.2.2.) During continuous snowfalls in February the successive darkening of the blue layer shadings illustrates a phase of consequent compaction, which actually lasts until March, when strong decreases in HS_{obs} -(and, thus, in HS) already start to activate the *Drenching module*. Still, runoff is not yet produced. Only in the second half of March the whole model--snowpack reaches ρ_{max} ("saturation"). The
 - 690 The snow depth record of Kössen from 2008/09 was also used to compare different ERMs and the ΔSNOW.MODEL to SWE observations (Fig. 4). These measurements (light blue circles) are part of the SWE_{val} sample and were manually made with snow sampling cylinders; one after the December 2008 snowfall, and another nine on a nearly weekly base between late January and late March 2009. Figure 4 also provides various model results and some respective key values are given in Table 4. Not surprisingly, thus evidently, the ERMs' *SWE* curves "follow" the snow depth curve (black dashed line). The ΔSNOW.MODEL

ablation phase is clearly distinguishable and lets the snowpack vanish quite fast until about April 10th, 2009.

- 695 (red line) does not get the first four measurements decently correct, the ERMs perform better in this illustrative case.-(By the way, the "jumpy behavior" of the model of Jonas et al. (2009), criticized by Pistocchi (2016), does not play a role.) But, after the stronger snowfalls of February; the picture changes indisputably in favor of the Δ SNOW.MODEL. This is a typical pattern, nothing special for Kössen 2008/09, (albeit it is quite pronounced in this illustrative example): The ERMs are too strongly tied to snow depth and, therefore, mostly (1) overestimate SWE_{pk} , (2) model its occurrence too early, and (3) – most importantly –
- force modeled *SWE* to reduce during pure compaction phases after snowfalls. All these points were discussed in detail earlier, Fig. 4 visualizes them. At the same time, the Δ SNOW.MODEL does a good job in modeling mean and maximum *SWE*, not only in Kössen 2008/09 but also on average (cf. Fig. 2 and Table 4). Evidently, the ability of the Δ SNOW.MODEL to "conserve" mass during the phases with dry metamorphism is its strongest point, not only in Kössen 2008/09 but also on average (cf. Fig. 2 and Table 4).

705 4 Discussion and Outlook

Model results clearly depend on the parameters. Their optimal values are subject toof calibration. The choice of the best-fitted values is rated and discussed in the following Sects. 4.1 to 4.5. Sections 4.6 to 4.8 cover possible future developments, accuracy issues, and the Δ SNOW.MODEL's applicability in remote sensing.²³

4.1 New Snow Density ρ_0

710 Being aware of both – the huge possible variations of new snow density depending on meteorological conditions during snowfalls and the possible cruciality of this parameter for *SWE* simulation by the Δ SNOW.MODEL – ρ_0 was chosen to be a constant in the framework of the model. $\rho_0 = 81 \text{ kg m}^{-3}$ turned out to be the best choice after calibration with *SWE*_{cal}. This

²³Sects. 4.1 to 4.5 basically mirror Sects. 3.1.1 to 3.1.5 of the initially submitted manuscript. Only minor changes were made during the revision.

value clearly lies within the broader frame of possible new snow densities (Table 3) and guite closely to Sturm and Holmgren (1998)'s 75 kg m^{-3} , but it is found in the lower part for "typical" new snow densities (e.g., Helfricht et al., 2018). A possible

- explanation could be that the SWE measurement records used for the calibration tend to underrepresent late winter and spring 715 conditions. Regular (weekly, biweekly) observations capture the short melt seasons worse than the longer accumulation phases. Therefore, SWE records might be biased towards early and mid winter new snow densities, which are lower (e.g., Jonas et al., 2009). Still, there are also some indications that using, e.g., $100 \,\mathrm{kg}\,\mathrm{m}^{-3}$ as constant new snow density when modeling SWE results in an overestimation of precipitation (up to 30% according to Mair et al., 2016). The calibrated value for ρ_0 can be 720 regarded as a reasonable result, even more when only considering it as a model parameter but not as a physical constant.
- The sensitivity analysis illustrated in Fig. 5 confirms the importance of a good choice of ρ_0 . Increasing ρ_0 quite fast leads to a decrease of the relative bias of seasonal SWE maxima (SWE_{pk}). Note the definition of the relative bias in Fig. 5's caption. In absolute values: too small ρ_0 cause too small SWE_{pk} , using higher values leads to an overestimation of SWE_{pk} . This behavior supports above-mentioned tendency to overestimate precipitation when choosing constant $100 \,\mathrm{kg}\,\mathrm{m}^{-3}$ as new snow
- density. As expected, the new snow density is the most crucial parameter of the Δ SNOW.MODEL (cf. Table 3). The median 725 relative bias of SWE_{pk} changes by -0.46% per +1 kg m⁻³, if the whole calibration range of ρ_0 is considered to calculate the sensitivity $(50 - 200 \text{ kg m}^{-3})$. This means a median change in SWE_{pk} of $+0.37 \text{ kg m}^{-2}$ when ρ_0 is risen by $+1 \text{ kg m}^{-3}$. If the limits are chosen tighter around the optimal value, the gradient is even steeper: -0.62% and +0.50 kg m⁻² per +1 kg m⁻³, respectively, when the gradient is approximated for the range $70 - 90 \text{ kg m}^{-3}$. Widely-used $\rho_0 = 100 \text{ kg m}^{-3}$, consequently,
- causes a median overestimation of SWE_{pk} of about 12% in the Δ SNOW.MODEL. Daily SWE show the same behavior (not 730 shown). Users should be aware of this. The suggestion clearly is to either use the best-fitted parameters of this study or recalibrate all parameters with appropriate SWE data, but not adjusting only single parameters. As the calibration data of this study are spread across various climates and altitudes, users can be quite confident to get good results if using $\rho_0 = 81 \text{ kg m}^{-3}$. This value seems to be a good compromise, at least at alpine areas. However, for very maritime, very dry, polar or tundra regions the optimized ρ_0 should be used with caution; if possible, recalibration is recommended.
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4.2 Maximum Density ρ_{max}

Of course, the maximum bulk snow density of a snowpack changes from year to year and site to site. For the Δ SNOW.MODEL simplicity and independence from meteorological variables outweigh precision. Even more so, when there are good arguments for the existence of a "typical" maximum bulk density ρ_{max} . Put simply, (not too old) seasonal and also ephemeral snowpacks melt away when they get water saturated. Before that, there is limited time for dry densification; dry winter snow's bulk

- 740 density is widely described as staying below about 350 kg m^{-3} (e.g., Paterson, 1998; Sandells et al., 2012). Accounting for the fact that volumetric liquid water content of about 10% marks the funicular mode of liquid distribution in old, coarsegrained snow (Denoth, 1982; Mitterer et al., 2011), this leads to the rough estimate of a typical maximum bulk density of about $\frac{9}{10} \cdot 350 + \frac{1}{10} \cdot 1000 = 415 \text{ kg m}^{-3}$. Convincingly, the fittest value for ρ_{max} in the Δ SNOW.MODEL turns out to be 401 kg m^{-3} ,
- which is close to that value and well situated within the range given in the literature (Table 3). Moreover, this is virtually 745

the same value like the median maximum seasonal density of the SWE_{val} data records (400 kg m⁻³, see box plot in Fig. 6), another indication why ρ_{max} could be regarded as a typical seasonal maximum of ρ_b .²⁴

Figure 5 illustrates the similarity between ρ₀ and ρ_{max} regarding their influence on SWE simulations. Keeping the other six ΔSNOW.MODEL parameters constant but increasing ρ_{max} leads to increased SWE_{pk} and vice versa – just like ρ₀. This
750 is not surprising, however reasonable. The ΔSNOW.MODEL is not as sensitive to changes in ρ_{max} than to changes in ρ₀: Raising ρ_{max} by +1 kg m⁻³ leads to a mean decrease of the relative bias of SWE_{pk} of -0.06%, which corresponds to an increase in absolute SWE_{pk} of +0.24 kg m⁻² per +1 kg m⁻³. The same argumentation like for ρ₀ in Sect. 4.1 lets users of the ΔSNOW.MODEL be quite sure when taking ρ_{max} = 401 kg m⁻³, the best-fitted value according to this study's calibration. Be aware that solely changing parameter ρ_{max} for an application of the ΔSNOW.MODEL elsewhere, without proper recalibration
755 of the other parameters, might lead to significant changes in the results for SWE.

4.3 Viscosity Parameters η_0 and k

Equation (2) represents the settlement and densification function of the Δ SNOW.MODEL. Two parameters η_0 and k act as adjustment screws and have to be calibrated. In this study best-fittet η_0 is 8.5×10^6 Pas and the optimized value for k is $0.030 \text{ m}^3 \text{ kg}^{-1}$. Both values are close to other studies' results and suggestions (Table 3).

As far as the Δ SNOW.MODEL's sensitivity to changes in the "viscosity parameters" η_0 and k is concerned, Fig. 5 shows that an isolated rise of the model snow viscosity – either by enhancing η_0 or k – increases the relative bias of SWE_{pk} , which means a decrease in absolute values of SWE_{pk} . This behavior is consistent, since higher viscosity reduces the densification rate and the model-snowpack tendentially stays deeper. Consequently, increases in observed snow depth tend to bring less new snow while the *New Snow module* is run (Sect. 2.2.1). Finally, simulated SWE_{pk} is reduced when η_0 or k are increased and vice versa.

4.4 Threshold Deviation au

The Δ SNOW.MODEL's parameter to cope with uncertainties in snow depth is τ . It is supposed to be not bigger than a few centimeters. In particular, it should avoid excessive production of snow mass in the model through too frequent simulation of new snow events (see Sect. 2.2). τ is kind of a peculiarity of the Δ SNOW.MODEL and therefore no bounds can be found in

1 literature. It was generously accepted to range between 1 and 20 cm for calibration and turned out to be optimal at $\tau = 2.4$ cm (Table 3). Given the wide range of possible values, this is very close to what it would be expected to be as a measure for HS_{obs} accuracy.

Model sensitivity to changes in τ turns out to be quite low for values in the order of a few centimeters, but the influence on simulated SWE_{pk} is strongly increasing if τ is chosen greater than about 5 cm (Fig. 5). This result makes a lot of sense, if τ is
seen as a measure of observation accuracy, because this is very likely to be better than 5 cm. Like changes in η₀ and k, changes

in τ are indirect proportional to changes in SWE_{pk} , for a closely related reason: The bigger τ the more often small new snow events are not counted as such because the *Scaling module* (Sect. 2.2.2) is more frequently activated at the cost of the *New*

²⁴Figure 3 of the initially submitted manuscript is Fig. 6 of the revised version.

Snow module (Sect. 2.2.1). Mass gains are tendentially modeled less frequently and, as a consequence, snow water equivalents get smaller.

780 **4.5** Overburden Parameters c_{ov} and k_{ov}

Aside τ , there are two more parameters that are peculiar to the Δ SNOW.MODEL. They are needed to simulate unsteady compaction by overburden load of new snow. Because of their presumed uniqueness in the snow model spectrum there is no information available on how to choose them (see Sect. 2.3). However, the calibration produces $c_{ov} = 5.1 \times 10^{-4} \text{ Pa}^{-1}$ and $k_{ov} = 0.38$ as fittest values (Table 3).

- As outlined in Sect. 2.2.1, the implementation of overburden strain in the ΔSNOW.MODEL is supposed to be an important aspect of the model. Still, the sensitivity of modeled SWE_{pk} to changes in either c_{ov} or k_{ov} are quite minor. (See Fig. 5 for c_{ov}. k_{ov} is not shown, because it is comparable, but with different sign.) The reason for this relative insensitivity of the model to changes in c_{ov} and k_{ov} could be the contradicting effects of these two "overburden parameters": Higher c_{ov} push overburden strain ε towards 1.0 (cf. Eq. (3)), which increases the role of overburden snow. hs* and HS* of Eq. (4) are reduced and, consequently, the new layer thickness and mass are increased (Eq. (5)). Higher c_{ov}, therefore, lead to higher SWE and SWE_{pk}.
- For k_{ov} it is the opposite, higher values of k_{ov} cause lower SWE.

4.6 Incorporating Rain-on-Snow and other possible improvements

In principle, the Δ SNOW.MODEL could deal with rain-on-snow events. Some questions remain. The treatment of rain-on-snow events surely is one of them. The Δ SNOW.MODEL can – in principle – deal with it. Unsteady compaction due to overburden

- ⁷⁹⁵ load, for example, is not restricted to new snow. It could also be triggered by the mass of rain water in nature, but also in the framework of the Δ SNOW.MODEL. Still, the respective feature is not implementedeoding is not finalized at the moment, because identifying criteria for rain-on-snow events based on pure snow depth records is very problematica problem, and its resolving (if at all possible) is beyond the scope of this paper. In case some meta information on, e.g., rain climate (maybe for a stochastic "rain generator") or on precipitation type and amount(no matter if liquid or solid) combined with information on
- 800 the snowfall line is available, it could quite easily be incorporated in the Δ SNOW.MODEL. Given the relative success of the Δ SNOW.MODEL in its current version—especially the reduction of the RMSE of SWE_{pk} by 50-70% compared to ERMs and *maybe* even down to the *SWE* error range of thermodynamic models— the probably very costly, but potentially often only very minor improvements when including rain-on-snow should be considered.

Another discussion point and eventual future development is the refinement of the density parameters ρ_0 and ρ_{max} since, firstly, the Δ SNOW.MODEL reacts quite sensitive on their changes and, secondly, some relations are well known, e.g., ρ_0 's dependence on the climatic aridness or ρ_{max} 's tendentious increase for aging(very) old snow. Setting ρ_{max} to a fixed value at about 400 kg m⁻³ actually disqualifies the Δ SNOW.MODEL for snow older than estimated 200 days. Additional calibrations could be performed for very maritime, very dry, polar, or tundra regions as well as for very long-lasting snowpacks. Note, however, all of these adaptions introduce more parameters to the Δ SNOW.MODEL and reduce its generality. Benefits should

810 be evaluated critically, and probably this evaluation should start with. Probably against the overburden load treatment of the

 Δ SNOW.MODEL. I, since it is possible that refining the density parameters is more valuable than the special treatment of unsteady compaction due to overburden loads Δ SNOW.MODEL....

4.7 SWE Accuracy

- Table 4 provides an overview of model uncertainties for SWE, also for thermodynamic models:- Vionnet et al. (2012) find a
- 815 root mean square error and bias of 39.7 kg m^{-2} and -17.3 kg m^{-2} , respectively, comparing 1722 manual samplings at Col de Porte (Chartreuse Mountains, France) and Crocus. Wever et al. (2015) and Sandells et al. (2012) come up with RMSEs of about 39.5 kg m^{-2} (SNOWPACK) and $30 - 49 \text{ kg m}^{-2}$ (SNOBAL), respectively. Langlois et al. (2009) find more optimistic values, however, based on much fewer data. On the contrary, Egli et al. (2009) give reason to expect higher RMSEs, but their study exclusively bases on data from snowy, high altitude station Weissfluhjoch (Switzerland), which intrinsically promotes higher
- 820 absolute errors. Essery et al. (2013)'s comprehensive simulation experiment results in a RMSE-range of $23 77 \text{ kg m}^{-2}$. These seem to be quite pessimistic values, since root mean square differences found by Langlois et al. (2009) are significantly lower. (However, they base on much fewer data.) Roughly summarized, *SWE* observations as well as "first-class" snow models' *SWE* simulations are associated with comparable uncertainties; RMSEs might be favorably approximated in the order of $10 \text{ to } 20 \text{ kg m}^{-2}$.
- As a synopsis of the study in hand, absolute *SWE* accuracies could be estimated as follows: (1) $2 \text{ to } 50 \text{ kg m}^{-2}$ for manual measurements, which are widely used as reference, (2) $30 \text{ to } 40 \text{ kg m}^{-2}$ for thermodynamic models, and (3) $40 \text{ to } 50 \text{ kg m}^{-2}$ for empirical regression models. In this respect, it is striking to find the Δ SNOW.MODEL's RMSE at 30.8 kg m^{-2} . "Maybe" is emphasized in the last paragraph because it leads to another discussion and outlook point: An intensive, multi-year model comparison should be performed; at some benchmark sites with fully equipped snow stations and very important different
- 830 methods of *SWE* measurements, including regular manual observations (with sampling cylinders etc.). Some of those data sets do already exist, however, a comprehensive comparison of techniques and methods to measure and, in particular, model *SWE* is lacking. Often the "target variable" is bulk density (not *SWE*), and relative (not absolute) numbers are the only information on observation accuracy one can get, although recently efforts are undertaken (e.g., in the framework of the ESSEM COST Action ES1404; Leppänen et al., 2018; López-Moreno et al., 2020,...) at least concerning measurements. Shouldn't there be more
- 835 studies, that also comprehensively quantify the abilities of various, especially thermodynamic snow models to simulate *SWE*? A top-quality comparison between the Δ SNOW.MODEL and thermodynamic snow models is actually difficult to achieve since hardly any numbers for *SWE* accuracy of thermodynamic models are available. *Maybe* they perform worse than the generously estimated RMSEs of 10 to 20 kg m⁻² of this study?

4.8 Application to Remote Sensing Data

840 LLast but not least, looking at current developments in deriving SWE from snow depths, that are monitored with lidar and photogrammetry, the Δ SNOW.MODEL should be considered as one of the "potential [...] other snow density models" (Smyth et al., 2019) that should be included in respective future research.—following Smyth et al. (2019) – one of the "potentially [applicable] other snow density models". Lidar and photogrammetry have errors in the order of 10 cm (Smyth et al., 2019), typically corresponding to SWE errors of 20 to 40 kg m⁻². This is in the order or somewhat more than the error of the Δ SNOW.MODEL

errors. Remote sensing derived snow depth data are discontinuous through time. The Δ SNOW.MODEL would have to be adapted to that, but for the benefit of upgrading the Δ SNOW.MODEL from a point model to a computationally fast distributed model. (which might not be a big issue), though, for the big benefit of being independent from meteorological data and models – and their errors.

5 Conclusions

A new method to simulate snow water equivalents (*SWEs*) is presented. It exclusively needs snow depths and their temporal changes as input, which is its major advantage compared to many other snow models. It is shown that basic snow physics, smartly implemented in a layer model, suffice to better calculate *SWE* than snow models relying on empirical regressions. Consequently, the study's null hypothesis (Sect. 1.3) is rejected.

RegularGapless snow depth records are used to stepwise model the evolution of seasonal snowpacks, focusing on their mass (i.e. *SWE*) and respective load. Snow compaction is assumed to follow Newtonian viscosity, unsteady stress for underlying snow layers by the overburden load of new snow is regarded separately, melted mass is distributed from upper to lower layers, and – eponymous for the model – the measured change in snow depth between two observations is used as a precious corrective, though by accounting for measurement uncertainties. The model steps are rather simple, however tricky in details, and all is frankly revealed in this article.

- The Δ SNOW.MODEL mainly bases on Martinec and Rango (1991) and Sturm and Holmgren (1998), and transforms them to a modern R-code, which is available through https://cran.r-project.org/package=nixmass (*nixmass* package). Aside snow depth, meteorological-Other meteorological (aside *HS*) and also geographical input is consequently avoided in the framework of the Δ SNOW.MODEL. Still, calibration of seven parameters is needed. To provide an optimal setting and utmost applicability, data from 14 climatologically different places in the Swiss and Austrian Alps are utilized. This is challenging, since calibra-
- tion needs multi-year *SWE* observations as well as consecutive (e.g. daily) snow depth readings from the same places. The Δ SNOW.MODEL is calibrated with 67-the help of 71 winters. The validation data set consists of another 7173 independent winters. Whereas calibration is rather complex, the application of the Δ SNOW.MODEL is cheap in terms of computational effort: Deriving a one-year *SWE* record from 365 snow depth values, e.g., only takes a few seconds with today's standard desktop CPUs and can certainly be speeded up significantly.
- 870 In this study it is argued that the Δ SNOW.MODEL is situated between sophisticated "thermodynamic snow models", necessitating lots of meteorological and other input, and modest "empirical regression models" (ERMs), relying on statistical relations between *SWE* and snow depth, date, altitude, and region. These key qualities of the Δ SNOW.MODEL are:
 - This "interposition" is true in terms of model low complexity: The Δ SNOW.MODEL is a semi-empirical multi-layer model with seven parameters. It only needs regular *HS* records as input. In some respect it is even less demanding than ERMs, because no information on date, altitude, or region is required. but only needs one input variable, which is *HS*.

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- Still, the ΔSNOW.MODEL is even less demanding than ERMs: It exclusively needs HS records, though gapless ones. No information on date, altitude, or region is required.
- In terms of high universality: The Δ SNOW.MODEL simulates individual *SWE* values like the important seasonal maximum *SWE*_{pk} comparably well as *SWE* averages., this shifts the Δ SNOW.MODEL close to thermodynamic models, and because the Δ SNOW.MODEL simulates individual *SWE* values like the important seasonal maximum *SWE*_{pk} as good as *SWE* averages, it can compete with thermodynamic models at the, e.g., daily level as well (which is not reasonable for ERMs).
- high accuracy: TNot least, the Δ SNOW.MODEL's performance in modeling *SWE* and *SWE*_{pk} is comparable to thermodynamic models and superior to ERMs.lies between thermodynamic models and ERMs, albeit close to the very sophisticated ones: Root mean square errors for *SWE*_{pk} are 36.3 kg m⁻² for the Δ SNOW.MODEL and about 70 to > 100 kg m⁻² for ERMs. (cf. Table 4) are at ca. 23 kg m⁻² (Δ SNOW.MODEL), at about 60 to 90 kg m⁻² (ERMs), and somewhere between 10 and 40 kg m⁻² (thermodynamic models).

Given these promising results, the Δ SNOW.MODEL's ancestors Sturm and Holmgren (1998)'s argument, whereby "snow load plays a more limited role in determining the compaction behavior in seasonal snow than grain and bond characteristics and temperature", might be disproved.

The development of the Δ SNOW.MODEL is application-driven. It is therefore not surprising that this study provides no significant new findings in snow physics. Still, the Δ SNOW.MODEL seems to be the first model (since long) that takes well known basic snow principles and arranges them in a physically consistent way, while consequently ignoring all potential information except snow depth. Not particularly innovative, but remarkably successful. Nevertheless, the synopsis of the Δ SNOW.MODEL and

measured data gives significant hints on two important snowpack features (at least for the Alps): Typical mean density for new snow (24 h) seems to be clearly below often assumed 100 kg m⁻³ and a characteristic average maximum bulk density for seasonal snow (also including ephemeral snowpacks from low-lying places) can rather be found around 400 kg m⁻³ than at often cited 500 to 600 kg m⁻³, which might be biased by "too alpine" snowpacks for many applications. The ΔSNOW.MODEL is widely usable, but first of all it can attribute snow water equivalents to all longterm and historic snow depth records, which are so valuable for climatological studies and extreme value analysis for risk assessment of natural hazards.

Code availability. R-code of the Δ SNOW.MODEL and some empirical regression models: https://cran.r-project.org/package=nixmass. (Phyton-code of the Δ SNOW.MODEL, ported by M. Theurl (Univ. of Graz, Austria): https://bitbucket.org/atraxoo/snow_to_swe.)

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Appendix A

905 A map with the stations used for calibration and validation of the Δ SNOW.MODEL is shown in Fig. A1. Table A1 provides details on the stations and the data.

Appendix B

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The *Scaling module* (Sect. 2.2.2) recalculates the "viscosity parameter" η_0 . This temporary $\eta_0^*(i,t)$ does not only depend on the point in time *t* (whenever the *Scaling module* is activated), but is also different for each layer *i*. The reason is described in the following.

The *Scaling module* aims for the condition, that today's model snow depth HS(t) equals today's observed snow depth $HS_{obs}(t)$.

$$HS(t) = \sum_{i=1}^{ly(t)} hs(i,t) \stackrel{!}{=} HS_{obs}(t)$$

It follows from Eq. (2) and substituting $x(i,t) = \Delta t \cdot \hat{\sigma}(i,t) \cdot e^{-k \cdot \rho(i,t)}$:

$$\sum_{i=1}^{ly(t)} hs(i,t) = \sum_{i=1}^{ly(t)} \frac{\eta_0^*(t) \cdot hs(i,t-1)}{\eta_0^*(t) + x(i,t)} \stackrel{!}{=} HS_{\text{obs}}(t), \tag{B1}$$

which is a rational function f of the form

$$f(\eta) = \sum_{i=1}^{N} \frac{\eta \cdot h_i}{\eta + x_i}$$

Because $f(\eta)$ has poles at $-x_1, \ldots, -x_N$, the equation $f(\eta) = HS_{obs}$ has multiple solutions. Consequently, this approach – with $\eta_0^*(t)$ being independent from layer i – shows a clear non-physical behavior making it necessary to calculate different $\eta_0^*(i,t)$ for each layer i based on Eq. (B1):

$$\eta_0^*(i,t) = \frac{x(i,t) \cdot hs(i,t)}{hs(i,t-1) - hs(i,t)}$$

The solution of this issue in the *Scaling module* of the Δ SNOW.MODEL bases on the assumption, that observed compaction between t - 1 and t can be approximated linearly for each layer:

$$\frac{hs(i,t)}{hs(i,t-1)} \stackrel{!}{\approx} \frac{HS_{\rm obs}(t)}{HS_{\rm obs}(t-1)}$$

920 The layer-individual viscosities can be calculated as

$$\eta_0^*(i,t) = \frac{x(i,t) \cdot HS_{\text{obs}}(t)}{HS_{\text{obs}}(t-1) - HS_{\text{obs}}(t)}$$

Substituting those values for η_0^* in Eq. (B1) fulfills its precondition, and the modeled equals the observed snow depth. The newly calculated $\eta_0^*(i,t)$ are different for each layer – in contrast to the fixed η_0 defined in Sect. 2.1, which is valid for the whole snowpack (outside the *Scaling module*). Note, that these new viscosities are only used temporarily in the *Scaling module*. They have no analog in reality and can also have negative values, but they are mathematically sound.

Appendix C: Example of Application – Snow Load Map of Austria 925

In this section an example is given how the Δ SNOW.MODEL can be used to attain a map of snow loads in Austria; snow load $SL = SWE \cdot g$. European Standards (e.g., European Committee for Standardization, 2015) define the "characteristic snow load" s_k as the weight of snow on the ground with an annual probability of exceedance of 0.02, i.e. a snow load that – on average - is exceeded only once within 50 years. Unfortunately, SWE is not measured on a regular basis at a reasonable number of 930 sites in Austria (and most other countries). The Δ SNOW.MODEL, however, can provide longterm Austrian SWE series from widely available HS series, which can in turn be used for a spatial extreme value model. No other snow model is capable of this in a comparable manner, since either SWE_{pk} is poorly modeled (ERMs) or more meteorological input would be needed (thermodynamic models). Among several possibilities to spatially model snow depth extremes like max-stable processes (see e.g. Blanchet and Davison, 2011), the smooth modeling approach of Blanchet and Lehning (2010) can be used when marginals instead of spatial extremal dependence is in focus.

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C1 Smooth Modeling

Extremes following a generalized extreme value distribution (GEV; Coles, 2001) with parameters μ , σ and ξ can be modeled in space by considering linear relations for the three parameters of the form

$$\eta(x) = \alpha_0 + \sum_{k=1}^m \alpha_k y_k(x) \tag{C1}$$

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at location x, where η denotes one of the GEV parameters, y_1, \ldots, y_m are the considered covariates as smooth functions of the location, and $\alpha_0, \ldots, \alpha_m \in \mathbb{R}$ are the coefficients. Assuming spatially independent stations, the log-likelihood function then reads as

$$l = \sum_{k=1}^{K} \ell_k \left(\mu(x_k), \sigma(x_k), \xi(x_k) \right),$$
(C2)

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where l only depends on the coefficients of the linear models for the GEV parameters. This approach was termed *smooth modeling* by Blanchet and Lehning (2010). A smooth spatial model for extreme snow depths in Austria was already presented in Schellander and Hell (2018), using longitude, latitude, and mean snow depth at 421 stations. Considering the strong correlation between snow depth and snow water equivalent, it would be natural to spatially model SWE extremes in the same manner.

C2 Fitting a Spatial Extreme Value Model

950 For this application 214 stations with regulargapless snow depth observations in and tightly around Austria of the National Weather Service (ZAMG) and the Hydrological Services are used. The dataset has undergone quality control by the maintaining institutions and covers altitudes between 118 and 2290 m.118 and 2290 m.a.s.l. The records have lengths of 43 years and cover winters from 1970/71 to 2011/2012.

In a first step the Δ SNOW.MODEL was applied to these snow depth series to achieve 214 data series of SWE across Austria.

- 955 (This indeed is the great strength of the Δ SNOW.MODEL and can hardly be done with other methods!) Then the linear models for the three GEV parameters according to Sect. C1 were defined via a model selection procedure. For that purpose a generalized linear regression was performed between the parameters and the covariates longitude, latitude, altitude, and mean snow depth, which were added in a stepwise manner. Using the Akaike information criterion (AIC; Akaike, 1974), the best linear model between a given full model ($\mu \sim$ all covariates) and a null model ($\mu \sim 1$) with the smallest AIC was selected. Using
- 960 these models and the covariates of the 214 stations, a smooth spatial model for the yearly maxima of the *SWE* values was fitted.

C3 Return Level Map of 50-year Snow Load in Austria

The spatial extreme value model developed in the previous section was applied to a grid provided by the SNOWGRID climate analysis (Olefs et al., 2013). It offers the necessary covariates longitude, latitude, altitude, and with yearly mean snow depths

- from 1961 to 2016. The grid features a horizontal resolution of 1×1 km. Some minor SNOWGRID pixels have unrealistically large mean snow depth values, arising from a poor implementation of lateral snow redistribution at high altitudes (18 pixels, i.e. 0.02% with values between 5 and 65 m). They are masked for the calculation of *SWE* return level maps. The return level map for a return period of 50 years can be seen in Fig. A2.
- As expected, due to the strong correlation of the *SWE* maxima with mean snow depth, the largest snow loads are located in the mountainous areas of Austria. Although the unrealistic mean snow depth values of SNOWGRID are masked, the model produces a number of 59 (0.06%) unrealistic snow load values larger than 25 kN m⁻² in an altitude range between 1500 and 3700 ma depth range between 1500 and 3700 m a.s.l.. For a model that would be seriously used e.g. in general risk assessment or structural design, this problem could possibly be tackled with a non-linear relation between *SWE* maxima and mean snow depth or altitude. This is, however, beyond the scope of this study. Note, that in the actual Austrian standard 975 (Austrian Standards Institute, 2018) there are no normative snow load values defined above 1500 m altitude.
- All but two locations of the Austrian SWE measurement series that were used for calibration and validation of the ΔSNOW.MODEL (see Sect. 2.3.1) are included in the dataset used to fit the spatial model in Sect. C2. Those two stations, Holzgau and Felbertauern with 14 years of SWE observations each, are used to qualitatively compare (1) the spatial model fitted in Sect. C2, (2) SWE extremes modeled from daily snow depths with the ΔSNOW.MODEL, and (3) extremes computed "directly"
 980 from (ca. weekly) observed SWE values. Figure A3 gives an idea of the model performance at stations Holzgau and Felber-
- tauern (see Figs. A1 and A2 for their locations)(see Fig. A2 for their locations). For the lower-lying station Holzgau (1100 m)

(1100 m a.s.l.) all three variants overlap very well. The 50-year return level is 4.65 kN m^{-2} for the smooth spatial model, 4.72 kN m^{-2} for the Δ SNOW.MODEL, and 4.8 kN m^{-2} for the observations. Note, that the latter stem from weekly observations and, therefore, not necessarily reflect the true yearly maxima, which naturally must be equal or slightly higher. By the way, the corresponding value of s_k from the Austrian snow load standard for Holzgau is 6.3 kN m^{-2} (Austrian Standards Institute (2018); accessible online at eHORA (2006)).

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For the higher station Felbertauern (1650 m)(1650 m a.s.l.) the agreement between *SWE* from the Δ SNOW.MODEL and observed values is again very good. However, their GEV fits differ significantly. While the fit to the observations shows a negative shape parameter of $\xi = -0.1$, the fit to the values modeled with the Δ SNOW.MODEL gives a positive shape parameter of $\xi = 0.1$, leading to much larger return levels for higher recurrence times due to the Fréchet-like distribution. It should be pointed out that the GEV fits based on Δ SNOW.MODEL simulations and observations are unreliable, given the short data sample of only 14 yearly maxima. Indeed, by using a sample size of 43 years and borrowing strength from neighboring stations, the spatial model provides the best fit to observations as well as modeled *SWE* values. The 50-year snow load return values are 6.4 kN m^{-2} for the spatial model, 6.8 kN m^{-2} for the Δ SNOW.MODEL, and 5.7 kN m^{-2} for the fit to the observations. No normative value is defined for Felbertauern because it is situated higher than $1500 \text{ m} \frac{1500 \text{ m} \text{ a.s.l.}}{1000 \text{ m} \text{ a.s.l.}}$ (Austrian Standards Institute,

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2018).

Author contributions. MW rose and led the project, structured and managed it. He was a key figure in developing and designing the snow model, and he did most of the writing. HS developed, coded and calibrated the model. He wrote the R-package and helped writing the paper, particularly the application example in the appendix. SG developed early versions of the model and its code.

1000 *Competing interests.* The authors declare no competing interests.

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Figure 1. Schematic figure of the Δ SNOW.MODEL's principles. See text for more details. At time t = 1 no snow is observed (black bullet at $HS_{obs}(1) = 0$) and – consequently – no snow is modeled. At t = 2 initially no snow is modeled. However, snow is observed ($HS_{obs}(2) > 0$) and, thus, model snow depth HS(2) is set to the observed value by the Δ SNOW.MODEL's *New Snow Module* (pink arrow) and a certain SWE (green boxes) is assigned to this new snow layer. For t = 3 densification by the *Dry Metamorphism Module* (Dry Met.) is modeled (light blue arrow). Snow density (ρ , shown as bluish shades) is enhanced, but SWE stays constant. Still, the model and observation slightly disagree ($|\Delta HS| < |\tau|$). The *Scaling Module* solves this issue (black arrow): At t = 3 (and also at t = 4,5,6,8, and 9) modeled HS(t) is scaled to equal $HS_{obs}(t)$ with respective consequences for the snow density, but not altering SWE. $HS_{obs}(7)$ is way higher than HS(7), the *New Snow Module* builds up a new layer and raises SWE accordingly. The *New Snow Module* also treats the unsteady and strong compaction of the underlying layer(s) due to overburden snow load (purple arrow). At t = 10 observed snow depth is significantly smaller than HS(10), the *Drenching Module* (brown arrows) wettens the layers from top to bottom until ρ_{max} is reached. Figuratively, the layers get water-saturated, however, at t = 10 not all layers reach ρ_{max} : No mass loss is requested by the model, and SWE stays constant. At t = 11 the Drenching Module necessitates mass loss by runoff (cyan arrows) as all layer densities would have to exceed ρ_{max} to fulfill $HS(t) = HS_{obs}(t)$, but this is not possible in the Δ SNOW.MODEL. All layers are set to ρ_{max} and they "get cut" by an appropriate amount of depth and mass, respectively, depending on their thickness: Thick layers contribute more to the mass loss than thin ones. In the end, at t = 12, no snow is observed anymore and final runoff is mod



Figure 2. Root mean square errors (RMSE) and biases (BIAS) between the Δ SNOW.MODEL and different empirical regression models and the SWE_{val} observations.Validation results for the model biases and root mean square errors (RMSE). The plots show the results for the models applied to the SWE_{val} data. The Δ SNOW.MODEL, Pistocchi (2016)'s and Guyennon et al. (2019)'s models, as well asPistoeehi (2016), Guyennon et al. (2019), and the "constant density approach" were calibrated with SWE_{cal} data (Δ snow, Pi16cal, Gu19cal, ρ_{278} ; upper panels, solid lines). Dashed lines indicate the Pistocchi (2016), the Guyennon et al. (2019), the Jonas et al. (2009), and the Sturm et al. (2010) models with their standard parameters (Pi16, Gu19, Jo09R6, Jo09R7, and St10). Jo09R6 and Jo09R7 together illustrate the maximum possible spread of the Jonas et al. (2009) model since Region 6 (R6) and Region 7 (R7) are characterized by the highest and lowest "regionspecific offset", respectively. The upper left panel shows RMSEs for all SWE_{val} values (short horizontal lines) as well as for three SWEclasses: $SWE \leq 75$, SWE > 150, and intermediate. Analogously for SWE_{pk} (upper right panel). The boxes for the biases (lower panels) encompass 774761 values (left panels, SWE) and 7173 values (right panels, SWE_{pk}) and spread from the 25%- to the 75%-quantile, the whiskers indicate 1.5 times the interquartile range.minima and maxima. The Δ SNOW.MODEL does not behave significantly better for the bias of all SWE (upper left), however, its performance for seasonal maxima SWE_{pk} (upper right) as well as for the mean errors (lower panels) is very convincing. (Note the different y-axes scalings.) Units are kg m⁻².



Figure 3. Winter of 2008/09 in Kössen (Northern Alps, Austria) The winter of 2008/09 in Kössen, a low-lying but snowy station at the Northern Alps, portrays density evolution (shades) as simulated by the Δ SNOW.MODEL. The regulargapless, daily snow depth record is used as only model input. The *New Snow module*, the *Scaling module* and the *Drenching module*, as well as the *Runoff submodule* Three (out of four) modules of the Δ SNOW.MODEL- are depicted in colors at the bottom, whenever activated: Drenching, New Snow, and Scaling (The Dry *Compaction module* is activated at every point in time.) Runoff is a subcategory of the *Drenching module*. Note, the Δ SNOW.MODEL is not intended to simulate individual layers, but to calculate daily *SWE*, *SWE*_{pk}, and *mean* daily bulk density as accurate as possible. This figure illustrates what happens during the modeling. However, the aim of the Δ SNOW.MODEL is to get daily *SWE* and *SWE*_{pk} right – i.e. *mean* daily bulk density, not layer-individual densities. Descriptions and discussions of some features are given in the text.



Figure 4. *SWE* simulations and observations (mostly weekly; *SWE*_{obs}) for the winter 2008/09 in Kössen (cf. Fig. 3). Details and Model abbreviations are given in the text (Sect. 3.2) and summarized in Fig. 2. See Table 4 for values, and consider the note in its caption. This plot is an illustration of the Δ SNOW.MODEL performance during a distinct winter and outlines important features, which are addressed in the text.



Figure 5. Sensitivity of SWE_{pk} to changes in model parameters. The "relative BIAS of SWE_{pk} " relative bias of SWE_{pk} is defined as the difference between SWE_{pk} with best-fitted values and SWE_{pk} with changed parameters (while all others are kept unchanged), divided by the best-fitted SWE_{pk} . The boxes comprise SWE_{pk} of all stations and all years of the validation data set SWE_{val} (7173 values) and display medians as well as 25% and 75% percentiles, the whiskers indicate 1.5 times the interquartile range.minimum and maximum biases. (Parameter k_{ov} behaves unremarkably – similar to c_{ov} – and is not shown here.) Details and analysis see text.



Figure 6. Scatter plot of all modeled bulk snow densities ρ_b versus all observed ρ_b from the validation data set. (*SWE*_{val}, 767 data pairs. Seven observations, which are higher than 600 kg m⁻³, were ignored due to implausibility.) Red cEircles reflect the 7173 observed yearly maxima (ρ_{max}^{obs}), most of them occur when also modeled snowpack isall modeled snow layers are saturated at $\rho_{max} = 401$ kg m⁻³. The box plot shows the distribution of ρ_{max}^{obs} with median, 25% and 75% percentiles, and whiskers at 1.5 times the interquartile range.: The median is at 400 kg m⁻³. (This round value is somewhat fortuitously and should not be taken too seriously.) The horizontal line compares it to the Δ SNOW.MODEL's maximum density at ρ_{max} .

Table 1. Different types of *SWE* models, categorized by their essential input. TD, SE, and ERMs are abbreviations for thermodynamic, semi-empirical, and empirical regression models, respectively.

essential input	TD	SE	ERMs
HS (single values)			х
HS (regular records)	x ^a	х	
one or more			
atmospheric variable(s)	х		
date	(x) ^b		х
location parameters ^c	$(\mathbf{x})^{\mathbf{b}}$		х

^aor another precipitation input

^bonly essential in some cases, e.g. for parameterizations

^caltitude, regional climate, etc.

Table 2. Summary of compaction processes and processes forcing mass changes that are integrated in the modules and submodules of the Δ SNOW.MODEL, and of processes that are ignored.

module	process
New Snow	significant rise of HS, enhanced compaction due to overburden load (Overburden submodule)
Dry Compaction	significant decline of HS due to dry metamorphism ^a and/or deformation ^a
Drenching	significant decline of HS due to wet metamorphism ^a , runoff through melt (Runoff submodule)
Scaling	adjustments to small changes of HS within threshold deviation $ au$
ignored:	snow drift compaction ^a and mass changes due to:
	rain-on-snow, runoff during snowfalls, wind drift, small snowfalls, sublimation and deposition

^aterminology follows Jordan et al. (2010)

Parameter	unit	optimal	calibration	literature	sensitivity
par		value	range	range	$\frac{\delta SW\!E_{\rm pk}[{\rm kgm^{-2}}]}{\delta par}$
$ ho_0$	${\rm kgm^{-3}}$	81	50-200	75 ^a , 10-350 (70-110) ^b	+0.37 (+0.50 [†])
$ ho_{ m max}$	${\rm kgm^{-3}}$	401	300-600	450°,217-598 ^d , 400-800 ^e	+0.24
η_0	$10^6\mathrm{Pas}$	8.5 8.51	1-20	$8.5^{a}, 6^{f}, 7.62237^{g}$	not calc.
k	${ m m}^3{ m kg}^{-1}$	0.030 0.0299	0.01-0.2	$0.011\text{-}0.08^{a}, 0.185^{h}, 0.023^{f,g}, 0.021^{i}$	not calc.
au	cm	2.4 2.36	1-20	-	not calc.
Cov	$10^{-4} \mathrm{Pa}^{-1}$	5.1 5.10	0-10	-	not calc.
$k_{ m ov}$	-	0.38 0.379	0.01-10	-	not calc.

Table 3. The seven parameters of the Δ SNOW.MODEL. The last column depicts model sensitivity to changes in the density parameters. The respective gradients are means over the whole calibration ranges. Detailed information is found in the text.

^aSturm and Holmgren (1998), ^bHelfricht et al. (2018) with range for means in brackets, ^cRohrer and Braun (1994), ^dSturm et al. (2010), ^ePaterson (1998), ^fJordan et al. (2010), ^gVionnet et al. (2012), ^hKeeler (1969), ⁱJordan (1991). See Sect. 2.3 for more details. [†]The value in brackets is the gradient taken from the smaller window between 70 and 90 kg m⁻³ (cf. Sect. 4.1).

Table 4. Overview on *SWE* accuracies of different modelsmethods and studies. The numbers of this study are the median values, which are also depicted in Fig. 3. The numbers in brackets represent the results for the example portrayed in Figs. 3 and 4 from station Kössen in 2008/09. Note, the performance of the Δ SNOW.MODEL of the example is quite ordinary, while other models do better on average. Units are kg m⁻² except last column, which is in days, TD is short for thermodynamic snow models. Model abbreviations see caption of Fig. 2.

source	model (version)	SWE BIAS	SWE RMSE	SWE MAE	SWE _{pk} BIAS	SWE _{pk} RMSE	SWE_{pk} BIAS [d]
this study	Δ snow.model	-3.0 - 4.0	30.8 23.9 (21)	21.9 19.5	0.32.3(-3)	36.3 23.1	θ
	Gu19cal	4.8 4.0	39.1 31.3 (43)	$27.6 \frac{24.4}{2}$	63.067.3(93)	85.6 70.8	-6
	Pi16cal	$5.6\frac{5.3}{5.3}$	39.4 32.9 (47)	28.125.6	70.3 71.0 (106)	80.8 72.2	-6
	Jo09R7	-3.2 - 2.0	39.4 26.6 (41)	27.3 21.9	52.056.5(74)	$70.2\frac{58.1}{}$	-2
	St10	$14.0\frac{17.6}{17.6}$	45.137.1(57)	32.628.5	91.195.1(154)	117.295.7	-11
	$ ho_{278}$	$10.6 \frac{14.8}{10.6}$	50.938.1(51)	36.331.2	45.2 47.7 (77)	$66.4\frac{53.5}{5}$	-16
Guyennon et al. (2019)	Gu19			49.2			
	Pi16cal			50.6			
	Jo09cal			48.5			
	St10cal			51.0			
Jonas et al. (2009)	Jo09		50.9 - 53.2				
Sturm et al. (2010)	St10 ("alpine")	29 ± 57					
Vionnet et al. (2012)	Crocus	-17.3	39.7				
Langlois et al. (2009)	Crocus	-7.9 to -5.4	10.8 - 12.5				
	SNTHERM	9 to 18.1	18.3 - 19.3				
	SNOWPACK	-0.1 to 5.6	7.4 - 14.5				
Egli et al. (2009)	SNOWPACK		56				
Wever et al. (2015)	SNOWPACK		ca. 39.5				
Sandells et al. (2012)	SNOBAL		30 - 49			$17 - 44^{a}$	
Essery et al. (2013)	various TD ^b		23 - 77				

^aThis is not RMSE of SWE_{pk} , but RMSE "from establishment of snowpack to SWE_{pk} ". ^bSee Essery et al. (2013)'s Table 10: RMSE for up to 1700 uncalibrated and calibrated simulations.



Figure A1. Locations of the stations used for calibration and validation. Austrian stations are operated by the Hydrographic Service of Tyrol (HD Tirol), the Swiss stations by the WSL Institute for Snow and Avalanche Research SLF. See Table A1 and text for more details.



Figure A2. 50-year return levels of snow load in Austria. Two stations with SWE observations are outlined for a qualitative validation. This map bases on 214 snow depth records, Δ SNOW.MODEL derived SWE, and smooth spatial modeling of their extremes. See text for details.



Figure A3. Return levels of snow load at stations Holzgau (left) and Felbertauern. Return periods in years are shown on the logarithmic x-axis. The blue line shows return levels obtained with the spatial extreme value model, pink bullets and lines depict yearly maxima and the GEV fit of *SWE* values modeled from daily snow depths with the Δ SNOW.MODEL, and green colors represent yearly *SWE* maxima and the corresponding GEV fit from (ca. weekly) observations.

Table A1. Overview of stations with daily snow depths record and about weekly/biweekly (Austria/Switzerland) manual *SWE* observations which were used for calibration and validation. $\#_{cal}^{SWE}$ and $\#_{val}^{SWE}$ give the numbers of respective manual *SWE* observations. Stations #1 to #6 are located in the Austrian province of Tyrol, #5 and #6 are in the sub-province of Eastern Tyrol; all operated by the Hydrographic Service of Tyrol. Swiss stations #7 to #15 are operated by the WSL Institute for Snow and Avalanche Research SLF. Compare Fig. A1. The data sources are Gruber (2014) and Marty (2017).

#	station name	lon [°]	lat [°]	alt [m]	$\#_{\mathrm{cal}}^{SWE}$	$\#^{SWE}_{ m val}$	calibration seasons ^a	valibration seasons ^a
1	Holzgau	10.333300	47.25000	1100	116	100	7 odd in 1999-2011	7 even in 1998-2010
2	Ladis	10.649200	47.09690	1350	83	66	7 odd in 1999-2011	67 even in 1998-2010 ^b
3	Obernberg	11.429200	47.01940	1360	105	88	7 odd in 1999-2011	7 even in 1998-2010
4	Koessen	12.402800	47.67170	590	87	70	7 odd in 1999-2011	67 even in 1998-2010 ^b
5	Felbertauern	12.505600	47.11810	1650	126	114	7 odd in 1999-2011	7 even in 1998-2010
6	Innervillgraten	12.375000	46.80830	1400	96	115	7 odd in 1999-2011	7 even in 1998-2010
7	Muerren	7.890193	46.55818	1650	37	27	2009,2012,2015,2017	2006,2011,2014,2016
8	Truebsee	8.395291	46.79121	1780	4	11	2016	2015,2017
9	Ulrichen	8.308283	46.50461	1350	24	23	2009,2013,2015,2017	2007,2011,2014,2016
10	Zermatt	7.751165	46.02340	1600	47	76	1961,1963 and	3 even 1960-1964,
							7 even in 2004-2016	7 odd in 2005-2017
11	Davos Flueelastr.	9.848163	46.81255	1560	8	19	2012	2008,2017
12	Klosters KW	9.895973	46.86058	1200	120	22 10	1999	1998,2017
13	San Bernardino	9.184634	46.46326	1640	11	14 13	2007	2006,2014
14	Sta.Maria	10.419344	46.59981	1415	0	8	-	1969
15	Zuoz	9.962676	46.60433	1710	24	21	2011,2013,2015,2017	2006,2012,2014,2016
Σ					780	774	67	71

^aIndicated years mark the start of respective winter seasons. ^b2006 is missing.