# Dear Editor,

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let me first of all thank you once more for handling our manuscript (MS). In line with the recommendation of the reviewers, our responses as well as your editor comment we revised our MS as outlined below:

- We clarified what we intended to say with the term *runoff generation* throughout the MS.
- We explained how a high or low rDUNE value could be interpreted as well as connect our results better to the physical properties of our chosen catchments (*page 8 line 1 6; page 16 line 8 15*).
  - We discussed and added the references given by the two reviewers (*page 2-3 line 30 12; page 16 line 8 24*).
  - We stressed that rDUNE is an energy-centered re-interpretation and enhancement of HAND and state clearly that we only test it against a subset of available topographic indices (*page 4 line 20 28; page 18 line 27-29*).
- 10 We mention that the absolute rDUNE values are sensitive to the chosen raster resolution, flow direction algorithm or DEM smoothing function however also stress that a sensitivity study is beyond the scope of this study (*page 14 line* 16-22).
  - We show in the appendix that the Jensen-Shannon divergence is not sensitive to the chosen bin width as long as the bin width is chosen in a meaningful range (*page 19 line 15 19*).
- 15 Append we added the revised MS (*changes in yellow*) as well as our detailed responses to the two reviewer comments. We furthermore added the page and line number at each response where we proposed to update our MS.

Again thank you for handling our MS.

20 Yours sincerely,

Ralf Loritz

# A topographic index explaining hydrological similarity by accounting for the joint controls of runoff formation

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**Abstract:** Surface topography is an important source of information about the functioning and form of a hydrological landscape. Because of its key role in explaining hydrological processes and structures, and also because of its wide availability

- 15 at good resolution in the form of digital elevation models (DEM), it is frequently used to inform hydrological analyses. Not surprisingly, several hydrological indices and models have been proposed to link geomorphic properties of a landscape with its hydrological functioning; a widely used example is the "Height Above the Nearest Drainage" (HAND) index. From an energy-centered perspective HAND reflects the gravitational potential energy of a given unit mass of water located on a hillslope, with the reference level set to the elevation of the nearest corresponding river. Given that potential energy differences
- 20 are the main drivers for runoff generation, HAND distributions provide important proxies to explain runoff generation in catchments. However, as expressed by the second law of thermodynamics, the driver of a flux explains only one aspect of the runoff generation mechanism, with the driving potential of every flux being depleted via entropy production and dissipative energy loss. In fact, such losses <u>dominate</u> when rainfall becomes runoff, and only a tiny portion of the driving potential energy is actually transformed into the kinetic energy of streamflow. In recognition of this, we derive a topographic index named
- 25 reduced dissipation per unit length (rDUNE) by re-interpreting and enhancing the HAND index following a straight forward thermodynamic argumentation. We compare rDUNE with HAND, and with the frequently used topographic wetness index (TWI), and show that rDUNE provides stronger discrimination of catchments into groups that are similar with respect to their dominant runoff processes. Our analysis indicates that accounting for both the driver and resistance aspects of flux generation provides a promising approach to linking the architecture of a system with its functioning and hence develop similarity indices
- 30 in Hydrology.

# 1. Introduction

The key role that surface topography plays in Hydrology has long been recognized (*e.g. Horton 1945*). Topography provides information about the interplay between uplift, weathering and erosion, and hence about the past morphological development of a landscape. Further, it provides a strong constraint for future hydrological and geomorphic changes and, importantly for

- 5 hydrology, is the key driver and control associated with runoff generation and several other hydrological processes. This insight about the past, present and future roles played by topography is surely one reason why almost all key landscape entities in Hydrology, such as *watershed boundaries, hillslopes* and *channel networks*, are derived from properties of the land-surface topography. In support of this, digital elevation models (DEM) are available at fairly high resolution across the globe (*Farr et al., 2007*), helping to fuel the growing popularity of spatially explicit hydrologic models (e.g. *Beven 2001*).
- 10 It is therefore no surprise that hydrology does not suffer from a lack of models or indices linking geomorphic properties of a landscape with its hydrological functioning. The most popular approach is arguably the topographic wetness index (TWI) proposed by *Kirkby (1975)* and *Beven and Kirkby (1979)*. As a function of the local slope with the upslope contributing area per contour length, the TWI was originally developed to classify areas of similar functioning within a catchment and has been applied (e.g. *Grabs et al. 2009*), refined (*e.g. Barling, et al. 1994*) and tested (*e.g. Rodhe and Seibert, 1999*) in numerous
- 15 studies.

However, other indices have also been proposed to link land surface topography with its runoff response. *Hjerdt et al.* (2004) developed the "down slope topographic wetness index" (also called the tan  $\beta$  index) that reflects the local hydraulic gradient in the case that flow is exclusively driven by gravity and under the assumptions of a fixed drop in elevation. They claimed that this index represents groundwater level gradients in a manner that is superior to the classical TWI approach, and showed it to

- 20 be less sensitive to the quality of the DEM used to estimate the local slope. Adopting a hydraulics framework, *Lyon and Troch* (2010) developed an index called the catchment Péclet number, that is a volume or area weighted version of the hillslope Péclet number. The latter was derived by *Berne et al* (2005) to characterize hillslopes by subsurface runoff formation, based on the relative importance of advective and diffusive flows, using the hillslope storage Boussinesq equation (*Troch et al., 2003*). *Lyon and Troch* (2010) showed that in a set of 400,000 synthetically generated and four real world catchments the catchment Péclet
- 25 number provided a meaningful link between hydrological response and the geomorphic properties of a landscape.

An approach that has recently gained considerable attention is the "*height above the nearest drainage*" index (HAND) developed by *Rennó et al.* (2008), and under a different name "elevation difference (DZ)" by *Crave and Gascuel-Odoux* (1997). This approach assumes that water follows the steepest descent along the surface topography and, based on these drainage paths, the corresponding elevation of each raster cell above the nearest corresponding river cell is estimated. HAND

30 has thereby been successfully applied and tested in numerous studies in a wide range of different landscapes. For instance, *Gharari et al. (2011)* compared a collection of hydrological similarity indices, their sensitivity to the DEM resolution as well as their ability to identify three visually pre-classified landscape types (wetlands, hillslopes, plateaus). Their results highlight the sensitivity of HAND to the chosen DEM resolution and show that HAND in combination with the slope lead to the "best" result with respect to match pre-classified observation points. Also *Gao et al. (2014)* used HAND in combination with the slope (additionally they also used the aspect) to identify hydrological similar areas in a model comparison study. They showed that a semi-distributed model setup which was based on a HAND landscape classification scheme outperformed a lumped and semi-distributed (based on the forcing data) hydrological model with respect to matching the hydrograph. The same leading

- 5 author (Gao et al., 2019) further exploited the role surface topography plays when rainfall becomes runoff and used HAND to infer model parameters of a conceptual hydrological model showing that their developed runoff generation module performed almost as good as fully calibrated models. Finally, Zehe et al. (2019) used HAND as a proxy for the gravity potential for calculating potential energy of soil water and showed that their approach is "well suited to distinguishing the typical interplay of gravity and capillarity controls on soil water dynamics in different landscapes."
- 10 The above mentioned studies highlight the large potential of the topographic index HAND and its relevance for hydrological research. From a theoretical point of view, HAND reflects thereby the gravitational potential energy of a given unit weight of water with the reference level set to the elevation of the nearest corresponding river. Given that differences in potential energy act as drivers for overland and subsurface storm flow, the distribution of HAND across a landscape represents a predominant control on the lateral distribution and redistribution of water in a catchment. However, because surface and subsurface water
- 15 flows are also highly dissipative (e.g. *Kleidon et al. 2013*), similarity with respect to HAND distribution is not sufficient to ensure similarity with respect to runoff generation. This is due to the fact that the driving potential is only one of the important factors, with every flux encountering frictional losses along its flow path.

This latter insight recognizes the essential role of the second law of thermodynamics, based on which Zehe et al. (2014) postulated that equifinality is inherent to most of our governing equations, because every flux is unavoidably the result of the

20 interplay between a driving potential and a resistance term. Accordingly, the overall flux through a system can remain unaffected when the driving potential is doubled if the corresponding frictional resistance losses are also doubled. From this perspective, only landscapes having similar combinations of characteristics controlling <u>both</u> the driver and resistance terms should satisfy a sufficiency condition for hydrological similarity (in terms of runoff generation).

In recent years the importance of thermodynamic principles has increasingly gained attention in Hydrology. The Oxford dictionary defines thermodynamics as a "branch of physical science that deals with the relations between heat and other forms of energy (such as mechanical, electrical, or chemical energy), and, by extension, of the relationships between all forms of energy." Given that all fluxes are driven by potentials, and that fluxes are necessarily "dissipative" (meaning that they produce entropy following the second law of thermodynamics (e.g. Kondepudi and Prigogine 2014) it seems logical that thermodynamic concepts are relevant in Hydrology. However, although an energy-centered view has been applied to a variety

30 of different issues in sub-disciplines such as groundwater hydrology (*Hubbert, 1940*) and soil physics (*Babcock and Overstreet, 1955*) it has not become established practice in classical rainfall-runoff centered surface water hydrology. This is likely due to the strong engineering context in which the understanding of surface hydrology was historically developed, with its overt focus on practical problem solving (*Sivapalan, 2018*).

One interesting early exception is the work of *Leopold and Langbein (1962)*, who showed that the concept of "entropy" in its probabilistic form (see *Koutsoyiannis, 2014*) can be used in combination with a random walk term to infer the most probable state of a drainage network. Along the same lines, *Howard (1990)* and *Rodríguez-Iturbe et al. (1992)* showed how thermodynamic optimality principles can be used to derive realistic synthetic river networks. Such work motivated *Hergarten et al. (2014)* and others to apply similar concepts to explain subsurface flow patterns.

- However, a thermodynamic perspective can be much more general, and is by no means limited to the explanation of optimal drainage densities. As examples, *Zehe et al. (2013)* showed that a thermodynamic optimum density of macropores maximizing dissipation of free energy during recharge events allowed an acceptable prediction of the rainfall-runoff response of a lower
- 10 the soil, Zhang and Savenije (2018) how salt and fresh water mixing in estuaries can be described in energetic terms and Zehe et al. (2019) discussed how an energetic perspective on soil water movement can improve our general understanding of catchment hydrology.

mesoscale catchment; *Hildebrandt et al. (2016)* used an energy-centered approach to explain how plants extract water from

The above discussion highlights the considerable potential of a thermodynamic perspective to improve our understanding of hydrological functioning across a range of important issues. One reason that an energy-centered perspective on runoff

- 15 generation remains the exception, rather than the rule, in catchment hydrology may be that the connection between the laws of thermodynamics and issues underlying questions of practical importance in hydrology is not always readily evident. A motivating rationale of this study is, therefore, to bridge this gap by showing how the fundamental concepts of thermodynamics can be applied to develop a solution to the classical hydrological question "*How can the geomorphic properties of a landscape be used to identify hydrological units that have similar hydrological functioning*".
- In this study, we propose a topographic index that accounts for both the driving potential energy difference and the accumulated dissipative loss along the flow path following straightforward thermodynamic arguments. Our index, (reduced dissipation per unit length index) is thereby an energy-centered re-interpretation and enhancement of the well-established topographic index HAND. In the following, we discuss its similarities to other geomorphic indices used in Hydrology and test whether it provides sufficient information to enable distinguishing between two landscapes which differ distinctly with respect to their dominate runoff processes. Furthermore, are we comparing our index against a small subset of topographic indices namely its origin
- HAND and the frequently used TWI. Based on our findings we conclude that one meaningful way to build similarity indices in Hydrology is to acknowledge both the driving potential and the resistance term separately and hence identify the driving potentials and dissipative losses separately.

# 2 Approach and methods

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30 Here, we derive a topographic index based on the energy balance associated with runoff generation from a hillslope. This involves two steps: i) inferring which properties of a DEM provide information about the forces driving runoff generation, and

ii) identifying how much resistance to the flow of water is offered by the landscape. As benchmarks for comparison, we briefly explain the well-established TWI and HAND indices.

#### 2.1 Energy balance of streamflow generation

One of the most important steps in any thermodynamic approach is a proper system definition. Given that hillslopes are often

- 5 described as the key landscape elements controlling runoff generation (e.g. *Bachmair and Weiler 2011*), a starting point to describe the runoff generation of an entire catchment is to examine the energy balance of a hillslope with respect to the total energy of all fluids located on that hillslope. The total energy relevant for streamflow generation at the hillslope scale is thereby the sum of the influx of potential energy by water J<sub>pot</sub> (energy flux in W), the export of kinetic energy by water J<sub>kin</sub> (W), and the amount of energy D (W) dissipated due to friction along the flow path to the river (*see Kleidon et al. (2013)*). In this regard,
- 10 it is interesting to note that typically observed kinetic energies associated with overland flow are quite small compared to their driving potential energies. To get a sense of this, imagine a catchment having an average height above the runoff recording gauge of 20 m and a typical flow velocity of 1 m/s. In this case, only 0.5 % of the average potential energy is transformed into kinetic energy, while by far the largest amount (99.5%) is dissipated due to friction at the fluid-solid interface along the flow path. This irreversible process implies an accumulative loss of free energy along its flow path, and hence a potential decrease
- 15 in the ability of the fluid to perform work (*Freeze and Cherry, 1979; A. Kleidon et al., 2013*). The reason for this is that the potential and kinetic energies primarily determine how the fluid moves, while temperature differences within the fluid are of only minor importance. Accordingly, streamflow generation is accompanied by the conversion of potential energy into kinetic energy, and finally into heat (*Currie, 2003; Song, 1992*).
- Fundamentally, the phenomenon of energy dissipation was first described through the second law of thermodynamics, which states that entropy can be produced but not consumed, implying that the sum of all processes in our universe proceed in a direction of entropy increase, meaning that they necessarily dissipate free energy and hence reduce the capacity of the system to perform work (*Schneider and Kay, 1994*). An elementary consequence of this is the negative sign in a diffusive flux law, which implies that heat flows from warm to cold temperatures, water flows downslope (more generally from higher to lower potential energy), and air moves from high-pressure to low-pressure. Mathematically this can be formulated as the flux gradient law, which states that any flux  $\vec{q}$  is the product of a gradient  $\nabla \varphi$  and the inverse of an effective resistance term *R* which hampers the flux.

(1) 
$$\vec{q} = -\nabla \varphi R^{-1}$$

This equation was the basis for the statement by *Zehe et al. (2014)* that when dealing with the identification of hydrologically similar landscape entities we must consider the driving potential and the resistance terms separately. In the subsequent sections

30 we explore each of these terms.

# 2.1.1 The driving potential

The main drivers for streamflow generation at the hillslope scale are the geo-potential differences between the upslope catchment areas and the stream channel, resulting from the gravitational energy of the mass of the water relative to its position (*Bear 1972; Kleidon 2016*). These potential energy differences driving streamflow generation are largely dependent on

5 topographic differences, and on the space-time pattern of precipitation (*Blöschl & Sivapalan, 1995*). If the topography of a catchment is known, we can (in theory) calculate the potential energy associated to all water on the surface of a hillslope simply by applying Newtonian mechanics:

$$(2) E_{pot} = mgh$$

where  $E_{pot}$  is the potential energy of the water on the hillslope (J), m its mass (kg), g represents the gravitational acceleration 10 (m s<sup>-2</sup>), and *h* is the relative height of the water above a reference (m). Given Eq. 2 we can compute the influx of potential energy by water associated with a grid cell of a DEM by accounting for the spatial extension of the grid cell and the precipitation accumulated over a given time period. Accordingly, for each grid cell *i* of a DEM, we replace the mass term by the volumetric flux of water multiplied by its density  $\rho$  (kg m<sup>-3</sup>), the former computed as the summed total precipitation depth per time  $P_i$  (m s<sup>-1</sup>) within that grid cell multiplied with its area  $A_i$  (m<sup>2</sup>):

$$(3) \qquad J_{pot,i} = P_i A_i \rho g h_i$$

 $J_{pot,i}$  quantifies the influx of potential energy for a given grid cell *i* and for a given time period. To finally calculate the influx of potential energy we need to set a reference level against which to quantify  $h_i$ . In this study, we will focus on catchments smaller than 50 km<sup>2</sup>, and will therefore treat all of them as being "hillslope dominated", implying that channel routing is of only minor importance in the development of runoff generation (*Kirkby, 1976; Robinson et al., 1995*). By neglecting the stream network and assuming that water follows the surface topography along the steepest gradient, we can set the reference level to zero at the point where the hillslope connects to the nearest drainage, and thereby estimate  $h_i$  for each cell in our DEM ( $h_i =$  HAND). To summarize  $J_{pot,i}$  quantifies the influx of potential energy by water within a given raster cell *i*, thereby providing an energy-centered interpretation of the well-established HAND concept. The sum of  $J_{pot,i}$  over a hillslope or catchment represents thereby the total influx of energy by water available to perform work in a given time period. It is hence straightforward to calculate  $J_{pot,i}$  associated with, for instance, the long-term climatic precipitation if relevant information

about the region of interest is available.

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# 2.1.2 Identifying the structures controlling dissipation

While differences in geo-potential energy drive runoff generation, most of the available potential energy is dissipated during runoff generation. At the land surface this is controlled mainly by surface roughness (i.e. friction per unit length), which in turn depends on the nature of the vegetation, soil texture and the micro-topography. On the other hand, frictional losses within the subsurface are controlled by soil hydraulic conductivity, soil water content and (in case of deep percolation) by bedrock

topography and conductivity. In both domains, additionally connected flow networks (such as rills, or vertical and lateral macropores) dramatically reduce frictional losses per flow volume, by providing a larger hydraulic radius (*Hergarten et al., 2014; Howard, 1990*).

The difficulty associated with estimating frictional losses, is that a variety of different runoff processes can occur within a

- 5 hydrological year, all having different occurrence probabilities that are in turn controlled by different landscape properties of the hillslope. It is precisely this diversity of different spatio-temporal controls that makes it so difficult to upscale small scale processes to the scale of the entire catchment (*Sivapalan, 2003*). However, despite this variability, dissipation remains accumulative along the flow path (*Rodríguez-Iturbe et al., 1992, Kleidon et al., 2013*), offering the opportunity to define a "dissipation length" as a surrogate for the macroscopic flow resistance in the flux-gradient relationship (Eq.1) as long as the
- 10 pedo-geological setting does not change significantly along the flow path.

For simplicity, we henceforth assume that the dissipation of the geo-potential energy during runoff production is proportional to the flow path length to the river. This assumption is in line with those made by *Rodríguez-Iturbe et al.* (1992) in the context of stream networks, and is based on the observation that the export of kinetic energy by water  $(J_{kin})$  is often negligible small compared to the influx of potential energy by water  $(J_{pot})$ . The majority of available potential energy is hence dissipated (*D*)

15 when rainfall becomes runoff:

(4) 
$$D = J_{pot}$$
 given  $J_{kin} \ll J_{not}$ 

A given mass of water traveling from a specific location (grid cell *i*) to the stream will dissipate its potential energy over its travel distance leading to:

(5) 
$$\frac{D_i}{l_i} = P_i A_i \rho g \frac{h_i}{l_i}$$

With  $l_i$  being the flow length of a given raster cell *i* to the nearest drainage (m) and  $h_i$  the height above the nearest drainage (m) of that raster cell *i*. To assure that the developed index depend exclusively on information about the topography stored within a DEM we normalize Eq. (5) by the mass flux of precipitation and divide it by the gravity constant *g*, the resolution  $A_i$ , and by the density of water  $\rho$ , to obtain a dimensionless index:

(6) 
$$rDUNE = \frac{h_i}{l_i}$$

25 This <u>reduced dissipation per unit length index (rDUNE)</u> is an estimate of the potential energy gradient at the surface topography of a given raster cell under the assumption of gravitational flow, and is similar to the index proposed by *Hjerdt et al. (2004)* but without the need to arbitrarily define the drop in elevation. Here we have chosen to multiply rDUNE with minus one as well as use the natural logarithm transformation to make the rDUNE more easily comparable with the TWI as well as to transform the skewed distributions to be more normally distributed and thereby make its patterns more easily interpretable.

(7) 
$$rDUNE = -ln(\frac{h_i}{l_i})$$

rDUNE is defined in a range from  $-\infty$  till  $\infty$ , is zero if the flow length and height to the nearest drainage are equal, positive if the flow length is larger and negative if the flow length is shorter than the height to the nearest drainage. High rDUNE values mean that the dissipation of potential energy is reduced compared to landscapes with lower rDUNE values. This reduction could for instance stem from higher hydraulic conductivities of the prevailing soils or from the occurrence of different forms of preferential flow paths.

# 2.2 Topographic wetness index (TWI) and height above the nearest drainage (HAND)

We compute the frequency distributions of grid cell TWI and HAND indices for comparison with the rDUNE distributions. The TWI is defined for each raster cell as:

10 (8)  $TWI = \ln(\alpha/\tan(\beta))$ 

where  $\alpha$  is the upslope accumulated area and tan( $\beta$ ) the local slope angle (the TWI is usually divided by the resolution of the DEM before the logarithm is taken, to make it dimensionless). Meanwhile HAND is based on the concept that water follows the steepest gradient along the surface topography, and hence both a river network and as a flow direction map are required for its calculation. To better compare HAND with TWI and rDUNE, we again use its natural logarithm (ln(HAND)).

#### 15 2.3 Measuring divergences between distributions

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Measuring the similarity or dissimilarity of frequency distributions without resorting to statistical moments is not straightforward. Here we use a less well known measure, called Jensen-Shannon divergence (JSD, *Lin*, *1991*) to estimate how similar catchments are with respect to their ln(HAND), TWI and rDUNE distributions. JSD is a non-negative, finite and bounded distance measure developed to quantify the divergence between probability distributions. It was introduced into

20 Hydrology by *Nicótina et al. (2008)* and is strongly, but not necessarily, motivated by Information Theoretic considerations (*for details on Information Theory please see Cover and Thomas, 2005*). JSD is based on the well-known Kullback-Leibler divergence (KLD; sometimes referred as relative entropy) defined as:

(9) 
$$D_{KL}(X ||Y) = \sum_{x \in X} p(x_i) \log_2 \frac{p(x_i)}{p(y_i)}$$

where  $p(x_i)$  and  $p(y_i)$  are the probabilities that X and Y are respectively in the states  $x_i$  and  $y_i$ . In brief, KLD quantifies the 25 information loss when the probability density function of Y is used in place of X, and has been applied in hydrology by *Weijs*, *Schoups, and van de Giesen (2010)* to evaluate hydrological ensemble predictions. However, because KLD is not a classical distance measure, being neither symmetric nor bounded (*Majtey et al., 2005*) it is not well suited to the simple comparison of distributions. To overcome this issue *Lin* (1991) and *Rao* (1982) developed a symmetric and bounded version of KLD that, when subjected to a square root transformation, satisfies the triangle inequality condition required of a distance metric (*Endres and Schindelin, 2003*). This is accomplished by computing the sum of the KLD of (X ||Y) and (Y || X), thereby making it symmetric, as was originally proposed by *Kullback and Leibler* (1951) as the "J divergence". In its general form for N distributions, the J divergence can be written as:

(10) 
$$J_{KL} = \sum_{i=1}^{N} (X_i || Y_i)$$

From this, the JSD is developed, by comparing each distribution to the "mid-point" distribution *M*, defined as:

(11) 
$$M = \frac{1}{N} \sum_{i=1}^{N} (X_i + Y_i)$$

Accordingly, the JSD represents the average divergence of N probability distribution from their mid-point distribution, defined 10 as:

(12) 
$$JSD = \frac{1}{N} \sum_{i=1}^{N} D_{KL}(X_i || M)$$

If we calculate the JSD using logarithms to the base 2 the JSD associated with two distributions is bounded between zero and unity, while for N distributions it is bounded between zero and the maximum entropy  $\log_2 N$  (*Jaynes, 1957*). This is because the mid-point distribution *M* converges to a uniform distribution in the case of maximum dissimilarity between the distributions.

#### 2.3.1 Derivation of probability distributions

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To calculate the JSD it is necessary to convert the frequency distributions of ln(HAND), TWI and rDUNE into probability density functions. This step requires a careful choice of bin width (*Gong et al., 2014*). Various guidelines to properly estimate the bin width have been proposed, one of the earliest and most frequently used having been proposed by *Scott (1979)*:

20 (13) 
$$W = 3.49 * \sigma * N^{-\frac{1}{3}}$$

where W is the bin width,  $\sigma$  is the standard deviation of the distribution and N is the number of available samples belonging to the distribution.

In our study, however, the optimal bin width turns out to be different for each distribution as a result of its shape and the number of samples (size of the catchment). This is inconsistent with the need to use the same binning for each case to facilitate

25 comparisons of the different distributions. Accordingly, we decided to use only the largest bin width calculated for each similarity index – which is 0.5 for the TWI distributions, 0.2 for the ln(HAND) distributions and 0.15 for the rDUNE distributions (please note the JSD values between the distributions change only slightly if calculated with the smallest bin size;

see appendix A1). Finally, as recommended by *Darscheid et al. (2018)*, for any bin indicating zero probability (no data samples are found to fall in that bin) we treated it as though it contained a single sample, thereby associating that bin with a very low probability of occurrence.

#### 3. Study area

- 5 The 288 km<sup>2</sup> Attert catchment, located in Luxembourg, has a mean annual precipitation of 850-1100 mm and mean monthly temperatures varying between 0°C in January to 18°C in July. Detailed descriptions of the climatology and hydrology of the catchment can be found in a series of studies (*e.g. Bos et al., 1996; Martínez-Carreras et al., 2012; Wrede et al., 2015, Jackisch, 2015*). An important and particularly relevant characteristic of the catchment is that it consists of two major geological formations. Devonian schists dominate the Ardennes massif in the northern and western part, and Triassic sandy marks
- 10 dominate the rest of the catchment, interrupted by several small areas of sandstone in the south and north-west. To test the functional discrimination ability of the rDUNE, TWI and ln(HAND) indices, we selected six headwater catchments of different sizes (see Fig. 1), three in the Schist area (Platen 40 km<sup>2</sup>; Colpach 19.4 km<sup>2</sup>; and Weierbach 0.45 km<sup>2</sup>), and three in the Marl area (Schwebich 30 km<sup>2</sup>; Niederpallen 32.2 km<sup>2</sup>; and Wollefsbach 4.4km<sup>2</sup>).

#### 3.1 Hydrological regimes and runoff generation

- 15 Important to this study is that the six catchments share similar hydro-climatic regimes (*Jackisch 2015*), which can be separated into winter and vegetation seasons, during which either runoff or evapotranspiration respectively are the dominant water fluxes leaving the catchments (*Loritz et al., 2017*). Annual runoff coefficients vary from 30- 60% indicating distinct differences between the years; this is most likely the result of annual climatic variations (*Pfister et al., 2017*).
- However, the way how the catchments transform rainfall to runoff, varies significantly between the different geological formations (*Bos et al. 1996*). The Schist region is characterized by a "*fill and spill*" runoff generation mechanism, wherein water flows along or within the bedrock comprise the dominant runoff process. On the other hand, in the Marl regions, saturated areas and preferential flow paths within macrospores and soil crack dominate how water is distributed.

Differences between the runoff regimes are highlighted in Fig. 2 for a series of rainfall-runoff events in the winter, summer, and autumn of 2012 and 2013. The runoff response in the Marl catchments is rather rapid and more peaked (but with less

- volume) than in the Schist catchments (*Loritz et al., 2017*). It is noteworthy that although all of the Marl catchments are of different size, they exhibit very similar patterns of runoff generation. On the other hand, the behaviours of the Schist catchments are quite different from each other, with Platen producing (over the long term) ~30% less discharge than the other two. A possible explanation for this is that ~30% of the Platen catchment belongs to a sandstone formation that tends to be less responsive with regards to runoff and has deeper groundwater stores (*Bos et al., 1996*). Despite these differences, and in spite
- 30 of the fact that their sizes differ by a factor of 10, the Schist catchments exhibit surprisingly similar runoff responses (with

Spearman rank correlations above 0.9). This is highlighted by the characteristic double peaked nature of the runoff events in all three Schist catchments during the winter (*Martínez-Carreras et al., 2016*).

To summarize, the two geological formations share rather similar hydro-climatic regimes, but differ significantly with respect to dominant runoff processes and hence how they transform rainfall to runoff. We should therefore expect that any catchment

- 5 similarity index, developed for the purpose of identifying and explaining differences in hydrological functioning (in terms of runoff generation), should be able to clearly distinguish these two geological areas from each other. It is important to note that we picked this set of catchments on purpose, because the climatic differences between the catchments are rather small and the corresponding catchments share a rather clear geological setting. This was possible due to the fact that the Attert catchment and sub-basins were setup for research purposes rather than for management reasons. Larger data sets with catchments
- 10 fulfilling the conditions of comparable climatic and geological settings are rare, making the definition of functional similarity challenging in catchment comparative studies as well as our assumption that the pedo-geological setting does not change significantly along the flow path.

#### 3.2 Spatial analysis and the stream network

For our topographic analyses we used a 5 m LIDAR digital elevation model, aggregated and smoothed to 10 m resolution. All
spatial analysis were conducted using GRASS GIS (Neteler et al., 2012) and the GRASS GIS extension r.stream\* (*Jasiewicz and Metz, 2011*). The latter was used to derive the distance-to-the-river and elevation-to-the-river (HAND) maps, used as the spatial basis for all subsequent analyses. Because the calculation of these maps is very sensitive to the extension and shape of the river network it is important to derive the stream network with care; for this analysis we used the stream network created by *Loritz et al.* (2018), by separately varying the minimum contributing area thresholds, depending on the geological setting,

20 to match the official stream network available from the Luxembourg Institute of Technology (LIST). In addition, the stream network was evaluated against orthophotos and manually adjusted in close collaboration with field hydrologists working in the Attert region.



Figure 1 Map of the Attert basin with the six selected headwater catchments. In the northern part of the Attert catchment the three schist dominated catchments (blue; Platen, Colpach, Weierbach) are highlighted and in the southern part, the three marl dominated catchments (green; Schwebich, Niederpallen, Wollefsbach).



Figure 2 Observed specific discharge and precipitation with different ordinate scales for a time period in summer 2012 autumn 2012 and winter 2013 in the six catchments (orange: marl catchments and blue schist catchments.). This figure highlights that the two geological formations have a distinctly different hydrological function with respect to how they transform rainfall to runoff throughout the year.

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# 4. Results

Fig. 3 displays the frequency distributions and corresponding cumulative density functions of TWI, ln(HAND) and rDUNE for the six catchments examined in this study. In general, the TWI distributions do not indicate strong differences between the two geologies. For all six catchments, the distributions tend to be approximately Gaussian, with mean values close to 8 (see

- 5 also Table 1). Visually, only the Platen and Colpach differ slightly from the other catchments, with distributions shifted somewhat to the left (lower means). That these six TWI distributions are indeed rather similar is also indicated by the JSD (Fig.4), the values of which are all rather small indicating low divergence between the distributions. This similarity of the TWI distributions in spite of geological differences may, on first glance seem somewhat surprising given that the Schist catchments are generally much steeper than the Marl ones. However, in the Marl regions the water flow along the surface tends to be much
- 10 less convergent, and consequently the flow accumulations tend to be lower than in the Schist regions.

The corresponding comparison of the ln(HAND) distributions indicates a greater degree of divergence between the two runoff regimes. In particular, the Platen and the Colpach catchments (both in the Schist region) differ from the other catchments with ln(HAND). This visual impression is reinforced by the average values of ln(HAND) (Table 1), with both the Colpach and the Platen catchments exhibiting similar average values close to 3 (ln(m)). In general, however, the index does not indicate a very

- 15 distinct separation between the two geologies, and does not clearly distinguish between the Weierbach (Schist) and Niederpallen (Marl) catchments. The JSD values further reinforce the fact that the differences between the distributions tend to be quite small. For instance, the Platen (Schist) and Schwebich (Marl) catchments have very small JSD values (~0.042), while the Wollefsbach catchment that is within the same geological formation (Marl) as the Schwebich has a JSD value of 0.11.
- 20 In contrast, the rDUNE distributions reveal a rather different picture. Visually, the rDUNE index clearly distinguishes between the two geologies. In particular, the shapes of the cumulative density functions indicate that the Marl catchments tend to have lower rDUNE values than the Schist catchments. The mean values of the rDUNE distributions (Table 1) are around 1.94-2.18 for the Schist catchments, and around 2.9-3.5 for the Marl catchments. Meanwhile, the JSD between all three Schist catchments are below 0.1, while being as large as 0.49 when computed against the Marl catchments.



Figure 3 Frequency distributions and cumulative density functions of the TWI, ln(HAND) and rDUNE for the six research catchments. In blue the schist catchments (Platen ,Colpach, Weierbach) and in green the marl catchments (Schwebich, Niederpallen, Wollefsbach).

					Ø TWI + std				Øl	$\emptyset \ln(\text{HAND}) + \text{std}$				Ø rDUNE + std					
						[-]				[ln(	(m)]			[-]					
	S	Schist																	
	I	Platen			$7.77 \pm 1.9$				$3.03\pm0.9$				2.	$2.18\pm0.5$					
	(	Colpach				$7.54 \pm 1.9$				$3.21\pm0.9$				1.	$1.94\pm0.6$				
	Weierbach				$8.05 \pm 1.6$				$2.85\pm0.7$				2.	$2.17\pm0.6$					
	1	Marl																	
	Niederpallen				$8.3 \pm 2$				$2.77\pm0.8$				2.	$2.93\pm0.7$					
	Schwebich				$8.1\pm1.9$				2.8	$2.88 \pm 1.0$				$2.9\pm0.7$					
	1	Wollefsbach			$8.67 \pm 1.8$				$2.66\pm0.6$			3.:	$3.52 \pm 0.6$						
																		_	10
a) TWI			b) ln(HAND)					c) DUNE								1.0			
col	0.006	1				col	0.014					col	0.046						0.8
wei	0.024	4 0.045				wei	0.12	0.16				wei	0.042	0.08					0.6 ISD
nie	0.026	6 0.052	0.011			nie	0.065	0.083	0.092			nie	0.23	0.32	0.27				0.4
sch	0.013	3 0.032	0.011	0.0043		sch	0.042	0.048	0.12	0.015		sch	0.18	0.27	0.22	0.014			0.2
wol	0.091	1 0.14	0.041	0.026	0.046	wol	0.16	0.2	0.058	0.059	0.11	wol	0.45	0.49	0.49	0.18	0.16		
	pla	col	wei	nie	sch		pla	col	wei	nie	sch		pla	col	wei	nie	sch		

Table 1	Average (Ø) and	d standard deviation	(std) of the	TWI, ln(HAND)	) and the rDUNE for	each experimental catchment.
			(		,	

Figure 4 JSD values for the six research catchments (Schist: Platen (pla), Colpach (col), Weierbach (wei); Marl: Niederpallen (nie),
Schwebich (sch), Wollefsbach (wol)). Panel a JSD of between the TWI distributions, b between the ln(HAND) distributions and c
between the rDUNE distributions. A high JSD value indicates a high divergence between the distributions with a maximum of 1.

# 5. Discussion

# 5.1 Potential energy differences as the driver for runoff generation

The reduced dissipation per unit length index (rDUNE) is a straightforward energy based enhancement of the frequently used

10 HAND approach (*Rennó et al., 2008*). The small, but significant, difference is that rDUNE is computed by dividing HAND by the flow path. This is motivated by the fact that almost all of the potential energy is dissipated within the runoff generation

process. Though this extension might seem incremental, rDUNE thereby accounts for both the driving potential energy difference and the dissipative energy losses associated with the production of runoff. The latter is likely of particular importance when examining environments having a distinct topography where runoff generation is not limited by the available potential energy but by dissipation, and therefore facilitating preferential flow structures dominate surface and subsurface

5 runoff generation. Accordingly, rDUNE should help to improve the classification of catchments into functionally similar spatial units, particularly for headwater catchments having moderate to steep topographies (*David R. Montgomery and Dietrich, 1988*).

The first indication that rDUNE is indeed a useful addition to the variety of available topographic indices in Hydrology is highlighted by our results which show that rDUNE distributions stronger discriminate catchments with two distinctly different

- 10 runoff regimes as it is the case using HAND or the TWI (Fig. 4). Furthermore, the fact that rDUNE values are on average higher in the marl region compared to schist catchments is physical reasonable considering the circumstances that soil cracks and worm burrows (in general preferential flow e.g. *Loritz et al.*, 2017) play an important role in the way how the catchments transform potential energy in kinetic energy. This is the case as these structures reduce the dissipation of energy along the flow path and higher rDUNE values are expected in landscapes in which the dominant runoff process are characterized by flow
- 15 through preferential flow path.

Despite these first promising results a full analysis of rDUNE and its sensitivity to different DEMs resolutions, flow direction algorithms as well as terrain smoothing functions is needed, as it has been shown in detail for the TWI and HAND by *Gharari et al.* (2011). However, as rDUNE is rather an extension and an energy-centered re-interpretation of HAND we would expect that the findings from *Gharari et al.* (2011) about the meaningful range of raster resolutions can be, at least partly, transferred

20 to rDUNE. Exactly this relationship between HAND and rDUNE is hence rather a strength as a weakness and it would be an interesting avenue to test how and if the landscape classifications and model results of *Gao et al. (2014)* and *Gao et al. (2019)* change if HAND is replace by rDUNE.

Additionally, we speculate that our energy-centered re-interpretation of HAND may, besides improving its theoretical underpinning, further open the possibility to dynamically classify landscapes over time. This is because the incoming potential

25 energy and the energy-centered foundation of rDUNE (Eq. 5 (J\*m<sup>-3</sup>)) can be instead calculated with a mass flux rather than with a total mass, for instance using an hourly precipitation time series. As discussed in *Loritz et al. (2018)* this kind of dynamic classification may provide the key to successfully partitioning a catchment into similar functioning landscape entities, as hydrological systems move from complex to organized states. As a consequence, rDUNE in its current time invariant form will always be limited to identifying hydrological similar landscape units.

#### 30 **5.2 Sensitivity to drainage density**

The fact that the rDUNE frequency distributions varied across the two geologies is clearly due to the fact that different accumulation values were used to derive the channel network in the different geologies. Changing the accumulation threshold

means that water will start to flow sooner or later at the surface and hence that the flow length and the elevation to the nearest drainage will increase or decrease. The origination point of the channel network is thereby controlled by a variety of structural and climatic controls, and often varies depending on the prevailing season (*Montgomery and Dietrich 1992*). However, varying the accumulation threshold within a reasonable range mainly changes the flow length in headwater catchments, and the flow

5 length and elevations along the main river (where we are rather certain about the position of the channel network) will not change dramatically.

Another point, more specific to our tested geological formations, is that flow directions are more parallel in the Marl regions as a result of the smoother topography. Therefore, water will start to flow later at the surface within the stream network even if we choose the same flow accumulation threshold in both geologies. This can, of course, depends on the chosen flow direction

10 algorithm (*Seibert and McGlynn, 2007*). Nevertheless, the fact that the accumulation area needed to form a channel is, in general, larger in the Marl region where slopes are more gentle compared to the Schist regions, matches the observation by *Montgomery and Dietrich (1988)* that there is a strong inverse relationship between the average length of a hillslope and its slope.

Finally, leaving aside the technical details of extracting a river network based on a DEM and the uncertainties that go along

- 15 with such an approach, we note that the stream network we use in this study was carefully extracted based on an official stream network, and on several visits to the area, and was checked using orthophotos. This means that we are confident that we have correctly captured the overall picture of the perennial channel network, even if we are not able to examine every location where water under typical conditions begins to form a channel. The fact that the drainage densities of a catchment provide important information about the hydrological functioning of a landscape has been shown by several studies (e.g. *Mutzner et al., 2016*).
- 20 This is because the extension of the stream network reflects the interplay of the climatic forcing and the hydro-pedological setting of a landscape and therefore the interaction of the driving potential of runoff generation and the resistance which works against it. This observation was previously made by *Montgomery and Dietrich (1988)*, who postulated that it is logical to use the information stored within the extension of a channel network and the average hillslope length and height (*slope*) for developing models that try to explain hydrological similarity based on the topography.

# 25 5.3 Topographic similarity and hydrological similarity

Our comparison of TWI, ln(HAND) and rDUNE indicates that the rDUNE is more able to detecting differences between the two runoff regimes tested here. However, there exist a variety of other topography-based indices in use which we do not test in this study, ranging from simple comparison of the mean slopes of a catchment to approaches based on assumptions that are rather similar to those made in this study. A prominent example is the work of *McGuire et al. (2005)* who used the median

30 flow path length (L), the median flow path gradient to the river (G) and the ratio of both (among other variables) to analyze how much of the inter-catchment variability of residence times of tracers can be explained by geomorphic properties. They found that the ratio of the flow path and the slope was superior to other variables in explaining hydrological dynamics. *McGuire*  et al. (2005) stated that "... the correlation of residence time with L/G is significantly better than the correlation of residence time with flow path length (L) or flow path gradient (G) individually. This suggests that both factors are important controls on residence time."

Interestingly, *Harman and Sivapalan (2009)* gave exactly this index, under a different name, a theoretical basis when they derived the Boussinesq equation within their hillslope similarity study. It is remarkable that their topographic index (L/G) is rather similar to the rDUNE or the *tan*  $\beta$  index (*Hjerdt et al., 2004*), although they use the median of the local slopes as proxy for the driving potential instead of the potential energy and further altered the ratio by dividing the flow path length by the gradient and not vice versa. The similarity between the three indices is, however, still evident as both include a surrogate for the driver of a flux and a surrogate for the friction term working against it.

- 10 In this context it is interesting to note that also the system properties represented in our governing equations are rarely independent but rather act in conjunction (*Bárdossy*, 2007). Because most similarity indices are derived upon those governing equations, we can find the aforementioned pattern in many other successful hydrological indices. For instance, also the TWI combines the driving potential (local slope) with an estimate of the conductivity of a given area (in the form of the upslope accumulation area). These assumptions might be appropriate for northern England (where TWI originally was developed) and
- 15 may also work in many other environments, but will likely fail if the driver or the resistance term are not appropriately estimated. This highlights the fact that the concept of combining system properties driving a flow with properties that hamper flow might indeed be one meaningful way to link the hydrological functioning of a system with its architecture (*Zehe et al., 2014*). As the physical foundation for this perspective is based on thermodynamics it might be an advantage to routinely consider runoff generation not exclusively as a mass flux but as an energy driven and dissipative process, as this perspective
- 20 may help us to better generalize our findings and identify the limitations of our concepts and models.

# 6. Conclusion

The dissipation per unit length index developed here is an energy-centered re-interpretation of the HAND index. Its use enabled us to use DEM data to detect differences between two sets of catchments having distinctly different dominant runoff processes and, in this regard exhibited superior performance to the TWI and HAND approaches. Our results indicate that a promising

25 way to link system architectures with their functioning is to identify system properties in such a way that we can account separately for both the drivers of a flux and the properties that act to resist it.

The general idea behind this study is thereby the observation that the majority of the incoming potential energy associated with water flow within a hillslope is dissipated and only a fraction of it reaches the stream network as kinetic energy. This highlights the important role energy dissipation plays when rainfall is transformed to runoff within a catchment. Establishing a proxy for

30 the structures that control energy dissipation is thus the key to functionally classifying environments that are not limited by the available potential energy and therefore have distinct topographies. Finally, by taking an energy-centered perspective on runoff generation, we can begin to address the question of why landscapes evolve in such a way that most of the potential energy is dissipated at the hillslope scale, although it is frequently reported that energy dissipation is minimized within river networks *(Kleidon et al., 2013; Rinaldo et al., 1992; Rodríguez-Iturbe et al., 1992; Zehe et al., 2010)*.

5 *Data availability.* The discharge observations as well as the digital elevation model were provided by the Luxembourg Institute of Science and Technology within the "Catchments As Organized Systems (CAOS)" research group (FOR 1598) funded by the German Science Foundation (DFG). Please contact Laurent Pfister or Jean-Francois Iffly.

Acknowledgements. This research contributes to the "Catchments As Organized Systems (CAOS)" research group (FOR 1598)

10 funded by the German Science Foundation (DFG ZE 533/11-1, ZE 533/12-1). Laurent Pfister and Jean-Francois Iffly from the Luxembourg Institute of Science and Technology (LIST) are acknowledged for organizing the permissions for the experiments and providing discharge data and the digital elevation model. We also thank the whole CAOS team of phase I & II. Finally, would we like to thank Steve Weijs for his detailed review and discussion of our previous work which motivated this study.

## Appendix A: Influence of different bin widths on the Jensen-Shannon divergence

15 In Fig. A1 we illustrate the influence of a different bin width when calculating the Jensen-Shannon divergence between the TWI, ln(HAND) and rDUNE distributions. Instead of using the largest bin width as described in Sect. 2.3.1 we use the smallest meaningful bin width which is 0.1 for the TWI, 0.03 for ln(HAND) and 0.05 for rDUNE. This figure in comparison to Fig. 4 highlights that the overall picture does persists even if we would have chosen the smallest statistical feasible bin width instead of the largest.



Figure A1: JSD values for the six research catchments (Schist: Platen (pla), Colpach (col), Weierbach (wei); Marl: Niederpallen (nie), Schwebich (sch), Wollefsbach (wol) ). Panel a JSD of between the TWI distributions, b between the ln(HAND) distributions and c between the rDUNE distributions. A high JSD value indicates a high divergence between the distributions with a maximum of 1. The difference between this figure and Fig. 4 is the chosen bin width when we estimated the JSD between the different

25 distributions.

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Reply to Referee #1 Hongkai Gao:

Hongkai Gao (HG): Summary and Recommendation: "Loritz and his colleagues attempted to propose a new topographic index, named dissipation per unit length (DUNE). Generally, this is an interesting paper. The authors wanted to increase our understanding between topography and hydrology, which I think is still a big treasure to be further exploited. The paper has

5 potential to be an important paper. But when I read the content, I found there are still several places need to be clarified before being considered for publication."

**Ralf Loritz** (**RL**): We would like to thank Hongkai Gao for his comments and the time he invested to review our Manuscript (MS). The revised MS will follow the reviewer's recommendations and include among other things a passage where we more extensively link DUNE with our system understanding. Furthermore will we carefully check the two references of the reviewer

10 and see whether they help to improve the argumentation in our MS.

#### **Comments:**

1. HG: Please plot precipitation data in Figure 2, otherwise we cannot see how runoff responses to rainfall events.
RL: Good idea. We will add precipitation to Figure 2 in a revised MS (page 12 line 5).

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2. HG: The hydrographs in six catchments present two interesting regimes: 3 marl catchments have sharp peak flow and fast recession, while 3 schist catchments have dampened peak flow and slow recession. If my understanding is correct, the authors wanted to conclude that topography controls the shape of hydrograph, rather than runoff generation (water balance), right? You probably intended to say runoff transfer or response or transition, right? What I want to say is that Figure 2 clearly shows that topography greatly impacts the shape of hydrography (runoff transfer/transition) rather than the amount of runoff (runoff (runoff transfer/transition)).

- *generation). The authors should clarify these two terms, which are very important to communicate your results.* **RL:** Exactly, the total amount of runoff over a longer time period is rather similar in the two geologies while the runoff generating processes and hence also the runoff reaction are not (e.g. Loritz et al. 2017). This is what we intended to say when we used the term runoff generation. In a revised MS we will rephrase the according section and explain in more detail what
- 25 we mean by runoff generation following your advice (*Changes throughout the MS*).

**3.** HG: Figure 3 showed that, comparing with TWI and HAND, DUNE has the best ability to distinguish different regimes of hydrograph response. The six DUNE curves show clearly two classes, which are well correlated with different hydrography regimes (Figure 2). This is an important conclusion, which is also the highlight of this research! From my understanding, the

30 catchments with less DUNE values (Platen ,Colpach, Weierbach) have steeper topography and larger dissipation per unit length (kind of gradient divided by length), and subsequently resulting in sharp peak flow and fast recession process, right? But the observed hydrograph in Figure 2 shows an opposite regime. How can you explain this contradiction? Are there other factors influencing the shape of different hydrographs? If topography cannot be used to interpret different shapes of hydrography, why shall we use DUNE to analysis the relation between topography and hydrological processes? Please correct me, if my understanding is wrong.

So I think it is really necessary to clarify the physical connection between DUNE and hydrographs.

- RL: Thank you, this is a very valuable comment. Indeed our chosen index name "dissipation per unit length (DUNE)" is
  5 misleading. For instance, a relative high dissipation per unit length value could be interpreted in a way that we would assume that the dissipation of potential energy is rather high given a unit flow length in a corresponding landscape. However, exactly the opposite is true and a landscape characterized by on average high DUNE values indicates low friction and hence a reduced dissipation per flow length (e.g. macorpores, high hydraulic conductivities etc) if compared to a landscape characterized by on average lower DUNE values.
- 10 Again thank you for this comment. In a revised MS we will explain in detail how a high or low index could be physically interpreted and also connect our results, the absolute values of DUNE, better to our two geological regimes. Furthermore will we change the name to of the index to "reduced dissipation per unit length (rDUNE)" to make the interpretation more straightforward (*page 8 line 1 6; page 16 line 8 15*).
- **4. HG:** The authors also mentioned the similar index from Hjerdt et al. (2014) and Harman and Sivapalan (2009). But I did not see the comparison between DUNE and these two indices. If you want to propose a new index, you should also compare it with its ancestors and show your advantage. Right?

**RL:** The similarity index from Hjerdt et al. (2014) is the ratio of HAND and an arbitrary drop in elevation. This means that it would reveal the same overall patterns as we have found using HAND (please see page 7 line 12).

20 The index developed by Harman and Sivapalan (2009) needs information that is not stored in a DEM, for instance the porosity and hydraulic conductivity. These can be neglected as long as we work in the same geological setting, however, this is not the case in our study.

We would also like to stress that DUNE is an energy centered re-interpretation and enhancement of HAND. This means that one of the major goals is to show that DUNE improves the ability of HAND to discriminate different landscapes exclusively

based on the information stored within a DEM. We will stress this in a revised MS (*page 4 line 20 - 28; page 18 line 27-29*).

5. HG: I found several very important relevant publications are missing in the reference list. I list some for your reference. Reference: Gao, H., Birkel, C., Hrachowitz, M., Tetzlaff, D., Soulsby, C. & H.H.G. Savenije (2019). A simple topographydriven and calibration-free runoff generation model. Hydrology and Earth System Sciences. DOI: 10.5194/hess-23-787-2019.

30 Gao, H., Hrachowitz, M., Fenicia, F., Gharari, S., and Savenije, H. H. G.: Testing the realism of a topography-driven model (flex-topo) in the nested catchments of the upper Heihe, china, Hydrology and Earth System Sciences, 18, 1895-1915, 10.5194/hess18-1895-2014, 2014.

**RL:** Thank you very much for pointing us to your studies. We will examine them in detail and consider adding these publications as references for our argumentation in our MS (*page 2-3 line 30 – 12; page 16 line 8 – 24*).

References:

Harman, C., & Sivapalan, M. (2009). A similarity framework to assess controls on shallow subsurface flow dynamics in
hillslopes. Water Resources Research, 45(1), 1–12. https://doi.org/10.1029/2008WR007067

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Reply to Referee #2 Shervan Gharai:

Shervan Gharari (SG): Summary and Recommendation: "The manuscript tries to introduce a new topographical index called DUNE which presumably can differentiate the runoff generation mechanisms in comparison with the other topographical indices. The paper is well structured and use of English language is good and sufficient. However the manuscript needs significant clarification and its scientific merit should be better justified.

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Overall, I am positive that this manuscript can be an interesting contribution to the field of hydrology and hydrological modeling, however I am not convinced in the current format the manuscript meets the applicability and reproducibility standards. I would therefore suggest major revision for this study and I am more than happy to receive the revised version of

10 the manuscript."

**Ralf Loritz** (**RL**): We would like to thank Shervan Gharari for the time and the effort he put into writing his review. Most of the points he raises are relevant and addressing them will help improving our manuscript (MS). We hope that after this discussion (as well as after we revised our manuscript) all issues he raises can be clarified.

## 15 Comments:

**1.** SG: Too broad and philosophical introduction, need more citation of relevant work, need more focus: I found the introduction very broad and philosophical. As a reader, I was not able to follow what massage the authors would like to convey in the introduction. Introduction ends with vague sentences such as "how can the geomorphic . . .". I am not an expert in the field of geomorphology but I assume there should be a significant body of literature on this topic. I believe the introduction

20 benefit significantly from focusing on the existing methods for extracting information from topography, their importance and relevance to hydrological modeling and possible need for a new index (DUNE).

**RL:** Respectfully, we do not agree with the assessment of the reviewer regarding our introduction and the use of references in our MS. We think that our introduction is appropriate for our MS, thereby also keeping in mind that it is intended as a contribution to the special issue "Thermodynamics and Optimality".

- 25 The first part of our introduction explicitly mentions the different indices already used in literature. The second part focusses more on the energy perspective since our approach focusses on energy dissipation, and thus on thermodynamics. We therefore think that it is appropriate to refer to other studies that also applied a thermodynamic perspective in their approach. We do not consider that as being "too philosophical". Furthermore, would we encourage SG to explicitly point out to us the "*important references*" he is referring to, as did the first reviewer in his assessment.
- 30

**2.** SG: Too few catchment for the study: I am very surprised by the number of few catchments that the authors have used in this study. I suggest the authors to include more catchments with more diverse forcing data. This needs to be address for applicability of the study. If there is a reason why only a handful of catchments are selected then it should be explained.

**RL:** Thank you for this comment and indeed this is an important point and we hence would like to highlight that we stated in our MS at page 10 line 15-23 :

"It is important to note that we picked this set of catchments on purpose, because the climatic differences between the catchments are rather small and the corresponding catchments share a rather clear geological setting. This was possible due

5 to the fact that the Attert catchment and sub-basins were setup for research purposes rather than for management reasons. Larger data sets with catchments fulfilling the conditions of comparable climatic and geological settings are rare, making the definition of functional similarity challenging in catchment comparative studies."

In short, to answer our question we need an ensemble of catchments which are as similar as possible with respect to climate etc. but dissimilar with respect to how they transform rainfall to runoff and with respect to their geology. Unfortunately such

10 data sets are rare and we hence have only a set of three catchments in each of the two geological groups. We hope that this clarifies the choice of catchments we made in our MS.

**3.** SG: Unfair comparison of the indices: I found the comparison of the topographical indices unfair. To my point of view, TWI was not intended to be used with very high resolution DEM as currently is available. The assumptions of the TWI is based on

- 15 the unit length and that is the case when the changes across length is minimal. This assumptions is not valid anymore when moving to a very high resolution DEM. For example, a very dry cell can be located near a very wet cell (Figure 13-d of Gharari et al., 2011). The suggestion from Hjerdt et al., 2004 is also aligned with this the mentality of relaxing the assumption of TWI from local slope to more of a transect slope which is more representative. Moreover, TWI, HAND etc are resolution dependent this should also be addresses. I am not asking the authors to repeat the study for different resolutions but it may
- 20 have some benefit mentioning this aspect in the discussion part.

**RL:** We agree with the second point in the comment related to the importance of resolution when we estimate HAND or DUNE. As DUNE is an energy-centered re-interpretation and enhancement of HAND the same findings already published which are valid for HAND should also be valid for DUNE. We furthermore agree that also the TWI is sensitive to the DEM resolution, similar as DUNE and HAND. We will mention this in the revised manuscript, but would like to stress that a

- 25 complete sensitivity study is beyond the scope of this MS (*page 14 line 16 22*). We, however, respectfully disagree with the first comment related to the chosen indices. We added the TWI to our MS as it is frequently used in numerous hydrological studies and is still one of the most prominent similarity indices in Hydrology. We don't think our comparison is unfair and even discuss that the TWI might be superior to DUNE or HAND in some environments. We would also like to highlight that the intention of the MS is not to show that one index is in general superior
- 30 to another but rather more or less appropriate for different environments and research questions. We will explain this better in a revised MS (*page 4 line 20 28; page 18 line 27-29*).

**4.** SG: Novelty and viability of DUNE is not justified: both HAND and DUNE and other topographical indices are valuable information on the catchment scale routing and runoff generation mechanism however the real applicability needs more justification and larger sample of headwater catchments. I am not sure if DUNE is a new concept.

RL: The novel aspect of DUNE is to merge the energetic driver i.e. the potential energy difference and an estimator of the dissipative losses along the flow path into the same index. In general, flow will not simply follow the steepest descent but the path which minimizes dissipative losses. The latter is of course a function of surface roughness and thus landuse or subsurface hydraulic conductivity along the flow path as well as the length of the flow path (please see page 6 line 15-22). As our study areas are located in a homogeneous geological and hydro-pedological setting, we can straightforwardly compare differences in dissipation by comparing different distances to the next drain. Rodríguez-Iturbe et al. (1992) and Hergarten et al. (2014)

10 argue along similar lines when setting energy expenditure in the stream/ preferential flow network as proportional to the total flow path length. In catchments with a diverse and complex geological setup this is not so straightforward. We will discuss our assumptions and the limitations better in the revised manuscript.

Furthermore, if the reviewer knows of a previously introduced index similar to DUNE, we will be happy to add and discuss this reference in detail in our study. However, we would like to stress once more that the merits of this MS are an energy-

15 centered re-interpretation and enhancement of HAND and we see DUNE rather as an extension to HAND (as we state twice in our MS) (*page 4 line 20 – 28; page 18 line 27-29*).

*SV:* To me, DUNE is as simple as slope and in fact it is a, kind of, average slope compared to the river network. The assumption to derive the DUNE are oversimplified. For example the available energy in the catchment is not the same as HAND. It would

- 20 be the case if the catchment was impervious with no soil, porous medium, and vegetation. However the state of catchment energy is less due to the negative pressure or potential energy of the unsaturated soil. This is explained by the authors in their recent work also (Zehe et al., 2019). Moreover, the authors have mentioned that the similar indices have previously been proposed. So my question is to what degree DUNE bring us closer to better estimation of the runoff regime. What is the added value? This needs to be clearly justified and embolden.
- 25 RL: Good point. We agree that not all forms of energy available in a catchment are considered in DUNE (such as chemical energy, or capillary energy stored in dry porous media etc). However, as we explain in section 2.1, DUNE is not about a complete energy balance of a catchment but is focused on the energy conversion associated with rainfall-runoff processes. We discuss this in detail in section 2.1 "Energy balance of streamflow generation" where we also introduce and explain which forms of energy we are considering to be relevant for DUNE. Furthermore, do we agree that the assumptions made in our MS
- 30 to derive DUNE amount to a strong simplification; these are necessary as our goal is to use only information about the topography stored within a DEM. However, we also think that our simplifications are founded on physical explanations that are discussed in detail in our MS (*page 7 line 9-10; page 11 line 11 12*).

5. SV: Correlation is not causation: Without going into too philosophical discussion here, I would say the inference of the result based on the correlation of the DUNE and its hydrological regime might not be a good estimator of the causation, in this case processes. Information theory (binning), with all it power, as used here is not really about the internal processes of the system. In this study also the authors are looking directly to the output of the system rather than the internal behavior. My

5 questions then how the internal behavior can be differentiated using DUNE. As an example, the internal preferential pathway might be different in the two geology; does DUNE able to reflect on those differences?

**RL:** This is an important point and valuable comment and also reviewer 1 raised a similar question (please see the answer to reviewer 1 comment no. 3 and 4.). In the revised MS we will discuss the connection of DUNE and the hydrological functioning of the catchments in more detail and explain how high or low DUNE-values could be interpreted. Furthermore we agree that

10 it only makes sense to use DUNE in landscapes in which the topography is a relevant control on the rainfall-runoff transformation as DUNE is based purely on information about the topography (please also see page 15 line 2-6). We will improve the according section (*page 8 line 1 – 6; page 16 line 8 - 15*).

6. SV: Applicability of the DUNE for practical purposes: I totally missed how DUNE can help hydrological modeling or hydrological understanding in comparison with other topographical indices. This needs to be further clarified. For example, pervious works from TU Delft group, have shown the added benefit of topography in inclusion of more hydrological knowledge (true or not that was proposed and implemented [I think it was right], Savenije 2010, Gharari et al., 2014, Gao et al., 2014). Similarly, TWI is used as a basis of the TOPMODEL. How do the authors would like to use DUNE in hydrological modeling? RL: Please see answer to comment no. 4.

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7. SV: Information content is all binning, how can information/distribution be compared with various bin sizes: To me information content is only about the binning of the available data into designed bins. Basically information gained by a data is to which degree we can justifiably discriminate them. Did the authors really looked into the information content here? I don't see that, so why they have discussed the bin sizes for this study? And do different bin sizes change the conclusion for the

25 various indices?

**RL:** We are sorry that this was not clear in our MS. We use the same bin sizes to compare the different distributions following the instructions given by Gong et al. 2014 and Scott 1979. We stated in our MS page 9 line 8 :

"In our study, however, the optimal bin width turns out to be different for each distribution as a result of its shape and the number of samples (size of the catchment). This is inconsistent with the need to use the same binning for each case to facilitate

30 comparisons of the different distributions. Accordingly, we decided to use only the largest bin width calculated for each similarity index "

We will add a section to the appendix where we show that by changing the bin size in a meaningful range we do not change our overall results and conclusion. Thank you for this comment (*page 19 line 15 - 19*).

8. SV: Other topographical indices: as the authors rightly mentioned there are more than DUNE out there. Can the authors tell the same story using another topographical index? For example, and to what I see from the Figure 2, does the average slope or river network density have the power to make the distinction between the hydrological regime similar to median of DUNE values? Visually it seems that it has.

5 **RL:** This is an important point and indeed there are plenty of other topographic indices, as we also stated in our introduction. In a revised MS we will stress that we do not claim to have tried all topographic indices available but once more that our goal is an energy-centered re-interpretation of HAND (*page 4 line 20 – 28; page 18 line 27-29*).

9. SV: No substantial conclusion: it seems that the conclusion can be written even without looking at the manuscript. The
10 conclusion is very general and it lack any point. I strongly would ask the authors to include bullet point conclusions that reflect the manuscript finding in a one to one fashion. One more time here, the authors have talks about the system architecture, my question how DUNE can reflect on the system architecture hypothesis. Any suggestion on that should be presented in the introduction, methodology and discussion part.

RL: Respectfully, we do not agree with this assessment of the second reviewer. We think that our conclusion is appropriate
and well justified for our MS. We appreciate the reviewers opinion and expertise, but prefer not to use bullet points in our conclusion.

10. SV: Literature review: Please include all the relevant work and their context in the study. A coherent story is needed instead of just mentioning a sentence from each study in isolation. Each sentence should also be discussed in its proper Section.

20 For example and after reading the paper I was surprised that the authors have mentioned more similar work that has been done in the past. Moreover, a more comprehensive literature review on the effect of soil, topography are needed for this study and its application. As an example, author can take a look at the reference of Gharari et al., 2011, 2014, Fang et al., 2019. RL: Thank you we will have a look at the proposed literature.

# 25 **References:**

Hergarten, S., Winkler, G. and Birk, S.: Transferring the concept of minimum energy dissipation from river networks to subsurface flow patterns, Hydrol. Earth Syst. Sci., 18(10), 4277–4288, doi:10.5194/hess-18-4277-2014, 2014. Rodríguez-Iturbe, I., Rinaldo, A., Rigon, R., Bras, R. L., Marani, A. and Ijjász-Vásquez, E.: Energy dissipation, runoff production, and the three-dimensional structure of river basins, Water Resour. Res., 28(4), 1095–1103,

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