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Complementary principle of evaporation: From original linear relationship to generalized nonlinear functions

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Abstract. The complementary principle is an important methodology for estimating evaporation. Throughout the 56-year development, related studies have shifted from adopting a symmetric linear complementary relationship (CR) to employing generalized nonlinear functions. Studies based on the linear CR have been maintained for a long time by rationally formulating the potential (E_{po}) and "apparent" potential evaporation (E_p) and/or employing an asymmetric parameter. These works have also advanced two types of generalized nonlinear complementary functions by invoking the boundary conditions. The first type inherits the concepts of three types of evaporation yet still requires the prognostic modelling of E_{po}. Polynomial functions are derived and tested for this type of function. Meanwhile, the second type does not involve E_{po}, yet requires a diagnostic modelling of actual evaporation by using the radiation and aerodynamic components of the Penman (1948) equation as inputs. A sigmoid function is derived by satisfying the boundary conditions based on physical considerations. The generalized nonlinear functional approach has improved the understandings on the complementary principle, and shows potential in advancing the evaporation research. Further studies may cover several topics including boundary conditions, analytical forms, parameterization, and application.

1 Introduction

The complementary principle provides a framework for estimating terrestrial land surface evaporation by adopting routinely observed meteorological variables, and offers strong potential applications (Brutsaert and Stricker, 1979; Morton, 1983; McMahon et al., 2016). In this paper, the terms "evaporation" and "evapotranspiration" are considered equivalent. As its underlying physical basis, this principle describes the feedback of areal evaporation on evaporation demand (Bouchet, 1963; Brutsaert, 2015) as illustrated by the fact that reducing areal evaporation can make the overpassing air hotter and drier (Morton, 1983). Based on the complementary principle, Bouchet (1963) first proposed a complementary relationship (CR) among three types of evaporation (Brutsaert, 2015), namely, the actual evaporation (E) from an extensive landscape under natural conditions by relating the apparent potential evaporation (E_{pa}) of a small saturated surface inside the landscape that does not affect the overpassing air and the natural evaporation process, and the potential evaporation (E_{po}) that occurs from the same large-size

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Hydrology and Earth System Sciences Discussions
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surface of E when it is saturated. The original symmetric linear "complementary" relationship (Bouchet, 1963; Brutsaert and Stricker, 1979; Morton, 1983) evolved into an asymmetric linear relationship (Brutsaert and Parlange, 1998; Pettijohn and Salvucci, 2006; Szilagyi, 2007). However, its development and applications are hindered by the use of complex formulations of E_{po} and E_{pa} to retain the linear CR.

Recent studies have adopted the "generalized" complementary principle, which employs nonlinear functions instead of the linear CR (Han et al., 2012; Brutsaert, 2015; Han and Tian, 2018a). The generalized complementary function comes in two types, with the first inheriting the three types evaporation of linear CR yet adopts a polynomial function to describe their relationship (Szilagyi et al., 2017; Crago et al., 2016; Brutsaert, 2015), while the other does not use the concept of E_{po} yet uses a sigmoid function to describe the relationship among E, Penman's potential evaporation (E_{Pen}), and its radiation term (E_{rad}) (Han and Tian, 2018a; Han et al., 2012). The generalized complementary principle has received much attention for its promising applications in estimating evaporation (Liu et al., 2016; Szilagyi et al., 2016; Ai et al., 2017; Brutsaert et al., 2017; Zhang et al., 2017; Han and Tian, 2018a). However, the boundary conditions and proper mathematical forms of the generalized complementary functions are still under study (Han and Tian, 2018a; Crago et al., 2016; Ma and Zhang, 2017; Szilagyi et al., 2017).

In this review, we summarize the 56-year development of the complementary principle with a specific focus on its evolution from a symmetric linear CR to generalized nonlinear functions. We also compare the two types of generalized complementary functions, and discuss their future development.

2 Symmetric complementary relationship

2.1 Concept of complementary relationship

The concept of CR is illustrated in Figure 1. When the water availability of the landscape is not limited, E is assumed to proceed at E_{pa} and $E = E_{pa} = E_{po}$. Given that the surface dries with constant available energy, E and E_{pa} depart from E_{po} with equal yet opposite changes in fluxes and exhibit a CR as follows:

$$E_{pa} - E_{po} = E_{po} - E \quad . \qquad (1)$$

E_{pa} and E_{po} should be specified in Eq. (1). Bouchet (1963) assumed E_{po} to be half the input solar radiation. Morton (1976) calculated E_{pa} by using the modified Penman's (1948) equation proposed by Kohler and Parmele (1967)(E_{Pen}^{KP}), in which a constant vapor transfer coefficient was used to replace the wind function, and calculated E_{po} by using the Priestley–Taylor's (1972) equation (E_{PT}) for an extensive saturated surfaced with vanished advection. This method has been used to calculate monthly evaporation in large areas.

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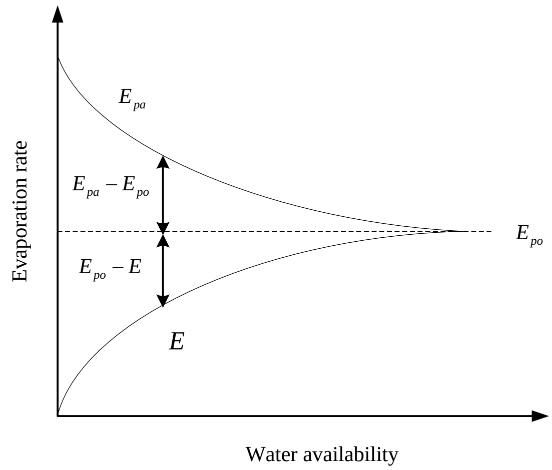

Figure 1. Schematic of symmetric CR

Table 1. Different formulations of E_{pa} and E_{po} in the CR

Types	E_{pa}	E_{po}	b	References
	E_{Pen}^{KP}	E_{PT}		Morton (1976)
	E_{Pen}	E_{PT}		Brutsaert and Stricker (1979)
	E_{Mor}	$E_{PT}^{T_p}$		Morton (1983)
Symmetric	$E_{Pen}^{r_s}$	$E_{Pen}^{r_s=0}$	1	McNaughton and Spriggs (1989)
	E_{Pen}	$E_{PT}+\|H_P\|$		Parlange and Katul (1992a)
	$E_{PM}^{r_{s\min}}$	E_{PT}		Pettijohn and Salvucci (2006)
	E_{Pen}	$E_{PT}^{T_{ws}}$		Szilagyi and Jozsa (2008)
	E_{Pan}	E_{PT}	b	Kahler and Brutsaert (2006)
	ET_0	E_{PT}	b	Han et al. (2014d)
Asymmetric	E_{MT}	E_{Pen}	$\gamma/\Delta(T_a)$	Granger (1989)
	E_{Pen}	E_{PT}	$\gamma/\Delta(T_s)$	Szilagyi (2007)
	E_{Pen}	$E_{PT}^{T_{ws}}$	$f(RH)$	Szilagyi (2015)

*The symbols can be referred to the main text.

Brutsaert and Stricker (1979) proposed the advection-aridity (AA) approach at a daily timescale, where E_{pa} and E_{po} are directly formulated by E_{Pen} and E_{PT}, respectively. Although various combinations of E_{pa} and E_{po} exist (Table 1), E_{po} is widely accepted to reflect the energy input while E_{pa} includes the drying power of air simultaneously (Bouchet, 1963; Morton, 1983; Lhomme and Guilioni, 2006). Therefore, the AA approach seems logical and convincing (Lhomme and Guilioni, 2006). This approach has been validated based on hourly (Parlange and Katul, 1992a; Crago and Crowley, 2005), daily (Brutsaert and Stricker, 1979; Ali and Mawdsley, 1987; Qualls and Gultekin, 1997), monthly (Xu and Singh, 2005; Lemeur and Zhang, 1990; Hobbins et al., 2001a), and annual (Ramirez et al., 2005; Yu et al., 2009) data from either plot-scale lysimeters and eddy-covariance meausresents or basin-wide water balance-derived results. By calculating E_{Pen} and E_{PT} using the standard meteorological data, the AA approach has been applied to estimate evaporation in various land covers and climatic regions (Hobbins et al., 2001a; Liu et al., 2006; Wang et al., 2011; Ozdogan and Salvucci, 2004). For instance, this approach has been applied in China from the Gobi Desert with a mean annual precipitation of less than 150 mm (Liu and Kotoda, 1998; Han et al., 2008; Lemeur and Zhang, 1990) to the humid Eastern China with an annual precipitation of approximately 1,800 mm (Xu and Singh, 2005). Note that however, the AA approach tends to overestimate E under wet environments but underestimate E under arid environments (Qualls and Gultekin, 1997; Hobbins et al., 2001a).

2.2 Proofs of complementary relationship

Bouchet (1963) and Morton (1965; 1970) approximately validated the CR by using annual and monthly data, respectively. At an annual scale, E and E_{pa} (which are represented by E_{Pen} or pan evaporation (E_{Pan})) were plotted against annual precipitation and their negative relationship was used as an evidence to support the reliable probability of the complementarity (Morton, 1983). Ramirez et al. (2005) tested the CR by using a composite of 192 data pairs from 25 basins across US, and claimed a direct observational evidence for the symmetry. Yu et al. (2009) examined the CR at 102 observatories across China and found the CR at low elevations. Su et al. (2015) also showed a negative correlation between E from atmospheric reanalysis data and E_{Pan} in the non-humid regions of China. The large scale irrigation development in an arid environment provides a large "natural" experimental area for validating the CR by the opposite changes in E and E_{pa} (Roderick et al., 2009). A study from Turkey revealed that the warm-season E_{pa} decreases progressively along with an increasing irrigation area (Ozdogan and Salvucci, 2004). Similar results were obtained from arid irrigation districts in Northwest China, where an increasing irrigation water consumption reduces E_{pa} (Han et al., 2014d) while a decreasing irrigation water consumption increases E_{pa} (Han et al., 2017). However, although these studies showed that E and E_{pa} move in opposite directions in most cases, there was not solid evidence to support the symmetric CR. A rigorous quantitative assessment of the symmetric CR was not conducted by Ramirez et al. (2005). Yu et al. (2009) found that the CR is asymmetric at high elevations. However, Ma et al. (2015) argued that the asymmetric CR in TP was mainly due to inappropriate parameterizations of the wind function in E_{Pen} , the wet environment air temperature and Priestley-Taylor coefficient in E_{PT} .

The plausibility of CR has also been validated on theoretical bases and has been mathematically rationalized by Bouchet (1963), Morton (1971, 1983), and Seguin (1975). The rationalization proposed by Morton (1971, 1983) considers governing the changes in the humidity and temperature of the equilibrium sublayer of the atmospheric boundary layer (ABL). Relaxing the assumption of Morton (1983), Szilagyi (2001) derived the CR by using the mass conservation equation for water vapor. However, by using a diagnostic model of the energy fluxes within a closed system, LeDrew (1979) argued that the CR proposed

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Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences Discussions
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by Morton (1971) is physically unrealistic and added that the complementarity of the negative relationship is not supported by any proof.

The physical basis of the CR has been further advanced by using climate models. McNaughton and Spriggs (1989) tested the CR by using a simple model of the atmospheric mixed layer with entrainment in which the latent heat of the surface is simulated by using the bulk mass transfer equation with bulk resistance. During the validation, E_{pa} is calculated via Penman's equation, which uses the temperature and humidity obtained from the results of the mixed-layer model corresponding to certain resistance ($E_{Pen}^{r_s}$), while E_{po} is calculated with the surface resistance set to zero ($E_{Pen}^{r_s=0}$). Kim and Entekhabi (1997) added the surface energy balance and atmospheric thermal radiation fluxes into the model to extend the study of McNaughton and Spriggs (1989). By using the Penman–Monteith equation to govern the areal latent heat flux at the surface, Lhomme (1997a) proposed a closed-box model with an impermeable lid at a fixed height while Lhomme (1997b) used a realistic open-box model of the ABL with entrainment to assess the CR. Sugita et al. (2001) tested the CR by using a modified version of Lhomme (1997b) model, which was calibrated by using a dataset obtained from the Hexi Corridor desert area in Northwest China. But a strict symmetric CR wss hardly confirmed by these studies.

3 Efforts in maintaining a linear complementary relationship

3.1 Rational formulation of E_{pa} and/or E_{po}

The imperfect asymmetric CR has inspired researchers to apply a rational formulation of E_{pa} and/or E_{po} for retaining the symmetry of CR. One direct method is to improve the formulations of E_{Pen} and/or E_{PT} based on the AA approach through calibration. For E_{Pen}, the empirical wind function was calibrated to improve the CR (Hobbins, 2001). However, Penman's wind function cannot work under wet and dry conditions simultaneously (Pettijohn and Salvucci, 2006). The wind function derived from Monin–Obukhov's similarity theory was then employed (Crago and Crowley, 2005; Parlange and Katul, 1992b; Pettijohn and Salvucci, 2006; Ma et al., 2015a). The surface roughness and surface albedo were also calibrated to improve the CR (Lemeur and Zhang, 1990). Meanwhile, for E_{PT}, the Priestley–Taylor coefficient (α) is regarded varying, thereby leaving a range for calibration (Han et al., 2006; Yang et al., 2012; Xu and Singh, 2005). In addition to E_{Pen} and E_{PT}, the mass-transfer type potential evaporation (van Bavel, 1966) (E_{MT}) was considered another formulation of potential evaporation (Granger, 1989). Different combinations of E_{pa} and E_{po}, (i.e., E_{Pen}, E_{PT}, and E_{MT}) were tested through the trial-and-error method proposed by Crago and Crowley (2005). The local re-parameterizations and/or calibrations have significantly improved the evaporation estimation (Hobbins et al., 2001b; Xu and Singh, 2005; Ma et al., 2015a). However, the calibration approach and trial-and-error process are deemed ineffective because of their high computation demand, which is a key stumbling block when applying the CR in large-scale (e.g., continental or global) E modelling (Ma et al., 2019).

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Given the conceptual problems in using E_{Pen} and E_{PT} to denote E_{pa} and E_{po} (Morton, 1983; Szilagyi and Jozsa, 2008), a better CR must be obtained by modifying the formulations of E_{Pen} and/or E_{PT} on physical basis. For E_{pa}, the net long-wave radiation depends on the land surface temperature; meanwhile, adjusting surface temperature with air temperature (T_a) to calculate solar radiation in E_{Pen} may be problematic (Morton, 1983). To address these limitations, Morton (1983) combined the energy balance and water vapor transfer equations by using an equilibrium temperature (T_p) and derived a Morton-type potential evaporation E_{Mor} to denote E_{pa}. By attributing the asymmetry to the assumption that E_{pa} conceptually includes a transpiration component, Pettijohn and Salvucci (2006) improved the asymmetry by replacing E_{Pen} with the Penman–Monteith equation with a minimum surface resistance ($E_{PM}^{r_{s\min}}$). Silimarly, the reference evapotranspiration (ET_0) was also used to replace E_{Pen} (Han et al., 2014d; Han et al., 2017).

E_{PT} was proposed by Priestley and Taylor (1972) to represent evaporation from extensive saturated surfaces and has been widely used (Brutsaert, 1982; Priestley and Taylor, 1972). This way it could be used to represent E_{po} (Brutsaert and Stricker, 1979). The AA approach calculates E_{PT} by using the atmospheric variables that correspond to the current natural landscape. However, the atmosphere in contact with the land surface will change if the land surface was brought into saturated (Morton, 1983; Brutsaert, 2015). Therefore, E_{PT} cannot represent the "true" E_{po} since the slope of the saturation vapor pressure at the current air temperature ($\Delta(T_a)$) is imperfect because the temperature corresponding to E_{po} is different from that corresponding to a non-saturated environment (Morton, 1983; Szilagyi and Jozsa, 2008). Moreover, E_{PT} does not fully consider the effects of advection, which are inevitable in reality (Morton, 1983, 1975; Parlange and Katul, 1992a). To this end, Morton (1983) derived E_{po} by using a modified Priestley–Taylor equation with net radiation and the slope of the saturation vapor pressure that is calculated at equilibrium temperature T_p ($E_{PT}^{T_p}$). The effects of advection were considered by an empirical correction factor in $E_{PT}^{T_p}$ (Morton, 1975, 1983). Parlange and Katul (1992a) attributed the asymmetry to the horizontal advection of dry air, which would make E_{Pen} larger than the available energy ($R_n - G$) (i.e., $R_n - G - E_{Pen} < 0$) and proposed to replace E_{PT} with $E_{PT} + |R_n - G - E_{Pen}|$ to improve the CR on an hourly basis. Szilagyi and Jozsa (2008) argued that Δ in E_{PT} should be calculated at the air temperature corresponding to the wet environment (T_{wa}) instead of actual air temperature. While it is not straightforward to derive T_{wa}, Szilagyi and Jozsa (2008) proposed an iterative approach based on the Bowen ratio method for a small wet patch to estimate the surface temperature under wet environments (T_{ws}). By assuming a negligible temperature gradient over such a small wet area, T_{wa} is approximately equal to T_{ws}. In this way, they replaced $\Delta(T_a)$ with the slope of the saturation vapor pressure curve at T_{ws} ($\Delta(T_{ws})$) in the Priestley–Taylor equation ($E_{PT}^{T_{ws}}$). Besides, $E_{PT}^{T_{ws}}$ was used to improve the symmetry of the CR in arid shrubland environments (Huntington et al., 2011) and in an alpine steppe of the

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences
Discussions

Tibetan Plateau (Ma et al., 2015a). The evaporation estimations across the US were also improved by applying the modified AA approach (Szilagyi and Jozsa, 2008; Szilagyi et al., 2009; Szilagyi, 2015).

3.2 Asymmetric linear complementary relationship

With E_{pa} and E_{po} denoted by E_{MT} and E_{Pen}, respectively, Granger (1989) proposed an alternative CR as follows:

$$(E_{MT} - E_{Pen}) = \frac{\Delta(T_a)}{\gamma}(E_{Pen} - E), \quad (2)$$

where γ is the psychrometric constant. Despite being identical to the surface energy balance, Eq. (2) has inspired researchers to examine whether the CR should be symmetric (Szilagyi, 2007; Pettijohn and Salvucci, 2006). By using pan evaporation to denote E_{pa}, Brutsaert and Parlange (1998) extended the symmetric CR as follows:

$$(E_{pa} - E_{po}) = b(E_{po} - E), \quad (3)$$

where b is the coefficient that denotes asymmetry. Kahler and Brutsaert (2006) clarified and tested the validity of Eq. (3) at a daily timescale and attributed the asymmetry to the nature of the heat transfer between the pan and its surroundings, which made the changes in E_{pan} larger than those in E. Szilagyi (2007) showed that the asymmetry is not limited only to E_{pan} but is inherently linked to the definition of E_{pa}. The asymmetric CR is widely used, and Brutsaert (2015) stated that asymmetry is an inherent characteristic of the CR. The parameter b is often considered a calibrated parameter in evaporation estimation (Ma et al., 2015a; Szilagyi, 2015), and its controlling factors were also detected (Szilagyi, 2015; Lintner et al., 2015; Szilagyi, 2007).

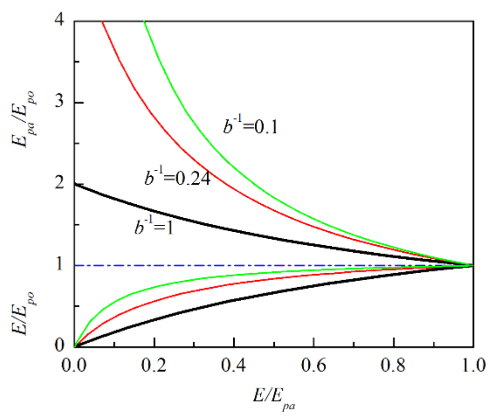

Figure 2. Scaled E_{pa} and E, which serve as functions of the evaporative moisture index E/E_{pa} and calculated on the basis of the asymmetric CR.

The asymmetric CR can be illustrated in a dimensionless form (Figure 2) (Kahler and Brutsaert, 2006). Normalized by E_{po}, E_{pa} and E can be scaled as

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences Discussions

$$\frac{E}{E_{po}}=\frac{(1+b)E/E_{pa}}{1+bE/E_{pa}} \text{ and } \frac{E_{pa}}{E_{po}}=\frac{1+b}{1+bE/E_{pa}}. \tag{4}$$

The scaled E_{pa} and E are both functions of the dimensionless variable E/E_{po}, while E/E_{po} serves as the evaporative surface moisture index. Compared with the original form (Eq. (1) and Figure 1), the CR here is illustrated without the appearance of the water availability explicitly.

4 Generalized complementary principle via nonlinear functions

4.1 Normalized complementary functions

Unlike the normalization by E_{po} (Kahler and Brutsaert, 2006), Han (2008) normalized Eq. (3) by using E_{pa} and found that E/E_{pa} is expressed as a linear function of E_{po}/E_{pa}. Normalized by E_{Pen} (Han et al., 2008), the AA approach can be expressed as

$$\frac{E}{E_{Pen}}=\alpha(1+\frac{1}{b})\frac{E_{rad}}{E_{Pen}}-\frac{1}{b}, \tag{5}$$

where E/E_{Pen} is regarded as a linear function of E_{rad}/E_{Pen}. The bias of the AA function under arid and wet environments can be easily understood in its dimensionless form, but the AA approach with a tuned *b* still underestimated evaporation in arid environments (Han et al., 2008). The work of Crago and Brutsaert (1992) in Kansas during FIFE 1987 revealed that the parameter *b* are obviously different for days with differing degrees of soil moisture. These studies imply that the CR may deviate from its linear characteristics.

The CR model proposed by Granger (1989) based on Eq. (2) has demonstrated promising application across different land covers and regional climate conditions (Carey et al., 2005; Granger, 1999; Granger and Gray, 1989b; Pomeroy et al., 1997; Xu and Singh, 2005). In fact, the relationship between relative evaporation and relative drying power plays a key role in reflecting the dryness of the surface (Granger and Gray, 1989a). Normalized by E_{Pen}, Granger's model is similar to the AA function in that E/E_{Pen} is expressed as a function of the relative magnitude of drying power to net radiation (Han et al., 2011). By synthesising the dimensionless forms of the AA function and the Granger's model, Han et al. (2011) proposed the following logistic function as an alternative:

$$\frac{E}{E_{Pen}}=\frac{1}{1+c_1 e^{d(1-\frac{E_{rad}}{E_{Pen}})}}, \tag{6}$$

where c_1 and *d* are the parameters. Eq. (6) approximates the linear AA function under normal conditions neither too wet nor too dry but amends its bias (Han et al., 2011).

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences
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EGU
Discussions

Actual evaporation can be estimated using routinely measured meteorological data by using the climatological resistance to parameterize the bulk surface resistance in the Penman–Monteith equation (Liu et al., 2012; Rana et al., 1997; Katerji and Perrier, 1983; Ma et al., 2015b). A linear relationship between the ratio of surface resistance to aerodynamic resistance and the ratio of climatological resistance to aerodynamic resistance was proposed by Katerji and Perrier (1983). Han et al. (2014c) integrated this linear relationship into the Penman–Monteith equation and derived a dimensionless form via normalization by E_{Pen}:

$$\frac{E}{E_{Pen}} = \frac{1}{1 + k(\frac{E_{Pen}}{E_{rad}} - 1) + l}, \quad (7)$$

where k and l are the empirical calibration parameters. With similar variables yet different mathematical formulations, Eq. (7) can also be considered a complementary function (Han et al., 2014c).

4.2 Sigmoid function relating E/E_{Pen} to E_{rad}/E_{Pen}

By synthesizing the aforementioned three functions, Han et al. (2012) generalized the CR as a function that relates E/E_{Pen} to E_{rad}/E_{Pen}:

$$E/E_{Pen} = f(E_{rad}/E_{Pen}). \quad (8)$$

Eq. (8) shares the same form of Penman's approach for estimating evaporation. The function of surface wetness that denotes the reduction of E to E_{Pen} is replaced by the function of E_{rad}/E_{Pen}, which is termed "atmospheric wetness" (Han and Tian, 2018b). Despite not explicitly exhibiting a CR, Eq. (8) holds the complementary principle that the land surface wetness is indirectly denoted by the drying power of air with a constant radiation energy input (Brutsaert, 1982). Accordingly, Eq. (8) is considered a "general form" of the CR (Han et al., 2014b) (hereinafter referred to as H12). The existing analytical forms of the function can be classified into linear, concave, or sigmoid (Table 1 in Han and Tian (2018a)). Studies on the complementary principle can be advanced by formulating a proper analytical form for H12.

The exact analytical form of H12 is inadequately understood at present. However, some of its characteristics can be detected from its boundary conditions under extremely arid and completely wet environments. Han et al. (2012) derived the zero-order and first-order boundary conditions for H12 as

$$\begin{cases} y_H = 0, \ x_H \to 0 \\ y_H = 1, \ x_H \to 1 \\ \frac{dy_H}{dx_H} = 0, \ x_H \to 0 \\ \frac{dy_H}{dx_H} = 0, \ x_H \to 1 \end{cases}, \quad (9)$$

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences
Discussions

where $x_H = E_{rad}/E_{Pen}$ and $y_H = E/E_{Pen}$. Han et al. (2012) proposed the following sigmoid function (hereinafter referred to as H2012):

$$\frac{E}{E_{Pen}} = \frac{1}{1+m(\frac{E_{Pen}}{E_{rad}}-1)^n}, \tag{10}$$

where *m* and *n* are parameters. The linear AA and nonlinear H2012 have been compared in a 2D space (E_{rad}/E_{Pen}, E/E_{Pen}) (Han et al., 2012). The results obtained from an extremely dry desert and a wet farmland reveal that the sigmoid H2012 corrects the bias of the linear AA and Equation (6) (Han et al., 2012); the application of this sigmoid function has also been recommended for an alpine meadow region of the Tibetan Plateau (Ma et al., 2015b).

Table 2. Different forms of the generalized complementary function, $y = f(x)$

Form	Specific function	E_{po}	x	y	Typical type	Reference
H12*	H2017	Not involved	$\frac{E_{rad}}{E_{Pen}}$	$\frac{E}{E_{Pen}}$	Sigmoid	Han et al. (2018)
	B2015	E_{PT}	$\frac{\alpha E_{rad}}{E_{Pen}}$	$\frac{E}{E_{Pen}}$	4-order polynomial	Brutsaert (2015)
B15*	C2016	$E_{PT}^{T_{ws}}$	$\frac{E_{PT}^{T_{ws}}/E_{Pen} - E_{PT}^{T_{ws}}/E_{MT}^{\max}}{1-E_{PT}^{T_{ws}}/E_{MT}^{\max}}$	$\frac{E}{E_{Pen}}$	Linear	Crago et al. (2016)
	S2017	$E_{PT}^{T_{ws}}$	$\frac{E_{Pen}^{\max}-E_{Pen}}{E_{Pen}^{\max}-E_{PT}^{T_{ws}}}\frac{E_{PT}^{T_{ws}}}{E_{Pen}}$	$\frac{E}{E_{Pen}}$	3-order Polynomial	Szilagyi et al. (2017)

*H12 and B15 denote $E/E_{Pen} = f(E_{rad}/E_{Pen})$ and $E/E_{pa} = f(E_{po}/E_{pa})$, respectively.

The zero-order arid boundary condition of H12 adopted in H2012 may be problematic in the sense that the aerodynamic term (E_{aero}) of E_{Pen} may not reach infinity under an arbitrary E_{rad} (Crago et al., 2016; Szilagyi et al., 2017; Kovács, 1987). Moreover, E_{rad}/E_{Pen} cannot easily approach unity because of advection (Kovács, 1987; Priestley and Taylor, 1972). Therefore, Han and Tian (2018a) brought in the minimum and maximum limits of E_{rad}/E_{Pen} ($x_{\min}$ and $x_{\max}$) under an assumed constant E_{rad} and re-derived the boundary conditions of H12 by adopting two widely accepted assumptions following Penman's combination theory, namely, $\partial E/\partial E_{Pen} = 0$ under extremely arid environments and $E = E_{Pen}$ under completely wet environments. The boundary conditions are set as follows:

https://doi.org/10.5194/hess-2019-545
Preprint. Discussion started: 25 November 2019

Hydrology and Earth System Sciences Discussions

$$\begin{cases} y_H = 0, \ x_H \to x_{\min} \\ y_H = 1, \ x_H \to x_{\max} \\ \dfrac{dy_H}{dx_H} = 0, \ x_H \to x_{\min} \\ \dfrac{dy_H}{dx_H} = 0, \ x_H \to x_{\max} \end{cases} . \quad (11)$$

Based on the boundary conditions, Han and Tian (2018a) found that the growth of E/E_{Pen} upon E_{rad}/E_{Pen} exhibits a sigmoid feature, which is a three-stage pattern in which E/E_{Pen} gradually increases along with E_{rad}/E_{Pen}, rapidly increases along with E_{rad}/E_{Pen} in the following stage, and then demonstrates a decelerated growth in the final stage. The sigmoid feature can be detected from the study by Han et al. (2012) in the arid Gobi-HEIFE site and the humid Wudaogou site in China. Han and Tian (2018a) further validated the sigmoid feature by using 22 eddy covariance towers from the FLUXNET (Baldocchi et al., 2001) dataset which includes representative biomes of grasslands, croplands, shrublands, evergreen needleleaf forests, deciduous broadleaf forests, and wetlands.

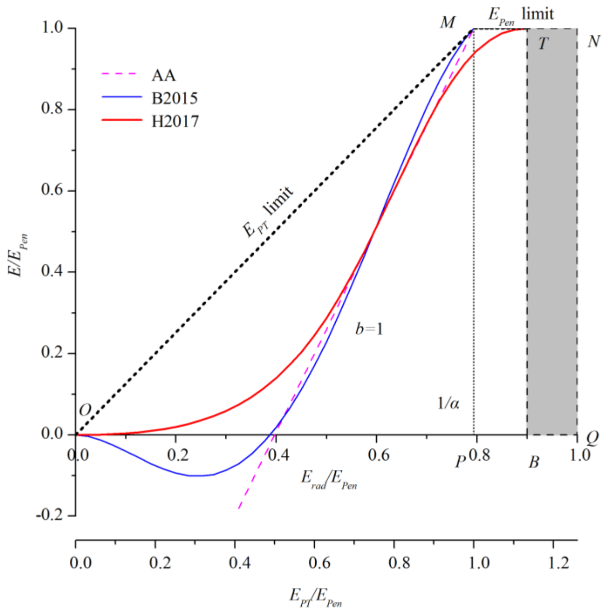

Figure 3. Generalized complementary functions in the state space (E_{rad}/E_{Pen}, E/E_{Pen}): linear AA, polynomial B2015, and sigmoid H2017, with $\alpha = 1.26$ and b=1. $x_{\min}$ and $x_{\max}$ are set to 0 and 0.9, respectively. *OM* is the edge at which $E = E_{PT}$, *M* corresponds to the condition of the minimal advection evaporation where $E_{PT} = E_{Pen}$, *MN* is the edge where $E = E_{Pen}$, and *N* corresponds to the condition of the equilibrium evaporation where $E_{Pen} = E_{rad}$.

Han and Tian (2018a) proposed the following new sigmoid function to accordance with the boundary conditions (hereinafter referred to as H2017):

Hydrology and Earth System Sciences
Discussions

$$\frac{E}{E_{Pen}} = \frac{1}{1 + m\left(\frac{x_{\max} - E_{rad}/E_{Pen}}{E_{rad}/E_{Pen} - x_{\min}}\right)^{n}}, \tag{12}$$

where E_{rad}/E_{Pen} adopts the feasible domain ($x_{\min}$, $x_{\max}$), which is a subdomain of (0, 1). Both the linear AA function and H2012 are special cases of H2017. Han and Tian (2018a) performed a first-order Taylor expansion of Eq. (12) at the point where $y = 0.5$ and set the linear equation equivalent to the linear AA function. Afterward, the parameters *m* and *n* of H2017 can be transformed from the Priestley–Taylor coefficient α and parameter *b* of the AA function.

4.3 Polynomial function relating E/E_{pa} to Epo/Epa

Inspired by Han *et al.* (2012), Brutsaert (2015) reformulated another general dimensionless form of the CR, $E/E_{pa} = f(E_{po}/E_{pa})$, and proposed its boundary conditions as follows:

$$\begin{cases} y_B = 0,\ x_B \to 0 \\ y_B = 1,\ x_B \to 1 \\ \dfrac{dy_B}{dx_B} = 0,\ x_B \to 0 \\ \dfrac{dy_B}{dx_B} = 1,\ x_B \to 1 \end{cases}, \tag{13}$$

where $x_B = E_{po}/E_{pa}$ and $y_B = E/E_{pa}$. The following fourth-order polynomial function was also derived to satisfy the boundary conditions:

$$\frac{E}{E_{pa}} = (2-c)\left(\frac{E_{po}}{E_{pa}}\right)^2 - (1-2c)\left(\frac{E_{po}}{E_{pa}}\right)^3 - c\left(\frac{E_{po}}{E_{pa}}\right)^4, \tag{14}$$

where *c* is a parameter. Brutsaert (2015) regarded Eq. (14) (hereinafter referred to as B15) as a generalization of the linear CR and referred to the corresponding methodology as the "generalized complementary principle."

B15 is essentially different from H12, with completely different normalized variables. The boundary conditions of H12 are derived for $x_H = E_{rad}/E_{Pen}$ and $y_H = E/E_{Pen}$, whilst those of B15 are derived for $x_B = E_{po}/E_{pa}$ and $y_B = E/E_{pa}$ (Table 2). B15 inherits all three types of evaporation dated from the original CR, while the validity of its boundary conditions depends on the proper definitions of E_{pa} and E_{po}. Therefore, B15 still faces the problem of the original CR, that is, formulating E_{po} and E_{pa}. By contrast, only the mostly accepted E_{Pen} appears in H12, and the knowledge on E_{po} is unnecessary in deriving the boundary conditions and the analytical form of H12. By doing so, the corresponding theoretical and practical difficulties can be prevented.

The application of Eq. (14) depends on specific formulations of E_{pa} and E_{po}. In the manner of the AA approach, Eq. (14) has been applied to estimate evaporation (Brutsaert et al., 2017; Liu et al., 2016; Szilagyi et al., 2016; Zhang et al., 2017;

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Ai et al., 2017). In this case, we refer to Eq. (14) in the manner of the AA approach as B2015 to avoid confusion. Although estimating E_{pa} by using E_{Pen} is widely accepted by the research community, prognostically predicting E_{po} based on E_{PT} remains a huge challenge considering the theoretical problems of the Priestley-Taylor coefficient. In addition, the lower limit of $x_B \to 0$ of B15 may not hold in the manner of the AA approach (Kovács, 1987; Szilagyi et al., 2017; Crago et al., 2016; Han and Tian, 2018a). To address these challenges, Szilagyi et al. (2017); Crago et al. (2016) used the maximum value of E_{pa} to rescale x_B and replaced E_{PT} with $E_{PT}^{T_{ws}}$, the latter of which is based on the air temperature in a wet environment. Crago et al. (2016) applied a mass transfer approach to calculate the maximum value of E_{pa} ($E_{MT}^{\max}$) and rescaled x_B as

$$x_C = \frac{E_{PT}^{T_{ws}}/E_{Pen} - E_{PT}^{T_{ws}}/E_{MT}^{\max}}{1 - E_{PT}^{T_{ws}}/E_{MT}^{\max}} . \quad (15)$$

Szilagyi et al. (2017) employed the Penman equation to calculate the maximum value of E_{pa} ($E_{Pen}^{\max}$) and proposed the following rescaled version:

$$x_S = \frac{E_{Pen}^{\max} - E_{Pen}}{E_{Pen}^{\max} - E_{PT}^{T_{ws}}} \frac{E_{PT}^{T_{ws}}}{E_{Pen}} . \quad (16)$$

x_C and x_S are essentially same (Szilagyi et al., 2017) except for the different formulations for the maximum value of E_{pa}. However, $E_{MT}^{\max}$ may became invalid under conditions with relatively strong available energy yet weak winds (Ma and Zhang, 2017). In the latest version of C16, $E_{MT}^{\max}$ is replaced with $E_{Pen}^{\max}$ (Crago and Qualls, 2018). After rescaling, Crago et al. (2016) proposed a new linear version of the generalized complementary function (C2016) (i.e., $y_B = x_C$; Table 2), while Szilagyi et al. (2017) used the third order polynomial function (S2017) by replacing B15 with c=0. With the same independent variable yet different functions (Table 2), C2016 and S2017 demonstrate improvements in their evaporation estimation performance (Szilagyi et al., 2017; Crago et al., 2016; Crago and Qualls, 2018).

4.4 Comparisons between different analytical forms

The linear AA, the polynomial B2015, and the sigmoid H2017 are three analytical forms of H12 (Table 1 in Han and Tian (2018a)). Han and Tian (2018a) compared them in the state space (E_{rad}/E_{Pen}, E/E_{Pen}) (Figure 3). The original CR adopts the limits of E_{pa} and E_{po} on E in a series ($E \le E_{po} \le E_{pa}$) (Brutsaert, 2015) while considering that the wet regional evaporation must always be smaller than the wet patch evaporation ($E_{po} \le E_{pa}$). The limits are deemed appropriate if the exact formulations for them have been derived. Following the AA approach, the limits on the actual evaporation are expressed as $E \le E_{PT} \le E_{Pen}$, which requires $E_{rad}/E_{Pen} \le 1/\alpha$. Thus, the curves of AA and B2015 are constrained by the limits *OM* and *MP* (Figure 3). However, considering that E_{Pen} may become smaller than αE_{rad} if a certain constant α is applied. In this case, the limits $E \le E_{PT} \le E_{Pen}$ (with a constant Priestley-Taylor coefficient) may be unreasonable. Han and Tian (2018a) showed that the upper limits of E_{Pen} and E_{PT} on evaporation must be in parallel, that is, $\begin{cases} E \le E_{Pen} \\ E \le E_{PT} \end{cases}$. Therefore, the complementary curves should be constrained by the limits of *OMN* (Figure 3). The limits of E_{Pen} and E_{PT} on E can be approximately satisfied by H2017 with the parameters transformed from the linear AA function (Han and Tian, 2018a). By contrast, B2015 with $c < -1$ produces a physically unreasonable E that is larger than E_{PT} (Han and Tian, 2018a).

In the state space (E_{rad}/E_{Pen}, E/E_{Pen}), the curve of the sigmoid H2017 exhibits a three-stage pattern. The linear AA and polynomial B2015 have one and two stages respectively. As it is difficult for one site to cover all the three stages with a wide range of wetness, the linear AA can effectively represent the complementary curve under normal conditions falling in the middle stage. The polynomial B2015 is effective if the first two stages exist. Given that the third stage under a wet environment is uncommon, the polynomial B2015 performs well with calibrated parameters (Brutsaert et al., 2017; Liu et al., 2016; Zhang et al., 2017; Han and Tian, 2018a). However, the sigmoid H2017 shows the best performance in estimating evaporation as validated by using data from FLUXNET (Han and Tian, 2018a).

4.5 Improved understanding on the correlation between actual and potential evaporation

Interpreting the changes in E based on the trends in E_{Pen} (or pan evaporation) greatly relies on the understanding of whether the correlation between E and E_{Pen} is positive or negative. The corresponding confusion has resulted in a discrepancy between the Penman hypothesis and the complementary principle (Yang et al., 2006) and encouraged debates on whether the increasing or decreasing trend in E corresponds to reductions in the observed pan evaporation in the past (Brutsaert and Parlange, 1998; Roderick and Farquhar, 2002; Roderick et al., 2009; Wang et al., 2017). According to the symmetric CR, E_{Pen} would be negatively correlated with E when the energy input is constant (Morton, 1983). Based on the asymmetric linear CR, Brutsaert and Parlange (1998) stated that the decreasing E_{pan} can be used to indicate an increasing E in water-limited regions. However, the interpretation is not general (Roderick et al., 2009) because of the inherent weakness of the linear CR (Han et al., 2014b).

A proper generalized complementary function offers advantages in assessing the correlation between E and E_{Pen} while considering the different impacts of E_{rad} and E_{aero} (Hobbins et al., 2004). Han et al. (2014b) proposed a systematic and analytical approach for evaluating the correlation between E and E_{Pen} by establishing a linear regression between them via H2012. Han et al. (2014b) also demonstrated that the relative variation of E_{rad} and E_{aero} (which significantly vary at different timescales) as well as water availability (which varies across different climate regions) are two factors that affect the correlation between E and E_{Pen}. With obvious variations in E_{rad} and a relative stable E_{aero}, which are commonly observed on a diurnal or intra-annual basis, the influence of E_{rad} becomes more significant and E is always positively correlated with E_{Pen}. Under conditions where the variations in E_{rad} and E_{aero} are comparable or when E_{aero} obviously varies (which tends to occur on a daily or annual basis), the influence of E_{aero} comes to force. As a result, the correlation between E and E_{Pen} changes from negative to positive along with increasing water availability. The theoretical results were validated in a grassland site in Northeast China, and can rationally interpreted the trends in E over China (Han et al., 2014b).

5 Future developments for the generalized complementary principle

5.1 Boundary conditions under completely wet condition

Boundary conditions are crucial to the derivations of the generalized complementary functions. Under completely wet environment, H12 adopts $dy_H/dx_H = 0$ for the first-order wet boundary condition, whereas B15 adopts $dy_B/dx_B = 1$, which results in the different types of H12 and B15. It should be noted that the boundary conditions of B15 were derived about theoretical wet patch evaporation E_{pa} and wet regional evaporation E_{po}, and their validity depends on proper definitions of E_{pa} and E_{po}. So, future studies can be conducted towards more proper formulations of E_{pa} and E_{po} to satisfy the boundary conditions of B15.

However, the boundary conditions of H12 were derived about E_{Pen} and it radiation term. The boundary conditions of B15 are only comparable to those of H12 if it is in the specific form of B2015. Han and Tian (2018a) found that the first-order wet boundary conditions of H12 and B2015 are two possible solutions of the assumption that actual evaporation proceeds at E_{Pen}, and they discarded each other. However, there was not perfect instance to demonstrate H12's first-order wet boundary condition. Thus, the controversies on the first-order wet boundary condition (Szilagyi and Crago, 2019; Han and Tian, 2019) require further studies from both the theoretical and practical aspects. Thus, the flux data of sites over lakes or wetlands need to be well examined.

5.2 Parameterizations of generalized complementary functions

Determining the parameters of the generalized complementary functions is the urgent work for the application of B2015 and H2017 for evaporation estimation, as well as the development of the generalized complementary principle. Given the variations in α, the AA, B2015 and H2012 all have two parameters. The linear AA with a default value of b=1 has achieved a great success in evaporation estimation. For the B2015, c was thought to be only applied to accommodate unusual situations (Brutsaert, 2015). In practice, c = 0 is adopted and the Priestley-Taylor coefficient is calibrated (Zhang et al., 2017; Brutsaert et al., 2017; Liu et al., 2016; Brutsaert, 2015). But the calibrated α is smaller than the widely accepted constant 1.26 or even smaller than the unit at several sites, which is physically unrealistic. Han and Tian (2018a) found that c corresponds to b in the AA by setting the B2015 approximately equal to the AA in the middle stage. However, the default value of c=0 corresponds

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Hydrology and Earth System Sciences Discussions

to b=4.16, not b=1. The consistency suggests that *c* needs to be calibrated. By calibrating both α and *c*, the B2015 performed well in estimating evaporation for 20 FLUXNET sites, and the value of α were more rational (Han and Tian, 2018a).

By contrast, two more parameters ($x_{\min}$ and $x_{\max}$) are added to H2017. Because the sigmoid complementary curve are insensitive to $x_{\min}$ and $x_{\max}$, Han and Tian (2018a) suggested that they could be treated them as constant parameters for application convenience. $x_{\min}$ and $x_{\max}$ may change along with E_{rad}, and were thought to vary with the time scales (Han and Tian, 2018a), $x_{\min}=0$ and $x_{\max}=1$ are appropriate at a daily scale for convenience, as have be evidenced by the well performances when compared to the flux measurements (Han and Tian, 2018a; Han et al., 2012). $x_{\min}$ and $x_{\max}$ are expected to be calculated by applying certain approaches to reduce the number of parameters of H2017 to two (Han and Tian, 2019).

Although α would vary in theory (Assouline et al., 2016), it is widely used with a constant value of 1.26 in practice (Priestley and Taylor, 1972). After calibrating, the variations of α is much less significant than those of the other parameters. Moreover, the calibrated α approaches 1.26, especially for the H2017. Thus, the constant $\alpha=1.26$ was suggested with acceptable weakening of the accuracy of *E* estimation (Han and Tian, 2018a; Han et al., 2012). In practice, α was also determined from the observed *E* values in wet condition when *E* is close to E_{Pen} and/or E_{PT} (Kahler and Brutsaert, 2006; Ma et al., 2015a; Wang et al., 2019). A novel method by using observed air temperature and humidity data under wet environment was proposed by Szilagyi et al. (2017) when measured E is lacking, and was successfully used for large-scale CR model applications (Ma and Szilagyi, 2019; Ma et al., 2019).

After determining α in advance, only a single parameter in the generalized complementary functions needs to be calibrated. As the parameters of the B2015 and H2017 can be transferred from the asymmetric parameter *b* of the original CR (Han and Tian, 2018a), the former studies on the characteristics of *b* could help its parameterization. The *b* in the desert was much smaller than those in the oases or irrigated farmlands (Han et al., 2008, 2012). *b* was thought to be related to the characteristics of the atmosphere, i.e., the atmospheric humidity (Szilagyi, 2015), the Clausius–Clapeyron relationship between saturation-specific humidity and temperature (Lintner et al., 2015), or the characteristics of the land surface, i.e., the surface temperature (Szilagyi, 2007), the water availability of the evaporating surface (Han and Tian, 2018b; Lhomme and Guilioni, 2010), or the ecosystem types (Wang et al., 2019). Szilagyi (2015) applied a sigmoid function of relative humidity to parameterize b^{-1}. Wang et al. (2019) used the ecosystem mean *b* values of 217 sites around the world in the B2017 with litter weakening of the evaporation estimation accuracy. However, the characteristics and determination methods of *b* need further studies toward a calibration-free evaporation estimation model.

5.3 Applications of generalized complementary principle

The generalized complementary functions have been validated or applied in evaporation estimation for many sites (Ai et al., 2017; Brutsaert et al., 2017; Zhang et al., 2017; Han and Tian, 2018a; Crago and Qualls, 2018), and several basins in

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Hydrology and Earth System Sciences Discussions

China (Liu et al., 2016; Gao et al., 2018). It should be noted that most, if not all, above mentioned CR applications need "prior" knowledge in E (either ground-measured or water-balance-derived) to calibrate the parameters. Recently, the calibration-free CR model of S2017 was applied for monthly evaporation estimation across the conterminous China (Ma et al., 2019) and United States (Ma and Szilagyi, 2019). A wide range of model evaluations against the plot-scale flux measurements and basin-scale water balance results suggested that the generalized complementary functions could serve as a benchmark tool for validating the large-scale E results simulated by those Land Surface models and Remote Sensing models (Ma and Szilagyi, 2019). However, further applications over the world are still needed to develop more long-term, high-resolution E datasets for use in hydrological and atmospheric communities.

Morton (1983) thought that the ability of the complementary principle to estimate actual evaporation by using meteorological variables only can significantly influence the science and practice of hydrology. However, the attempts in using the complementary principle for hydrological modelling (Oudin et al., 2005; Barr et al., 1997; Nandagiri, 2007) have been suspended, while those attempts in applying such principle in drought assessment (Kim and Rhee, 2016; Hobbins et al., 2016) are still in their infancy. Moreover, the potential applications in agriculture water management are limited in the sense that the irrigation-induced changes in potential evaporation at an annual timescale (Ozdogan et al., 2006; Han et al., 2014a; Han et al., 2017). Apparently, the complementary principle did not develop to its full capacity via the linear CR, which leaves a broad space for further applications of the generalized complementary functions.

6 Conclusion

The complementary principle conceptualizes the feedbacks of surface evaporation on potential evaporation and offers advantages in evaporation research. In the CR, both E and E_{pa} are thought to converge to E_{po}. An elaborated prognostic simulation of E_{po} is crucial in studying the CR (Parlange and Katul, 1992a; Szilagyi and Jozsa, 2008). Several efforts have attempted to retain the linear CR by rationally formulating E_{pa} and/or E_{po} and by employing an asymmetric parameter.

Inheriting the concepts of three types of evaporation, the linear CR has evolved into the generalized nonlinear function B15, $E/E_{pa} = f(E_{po}/E_{pa})$. The proper definitions of E_{pa} and E_{po} remain crucial in the application. In the manner of the AA approach, B15 has been used for estimating evaporation with calibration under various conditions, and has also been advanced with rescaled dimensionless variables in C2016 and S2017 (Szilagyi et al., 2017; Crago et al., 2016). By contrast, H12, $E/E_{Pen} = f(E_{rad}/E_{Pen})$, follows another perspective that does not involve E_{po}. Previous studies have attempted to find the mathematical form of H12 with calibrated parameters. The sigmoid function H2017 was derived by invoking boundary conditions. The generalized complementary function approach has enhanced our understanding of the complementary principle and offers potential in estimating the actual evaporation by using simple and standardized procedures. The main challenge lies in the derivation of suitable analytical forms and in determining the main factors that influence its parameters. However, this function requires further study by using multi-scale measured E data such as the flux data (e.g., FLUXNET) and the long-term water balance data from numerous catchments.

425 **Appendix**

List of symbols

E	Actual evaporation
E_{pa}	Apparent potential evaporation in CR
E_{po}	Potential evaporation in CR
b	Symmetry parameter of the CR
E_{Pan}	Pan evaporation
E_{Pen}	Penman's potential evaporation(Penman, 1948)
E_{rad}	Radiation term of E_{Pen}
E_{aero}	Aerodynamic term of E_{Pen}
E_{Pen}^{KP}	Modified Penman's equation by Kohler and Parmele (1967)
$E_{Pen}^{r_s}$	Penman's potential evaporation with temperature and humidity calculated from the ABL model corresponding to certain surface resistance (rs)
$E_{Pen}^{r_s=0}$	Penman's potential evaporation with temperature and humidity calculated from the ABL model corresponding to $rs=0$
$E_{PM}^{r_{s\min}}$	Penman–Monteith(Monteith, 1965) evapotranspiraiton with a minimum surface resistance
ET_0	Reference evapotranspiration(Allen et al., 1998)
E_{MT}	Mass-transfer type potential evaporation(van Bavel, 1966)
E_{Mor}	Morton's(Morton, 1983) potential evaporation
E_{PT}	Priestley-Taylor's(Priestley and Taylor, 1972) minimal advection evaporation
$E_{PT}^{T_p}$	Morton's modified Priestley-Taylor's minimal advection evaporation (Morton, 1983)
α	Priestley-Taylor coefficient (Morton, 1983)
$E_{PT}^{T_{ws}}$	Szilagyi and Jozsa (2008)'s modified Priestley-Taylor's minimal advection evaporation
T_a	Air temperature
T_s	Surface temperature
T_{ws}	Surface temperature under wet environment (Szilagyi and Jozsa, 2008)
T_p	Equilibrium temperature (Morton, 1983)
Δ	Slope of the saturation vapor curve
γ	Psychrometric constant
R_n	Net radiation
G	Ground heat flux
RH	Relative humidity

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