



Combining analytical solutions of Boussinesq equation with the modified Kozeny-Carman equation for estimation of catchment-scale hydrogeological parameters

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Abstract: Saturated hydraulic conductivity (K), drainable porosity (f), and effective aquifer
15 thickness (D) are essential hydrogeological parameters for hydrologic modelling and
predicting. Streamflow recession analysis using analytical solutions of Boussinesq
equation can yield estimated values for two of these three hydrogeological parameters
when one is known a priori. In this study, we improved the inverse method for parameters
estimation by combining the modified Kozeny–Carman equation with analytical solutions
20 of Boussinesq equation to express the three hydrogeological parameters (K , f , and D) in
relation to catchment characteristics and recession constants in a sloping aquifer. Here, the
three parameters can be estimated simultaneously from streamflow recession analysis.
Results of the estimated parameters are compared with the field measurements and the soil-
texture based estimations in four small experimental catchments. It shows that our
25 estimated values of these catchment-scale parameters can represent equivalent values in
the measured aquifer profiles/sites. In hilly areas, the slope aquifer takes a vital effect on
the estimates of K and f . Neglecting the sloping effect can lead to overestimation of K and
underestimation of f in 1 ~ 2 orders of magnitude in the study catchments. However, even
in the hilly catchments, the estimated aquifer thickness D is much greater than that from
30 measurements on hillslopes while it approaches riparian thickness, indicating that the
riparian zone takes a vital role on flow recession and the parameter estimations.



1 Introduction

Saturated hydraulic conductivity (K), drainable porosity (f), and aquifer thickness (D) are
35 basic physical attributes of catchment and essential parameters for physically based
hydrologic models as well as for the hydrologic analysis of ungauged basins (Ali et al.,
2014; Bogaart et al., 2016; Carrillo et al., 2011; Li et al., 2014; Vannier et al., 2016).
Extensive measurements of these parameters to represent their intrinsic variations in a
catchment are costly and time-consuming. Thus, indirect methods, based on other more
40 easily measurable soil properties (such as soil porosity or texture) and/or regular
hydrologic observation data (such as streamflow) are sometimes preferred.

The Boussinesq equation is a general differential equation for groundwater flow in
unconfined aquifers of hillslope areas. The analytical and numerical solutions of the
Boussinesq equation illustrates streamflow behaviors in relation to catchment properties
45 like hillslope gradients, shapes, and hydrogeologic parameters of the aquifer (Troch et al.,
2013). Such analytical solutions have been widely used for estimating catchment-scale
hydrogeological parameters since the parameter estimation only needs the regular
streamflow observations (Brutseart and Nieber, 1977; Brutseart and Lopez, 1998; Zhang
et al., 2009).

50 The analytical solutions of Brutseart and Nieber (1977) for hydrogeological parameter
estimations were derived based on assumptions of specific initial and boundary conditions
of an ideal aquifer (Szilagyi and Parlange, 1998) in relatively humid and slowly draining
catchments settings with moderate topography. Many efforts have been made to improve
the applicability of the analytical solutions to a broader range of conditions (i.e., finite
55 length aquifer, vertical heterogeneity aquifer, sloping aquifer, and arid setting). For



example, Parlange et al. (2001) derived a single analytical approximate solution for Boussinesq equation that unifies the early-time and late-time solutions for finite length aquifer. It was then further improved by Mendoza et al. (2003) to derive the non-linear solutions in a drier and steeper landscape, where they used it for estimating hydraulic parameters at semi-arid mountainous watersheds in Mexico (Mendoza et al., 2003). Rupp and Selker (2005) derived an analytical solution for the vertically heterogeneous aquifer on assumption of a power-law decreasing of K from aquifer surface to bottom. Gao et al. (2013) found that considering the high variability of vertical K could increase the reliability of the estimated values of f and vertical mean K at catchment scale. As an important component of hydraulic gradient, the slope of the aquifer is also crucial to accurately express the groundwater flow behavior. Extended analytical solutions from horizontal to sloping aquifer were derived by Brutsaert (1994) and Hogarth et al. (2014) in different ways. Pauritsch et al. (2015) claimed that analytical models for sloping aquifers can narrow the ranges of the estimated values of K and D compared to that for horizontal aquifers in complex and heterogeneous aquifer systems in an alpine catchment.

Streamflow recession analysis for determining hydrogeological parameters is popularly based on two-stages flow recessions (Brutsaert, 2005; Troch et al., 2013) in two distinct flow regimes: the early-time and the late-time regimes. Correspondingly, two equations are derived to describe the flow recessions. Their solutions could be only used to estimate two of the three unknown parameters (K , D , and f). It means that one parameter must be given a priori. For example, Zhang et al. (2009) estimated catchment-scale K and f with given D . Vannier et al. (2014) estimated K and D with given f . In most of the catchments, all of the parameters are unknown. If the parameters (usually D or f) are set in advance with certain



arbitrary values, it would bring great uncertainty of the estimated parameters.

80 Among the three parameters, K and f are dependent on soil properties. The drainable porosity (f), defined as volume of water that a saturated soil will yield by gravity to the total volume of the rock or soil (Bear, 1972), is directly dependent on soil pore connected, soil particle-size distribution, and the soil pore structures (Ding et al., 2016). Saturated hydraulic conductivity (K) can be expressed as a function of the hydraulic radius of soil
85 pore (defined as soil pore volume over the pore-solid surface area), indicating that K depends on soil particle size, soil porosity, and pore shape factor and tortuosity (Zhang and Schaap, 2019). Evidently, both K and f can be estimated from easy-to-measure soil properties, like soil texture, bulk density, and organic content (Saxton and Rawls 2006; Stephens et al., 1998; Zhang et al., 2016), popularly using the pedotransfer functions (PTFs)
90 (Saxton and Rawls 2006).

As both K and f depend on soil physical properties, K and f are significantly correlated. Such hypothesis has been validated from field experiments (Nyberg, 1995 and Espeby, 1990) and physically based equations, including the Kozeny–Carman equation (Carman, 1937) and its modified equations (Ahuja et al., 1984, 1989; Rawls et al. 1993), which relate
95 K to effective porosity (equal to saturated water content minus field capacity) that is viewed identical to f for shallow homogenous unconfined aquifers (Bear, 1972). Therefore, if the analytical solutions of Boussinesq equation combine the modified Kozeny–Carman equation, the three unknown parameters (K , D , and f) can be determined simultaneously by using streamflow recession analysis.

100 The aim of this study is to extend the estimations of catchment-scale hydrogeological parameters (K , f , and D) from two to three parameters by using assessable information of



catchment characteristics and observed streamflow. It is achieved by an approach deriving the analytical expressions of the parameters K , f , and D in terms of the analytical solutions of Boussinesq equations combining with the modified Kozeny–Carman equation. The approach is then applied to four experimental catchments with detailed observations of catchment properties and measurements of the parameters. The sensitivity analysis is executed to reveal the main controlling factors in the determination of the three hydrogeological parameters.

2 Methodology

2.1 Analytical solutions of Boussinesq equation for sloping aquifer

In hilly areas, a catchment can be viewed as an assembly of a series of sloping hillslopes along with river networks (Fig. 1a). To analytically estimate catchment-scale hydrogeological parameters, the non-uniformly distributed sloping aquifers can be simplified as an equivalently homogeneous aquifer (Fig. 1b).

The one-dimensional subsurface flow on the sloping aquifer (Fig. 1b) can be described by

$$q_x = -K\eta(\cos(\alpha)\frac{\partial\eta}{\partial x} + \sin(\alpha)) \quad (1)$$

where q_x is the flow rate per unit width of the aquifer, η is water table, K is the saturated hydraulic conductivity, α is the hillslope gradient, and x is the distance from the river to hillslope ridge.

Assuming that aquifer is isotropic and homogeneous, substituting Eq. (1) into the continuity equation yields:

$$f\frac{\partial\eta}{\partial t} = K\left[\cos(\alpha)\frac{\partial}{\partial x}\left(\eta\frac{\partial\eta}{\partial x}\right) + \sin(\alpha)\frac{\partial\eta}{\partial x}\right] \quad (2)$$

where t is time and f is the drainable porosity.



125 Brutsaert (1994) derived the analytical solution of flow rate in the hillslope with specific
 conditions, including: initially fully saturation of the aquifer, a sudden drawdown of the
 water table (hydraulic head η) to the channel as outflow starts for the flow condition at $x=0$
 (Fig. 1b), and the infinitely wide aquifer that ensures negligibility of draining flow
 influence on the upper boundary of the aquifer. With that, the analytical solution of Eq. (2)
 130 for unit width hillslope is

$$q_{xf} = [(p/\pi \cos(\alpha)KfD^3)]^{1/2}t^{-1/2} \quad (3)$$

where q_{xf} is fast flow in the early-time recession, p is a constant equal to 0.3465, and D is
 the aquifer thickness.

When the water table at the boundary ($x=B$, where B is the hillslope length) starts falling,
 135 the analytical solution of slow flow (q_{xs}) in the late-time recession is

$$q_{xs} = \frac{2p\cos(\alpha)KD^2}{B} \frac{z_1^2[1-2\cos(z_1)\exp(Hi/2)]}{(z_1^2+Hi^2/4+\frac{Hi}{2})} \exp\left[-\frac{(z_1^2+Hi^2/4)p\cos(\alpha)KD}{fB^2}t\right] \quad (4)$$

where $Hi = B \tan \alpha / (pD)$; $z_1 = \pi/2$ for small Hi in gentle hillslopes and thick aquifers,
 while $z_1 = \pi$ for large Hi in steep hillslopes and thin aquifers.

2.2 Recession analysis

140 In the flow recession period, the relationship between streamflow (Q) and change of
 streamflow ($-dQ/dt$) was proposed by Brutsaert and Nieber (1997)

$$-\frac{dQ}{dt} = f(Q) \quad (5)$$

The function $f(Q)$ is often expressed by power-law equation as:

$$-\frac{dQ}{dt} = aQ^b \quad (6)$$

145 where a and b are constants for a specific catchment. As streamflow in catchment outlet
 can be viewed as the collection of flow from hillslopes along the river, Q is the multiple of



two-fold q_x and river length (L). Inserting Eq. (3) into Eq. (6), the constants a (a_f) and b (b_f) for the early-time recession are obtained as:

$$a_f = (8p/\pi \cos \alpha)^{-1} (KfD^3L^2)^{-1} \quad (7)$$

$$150 \quad b_f = 3 \quad (8)$$

Inserting Eq. (4) into Eq. (6), the constants a (a_s) and b (b_s) for the late-time recession are

$$a_s = \pi^2 p \cos \alpha \left(4 + \left(\frac{Hi}{\pi}\right)^2\right) KD(L/A)^2 / f \quad (9)$$

$$b_s = 1 \quad (10)$$

When the term $\left(\frac{Hi}{\pi}\right)^2$ is much larger than 4, the subordinate term of 4 in the bracket in the
 155 right side of Eq. (9) can be ignored. Thus, Eq. (9) can be expressed as

$$a_s = \pi^2 p \cos \alpha \left(\frac{Hi}{\pi}\right)^2 KD(L/A)^2 / f \quad (11)$$

For capturing recession processes from the streamflow hydrographs, the selected flow events should recess markedly at least 4 days after rainfall ceases. Recession data in the first day are excluded for eliminating the influence of direct and surface flow. Because the
 160 observed recession rate $-\frac{dQ}{dt}$ is one to two orders less than streamflow Q , discretization errors on $-\frac{dQ}{dt} \sim Q$ plot are vulnerable to the measurement noise, especially in log-log space (Gao et al., 2017). The time-derivative of streamflow $\frac{dQ}{dt}$ and the concurrent streamflow Q between the time interval ($i, i-j$) are calculated in terms of the method proposed by Rupp and Selker (2006)

$$165 \quad -\frac{dQ}{dt} \approx \frac{Q(i)-Q(i-j)}{t(i)-t(i-j)}, \quad i = 2, 3, \dots, N; \quad 1 \leq j \leq i - 1 \quad (12)$$

$$Q \approx \frac{1}{(j+1)} \sum_{k=i-j}^i Q(k) \quad (13)$$

where i represents data points taken at discrete time increments and j is the number of time



increments over which $-\frac{dQ}{dt}$ is calculated. Thus, a variable time interval $t(i) - t(i - j)$ is used to properly scale the observed drop in streamflow in order to avoid artifacts in data.

170 Additionally, if values of recession segments in contiguous streamflow data are equal, only the latest data are involved in the calculations using Eqs. (12) and (13).

2.3 Modified Kozeny–Carman equation relating K to f

Kozeny (1927) derived a power-law function that relates the saturated hydraulic conductivity K to soil porosity based on the Hagen-Poiseuille's equation. It was later
175 modified as the Kozeny–Carman equation after Carman (1937, 1956), which accounts for tortuosity in tube flow. Ahuja et al. (1984) proposed a modified Kozeny-Carman equation relating K to effective porosity, which is identical to drainable porosity f in this study. The modified Kozeny-Carman equation is in the form of $K = Cf^m$, where C and m are constants. To obtain estimated values for coefficients C and m , one needs to fit the equation
180 with measured data of K and f at a variety of catchment sites. Rawls et al. (1998) redefined the exponent m as 3 minus the Brooks-Corey pore size distribution index (λ):

$$K = \gamma \times f^{3-\lambda} \quad (14)$$

where γ is equal to 5.36×10^{-4} for K in a unit of m/s, obtained by fitting the measured data for 26 soil texture/porosity classes. The value λ ranges 0.165 ~ 0.694, estimated by
185 fitting a log-log curve between water content and pressure head using the –33 and –1500 kPa water contents (Rawls et al., 1993), according to USDA soil texture classes and over 900 measurements. Eq. (14) has been fitted by a variety of soils, making the equation universally applicable.

2.4 Estimation of catchment-scale hydrogeological parameters

190 Substituting Eq. (14) into Eq. (7) gives



$$a_f^{-1} = C_f K^{1+\beta} D^3 \quad (15)$$

where $C_f = 8p/\pi \cos \alpha L^2 \gamma^{-\beta}$, and $\beta = 1/(3 - \lambda)$. Similarly, substituting Eq. (14) to Eq. (9) gives

$$a_s = C_{s1}(4 + C_{s2}D^{-2})K^{1-\beta}D \quad (16)$$

195 where $C_{s1} = B^{-2}\gamma^\beta \pi^2 p \cos \alpha / 4$, and $C_{s2} = B^2 \tan^2 \alpha / (\pi^2 p^2)$. Combining Eq. (15) with Eq. (16), K and D can be obtained from implicit equations as follows

$$4D^{\frac{-2+4\beta}{1+\beta}} + C_{s2}D^{\frac{-4+2\beta}{1+\beta}} = (a_s C_{s1}^{-1})(a_f C_f)^{\frac{1-\beta}{1+\beta}} \quad (17)$$

$$4K^{\frac{2-4\beta}{3}} + (a_f^{\frac{2}{3}} C_f^{\frac{2}{3}}) C_{s2} K^{\frac{4-2\beta}{3}} = (a_s C_{s1}^{-1})(a_f C_f)^{\frac{1}{3}} \quad (18)$$

According to Eq. (14), drainable porosity (f) can be expressed as

200
$$f = r^{-\beta} K^\beta \quad (19)$$

Hence, f can be obtained easily when K is estimated from Eq. (18). This proves that the essential catchment-scale hydrogeological parameters (K , f , and D) can be estimated simultaneously by combining the analytical solutions from Boussinesq equation with the modified Kozeny–Carman equation (Eqs. (17), (18), and (19)).

205 **3 Study catchments and data**

The proposed approach for catchment-scale hydrogeological parameter estimation was tested in four experimental catchments in the northern hemisphere. These include, the relict Schöneben Rock Glacier (SPG) in Austria, the Hemuqiao experimental catchment (HMQ) in China, the Panola Mountain Research Watershed (PMRW) and the WS10 from HJ
 210 Andrews Experimental Forest in the USA (Fig. 2). All of the four catchments are located in humid climate region with mean annual precipitation greater than 1000 mm. The experimental catchments are small and steep, with areas ranging from 0.102 to 1.35 km²



and average catchment hillslope gradients varying from 10.2° to 29° . The uniformly distributed soil deposits in all four catchments falls under typical soil types (Table 1).

215 The daily observation data of streamflow and precipitation at PMRW, HMQ, and WS10 are available. The observation data at PMRW are available during 1986-2016, (<https://doi.org/10.5066/P94JC2PD>). As a part of HJ Andrews catchment, WS10 has observation data during 2000 ~ 2011 (<https://andrewsforest.oregonstate.edu/>). Located in one of the famous hydrologic experimental catchments (Jiangwan) in China before the
220 1970s, HMQ has observation data during 1957 ~1958 published in China hydrologic year book. As to SPG, data published in a study by Pauritsch et al. (2015) are directly used in our analysis.

All the four catchments have field measures of K and D , which are used for validating the estimates of our proposed methods. In PMRW, soil K values were measured at two soil
225 pits using constant head permeameters method by McIntosh et al. (1999), while K values for saprolite and bedrock were measured by falling head permeameter (White et al., 2002). In HMQ, the K values were measured at three typical soil pits by using falling head permeameter (Han et al., 2016). In WS10, vertical K values were measured at ten soil pits by using constant head permeameter (Harr, 1977). For SPG, the catchment equivalent
230 values of K , f , and D were obtained from Pauritsch et al. (2015) according to field measurements (Winkler et al., 2016).

Although the aquifer deposits in each of these catchments are dominated by one type of soil textures (Table 1), the point scale hydrogeological parameters observations show great heterogeneity, especially for saturated hydraulic conductivity K and soil/saprolite thickness
235 D (Table 2). K shows highly vertical variations. The value of K at the upper soil layer can



be one order larger than the bottom one or the underlain saprolite (Table 2). Across catchments, the soil thickness D differs but the riparian areas much thicker than that the hillslopes for all catchments. For example, the soil thickness D for the typical soil profiles in PMRW ranges 0.6 ~ 1.6 m on hillslope but reaches 5 m at the riparian area (Peters et al.,
240 2003). The average soil thickness on a typical hillslope in HMQ is 0.8 m (Han et al., 2018) while the soil in riparian with colluvium and residuum deposits can reach up to 6 m (Han, et al., 2016). Saprolite or regolith are an important part of groundwater storage (Hale et al., 2016), they can be much thicker than the upper soils and more spatially heterogeneous. In WS10, the thickness of saprolite is 3.7 m in average, varying from 1 to 7 m (McGuire et al.,
245 al., 2010), nearly three times greater than the average thickness of soils (1.30 m). In PMRW, spatial variability of the saprolite thickness (0 ~ 5 m) over granodiorite (Peters et al., 2003) is much greater than that of the upper soil thickness (0.6 ~ 1.6 m).

Additionally, values of K and f for each catchment are estimated in terms of soil texture by using pedotransfer function proposed by Saxton and Rawls (2006) (Table 2).

250 4 Results

4.1 Estimated recession constants

Plots of recession data in the form of $-\frac{dQ}{dt} \sim Q$ for each of the four catchments are shown in Fig. 3. In PMRW, HMQ, and WS10, the lower envelopes with 10% of data points are excluded in order to remove the effect of outliers on the envelope lines (Brutsaert and
255 Lopez, 1977). For SPG, the recession data are adopted directly from Pauritsch et al. (2015). Here, only winter data are used to fit the lower envelop lines as suggested by Pauritsch et al. (2015). The lower envelop lines with slopes of 3 and 1 are used to derive the recession intercepts for early- and late-time recessions (a_f and a_s), respectively.



The estimated constants of a_f and a_s in all catchments are listed in Table 3. The values
260 of a_f range in $3.0 \times 10^{-4} \sim 9.11 \times 10^{-2} \text{ m}^{-6} \text{ s}$, where the largest is estimated for WS10 and
smallest is estimated SPG. The value of a_f in WS10 is about one order of magnitude larger
than the value for SPG and PMRW, and twice larger than the values for HMQ. The values
of a_s range in $2.36 \times 10^{-7} \sim 1.43 \times 10^{-6} \text{ s}^{-1}$ (8.1 ~ 49.1 days), the largest in HMQ and smallest
in SPG. The timescales of late-time recessions ($1/a_s$) in SPG, PMRW, HMQ, and WS10
265 are 49.0, 35.6, 8.1, and 23.1 days, respectively. Brutseart (2008) claimed that the timescale
of the late-time recession commonly ranges 45 ± 15 days for mesoscale catchments.
Compared to this range, the late-time recessions are relatively fast in our catchments except
for SPG where the soil deposits are much thicker.

4.2 Estimated catchment-scale hydrogeological parameters

270 Given catchment physical properties (area, river length, and slope in Table 1), the estimates
of a_f and a_s as well as the soil texture-based λ (Table 3), the catchment-scale
hydrogeological parameters (K , D , and f) are calculated according to solutions from
combinations of Eqs. (17), (18), and (19). The estimated hydrogeological parameters vary
significantly across catchments, especially for K and D in Table 3. The estimated values of
275 K range from 1.75×10^{-6} to $2.36 \times 10^{-5} \text{ m/s}$. The smallest K value was observed in WS10, it
is about one order of magnitude smaller than that of PMRW, SPG, and HMQ. The
estimated f ranges in $0.13 \sim 0.28$, and aquifer thickness D ranges in $2.99 \sim 18.7 \text{ m}$ for the
four catchments. Three of the catchments have the thickness D in several meters except for
SPG where the aquifer is much thicker ($D = 18.7 \text{ m}$). The differences of either K or f values
280 in the four catchments are highly attributed to the soil texture. As expected, clay loam in
WS10 has the smallest K and f while equivalent loamy sand in SPG has the largest K and f



(Table 1).

To validate the reliability of our catchment-scale hydrogeological parameter estimation approach, the estimated values of K , f , and D are compared to the measured values in all
285 four catchments. Additionally, the estimated values of K and f are compared to the estimates from soil texture (Saxton and Rawls 2006) (Table 2 and Fig. 4). For SPG, the estimated catchment-scale K (2.36×10^{-5} m/s) is close to the catchment equivalent value (1.00×10^{-5} m/s) given by Pauritsch et al. (2015) in terms of the order of magnitude. The estimated value of K for PMRW (1.90×10^{-5} m/s) is within the range of measured values
290 between upper soil (1.79×10^{-4} m/s) and underlying saprolite (5.0×10^{-6} m/s). In HMQ, the estimated value of K (1.20×10^{-5} m/s) is within the range of measured values in the whole soil profile ($2.5 \times 10^{-6} \sim 5.09 \times 10^{-5}$ m/s) but closer to the measures within the profile depths of 30 ~ 90 cm. The estimated catchment-scale K is 75% larger than the geometric mean of the measured K in the vertical profile for these three catchments. An exception is that the
295 estimated K value (1.75×10^{-6} m/s) in WS10 is much below the lower limit of the measured values in a shallow soil profile ($4.4 \times 10^{-5} \sim 1.1 \times 10^{-3}$ m/s). The two orders of magnitude smaller K value estimated in WS10 could be attributed to the fact that the measurements were only taken from the upper soils (1.5 m in maximum) with abundant macropores (Harr, 1977), which could be much greater than that in the soil below (e.g. ~5.77 m thickness in
300 our estimation). Fig. 4b shows the estimated catchment-scale K in this study vs. the estimated K from soil texture. Both estimated K values are close to the 1:1 line, indicating that both estimates are in agreement. The f values from our study method are close to the estimated f from soil texture with slight overestimation by 4% ~ 47% (Fig. 4c, an average of 23.7%).



305 The estimated aquifer thickness D values in our study are comparable to the measured
 values in SPG (Fig. 4d). But PMRW and HMQ showed significantly larger than soil
 thickness on the hillslope, the magnitudes are within the range of mean soil thickness on
 hillslopes and the maximum soil thickness in riparian. In WS10, the estimated D is close
 to the measured D when both soil and saprolite are considered as effective groundwater
 310 aquifers.

4.3 Sensitivity analysis for the hydrogeological parameters

As $\left(\frac{Hi}{\pi}\right)^2$ in Eq. (9) is proven to be much larger than 4 (Table 3), Eq. (10) is adopted by
 neglecting the subordinate term on the right side of Eq. (9). Thus, the hydrogeological
 parameters (K , D , and f) in Eqs. (17) ~ (19) can be explicitly expressed as

$$315 \quad D = \left(C_D a_f^{-(1-\beta)} a_s^{-(1+\beta)} (\tan \alpha)^2 (\sin \alpha)^{2\beta} \gamma^{2\beta} L^{-2(1-\beta)} \right)^{\frac{1}{4-2\beta}} \quad (20)$$

$$K = \left(C_K a_f^{-1} a_s^3 (\tan \alpha)^{-2} (\sin \alpha)^{-4} \gamma^{-2\beta} L^{-2} \right)^{\frac{1}{4-2\beta}} \quad (21)$$

$$f = \left(C_K a_f^{-1} a_s^3 (\tan \alpha)^{-2} (\sin \alpha)^{-4} \gamma^{-4} L^{-2} \right)^{\frac{\beta}{4-2\beta}} \quad (22)$$

where $C_D = (8p/\pi)^{-1+\beta} (4p)^{-(1+\beta)}$, $C_K = 8\pi p^2$, and $\beta = \frac{1}{3-\lambda}$ ranging in 1/3 and 1/2.

Eqs. (20) ~ (22) show that the hydrogeological parameters are controlled by not only the
 320 variables of streamflow recessions (a_s and a_f) but also catchment topography and
 topological properties (slope α and river length L) as well as parameters of the modified
 Kozeny–Carman equation (γ and λ). Given a 10% change of each influence variable relative
 to the estimated or known values in Tables 1 ~ 3, the sensitivity of D , K , and f to the
 influencing variables can be estimated. Here, as an example, the sensitivity analysis is done
 325 for HMQ catchment (Fig. 5). It clearly shows that K and f decrease with increases of the
 independent parameters except for a_s . D decreases with a_f , a_s , and L , but it increases with



α , λ , and γ . Among all the independent variables, the catchment slope α is the most sensitive variable to the three hydrogeological parameters (Fig. 5). The next sensitive variable is late-time recession rate (a_s) for K and D , and γ for f . The most insensitive variables are λ and a_f for f and D , and, λ and γ for K . Among the three parameters, K is the most sensitive parameter to the independent variables (α , a_s , a_f , and L).

Overall, accurate representation of catchment aquifer slope α and analysis of late-time recession could increase the reliability of the three estimated parameters, especially for K .

5 Discussions

5.1 Comparison of parameter estimates from analytical solutions in horizontal and sloping aquifers

Based on our sensitivity analysis, in the presented simultaneous estimation approach of the three hydrogeological parameters, catchment slope (α) exerts considerable influence on the parameters. To evaluate effects of the catchment slope on recession analysis, we compared the estimated values of the results from horizontal aquifer assumption. For a horizontal aquifer, where K and f can be estimated from equations as follows (Brutseart, 2005):

$$K = 0.5757(L^2/A)^{-1}D^{-2}(a_s/a_f)^{1/2} \quad (23)$$

$$f = 1.9688(DA)^{-1}(a_s a_f)^{-1/2} \quad (24)$$

Combing Eqs. (7) with (9), K and f for sloping aquifer can be expressed as

$$K = 0.5757(L^2/A)^{-1}D^{-2}\alpha(4 + (\frac{H^i}{\pi})^2)^{-1/2}(a_s/a_f)^{1/2} \quad (25)$$

$$f = 1.9688(DA)^{-1}(4 + (\frac{H^i}{\pi})^2)^{1/2}(a_s a_f)^{-1/2} \quad (26)$$

Thus, K and f varying with D can be described by Eqs. (23) and (24) for horizontal aquifers and by Eqs. (25) and (26) for sloping aquifers. Here, K and f related to D are given in two of the catchments, HMQ and PMRW, where catchment slopes are taken as representatives



350 from the mild (10.2° in PMRW) to the steep (20° in HMQ). As shown in Fig. 6, K and f
decrease with D for both sloping and horizontal aquifers. The differences of f and K
between sloping and horizontal aquifers are tremendous. The estimated values of f from
the horizontal aquifer are about one order of magnitude smaller than those from sloping
355 magnitude greater than those from a sloping aquifer. Estimates of catchment-scale f from
horizontal aquifer assumption are too small to be reasonable. For example, f is 0.0083 for
 $D=2.99$ m in HMQ, and 0.019 for $D=4.84$ in PMRW. Meanwhile, K estimated from
horizontal aquifer assumption reaches a larger magnitude of over 10^{-4} m/s when D is less
than 5.5 m in HMQ and less than 8 m in PMRW. This order of magnitude of K value
360 approaches the macropore flow velocity (in the order of $10^{-4}\sim 10^{-3}$ m/s) at hillslope scale
according to the summary from 110 observations by Gao et al (2018). Whilst the measured
 K values are in the order of magnitude of 10^{-5} and 10^{-6} m/s in these four catchments. Thus,
 K could be extremely overestimated by horizontal aquifer assumption.

5.2 Effect of vertical heterogeneity of soils on the determination of K and f

365 For natural soils, K usually decreases with soil depth (Brooks and Boll, 2004) and field
measurements also indicate that f could decrease with soil depth as well (Harr, 1977;
McGuire et al., 2007). The decrease of K with depth could reduce the late-time recession
rate as the water level becomes lower. The depth dependent K affects estimates of
hydrogeological parameters from streamflow recessions. According to our previous study
370 (Gao et al., 2013), the estimated average K in a vertical profile increases with vertical
variability of K in terms of the analytical solutions of Boussinesq equation considering the
vertical decrease of K (Rupp and Selker, 2005). The measured K in our selected catchments



(e.g. PMRW, HMQ, and WS10) shows that K at the upper soil can be 1 ~ 2 orders of magnitude greater than that in the deep deposits. The estimated K from our proposed
375 method can be regarded as an equivalent value that represents K in a specific range of the soil profile. For example, in PMRW, the estimated value of K is more likely to represent the equivalent value between soil and saprolite. In HMQ, the estimated value of K is closer to the measures within the soil profile at 30 ~ 90 cm depths even though the soil profiles on the hillslope can be as deep as 1.2 ~ 4.4 m (Fig. 4a). The ranges of the measured K cover
380 our estimated values in three catchments, indicating that the measured ranges of K can be used to restrict parameter variations in model calibration. However, it should be mentioned that the selected profiles and sites for measures should be representative of a catchment. This study shows that the field measurements only taken at the upper soils (1.5 m in the maximum depth) cannot capture the whole profile values of 5.77 m in depth in WS10.

385 **5.3 Effect of the riparian zone on flow recessions**

Burns et al. (2001) demonstrated that the hillslopes contribute flow mainly during large storm event while the riparian zone in the gentle areas with thick deposits of sediment takes a vital role on runoff generation and flow regulation (Clark et al., 2009; McGlynn and McDonnell, 2003; Klaus et al., 2015). A riparian zone can generate a vast majority of runoff
390 during small event and between events (recession processes) (Burns et al., 2001; McGlynn and McDonnell, 2003), and contribute a primary flow component throughout the recessions (Clark et al., 2009). Our estimated catchment-scale aquifer thickness ranges between the measured thicknesses in hillslope and riparian zone but is much thicker than the thickness in hillslope areas. It implies that thicker deposits in the riparian zone have involved to slow
395 down flow recessions since the thicker aquifer reduces hydraulic conductivity K as shown



in Fig. 6. Thus, detail investigations on the riparian zone properties, such as the deposit thickness and hydrologic connectivity with hillslopes and streams are vital for appropriate analysis of flow recession behaviors.

6 Conclusions

400 Catchment-scale hydrogeological parameters (K , f , and D) are important for acquiring catchment hydrologic properties and simulating hydrologic processes, especially low flow processes. In our study, a new method that combines analytical solutions of Boussinesq equation with the modified Kozeny–Carman equation is proposed to estimate three key hydrogeological parameters (K , f , and D) simultaneously at catchment scale. Four
405 experimental catchments are used to test the validity of the proposed method.

The new method only needs streamflow observations and catchment geometric characteristics to estimate catchment-scale parameters. Compared to the field measurements and the estimated values from soil texture, our estimated results range within the measured values at various sites/profiles in the catchments and can be regarded as the
410 equivalent values in specific depths of the aquifer. It indicates that our estimations can be used as model parameters for simulation of the catchment-scale hydrologic processes, particularly baseflow processes.

Details of catchment geometric characteristics can affect the parameter estimation accuracy. The sensitivity analysis of the parameters to the catchment characteristics and
415 recession constants showed that catchment slope is the most sensitive variable to the three hydrogeological parameters. Neglecting effects of the catchment slope on streamflow recession analysis would result in significant overestimation in saturated hydraulic conductivity and underestimation in drainable porosity. Additionally, the estimated aquifer



thickness is much greater than the measured thickness on hillslopes but close to the
420 thickness in the riparian zone, indicating that riparian zone could take an important role on
flow recessions even in the hilly catchments.

Data availability. The observation data of streamflow and precipitation at PMRW and
WS10 catchments are available at <https://doi.org/10.5066/P94JC2PD> and
425 <https://andrewsforest.oregonstate.edu/>, respectively. The original observation data of
streamflow and precipitation in SPG and HMQ catchments cannot be publicly accessed.

Author contributions. MG and XC designed research; MG performed research; JL provided
data and involved in derivation of the theoretical equations. MG wrote the paper. XC and
430 JL extensively edited the paper.

Competing interests. The authors declare that they have no conflict of interest.

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References



- Ahuja, L., Cassel, D., Bruce, R., and Barnes, B.: Evaluation of spatial distribution of hydraulic conductivity using effective porosity data, *Soil Sci.*, 148(6), 404-411, 1989.
- Ahuja, L. R., Naney, J. W., Green, R. E., and Nielsen, D. R.: Macroporosity to Characterize
445 Spatial Variability of Hydraulic Conductivity and Effects of Land Management, *Soil Sci. Soc. Am. J.*, 48(4), 699-702, doi:10.2136/sssaj1984.03615995004800040001x, 1984.
- Ali, M., Ye, S., Li, H. Y., Huang, M. Y., Leung, L. R., Fiori, A., and Sivapalan M.: Regionalization of subsurface stormflow parameters of hydrologic models: Up-scaling from physically based numerical simulations at hillslope scale, *J. Hydrol.*, 519, 683-698,
450 2014.
- Bear, J.: *Dynamics of fluids in porous media*. Dover Publication Inc., New York, 1972.
- Bogaart, P. W., van der Velde, Y., Lyon, S. W., and Dekker, S. C.: Streamflow recession patterns can help unravel the role of climate and humans in landscape co-evolution, *Hydrol. Earth Syst. Sc.*, 20(4), 1413-1432, 2016.
- 455 Brooks, E. S., Boll, J., and McDaniel, P. A.: A hillslope-scale experiment to measure lateral saturated hydraulic conductivity, *Water Resour. Res.*, 40(4), 2004.
- Brutsaert, W.: The unit response of groundwater outflow from a hillslope. *Water Resour. Res.*, 30(10), 2759-2763, 1994.
- Brutsaert, W.: *Hydrology: an introduction*, Cambridge University Press, 2005.
- 460 Brutsaert, W.: Long-term groundwater storage trends estimated from streamflow records: Climatic perspective. *Water Resour. Res.*, 44(2), 2008.
- Brutsaert, W., and Lopez, J. P.: Basin-scale geohydrologic drought flow features of riparian aquifers in the southern Great Plains, *Water Resour. Res.*, 34(2), 233-240, 1998.



- Brutsaert, W., and Nieber J. L.: Regionalized drought flow hydrographs from a mature
465 glaciated plateau, *Water Resour. Res.*, 13(3), 637-643, 1977.
- Burns, D. A., McDonnell, J. J., Hooper, R. P., Peters, N. E., Freer, J. E., Kendall, C., and
Beven K.: Quantifying contributions to storm runoff through end-member mixing
analysis and hydrologic measurements at the Panola Mountain Research Watershed
(Georgia, USA), *Hydrol. Process.*, 15(10), 1903-1924, 2001.
- 470 Carman, P. C.: Fluid flow through granular beds. *Trans. Inst. Chem. Eng.*, 15, 150–166,
1937.
- Carman, P. C.: *Flow of Gases through Porous Media*. Academic Press Inc., New York,
1956.
- Carrillo, G., Troch, P. A., Sivapalan, M., Wagener, T., Harman, C., and Sawicz, K.:
475 Catchment classification: hydrological analysis of catchment behavior through process-
based modeling along a climate gradient, *Hydrol. Earth Syst. Sc.*, 15(11), 3411-3430,
2011.
- Clark, M. P., Rupp, D. E., Woods, R. A., Tromp-van Meerveld, H. J., Peters, N. E., and
Freer J. E.: Consistency between hydrological models and field observations: linking
480 processes at the hillslope scale to hydrological responses at the watershed scale, *Hydrol.*
Process., 23(2), 311-319, 2009.
- Ding, D., Zhao, Y., Feng, H., Peng, X., and Si, B.: Using the double-exponential water
retention equation to determine how soil pore-size distribution is linked to soil texture,
Soil Tillage Res., 156, 119-130, doi:10.1016/j.still.2015.10.007, 2016.
- 485 Espeby, B.: An analysis of saturated hydraulic conductivity in a forested glacial till slope,
Soil Sci., 150(2), 485-494, 1990.



- Gao, M., Chen, X., Liu, J. T., Zhang, Z. C., and Cheng, Q. B.: Using two parallel linear reservoirs to express multiple relations of power-law recession curves, *J. Hydrol. Eng.*, 22(7), 04017013, doi:10.1061/(ASCE)HE.1943-5584.0001518, 2017.
- 490 Gao, M., Chen, X., and Zhang, Z. C.: Recession flow analysis on vertical-heterogeneous aquifer and the method to estimate hydrogeological parameters, *Geotech. Invest. Surv.*, 6, 43-47+55, 2013. (In Chinese)
- Gao, M., Li, H. Y., Liu, D., Tang, J., Chen, X., Chen, X., Blöschl, G., and Leung, L. R.: Identifying the dominant controls on macropore flow velocity in soils: A meta-analysis, 495 *J. Hydrol.*, 567, 590-604, 2018.
- Han, X. L., Liu, J. T., Mittr, S., Li, X. P., Srivastava, P., Guzman, S. M., and Chen X.: Selection of optimal scales for soil depth prediction on headwater hillslopes: A modeling approach, *Catena*, 163, 257-275, 2018.
- Han, X. L., Liu, J. T., Zhang, J., and Zhang, Z. Z.: Identifying soil structure along 500 headwater hillslopes using ground penetrating radar based technique, 13(3), 405-415, doi:10.1007/s11629-014-3279-7, 2016.
- Harr, R. D.: Water flux in soil and subsoil on a steep forested slope, *J. Hydrol.*, 33(1-2), 37-58, 1977.
- Hogarth, W. L., Li, L., Lockington, D. A., Stagnitti, F., Parlange, M. B., Barry, D. A., 505 Steenhuis, T. S., and Parlange, J. Y.: Analytical approximation for the recession of a sloping aquifer, *Water Resour. Res.*, 50(11), 8564-8570, 2014.
- Klaus, J., Chun, K. P., McGuire, K. J., and McDonnell, J. J.: Temporal dynamics of catchment transit times from stable isotope data, *Water Resour. Res.*, 51(6), 4208-4223, 2015.



- 510 Kozeny, J.: Uber kapillare leitung der wasser in boden. Royal Academy of Science, Vienna,
Proc. Class I, 136, 271–306, 1927.
- Li, H. Y., Sivapalan, M., Tian, F. Q., and Harman, C.: Functional approach to exploring
climatic and landscape controls of runoff generation: 1. Behavioral constraints on runoff
volume, *Water Resour. Res.*, 50(12), 9300-9322, 2014.
- 515 McGuire, K. J., and McDonnell, J. J.: Hydrological connectivity of hillslopes and streams:
Characteristic time scales and nonlinearities, *Water Resour. Res.*, 46(10),
<https://doi.org/10.1029/2010WR009341>, 2010.
- McGuire, K. J., Weiler, M., and McDonnell, J. J.: Integrating tracer experiments with
modeling to assess runoff processes and water transit times, *Adv. Water Resour.*, 30(4),
520 824-837, 2007.
- McIntosh, J., McDonnell, J. J., and Peters N. E.: Tracer and hydrometric study of
preferential flow in large undisturbed soil cores from the Georgia Piedmont, USA,
Hydrol. Process., 13(2), 139-155, 1999.
- Mendoza, G. F., Steenhuis, T. S., Walter, M. T., and Parlange, J. Y.: Estimating basin-wide
525 hydraulic parameters of a semi-arid mountainous watershed by recession-flow analysis,
J. Hydrol., 279(1-4), 57-69, 2003.
- McGlynn, B. L., and McDonnell, J. J.: Quantifying the relative contributions of riparian
and hillslope zones to catchment runoff, *Water Resour. Res.*, 39(11), 2003.
- Nyberg, L.: Water flow path interactions with soil hydraulic properties in till soil at
530 Gårdsjön, Sweden, *J. Hydrol.*, 170(1-4), 255-275, 1995.



- Parlange, J. Y., Stagnitti, F., Heilig, A., Szilagyi, J., Parlange, M., Steenhuis, T., Hogarth, W., Barry, D. A., and Li, L.: Sudden drawdown and drainage of a horizontal aquifer, *Water Resour. Res.*, 37(8), 2097-2101, 2001.
- Pauritsch, M., Birk, S., Wagner, T., Hergarten, S., and Winkler, G.: Analytical
535 approximations of discharge recessions for steeply sloping aquifers in alpine catchments, *Water Resour. Res.*, 51(11), 8729-8740, 2015.
- Peters, N. E., Freer, J., and Beven, K.: Modelling hydrologic responses in a small forested catchment (Panola Mountain, Georgia, USA): a comparison of the original and a new dynamic TOPMODEL, *Hydrol. Process.*, 17(2), 345-362, 2003.
- 540 Rawls, W., Gimenez, D., and Grossman, R.: Use of soil texture, bulk density, and slope of the water retention curve to predict saturated hydraulic conductivity, *Trans. the ASAE*, 41(4), 983-988, 1998.
- Rawls, W. J., Brakensiek, D., and Logsdon, S.: Predicting saturated hydraulic conductivity utilizing fractal principles, *Soil Sci. Soc. Am. J.*, 57(5), 1193-1197, 1993.
- 545 Rupp, D. E., and Selker, J. S.: Drainage of a horizontal Boussinesq aquifer with a power law hydraulic conductivity profile, *Water Resour. Res.*, 41(11), 2005.
- Rupp, D. E., and J. S. Selker (2006), Information, artifacts, and noise in dQ/dt - Q recession analysis, *Adv. Water Resour.*, 29(2), 154-160.
- Saxton, K. E., and Rawls, W. J.: Soil water characteristic estimates by texture and organic
550 matter for hydrologic solutions, *Soil Sci. Soc. Am. J.*, 70(5), 1569-1578, 2006.
- Stephens, D. B., Hsu, K. C., Prieksat, M. A., Ankeny, M. D., Blandford, N., Roth, T. L., Kelsey, J. A., and Whitworth, J. R.: A comparison of estimated and calculated effective porosity, *Hydrogeol. J.*, 6(1), 156-165, 1998.



- Szilagyi, J., and Parlange, M. B.: Baseflow separation based on analytical solutions of the
555 Boussinesq equation, *J. Hydrol.*, 204(1-4), 251-260, 1998.
- Troch, P. A., Berne, A., Bogaart, P., Harman, C., Hilberts, A. G., Lyon, S. W., Paniconi,
C., Pauwels, V. R. N., Rupp, D. E., Selker, J. S., Teuling, A. J., Uijlenhoet, R., and
Verhoest, N. E. C.: The importance of hydraulic groundwater theory in catchment
hydrology: The legacy of Wilfried Brutsaert and Jean-Yves Parlange, *Water Resour. Res.*,
560 49(9), 5099-5116, 2013.
- Vannier, O., Anquetin, S., and Braud I.: Investigating the role of geology in the
hydrological response of Mediterranean catchments prone to flash-floods: Regional
modelling study and process understanding, *J. Hydrol.*, 541, 158-172, 2016.
- Vannier, O., Braud, I., and Anquetin, S.: Regional estimation of catchment-scale soil
565 properties by means of streamflow recession analysis for use in distributed hydrological
models, *Hydrol. Process.*, 28(26), 6276-6291, 2014.
- White, A., Blum, A. E., Schulz, M. S., Huntington, T. G., Peters, N. E., Stonestrom, D. A.:
Chemical weathering of the Panola Granite: Solute and regolith elemental fluxes and the
weathering rate of biotite. In *Water-Rock Interactions, Ore Deposits, Environmental*
570 *Geochemistry: A Tribute to David A. Crerar*, Hellmann R., Wood S. A. (Eds.). The
Geochemical Society, Special Publication No. 7, 37-59, 2002.
- Winkler, G., Wagner, T., Pauritsch, M., Birk, S., Kellerer-Pirklbauer, A., Benischke, R.,
Leis, A., Morawetz, R., Schreilechner, M. G., and Hergarten, S.: Identification and
assessment of groundwater flow and storage components of the relict Schoneben Rock
575 Glacier, Niedere Tauern Range, Eastern Alps (Austria), *Hydrogeol. J.*, 24(4), 937-953,
2016.



Zhang, L., Chen, Y. Q. D., Hickel, K., and Shao, Q. X.: Analysis of low-flow characteristics for catchments in Dongjiang Basin, China, *Hydrogeol. J.*, 17(3), 631-640, 2009.

580 Zhang, Y., and Schaap, M. G.: Estimation of saturated hydraulic conductivity with pedotransfer functions: A review, *J. Hydrol.*, 575, 1011-1030, 2019.

Zhang, Y. G., Schaap, M. G., Guadagnini, A., and Neuman S. P.: Inverse modeling of unsaturated flow using clusters of soil texture and pedotransfer functions, *Water Resour. Res.*, 52(10), 7631-7644, 2016.

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Tables

Table 1. The landscape and climate properties of the four experimental catchments

Catchment	Location	Area (km ²)	Slope (°)	River length (km)	Elevation (m)	Precipitation (mm/a)	Soil	Geology
1 Schoneben rock glacier (SPG)	Styria, Austria	0.67	15	0.30*	1715 ~ 2295	1190	Loamy Sand**	Coarse-grained gneissic sediments
2 Panola Mountain Research Watershed (PMRW)	Georgia, USA	0.41	10.2	0.48**	222 ~ 279	1225	Sandy loam	Granodiorite (predominantly)
3 Hemuqiao Experimental Catchment (HMQ)	Zhejiang, China	1.35	20	2.50**	160 ~ 600	1580	Silt loam	Quartz sandstone
4 HJ Andrews Experimental Forest (WS10)	Oregon, USA	0.10	29	0.58**	473 ~ 680	2350	Clay loam	Volcaniclastics (predominantly)

Comments: * Equivalent value from Pauritsch et al. (2015).

** Measured from map in literature of Clark et al. (2009) for PMRW, Han et al. (2018) for HMQ, and Klaus et al. (2015) for WS10.

** The SPG surface sediment characteristics is relative coarse and is even described as “covered by coarse-grained, blocky material consisting of gneissic rocks ranging from cubic decimeters to a few cubic meters” (Pauritsch et al., 2015). Its soil texture is set as loamy sand combining this description with comparison of the hydraulic properties (K and f) measured with related soil.



Table 2. Values of the hydrogeological parameters from field measurements and estimates from soil texture in the four catchments

Catchments	K (m/s)		D (m)	$f(-)$ (Estimated from soil texture)
	Measured in field	Estimated from soil texture		
1 SPG	1.00×10^{-5} *	3.00×10^{-5}	30 *	0.2 *
2 PMRW	1.79×10^{-4} (soil);	1.39×10^{-5}	1.1 (hillslope) ~ 5 (riparian area)	0.27
	5.0×10^{-6} (saprolite); 1.6×10^{-6} (bedrock)			
3 HMQ	5.09×10^{-5} (soil surface)	4.47×10^{-6}	0.8 (hillslope) ~ 6 (riparian area)	0.17
	5.4×10^{-6} (30 ~ 90 cm) 2.5×10^{-6} (120 ~ 440 cm)			
4 WS10	3×10^{-4} (soil mean) (range: 4.4×10^{-5} ~ 1.1×10^{-3})	1.19×10^{-6}	5 (hillslope and saprolite)	0.12

Comment: * These values are equivalent values obtained from Pauritsch et al. (2015).



Table 3. Estimated hydrogeological parameters at catchment scale

	Catchments	λ (-)	a_f (m ⁻⁶ s)	a_s (s ⁻¹)	Estimated hydrogeological parameters			H_i
					K (m/s)	D (m)	f (-)	
1	SPG	0.55	3.0×10^{-4}	2.36×10^{-7}	2.36×10^{-5}	18.7	0.28	16.0
2	PMRW	0.38	8.50×10^{-3}	3.25×10^{-7}	1.90×10^{-5}	4.84	0.28	57.7
3	HMQ	0.23	2.40×10^{-3}	1.43×10^{-6}	1.20×10^{-5}	2.99	0.25	95.2
4	WS10	0.24	9.11×10^{-2}	5.00×10^{-7}	1.75×10^{-6}	5.77	0.13	24.4



Figure captions

Figure 1. Schematic representation of (a) catchment and (b) the cross section of an
595 unconfined aquifer lying on a sloping impermeable layer.

Figure 2. Locations of the four experimental catchments adopted in this study.

Figure 3. Recession analysis for estimation of recession parameters a_f and a_s in (a) SPG,
600 (b) PMRW, (c) HMQ, and (d) WS10 (the figure (a) is modified from Fig. 3 in Pauritsch et
al. (2015)).

Figure 4. The estimated catchment-scale hydrogeological parameters in this study vs. the
605 measured values in field and the estimated values from soil texture. Note: (a) estimated K
in this study vs. measured K , (b) estimated K in this study vs. estimated K from soil texture,
(c) estimated f in this study vs. estimated f from soil texture, and (d) estimated D in this
study vs. measured D .

Figure 5. Sensitivity analysis of the estimated parameters (K , D , and f) in terms of the
610 relative changes of the parameters caused by 10% increase of any independent variable in
HMQ.

Figure 6. Comparison of the estimated K and f from sloping and horizontal aquifer with
changing D in (a) PMRW and (b) HMQ.

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Figures

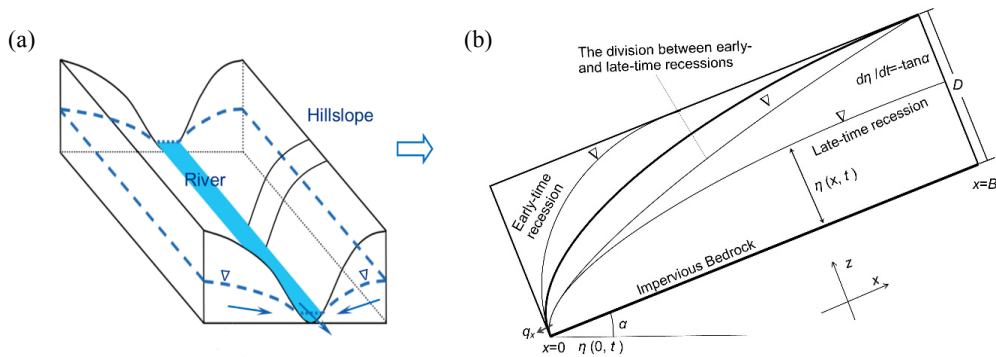


Figure 1. Schematic representation of (a) catchment and (b) the cross section of an

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unconfined aquifer lying on a sloping impermeable layer.

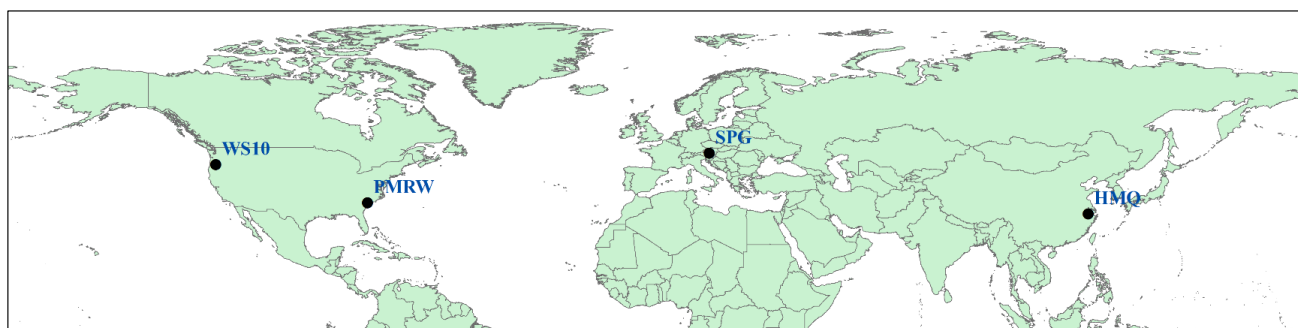


Figure 2. Locations of the four experimental catchments adopted in this study.



(a)

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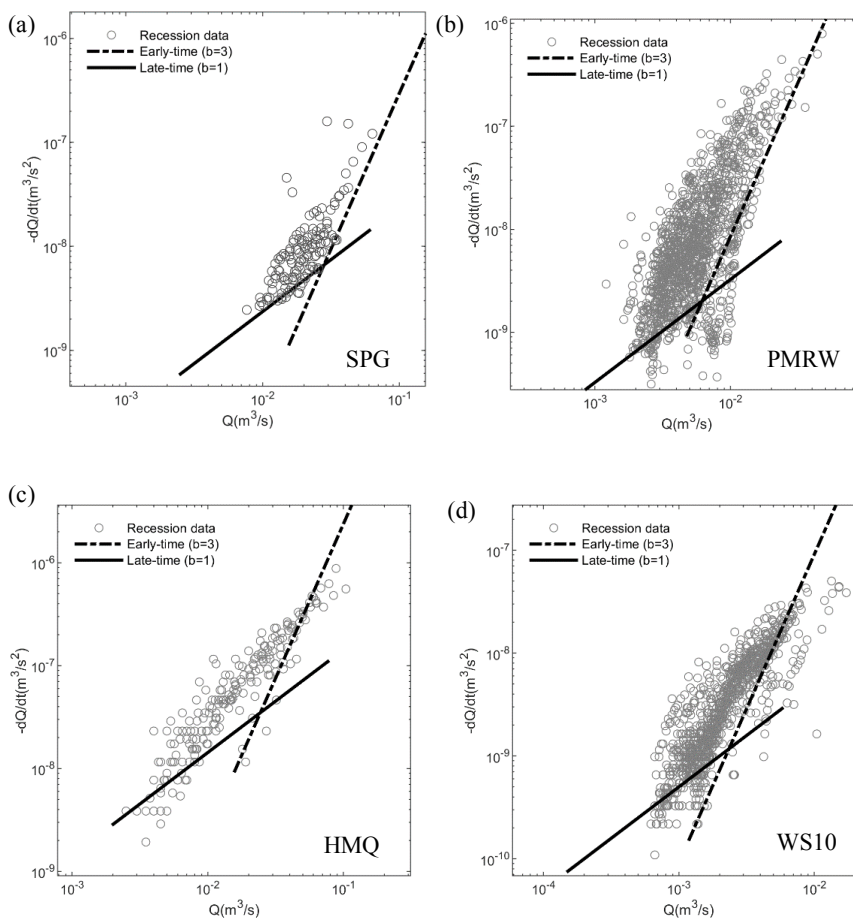
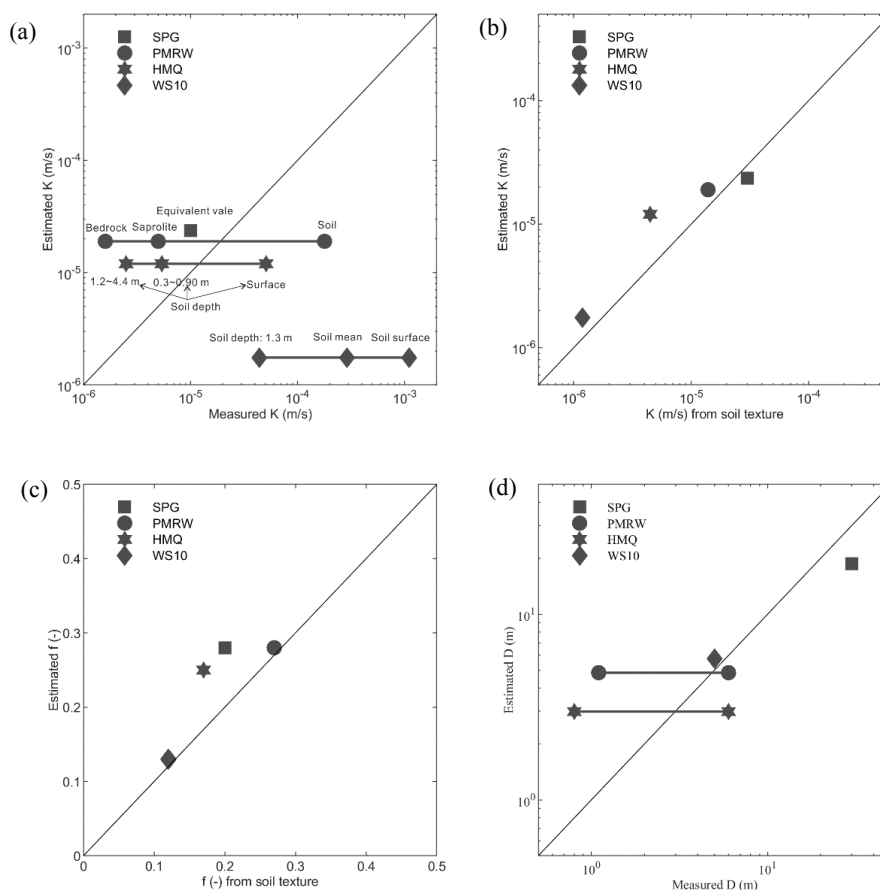


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625 al. (2015)).



630

Figure 4. The estimated catchment-scale hydrogeological parameters in this study vs. the measured values in field and the estimated values from soil texture. Note: (a) estimated K in this study vs. measured K , (b) estimated K in this study vs. estimated K from soil texture, (c) estimated f in this study vs. estimated f from soil texture, and (d) estimated D

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in this study vs. measured D .

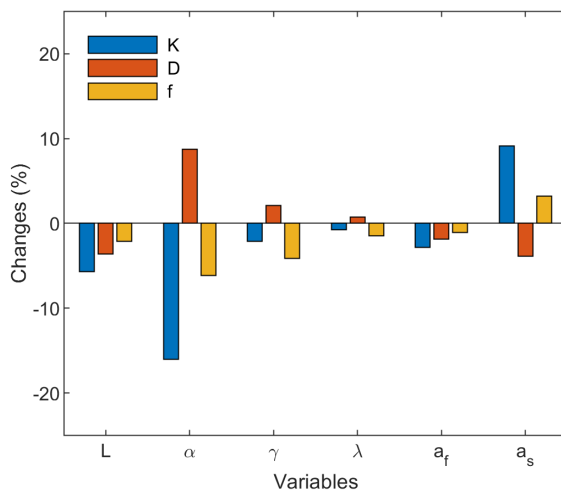
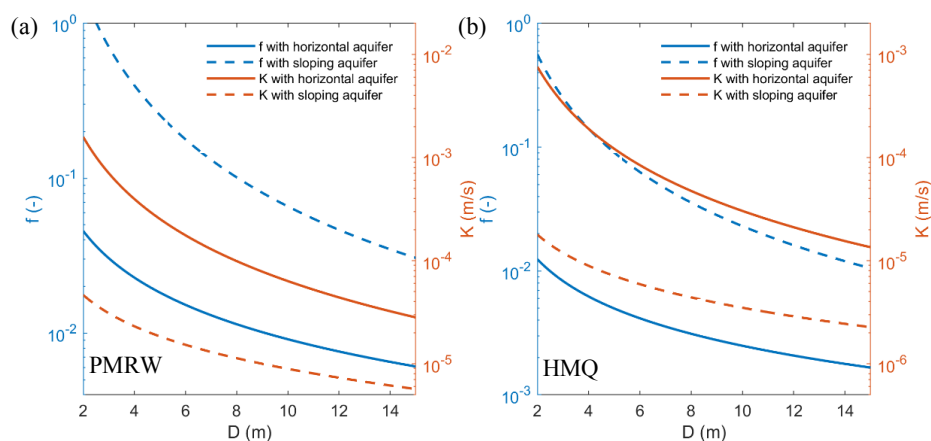


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changing D in (a) PMRW and (b) HMQ.