Reply to the comments from the reviewer #2:

Anonymous Referee #2

Received and published: 23 October 2019

Title: Combining analytical solution of Boussinesq equation with the modified KozenyCarman equation for estimation of catchment-scale hydrogeological parameters

Ref. MS #HESS-2019-453

Overview:

The authors claim to develop a novel methodology to estimate the catchment-scale hydrogeological parameters of saturated hydraulic conductivity, K; drainable porosity, f; and the soil depth, D by combining the existing analytical solution of the Boussinesq equation and the Kozeny-Carman equation. Subsequently, the developed approach is tested in four real-world study sites to conclude that the obtained soil parameters are well within the acceptable range. Although solutions to both the Boussinesq and the Kozeny-Carman equations exist in the literature, the authors' idea to combine both the solutions for estimating of aquifer property seems novel and interesting. It is worth mentioning that in earlier attempts to model the low flows from the delayed hillslope discharge, the soil depth, D is considered as a calibration parameter apart from K and f (e.g., Matonse and Kroll, 2009). The theoretical advancement, when established, could be helpful for modelling of the hydro-geologically ungauged basins wherein only streamflow data is available. However, there are several issues for which I am negative in recommending the paper for acceptance. Looking at the merits of the theoretical approach, the authors may be asked for a fully revised manuscript for resubmission.

We are very grateful for your comments that help us to improve the quality of the manuscript. The whole paper has been thoroughly revised. The point-by-point responses are provided below.

Specific comments:

1. Getting first-hand information on spatial distribution of soil depth, D is easy in comparison to K and f. Unless the catchment under study is strictly ungauged and inaccessible, it can be obtained from the available well-logs directly and by vertical electrical sounding experiment indirectly that is neither costly nor time consuming. How would the authors justify the necessity of estimating the soil depth by analytical or empirical methods? This needs to be clearly justified in Introduction.

Response: We agree that the soil thickness can be obtained from the available well-logs and by geophysical surveys, but it is still time consuming for the details of the soil depth distributions in a catchment. The detailed measurements of the soil thickness are only available in a few experimental catchments. Even these measured thicknesses are available, they cannot be directly used to represent the "effective depth" of flow dynamic domain.

We described the necessity in the revised manuscript.

2. The title and the spatial scale of catchment chosen seem to be contradictory. Although, the authors claim for catchment-scale estimation, the study areas chosen do not reflect

the same as all the four areas have the extent of 0.102 – 1.35 km2, which are only at the hillslope-scale. It is also reflected in the results obtained (Line #265- 270), where the author state that the late-time recession is relatively fast except for Schöneben rock glacier (SPG) catchment. It could be due to the fact that a small hillslope would recede fast. The authors need to rethink and either change the title or test the approach at a suitable scale.
Response: We revised the title to focus on hydrogeological parameters in small headwater catchments. The four small catchments can be conceptualized as hillslopes. As to the fast recession at small catchments, it is attributed to the steep slope and limited plain riparian area.

3. The delayed recession from the SPG could have resulted due to delayed release from snow and glacier melt. If this is the case, choosing this area poses a serious question as the Boussinesq equation and its solution deals with the draining hillslope aquifers, and not the glaciers. The authors can refer Winkler et al. (2016) for more details on the SPG.
Response: According to Winkler et al. (2016), the SPG is a catchment with relict rock glaciers, which is a kind of rock glaciers and ice has disappeared. Thus, there is no influence from glaciers. In our study, the recession data were selected in winter when the meltwater from snow and recharge from other sources is limited as temperature is below zero.

4. It is good to see that the authors have considered both the early-time and late-time recessions; however, plotting at least one season of discharge data for each catchment would be more informative.
Response: We plotted each recession segment in $Q{\sim}t$ and $-\mathrm{d}Q/\mathrm{d}t{\sim}Q$ forms in different colors as shown in Fig. 1 below and calibrated the parameters of each recession in the revised manuscript (Table 1 and Figure 2 below).

5. Form Fig. 3, it seems that the early-time and late-time recessions cannot be inferred from the analyzed recession data. Hence, a longer time series need to be analyzed with clear recession events (e.g., Rupp et al., 2009). As mentioned in Lines #257-258 and Fig. 3, it is not clear what is the physical basis of choosing the lower envelop lines with b=1 and b=3 to derive the recession intercepts. The range of this value looks too high.
Response: We agree that it not visual to infer the early-time and late-time recessions for aggregated data points of $-\mathrm{d}Q/\mathrm{d}t{\sim}Q$ and a longer time series is helpful. Considering the reviewer's comments, we further analyzed the individual recessions of $Q(t){\sim}t$ in semi-logarithm space as a complement and construct a master recession. Individual recessions lasting at least 6 days after rainfall ceases are selected and the first two days data are removed to exclude the influence of surface flow.

In the analyzed catchments (PMRW and WS10), the individual events are selected for separating the early-time and the late-time recessions (Fig 1). The early-time recessions of individual segments can be analyzed by the following nonlinear equation (Brutsaert and Nieber, 1977; Tallaksen, 1995):

$$Q(t) = \left[Q(0)^{1-b_f} - a_f(1-b_f)t\right]^{\frac{1}{1-b_f}} \quad (1)$$

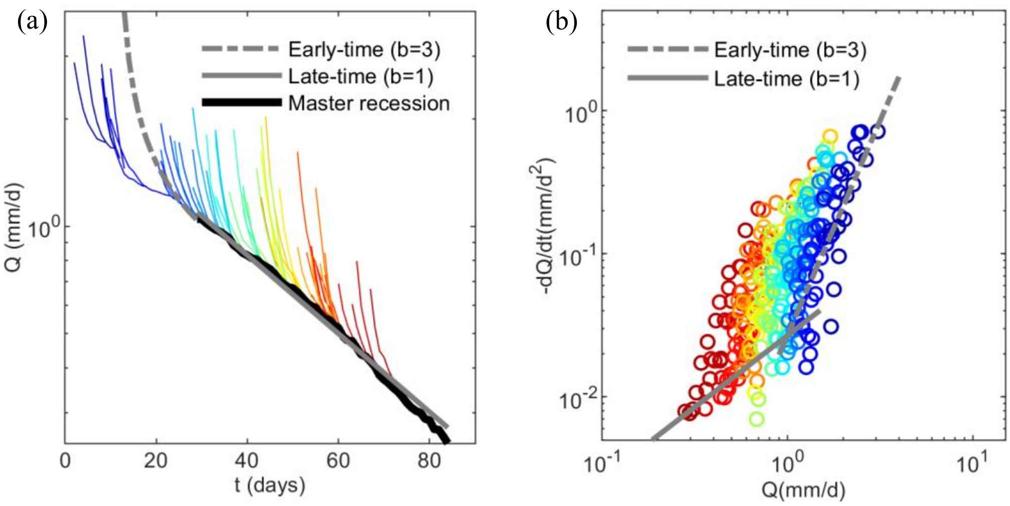
where Q is the discharge, $Q(0)$ is the initial discharge prior recession at t=0. This equation is equivalent to $-\mathrm{d}Q(t)/\mathrm{d}t = a_f Q^{b_f}$. Fig. 1 indicates that the parameter a_f depends on initial discharge $Q(0)$. The parameters a_f and b_f are estimated by fitting the early recession segments in the first four days in this study. To meet the condition of b_f=3 for Eq. (15) in the manuscript, we selected the early recession segments that the slopes approach to 3, and the corresponding a_f are listed in Table 1.

The tails of the late-time recessions of individual segments of $Q(t)\sim t$ in semi-logarithm space concentrate to a line (the master recession curve) in Fig. 1 (a, c). It indicates b_s=1 for the equation -$\mathrm{d}Q(t)/\mathrm{d}t = a_s Q^{b_s}$. The lower envelope with a slope of 1 is proved by Wang (2011) for the low discharges at the PMRW. Fitting the line of the master recession with slope b_s=1, we obtained the intercept of the line a_s in Table 1.

Table 1 Properties of individual recession segments in two catchments

Catchments	Numbers of recessions	Initial discharge (mm/d)	a_s (s^{-1})	a_f ($m^{-6}s$)	R^2
PMRW	48	0.67~3.43	2.33×10^{-7}	1.5×10^{-2}~3.2×10^{-1} (8.0×10^{-2})	0.995
WS10	53	0.79~6.93	4.34×10^{-7}	8.48×10^{-2}~6.97 (0.29)	0.990

Comments: the value in the bracket refers to the mean value. R^2 is the mean coefficient of determination for all the fitted recessions.



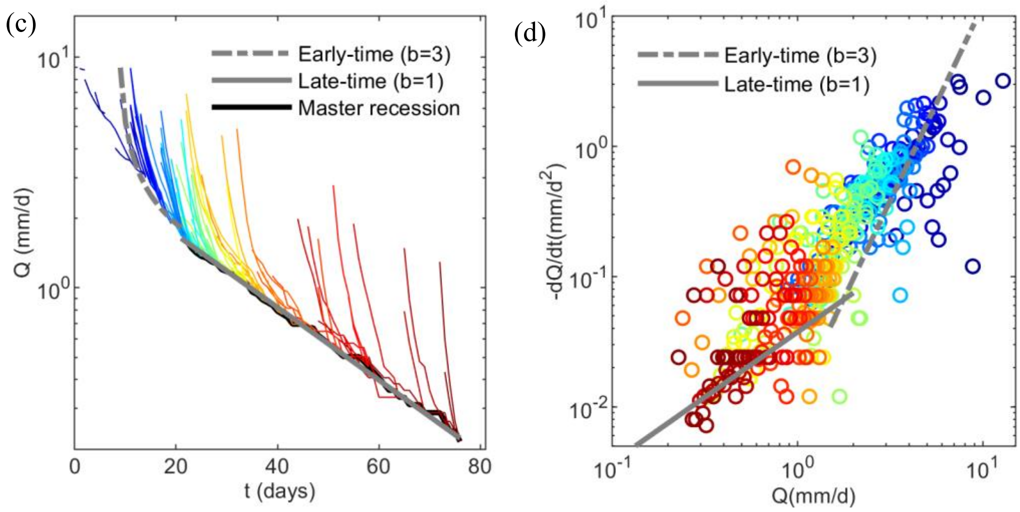


Figure 1. Individual recessions of (a, c) $Q \sim t$ and (b, d) $-dQ/dt \sim Q$ for (a, b) PMRW and (c, d) WS10 (Recessions with different initial discharges are presented in different colors)

Then according to the analytical solution of one-dimensional subsurface flow from the sloping aquifer (Brutsaert, 2005), K and D can be obtained from implicit equations as follows (refer to the derivations in the previous manuscript):

$$4D^{\frac{-2+4\beta}{1+\beta}} + C_{s2}D^{\frac{-4+2\beta}{1+\beta}} = (a_s C_{s1}{}^{-1})(a_f C_f)^{\frac{1-\beta}{1+\beta}} \tag{2}$$

$$4K^{\frac{2-4\beta}{3}} + (a_f^{\frac{2}{3}} C_f^{\frac{2}{3}})C_{s2}K^{\frac{4-2\beta}{3}} = (a_s C_{s1}{}^{-1})(a_f C_f)^{\frac{1}{3}} \tag{3}$$

where $C_f = 8p/\pi\cos\alpha L^2\gamma^{-\beta}$, $\beta = 1/(3-\lambda)$, $C_{s1} = B^{-2}\gamma^{\beta}\pi^2 p\cos\alpha/4$, $C_{s2} = B^2\tan^2\alpha/(\pi^2 p^2)$, α is slope, L is river length, B is aquifer length, $p = 0.3465$, γ and λ are the parameters in pedotransfer function. Combing the modified Kozeny–Carman equation relates K to f

$$f = \gamma^{-\beta}K^{\beta} \tag{4}$$

The catchment-scale hydrogeological parameters (K, f, and D) can be estimated simultaneously for each of the individual recessions.

For PMRW, the estimated K values from various recessions are in the range of the field measurements (Fig. 2(a)). The estimated median value of K from the individual recessions in WS10 is close to that from soil texture but is much smaller than the measured values (Fig. 2(a)). This could be attributed to the fact that the measurements were only taken at the upper soils (1.5 m in maximum) with abundant macropores (Harr, 1977), while the baseflow occurred at the underlying saprolite (McGuire and McDonnell, 2010) where K is much small, such as 5×10^{-6} m/s for the saprolite at PMRW (White et al., 2002).

Similarly, the estimated D is mostly within the range of the measurements of the soil thickness for PMRW catchment. The range of the estimated D are reasonable since the estimated D represents an active thickness of water table variations in the deposits while the measured D represents the entire thickness of deposits. The estimated median value of D from individual recessions is close to the measured soil thickness in WS10. The estimated *f* from soil texture approaches the maximum value of the estimated *f* from the individual recessions in PMRW and WS10.

Besides, the estimated hydrogeological parameters of *K* and *f* increase with $Q(0)$ (Fig. (3)). It indicates that the permeability and effective storage decrease with depths. Thus, the hydrogeological parameters analyzed from individual recessions reflect effect of vertical heterogeneity on baseflow recessions.

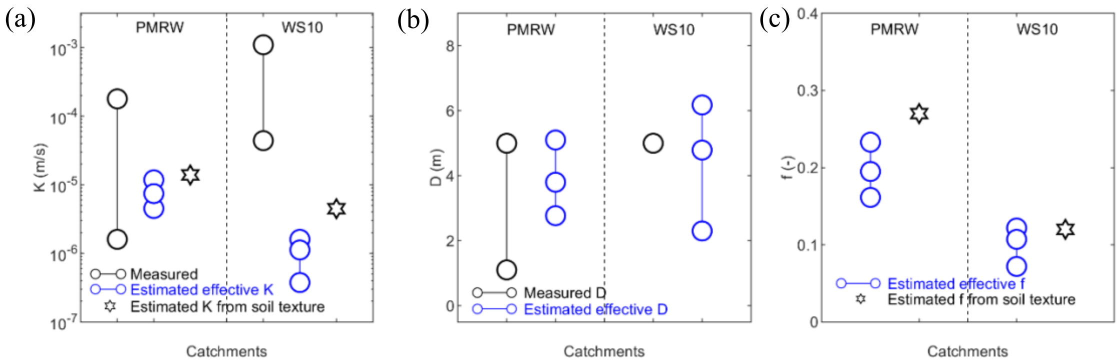


Figure 2. The estimated hydrogeological parameters of (a) *K*, (b) *D*, and (c) *f* from the individual recessions compared to the field measurements and the estimated values from soil texture. Note: the upper, middle, and lower circles in blue color represent the maximum, median, and minimum values of the estimated from individual recessions.

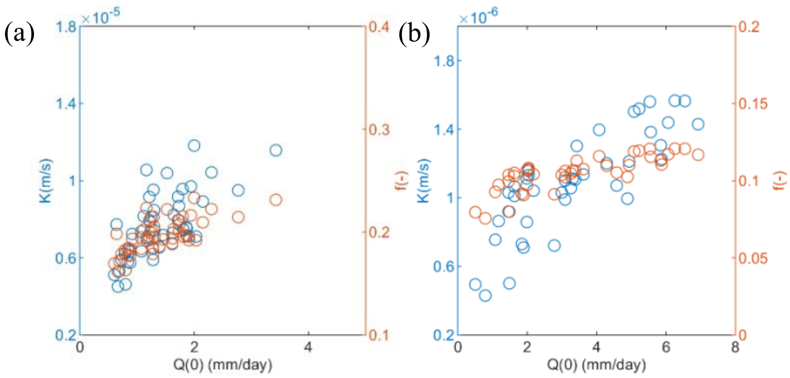


Figure 3. The relationships between initial discharge ($Q(0)$) and estimated effective parameters *K* and *f* for (a) PMRW and (b) WS10

6. The sensitivity analysis is not sufficient with only 10% change of independent variable (Fig. 5). Moreover, analysis for at least one more site would be informative.

Response: We did sensitivity analysis to all these four headwater catchments and conducted 10%, 20%, and 50% changes of each variable for WS10 to make the results more informative as shown in Fig. 4. We added more analyses in the section 4.3 in the revised manuscript.

Besides, these analytical solutions Eqs. (20) ~ (22) in the previous manuscript also show the importance of independent factors to these hydrogeological parameters. The sensitivity analysis makes the results visualization.

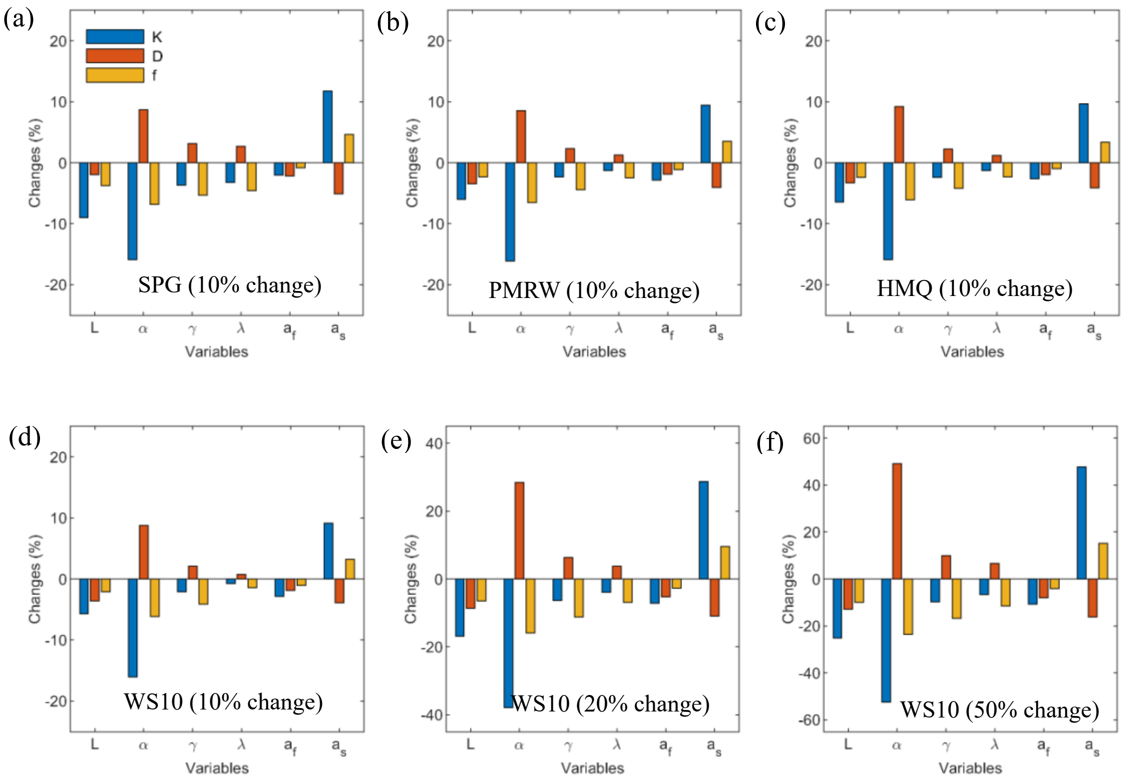


Figure 4. Sensitivity analysis of the estimated effective parameters (*K*, *D*, and *f*) in terms of the relative changes caused by changes of any variables by (a) 10% in SPG, (b) 10% in PMRW, (c) 10% in HMQ, (d) 10% in WS10, (e) 20% in WS10, and (f) 50% in WS10.

7. Following Rupp and Selker (2006), consideration of variable time interval for recession analysis is interesting. The authors should mention the range of time interval considered for arriving at Fig. 3. Further, is this range same for all the four catchment or different?

Response: The ranges of time interval are different for the four catchments. The range depends on the length of individual recessions. We revised the method to generate recession data points in -d*Q*/d*t*~*Q* form. An improved method proposed by Roques et al. (2017) are used instead of Rupp and Selker (2006).

8. Estimation of the hydraulic parameters considering both early- and late- time recession does not represent the same zone of aquifer that contribute to recession flow. Hence, it would not be better to say effective K, effective f rather than K and f.

Response: We agree with the reviewer's suggestions, the names of effective *K*, effective *f* rather than *K* and *f* are more suitable. We revised these in the manuscript as the reviewer's suggestions.

9. Page 14: The field application results show that there is a huge gap between the estimated and observed soil hydraulic parameters, which may result in significant uncertainty in estimating the subsurface flux. Therefore, it is always advisable to calibrate the parameters for their field use. An uncertainty analysis could strengthen the

outcome of these results.

Response: In the revised manuscript, we obtained the ranges of the estimated effective parameters from the analysis of individual recessions. The ranges of the estimated *D* are narrower than the measured *D*. This is reasonable since the estimated effective *D* represents an active thickness of water table variations in the deposits while the measured *D* represents the entire thickness of deposits. Surely, there is still a gap between estimated and observed *K*. The significant uncertainty can come from the accuracy of observed streamflow, heterogeneity in catchment landscape, and the linearization of analytical solutions. Our estimated effective parameter could be viewed as a representative parameter set used as the initial ranges of these parameters in hydrological models. We expressed these in the revised manuscript.

We cannot directly analyze the uncertainty in terms of our method and data, but the sensitivity analysis and the comparison between the estimated and measured values partly reflect the uncertainty in our analysis. We discussed it in the revised manuscript.

10. Eqs. (23)-(26) and Fig. 6: These are the ideal aquifer cases where K and f decrease with increase in the aquifer thickness, D. Therefore, these Eqs. could be far from the real hillslope cases.

Response: In the cases of Eqs. (23)~(24) and (25)~(26) in the previous manuscript, two equations express the three hydrogeological parameters. So at least one variable such as *D* is dependent on the other two, as shown in Fig 6. This figure shows relationships of *K* and *f* with increase in the aquifer thickness, *D*. It does not mean the spatial variations of *K* and *f* with *D* in the real hillslope cases. Actually, *K* and *f* are independent of *D* in our study when the third equation (the modified Kozeny–Carman equation) is introduced as shown in Eqs. (17) ~ (18) and (20) ~ (22) in the previous manuscript.

11. The ability of the present approach in estimating the hydrogeological parameters can be tested fully by modelling the streamflow with the Boussinesq equation-based models with the estimated parameters instead of calibrating the model. I hope this would be the authors' next plan. However, rather than comparing the estimated parameters with the pedo-transfer function-based estimated results, testing the parameters in real modelling case would strengthen the claim in discussion.

Response: We agree that it would be great important to test effectivity of these estimated parameters by modelling the streamflow. Actually, the estimated catchment-scale parameters based on recession analysis of the Boussinesq equation have been applied for hydrological modeling. For example, Vannier et al. (2016) estimated aquifer thickness and saturated hydraulic conductivity based on Brutsaert-Nieber method, which were applied for the hydrological model parameters and achieved a high simulation accuracy.

We discussed this aspect of application in hydrological modelling in the revised manuscript.

Editorial:

1. Some long sentences need to be fragmented for clearer meaning: Lines #236-238,

#242-243, #306-308
Response: We revised these sentences as suggested.

2. Line #115: Should be '. . . one-dimensional subsurface flow from the sloping aquifer' not 'on the'.
Response: We revised the manuscript as suggested.

3. Reference for Eq. 1?
Response: We added a reference to the equation.

4. Line #119: From Fig. 1, the distance from river to ridge is B.
Response: We revised this sentence.

5. References for Eqs. 3 and 4?
Response: We added references to these equations.

6. Line #157: The verb form of the term should be 'recede', not 'recess'. Change accordingly at all subsequent appearances.
Response: We revised it as suggested.

7. Lines: #157-160: It will be better to mention that the time duration between rainfall excess and beginning of recession depends upon catchment characteristic, extent, topography and depression storage.
Response: We added a sentence to mention this content.

8. Line #163: Replace '. . .in terms of. . .' by '. . .as per. . .'.
Response: We revised it as suggested.

9. Line #200: Typo in Eq. 19. Replace 'r' by gamma.
Response: We revised it as suggested.

10. Line #214: Change to '. . .fall under. . .'.
Response: We revised it as suggested.

11. Line #221: Change to '. . .For SPG, the data published. . .'
Response: We revised it as suggested.

12. Lines #234-235: Change to '. . .saturated hydraulic conductivity, K and soil/saprolite thickness, D. . . '. Change likewise at all other places.
Response: We revised it as suggested.

13. Line #396: Change to 'Thus, detailed. . .,'. 14. Line #410: Change to '. . .equivalent values at. . .'.
Response: We revised it as suggested.

References

Brutsaert, W.: The unit response of groundwater outflow from a hillslope, Water Resour. Res., 30(10), 2759-2763, 1994.

Brutsaert, W.: Hydrology: an introduction, Cambridge University Press, 2005.

Brutsaert, W., and Nieber J. L.: Regionalized drought flow hydrographs from a mature glaciated plateau, Water Resour. Res., 13(3), 637-643, 1977.

Harr, R. D.: Water flux in soil and subsoil on a steep forested slope, J. Hydrol., 33(1-2), 37-58, 1977.

McGuire, K. J., and McDonnell, J. J.: Hydrological connectivity of hillslopes and streams: Characteristic time scales and nonlinearities, Water Resour. Res., 46(10), 2010.

Roques, C., Rupp, D.E. and Selker, J. S.: Improved streamflow recession parameter estimation with attention to calculation of -dQ/dt, Adv. Water Resour., 108, 29-43, 2017.

Rupp, D. E., and Selker, J. S.: Information, artifacts, and noise in -dQ/dt-Q recession analysis, Adv. Water Resour., 29(2), 154-160, 2006.

Tallaksen, L. M.: A review of baseflow recession analysis. J. Hydrol., 165(1-4), 349-370, 1995.

Triska, F. J., Sedell, J. R., Cromack Jr, K., Gregory, S. V., McCorison, F. M.: Nitrogen budget for a small coniferous forest stream. Ecological Monographs, 54(1), 119-140, 1984.

Vannier, O., Anquetin, S., and Braud, I.: Investigating the role of geology in the hydrological response of Mediterranean catchments prone to flash-floods: Regional modelling study and process understanding. Journal of Hydrology, 541, 158-172, 2016.

Wang, D.: On the base flow recession at the Panola mountain research watershed, Georgia, United States, Water Resour. Res., 47(3), 2011.

White, A., Blum, A. E., Schulz, M. S., Huntington, T. G., Peters, N. E., Stonestrom, D. A.: Chemical weathering of the Panola Granite: Solute and regolith elemental fluxes and the weathering rate of biotite. In *Water-Rock Interactions, Ore Deposits, Environmental Geochemistry: A Tribute to David A. Crerar,* Hellmann R., Wood S. A. (Eds.). The Geochemical Society, Special Publication No. 7, 37-59, 2002.

Winkler, G., Wagner, T., Pauritsch, M., Birk, S., Kellerer-Pirklbauer, A., Benischke, R., Leis, A., Morawetz, R., Schreilechner, M.G., Hergarten, S., 2016. Identification and assessment of groundwater flow and storage components of the relict Schöneben Rock Glacier, Niedere Tauern Range, Eastern Alps (Austria). Hydrogeol. J. 24, 937– 953. doi:10.1007/s10040-015-1348-9.