Reply to the comments from the reviewer #1:

Anonymous Referee #1

Received and published: 7 October 2019
Title: Combining analytical solutions of Boussinesq equation with the modified Kozeny–Carman equation for estimation of catchment-scale hydrogeological parameters
MS No.: hess-2019-453

GENERAL COMMENTS

This paper presents a means to constrain values of hydraulic conductivity, effective porosity, and aquifer depth estimated from recession analysis by using an empirical relationship between hydraulic conductivity and drainable porosity that is a function of the pore size distribution, the latter being estimated from soil texture properties.

I think the concept has merit, is worth pursuing, and could lead to improved estimations of aquifer parameters. The theoretical part of the paper could be published but requires a better explanation of the assumptions that underlie the analytical solutions and a discussion of the implications of these assumptions when applying the parameter estimation technique to real world situations.

Where the paper falls short, and why I recommend this paper not be published, is in the application to real data. To claim that an early-time with b = 3 and late time regime with b = 1 exist in the aggregate of the data from any of the four watersheds is a very large stretch. An examination of individual recessions in the dQ/dt vs Q would likely reveal that such behavior (b = 3 to b = 1) is simply not present in the receding limbs of the hydrograph. Studies examining individual recessions have begun to show that the pattern in aggregated data, including apparent lower envelopes, does not represent constant aquifer properties (e.g., Biswal and Marani, 2010; Shaw and Riha, 2012; Mutzner et al., 2013; McMillan et al., 2014; Basso et al. 2015; Karlsen et al,. 2019; Santos et al., 2019). Jachens et al. (2019) in particular demonstrate the fallacy that the apparent pattern of the aggregated data in dQ/dt vs Q space (e.g., envelopes of b = 3 to 1 or other values of b estimated directly from aggregate) represent aquifer properties but rather arise from properties of the climate (i.e., magnitude and interarrival times of recharge events).

Response: We have revised the manuscript according to your suggestions. We described the assumptions of the analytical solutions derived from the Boussinesq equation in details in the revised manuscript.

We agree that the slopes of $-\mathrm{d}Q/\mathrm{d}t{\sim}Q$ from individual recessions could deviate the slopes of b=3 and 1 for the early-time and late-time recessions, respectively. According to reviewer's suggestions, we further analyzed the individual recessions of $Q(t){\sim}t$ in semi-logarithm space as a complement. Individual recessions lasting at least 6 days after rainfall ceases are selected and the first two days data are removed to exclude the influence of surface flow.

In the analyzed catchments (PMRW and WS10), the individual events are selected for separating the early-time and the late-time recessions (Fig 1). The early-time recessions of individual segments can be analyzed by the following nonlinear equation (Brutsaert and Nieber, 1977; Tallaksen, 1995):

$$Q(t) = \left[Q(0)^{1-b_f} - a_f(1-b_f)t\right]^{\frac{1}{1-b_f}} \tag{1}$$

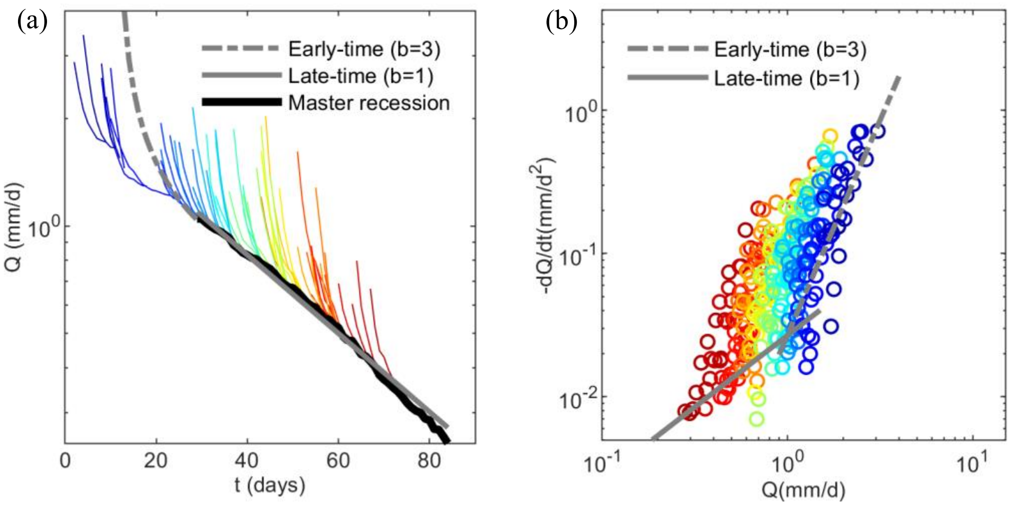
where Q is the discharge, $Q(0)$ is the initial discharge prior recession at t=0. This equation is equivalent to $-\mathrm{d}Q(t)/\mathrm{d}t = a_f Q^{b_f}$ $(b_f \neq 1)$. Fig. 1 indicates that the parameter a_f depends on initial discharge $Q(0)$. The parameters a_f and b_f are estimated by fitting the early recession segments in the first four days in this study. To meet the condition of b_f=3 for Eq. (15) in the previous manuscript, we selected the early recession segments that the slopes approach to 3, and the corresponding a_f are listed in Table 1.

The tails of the late-time recessions of individual segments of $Q(t)\sim t$ in semi-logarithm space concentrate to a line (the master recession curve) in Fig. 1 (a, c). It indicates b_s=1 for the equation -d$Q(t)/\mathrm{d}t = a_s Q^{b_s}$. The lower envelope with a slope of 1 is proved by Wang (2011) for the low discharges at the PMRW. Fitting the line of the master recession with slope b_s=1, we obtained the intercept of the line a_s in Table 1.

Table 1 Properties of individual recession segments in two catchments

Catchments	Numbers of recessions	Initial discharge (mm/d)	a_s (s^{-1})	a_f (m^{-6}s)	R^2
PMRW	48	0.67~3.43	2.33×10^{-7}	1.5×10^{-2}~3.2×10^{-1} (8.0×10^{-2})	0.995
WS10	53	0.79~6.93	4.34×10^{-7}	8.48×10^{-2}~6.97 (0.29)	0.990

Comments: the value in the bracket refers to the mean value. R^2 is the mean coefficient of determination for all the fitted recessions.



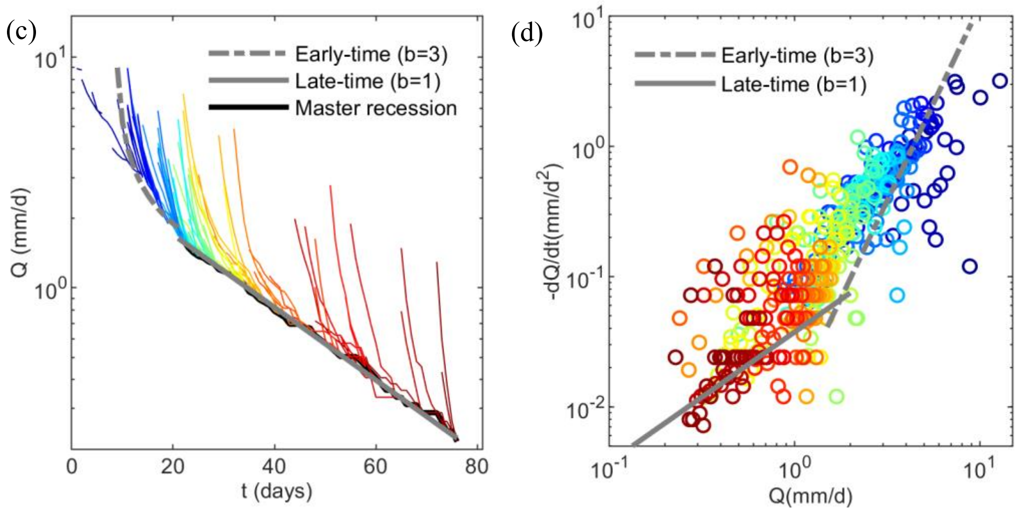


Figure 1. Individual recessions of (a, c) $Q{\sim}t$ and (b, d) $-dQ/dt{\sim}Q$ for (a, b) PMRW and (c, d) WS10 (Recessions with different initial discharges are presented in different colors)

Then according to the analytical solution of one-dimensional subsurface flow from the sloping aquifer (Brutsaert, 2005), K and D can be obtained from implicit equations as follows (refer to the derivations in the previous manuscript):

$$4D^{\frac{-2+4\beta}{1+\beta}} + C_{s2}D^{\frac{-4+2\beta}{1+\beta}} = (a_s C_{s1}{}^{-1})(a_f C_f)^{\frac{1-\beta}{1+\beta}} \tag{2}$$

$$4K^{\frac{2-4\beta}{3}} + (a_f^{\frac{2}{3}} C_f^{\frac{2}{3}})C_{s2}K^{\frac{4-2\beta}{3}} = (a_s C_{s1}{}^{-1})(a_f C_f)^{\frac{1}{3}} \tag{3}$$

where $C_f = 8p/\pi\cos\alpha L^2\gamma^{-\beta}$, $\beta = 1/(3-\lambda)$, $C_{s1} = B^{-2}\gamma^{\beta}\pi^2 p\cos\alpha/4$, $C_{s2} = B^2\tan^2\alpha/(\pi^2p^2)$, α is slope, L is river length, B is aquifer length, $p = 0.3465$, γ and λ are the parameters in pedotransfer function. Combing the modified Kozeny–Carman equation relates K to f

$$f = \gamma^{-\beta}K^{\beta} \tag{4}$$

The catchment-scale hydrogeological parameters (K, f, and D) can be estimated simultaneously for each of the individual recessions.

For PMRW, the estimated K values from various recessions are in the range of the field measurements (Fig. 2(a)). The estimated median value of K from the individual recessions in WS10 is close to that from soil texture but is much smaller than the measured values (Fig. 2(a)). This could be attributed to the fact that the measurements were only taken at the upper soils (1.5 m in maximum) with abundant macropores (Harr, 1977), while the baseflow occurred at the underlying saprolite (McGuire and McDonnell, 2010) where K is much small, such as 5×10^{-6} m/s for the saprolite at PMRW (White et al., 2002).

Similarly, the estimated D is mostly within the range of the measurements of the soil thickness for PMRW catchment. The range of the estimated D are reasonable since the estimated D represents an active thickness of water table variations in the deposits while the measured D represents the entire thickness of deposits. The estimated median value of D from individual recessions is close to the measured soil thickness in WS10. The estimated f from soil texture approaches the maximum value of the estimated f from the individual recessions in PMRW and WS10.

Besides, the estimated hydrogeological parameters of K and f increase with $Q(0)$ (Fig. (3)).

It indicates that the permeability and effective storage decrease with depths. Thus, the hydrogeological parameters analyzed from individual recessions reflect effect of vertical heterogeneity on baseflow recessions.

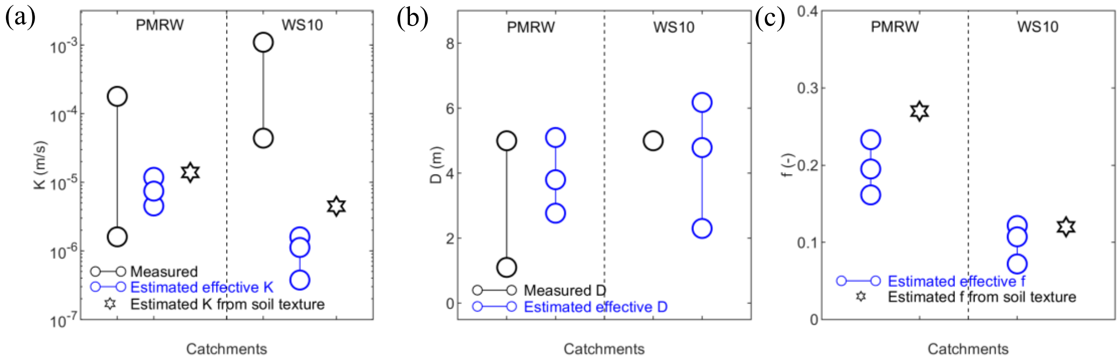


Figure 2. The estimated hydrogeological parameters of (a) *K*, (b) *D*, and (c) *f* from the individual recessions compared to the field measurements and the estimated values from soil texture. Note: the upper, middle, and lower circles in blue color represent the maximum, median, and minimum values of the estimates from individual recessions.

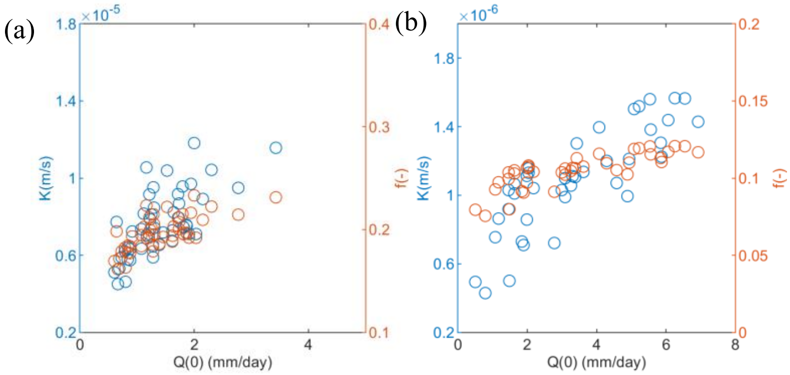


Figure 3. The relationships between initial discharge ($Q(0)$) and estimated effective parameters K and f for (a) PMRW and (b) WS10

Even to the extent the patterns do reflect aquifer properties, the authors do not show that the single Boussinesq aquifer (much less the simplifying assumptions required to achieve the analytical solutions) is a “good enough” representation of complex watershed made of multiple hillslopes and landscape scale heterogeneity in hydraulic properties to allow them to estimate aquifer properties using the proposed technique.

A more appropriate first test of the technique would be in a laboratory setting where the aquifer properties are consistent with a “Boussinesq” aquifer and the aquifer boundary conditions and initial conditions conform to those for which the analytical solutions were derived. Another appropriate test could be for a single and well-instrumented hillslope where the boundary conditions and initial conditions can at least be measured, if not controlled. Datasets exists for both situations.

Response: We agree that catchment flow comes from multiple hillslope flows and thus the landscape scale heterogeneity in hydrogeological properties could make difficulty to estimate aquifer properties using the analytical solutions of the Boussinesq equation. Ideally, the numerical models that calibrated against the observation water tables and discharges will offer reliable estimations of the hydrogeological parameters in a heterogeneous catchment. However, such observations are sparse in the mountainous areas. In our selected catchments, the catchments are located in the headwater catchments and areas are small (i.e. less than 1.5 km^2), which could reduce the spatial heterogeneity of landscapes and composition of multiple hillslopes on the baseflow analysis. WS10 has proven to represent a catchment dominated by hillslopes with negligible storage of water in riparian sediments (Harr, 1977; Triska et al., 1984).

The analytical solutions proposed by Brutsaert (1994) have been tested in the experiment sites and by the numerical simulations (Pauritsch et al., 2015; Rupp and Selker, 2006). Although analytical solutions of Boussinesq equation are derived from the simplified aquifer, they have been widely used to analyze baseflow characteristics and estimate catchment parameters (Brutsaert and Hiyama, 2012; Lyon et al., 2009; Pacheco and Van der Weijden, 2012; Sánchez-Murillo, et al., 2015; Thomas et al., 2015; Vannier et al., 2014; Zhang et al., 2009).

In our analysis, we selected the experimental catchments with relatively detail information of catchment properties, such as the measured hydraulic conductivities, soil thickness and porosity, in order to valid the estimated parameters from our derived equations.

SPECIFIC COMMENTS

53: Assumptions of the analytical solutions of Brutsaert and Nieber (1977) do not precisely include a relatively humid setting. Precisely, the solutions assume an initial saturated thickness of equal depth along the length of the hillslope. Mathematically, this could result from a spatially uniform pulse recharge to an initially dry aquifer.

Response: We agree with the reviewer's opinion that the analytical solutions do not restrict to humid setting. We expressed assumptions of the derived solutions in more details in the revised manuscript.

58-59: This statement is incorrect. Mendoza et al (2003) did not improve the solution of Parlange et al. (2001). They simply tweaked with the lower envelope fitting technique because they didn't observe a b = 3 regime.

Response: We revised this sentence in the manuscript.

87-89: How appropriate are these pedotransfer functions for fractured bedrock and less-weathered saprolite?

Response: The pedotransfer functions are derived from soils. In this study, the estimated parameters derived by the pedotransfer functions are regarded as the values of the hydrogeological parameters in the porous saprolite equivalent to a specific soil. For example, in PMRW and W10, K estimated from the soil texture is equivalent to K in silt loam and clay loam, respectively. We discussed this aspect in the revised manuscript.

119: "x" is not the distance from the river to the hillslope ridge. This would be "B".

Response: Here, x represents the distance from river to a specific position on the hillslope. We revised this expression in the manuscript.

125-134: The authors leave out mentioning that linearization of Eq (2) is required to obtain the solutions presented here, and do not say what that linearization implies physically.

Response: This expression has been revised according to your suggestions.

131: Eq. (3) is true as time goes to zero, or if advection is excluded, as Brutsaert (1994) states. The authors don't mention this.

Response: We added this condition in the statement of analytical solution from Brutsaert (1994).

136: I don't see Eq. (4) in Brutsaert (1994). Can the authors say what equation in Brutsaert (1994) this is meant to be?

Response: The analytical solution in Eq. (4) refers to the first term of Eq. (17) proposed by Brutsaert (1994). We revised the expression.

153: How do the authors reconcile that fact that the b = 1 here is an artifact of the linearization of the sloping Boussinesq equation and does not occur if the equation is not linearized (e.g., Bogaart et al. 2013)?

Response: Yes, the b = 1 could be an artifact of the linearization of the sloping Boussinesq equation. However, Pauritsch et al. (2015) found that it is more convenient to use the analytical solution from Brutsaert (1994) to estimate hydrogeological parameters, particularly at slope angles greater than 10°, by comparing different analytical solutions with numerical solution. We adopted this statement.

162-169: The authors may want to see Roques et al. (2017), who present an improvement to Rupp and Selker (2006a).

Response: We adopted the improvement method from Roques et al. (2017) to generate more reliable recession data points in the revised manuscript.

213-214: What does these slopes, along with the aquifer depth and length, imply for the validity of the sloping and non-sloping Boussinesq and their various solutions? The "Hi" term is useful dimensionless number in this regard. See, for example, Brutsaert (2005) and Rupp and Selker (2006b).

Response: As it is shown in Eqs. (4) and (9) in the previous manuscript, the dimensionless parameter Hi, represents the relative magnitude of the slope term, i.e. the effect of gravity, versus the diffusion term. If Hi/2 greater than π (tending to sloping), the gravity term is dominant, otherwise the diffusion term is the dominant one (tending to non-sloping).

226: Vertical vs horizontal K can be very different in bedrock. A falling head permeameter and the assumption about the shape of the wetting front made in estimating K make it not a suitable instrument for determining this. The authors should comment on possibly large errors in estimating K.

Response: We added sentence on possible errors between our estimates and the measurements in the section of discussions.

248-249 and Table 2: Are these values of drainable porosity high for bedrock and some saprolite?

Can the authors compare with directly measured "f" for bedrock from other locations, to see if these values derived from the pedotransfer functions are unusually high?

Response: The drainable porosity of saprolite and shallow weathered bedrock can be large or even approximate that of soil. For example, Hubbert et al. (2001) showed that the drainable porosity of weathered granitic bedrock at 0.8~1.6 m can be larger than 0.2 (Fig. 2 in the his published paper); according to measurements by Graham et al., (1997), the porosity of macro-void for weathered granitic bedrock is greater than 0.1 mm in upland areas of California, leading to the effective porosity in a range of 0.089~0.149 at entisol site and 0.068~0.115 at alfisol site.

We added these comparisons in the revised manuscript.

252-253 and Figure 3: The recessions in Figure 3 do not support convergence to a value of 1 at late time. How do the authors reconcile the lack of data to support a lower envelope with a slope of 1 with their subsequent estimation of the aquifer parameters? Clark et al. (2009) and Wang (2011) propose different conceptual models for PMRW, with at least two water sources contributing to the streamflow. Both are able to mimic the observed dQ/dt vs dQ pattern much better than can the single homogeneous Boussinesq aquifer assumed by the authors.

Response: Fig. 1 shows that individual recessions of $Q \sim t$ in semi-logarithm space approach to a line with b=1 in our study.

In our re-analyzed results, the fitting of the early-time and late-time individual recessions obtained fast flow recession and slow flow recession, respectively, indicating that two water sources contribute to the streamflow.

267-268: I would say that 35.6 days at PMRW is not relatively fast compared to 45 +/-15 days, but well within that range. PMRW has a similar D to HMQ and WS10, so D does not explain why PMRW is "slower" than HMQ and WS10. Yet the authors use D to try to explain why SPG is

Response: These sentences have been revised as suggested.

356-357: Can the authors comment on how these values for f compare to those in Brutsaert and Nieber (1977) and Brutseart and Lopez (1998)? They also calculated values for f.

Response: The estimated values of f from Brutsaert and Nieber (1977) and Brutsaert and Lopez (1998) are around 0.02 in terms of the geometric mean value, which are underestimated for soils. The reason may be their overestimations of K based on horizontal aquifer assumption.

385-499: It is good that the authors consider to what degree discrepancies are due to riparian area impacts. Can the authors estimate the volume of water that must be stored to explain such discrepancies. Does it exceed riparian storage?

Response: We added this component in our revised manuscript. For example, at PMRW where the riparian, hillslope, and bedrock outcrop area consist the catchment area of 15%, 75%, and 10%, respectively. The soil thickness is about 1 m at hillslope and can reach 5 m in riparian area. When the riparian aquifer is the fully saturated, the water storage capacity for the riparian area can be calculated by multiplying the aquifer thickness (5 m) and drainable porosity (0.27). When assuming groundwater in the whole catchment only stored in riparian area, water storage of the whole catchment is 202.5 mm (0.15×5 m $\times 0.27$).

The storage for each event in the catchment aquifer can be calculated from discharge as $S = \int_0^T Q(t)dt + Q(T)/a_s$, where T is the transition time from the early-time recession to the late-time recession. As listed in Table 1, the largest initial discharge is 3.43 mm/day, and the corresponding water storage estimated in the whole catchment is 84.3 mm. Thus, riparian storage (202.5 mm) can satisfy the groundwater draining storage (84.3 mm).

TECHNICAL/EDITORIAL CORRECTIONS

84: …of a soil pore. Or maybe better it is more correct to say the distribution of hydraulic radii of soil pores.

Response: It has been revised as suggested.

118: …, eta is the water table height above an impermeable layer, …

Response: It has been revised as suggested.

171-172: "… only the latest data are involved in the calculations …" It is not clear what this means.

Response: We revised this sentence to make it clear.

200: The gamma term appears as an "r" in Eq. (19) in my pdf file.

Response: We revised this.

219: Referring to any catchment as "famous" in this paper is unnecessary.

Response: We revised this sentence.

233: textures should be texture.

Response: We corrected this word as suggested.

254-255: Should be either Brutsaert and Nieber (1977) or Brutsaert and Lopez (1998)

Response: It a mistake. We revised this.

256 and 257: envelop should be envelope in both instances.

Response: We corrected this word.

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