Dear Prof. Erwin Zehe:

Many thanks for your work. I have revised the manuscript again as you and the reviewer suggested. I accepted almost all of the reviewer’s suggestions. Major revisions include: 1) I have removed almost all of the second part of the results sections as the reviewer suggested; 2) Table 4 was displaced into the supplement. I think the revisions can better streamline the paper. In addition, I added the $R$ codes in the supplement.

Best regards

Mingguo Zheng

2020-3-29

Response to Reviewer

Many thanks for your comments. I accepted almost all of your suggestions this time. My responses are given as follows.

The new title "A mathematically precise method to partition climate and catchment effects on runoff" is less specific than the old one which I prefer. I have revised the title as suggested.

The second part of the results sections is concerned with the sensitivities, which I think is not so strongly related to the main message of this paper. For the sake of brevity, I suggest to remove it. This would focus the paper, allow to reduce the number of figures and avoid possible distraction of the reader. As suggested, I have removed all of the second part and only remained Fig. 8.

In my first review I suggested to remove or adapt the results of catchments 10,11 since these only have 3 years for their base period, while the evaluation period is > 10 yrs. This short period is probably insufficient to allow stationary conditions, which are essential for such a method. The argument of the author to keep these catchments only because they have been published by another paper does not help here. As suggested, I have removed the catchments from the analyses and made revisions in all related tables, figures and texts.

Minor comments:
Figures 1-3 show different axes although I thought that they intend to show the same relationships. I suggest to make this more consistent. I think the comments over, but I am sorry that the figures cannot have the same axes.
Figure 2: the annotation in the figure seems to be switched; also What represents the continuous lines?

Many thanks for your careful examination. The annotation was indeed switched. I have made revisions.

L653 "In this case, the integral path of the LI method can be considered as the broken line AB+BC in Fig. 3" There is no broken line in Fig.3
I have revised it as “path ABC”.
A line integral-based mathematically precise method to partition climate and catchment effects on runoff

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Abstract

It is a common task to partition the synergistic impacts of drivers in the environmental sciences. However, there is no mathematically precise solution to this partition process task. Here I presented a line integral-based method, which addresses the sensitivity to the drivers throughout the drivers’ evolutionary paths so as to ensure a precise partition. The method reveals that the partition depends on both the change magnitude and pathway (timing of the change), but not on the magnitude alone unless used for a linear system. To illustrate this method, I used applied the Budyko framework to partition the effects of climatic and catchment conditions on the temporal change in the runoff for 21 catchments from Australia and China. The proposed method reduces to the decomposition method when assuming a path in which climate change occurs first, followed by an abrupt change in catchment properties. The proposed method re-defines the widely-used sensitivity at a point as the path-averaged sensitivity. The total differential and the complementary methods simply concern the sensitivity at the initial or/and the terminal state, so they cannot give precise results. Although the path-averaged sensitivities varied greatly among the catchments, they can be readily predicted within the Budyko framework. The path-averaged sensitivity of water yield to climate conditions was found to be stable over time. Space-wise, moreover, the sensitivity can be readily predicted even in the absence of streamflow observations, which facilitates the evaluation of future climate effects on streamflow. As a mathematically accurate solution, the proposed method provides a generic tool to conduct quantitative attribution analyses.

Keywords: Runoff; Climate change; Human activities; Attribution analysis; Budyko
1 Introduction

The impacts of certain drivers on observed changes of interest often require quantification in environmental sciences. In the hydrology community, both climate and human activities have posed global-scale impact on hydrologic cycle and water resources (Barnett et al., 2008; Xu et al., 2014; Wang and Hejazi, 2001). Diagnosing their relative contributions to runoff is of considerable relevance to the researchers and managers. Unfortunately, performing a quantitative attribution analysis of runoff changes remains a challenge (Wang and Hejazi, 2001; Berghuijs and Woods, 2016; Zhang et al., 2016); this is to a considerable degree due to a lack of a mathematically precise method to decouple synergistic and often confounding impacts of climate change and human activities.

Numerous studies have detected the long-term variability in runoff and attempted to partition the effects of climate change and human activities through various methods (Dey and Mishra, 2017); these include the paired-catchments method and the hydrological modeling method. The paired-catchment method can filter the effect of climatic variability and thus isolate the runoff change induced by vegetation changes (Brown et al., 2005). However, this method is capital intensive; moreover, it generally involves small catchments and experiences difficulties when extrapolating to large catchments (Zhang et al., 2011). The physical-based hydrological models often have limitations such as a high data requirement, labor-intensive calibration and validation processes, and inherent uncertainty and interdependence in parameter estimations (Binley et al., 1991; Wang et al., 2013; Liang et al., 2015). Conceptual models such as Budyko-type equations (see Section 2.1) have consequently gained interest in recent years (see Section 2.1).

Within the Budyko framework, studies (Roderick and Farquhar, 2011; Zhang et al., 2016) have used the total differential of runoff as a proxy for the runoff change and the partial derivatives as the sensitivities (hereafter called the total differential method). The total differential, however, is simply a first-order approximation of the observed change (Fig. 1(a)). This approximation has caused an error in the calculation of climate impact on runoff, with the deviation ranging from 0 to 20 $10^3$m (or -118 to 174%) in China (Yang et al., 2014). The elasticity method proposed by Schaake (1990) is also based on the total differential expression (Sankarasubramanian et al., 2001; Zheng et al., 2009). The method uses the “elasticity” concept to assess the climate sensitivity of runoff. The elasticity coefficients, however, have been estimated in an empirical way and are not physically sound (Roderick and Farquhar, 2011; Liang et al., 2015).

The so-called decomposition method developed by Wang and Hejazi (2011) has also been widely used. The method assumes that climate changes cause a shift along a Budyko curve and then human interferences cause a vertical shift from one Budyko curve to another (Fig. 2). Under this assumption, the method extrapolates the Budyko models that are calibrated using observations of the reference period, in which human impacts remain minimal, to determine the human-induced runoff changes that occur during the evaluation period.

Recently, Zhou et al. (2016) established a Budyko complementary relationship for runoff and further applied it to partitioning the climate and catchment effects. Superior to the total differential method, the complementary method culminates by yielding a no-residual partition. Nevertheless, this method depends on a given weighted factor that is determined in an empirical but not a precise way.
Furthermore, Zhou et al. (2016) argued that the partition is not unique in the Budyko framework because the path of the climate and catchment changes cannot be uniquely identified.

Obtaining a precise partition remains difficult, even when giving a precise mathematical model. This difficulty can be illustrated by using a precise hydrology model \( R = f(\Delta x, \Delta y) \), where \( R \) represents runoff, and \( x \) and \( y \) represent the climate factors and catchment characteristics, respectively. We assumed that \( R \) changes by \( \Delta R \) when \( x \) changed by \( \Delta x \) and \( y \) changes by \( \Delta y \), i.e.,

\[
\Delta R = f(x + \Delta x, y + \Delta y) - f(x, y)
\]

To determine the effect of \( x \) on \( \Delta R \), i.e., \( \Delta R_x \), a common practice is to assume that \( y \) remains constant when \( x \) changes by \( \Delta x \). We thus obtain:

\[
\Delta R_x = f(x + \Delta x, y) - f(x, y)
\]

Similarly, we can obtain:

\[
\Delta R_y = f(x, y + \Delta y) - f(x, y)
\]

Although this derivation seems quite reasonable, it is problematic as \( \Delta R_x + \Delta R_y \neq \Delta R \). A further examination shows that a variable’s effect on \( R \) seems to differ depending on the changing path (timing of the change). For example, \( \Delta R_x = f(x + \Delta x, y) - f(x, y) \) and \( \Delta R_y = f(x, y + \Delta y) - f(x, y, y) \) if \( x \) changes first and \( y \) subsequently changes (Note that the partition is precise with \( \Delta R_x + \Delta R_y = \Delta R \) at this moment). If \( y \) changes first and \( x \) subsequently changes, the partition then becomes:

\[
\Delta R_x = f(x + \Delta x, y + \Delta y) - f(x, y + \Delta y)
\]

and \( \Delta R_y = f(x, y + \Delta y) - f(x, y) \). In the case of \( x \) and \( y \) changing simultaneously, unfortunately, current literature seems not to provide a mathematically precise solution.

The aim of this study is to propose a mathematically precise method to conduct a quantitative attribution to drivers. The method is based on the line integer (called the LI method hereafter) and takes account of the sensitivity throughout the evolutionary path of the drivers rather than at a point as the total differential method does. In this way, the proposed method revises the widely used concept of sensitivity at a point as the path-averaged sensitivity. To present and evaluate the proposed method, I decomposed the relative influences of climate and catchment conditions on runoff within the Budyko framework using data from 21 catchments from Australia and China. I also examined the spatio-temporal variability of the path-averaged sensitivities and assessed their spatio-temporal predictability.

**2 Methodology**

2.1 Budyko Framework and the MCY equation

Budyko (1974) argued that the mean annual evapotranspiration (\( E \)) is largely determined by the water and energy balance of a catchment. Using precipitation (\( P \) ) and potential evapotranspiration (\( E_0 \)) as proxies for water and energy availability, respectively, the Budyko framework relates evapotranspiration losses to the aridity index defined as the ratio of \( E_0 \) over \( P \). The Budyko framework has gained wide acceptance in the hydrology community (Berghuijs and Woods, 2016; Sposito, 2017). In recent decades, several equations have been developed to describe the Budyko framework. Among them, the Mezentsev-Choudhury-Yang’s equation (Mezentsev, 1955; Choudhury, 1999; Yang et al., 2008) (Called the MCY equation hereafter) has been widely accepted and was used in this study:
where \( n \in (0, \infty) \) is an integration constant that is dimensionless, and represents catchment properties. Eq. (3) requires a relatively long time scale whereby the water storage of a catchment is negligible and the water balance equation reduces to be \( R = P - E \). Here I adopted a “tuned” \( n \) value that can obtain an exact accordance between the calculated \( E \) by Eq. (1) and that actually encountered \( (= P - R) \).

The partial differentials of \( R \) with respect to \( P, E_0, \) and \( n \) are given as:

\[
\frac{\partial R}{\partial P} = R_0(P, E_0, n) - \frac{E_0}{(P^* + E_0^{*})^{1/n}} \\
\frac{\partial R}{\partial E_0} = R_0(P, E_0, n) - \frac{P^{*+1}}{(P^* + E_0^{*})^{1/n}} \\
\frac{\partial R}{\partial n} = R_0(P, E_0, n) - \frac{-E_0Pn^{-1}}{(P^* + E_0^{*})^{1/n}} \left[ \frac{\ln(P^* + E_0^{*})}{n} - \frac{P^* \ln P + E_0^n \ln E_0}{P^* + E_0^n} \right]
\]

2.2 Theory of the line integral-based method

We start by considering an example of a two-variable function \( z = f(x, y) \) and assumed that \( x \) and \( y \) are independent. The function has continuous partial derivatives \( \partial z / \partial x = f_x(x, y) \) and \( \partial z / \partial y = f_y(x, y) \).

Suppose that \( x \) and \( y \) vary along a smooth curve \( L \) (e.g., \( AC \) in Fig. 3) from the initial state \((x_0, y_0)\) to the terminal state \((x_N, y_N)\), and \( z \) co-varies from \( z_0 \) to \( z_N \). Let \( \Delta z = z_N - z_0, \Delta x = x_N - x_0, \) and \( \Delta y = y_N - y_0 \).

Our goal is to determine a mathematical solution that quantifies the effects of \( \Delta x \) and \( \Delta y \) on \( \Delta z \), i.e. \( \Delta z \) and \( \Delta z_0 \), \( \Delta z_2 \) and \( \Delta z_3 \) should be subject to the constraint \( \Delta z_0 + \Delta z_3 = \Delta z \).

As shown in Fig. 3, points \( M_1(x_1, y_1), \ldots, M_N(x_N, y_N) \) partition \( L \) into \( N \) distinct segments. Let \( \Delta x_i = x_{i+1} - x_i, \Delta y_i = y_{i+1} - y_i, \) and \( \Delta z_i = z_{i+1} - z_i \). For each segment, \( \Delta z_i \) can be approximated as \( d\Delta z_i \):

\[
\Delta z_i \approx \Delta z_i = f_i(x_i, y_i)\Delta x_i + f_i(x_i, y_i)\Delta y_i.
\]

We then have: \( \Delta z = \sum_{i=1}^{N} \Delta z_i = \sum_{i=1}^{N} f_i(x_i, y_i)\Delta x_i + \sum_{i=1}^{N} f_i(x_i, y_i)\Delta y_i \). We thus obtain the following respective approximation of \( \Delta z_0 \) and \( \Delta z_3 \): \( \Delta z_0 \approx \sum_{i=1}^{N} f_i(x_i, y_i)\Delta x_i \) and

\[
\Delta z_3 \approx \sum_{i=1}^{N} f_i(x_i, y_i)\Delta y_i.
\]

Next, define \( \tau \) as the maximum length among of the \( N \) segments. The smaller the value of \( \tau \), the closer to \( \Delta z \) the value of \( d\Delta z_i \), and then the more accurate the approximations are. The approximations become exact in the limit \( \tau \to 0 \). Taking the limit \( \tau \to 0 \) then converts the sum into integrals and gives a precise expression (this is an informal derivation and please see Appendix A for a formal one):

\[
\Delta z = \lim_{\tau \to 0} \sum_{i=1}^{N} f_i(x_i, y_i)\Delta x_i + \lim_{\tau \to 0} \sum_{i=1}^{N} f_i(x_i, y_i)\Delta y_i = \int_L f_i(x, y)dx + \int_L f_i(x, y)dy,
\]

where

\[
\int_L f_i(x, y)dx = \lim_{\tau \to 0} \sum_{i=1}^{N} f_i(x_i, y_i)\Delta x_i \quad \text{and} \quad \int_L f_i(x, y)dy = \lim_{\tau \to 0} \sum_{i=1}^{N} f_i(x_i, y_i)\Delta y_i
\]

denote the line integral of \( f_i \) and \( f_j \).
along $L$ (termed integral path) with respect to $x$ and $y$, respectively. \( \int_L f_i(x, y)dx \) and \( \int_L f_i(x, y)dy \) exist provided that \( f_i \) and \( f_t \) are continuous along $L$. We thus obtain a precise evaluation of \( \Delta z_i \) and \( \Delta z_{t_i} \):

\[
\Delta z_i = \int_L f_i(x, y)dx \\
\Delta z_{t_i} = \int_L f_t(x, y)dy .
\]

Unlike the total differential method, the sum of \( \Delta z_i \) and \( \Delta z_{t_i} \) persistently equals \( \Delta z \) (Appendix B). If \( f(x, y) \) is linear, then \( f_i \) and \( f_t \) remain constant. Defining \( f_i(x, y) \) and \( f_t(x, y) \) respectively, then \( \Delta z_i = C_i \Delta x \) and \( \Delta z_{t_i} = C_i \Delta y \). \( \Delta z_i \) and \( \Delta z_{t_i} \) are thus independent of $L$. If \( f(x, y) \) is non-linear, however, both \( \Delta z_i \) and \( \Delta z_{t_i} \) vary with $L$, as is exemplified in Appendix C. Hence, the initial and the terminal states, together with the path connecting them, determine the resultant partition unless \( f(x, y) \) is linear.

The mathematical derivation above applies to a three-variable function as well. By doing the line integrals for the MCY equation, we obtain the desired results:

\[
\Delta R_p = \int_L \frac{\partial R}{\partial P} dP \\
\Delta R_{E_0} = \int_L \frac{\partial R}{\partial E_0} dE_0 \\
\Delta R_n = \int_L \frac{\partial R}{\partial n} dn
\]

where \( \Delta R_p, \Delta R_{E_0}, \) and \( \Delta R_n \) denote the effects on runoff change of \( P, E_0, \) and \( n \), respectively. The sum of \( \Delta R_p \) and \( \Delta R_{E_0} \) represents the effect of climate change, and \( \Delta R_n \) is often related to human activities although it probably includes the effects of other factors, such as climate seasonality (Roderick and Farquhar, 2011; Berghuijs and Woods, 2016). $L$ denotes a three-dimensional curve along which climate and catchment changes have occurred. I approximated $L$ by a series of line segments. \( \Delta R_p, \Delta R_{E_0}, \) and \( \Delta R_n \) were finally determined by summing up the integrals along each of the line segments (see Section 2.3).

2.3 Using the LI method to determine \( \Delta R_p, \Delta R_{E_0}, \) and \( \Delta R_n \) within the Budyko Framework

1) Determining \( \Delta R_p, \Delta R_{E_0}, \) and \( \Delta R_n \) assuming a linear integral path

A curve can be approximated as a series of line segments. Hence, we can first handle the case of a linear integral path. Given two consecutive periods and assuming that the catchment state has evolved from \((P_1, E_{01}, n_1)\) to \((P_2, E_{02}, n_2)\) along a straight line $L$, let \( \Delta P = P_2 - P_1, \Delta E_0 = E_{02} - E_{01}, \) and \( \Delta n = n_2 - n_1; \) then the line $L$ is given by parametric equations: \( P = \Delta P t + P_1, \ E_0 = \Delta E_0 t + E_{01}, \ n = \Delta n t + n_1, \ t \in [0,1] \). Given these equations, Eq. (2) becomes a univariate function of $t$, i.e., \( \partial R / \partial P = \dot{R}_0(t), \partial R / \partial E_0 = \dot{R}_{E_0}(t), \) and \( \partial R / \partial n = \dot{R}_n(t) \). Then, \( \Delta R_p, \Delta R_{E_0}, \) and \( \Delta R_n \) can be evaluated as:

\[
\Delta R_p = \int_0^1 \frac{\partial R}{\partial P} dP = \int_0^1 \dot{R}_0(t) d(\Delta P t + P_1) = \Delta P \int_0^1 \dot{R}_0(t) dt
\]
\[ \Delta R_{E_0} = \int_0^1 \frac{\partial R}{\partial E_0} dE_0 = \int_0^1 R_{E_0}(t) d(\Delta E_0 + E_0) = \Delta E_0 \int_0^1 R_{E_0}(t) dt \quad (5b) \]

\[ \Delta R_e = \int_0^1 \frac{\partial R}{\partial E} dE = \int_0^1 R_e(t) d(Ant + n) = \Delta n \int_0^1 R_e(t) dt \quad (5c) \]

Unfortunately, I could not determine the antiderivatives of \( R(t) dt \), \( R_{E_0}(t) dt \) and \( R_e(t) dt \) and had to make approximate calculations. As the discrete equivalent of integration is a summation, we can approximate the integration as a summation. I divided the \( t \in [0,1] \) interval into 1000 subintervals of the same width, i.e., setting \( dt \) identically equal to 0.001, and then calculated \( R(t) dt \), \( R_{E_0}(t) dt \) and \( R_e(t) dt \) for each subinterval. Let \( t_i = 0.001i \), \( i \in [0,999] \) and is integer-valued, \( \Delta R_{E_0}, \Delta R_e \), and \( \Delta R_e \) are approximated as:

\[ \Delta R_{E_0} \approx 0.001 \Delta P \sum_{i=0}^{999} R_{E_0}(t_i) \quad (6a) \]

\[ \Delta R_e \approx 0.001 \Delta E \sum_{i=0}^{999} R_e(t_i) \quad (6b) \]

\[ \Delta R_e \approx 0.001 \Delta n \sum_{i=0}^{999} R_e(t_i) \quad (6c) \]

2) Dividing the evaluation period into a number of subperiods

I first determined a change point and divided the whole observation period into the reference and evaluation periods. To determine the integral path, the evaluation period was further divided into a number of subperiods. The Budyko framework assumes a steady state condition of a catchment and therefore requires no change in soil water storage. Over a time period of 5-10 years, it is reasonable to assume that changes in soil water storage will be sufficiently small (Zhang et al., 2001). Here, I divided the evaluation period into a number of 7-year subperiods with the exception for the final subperiod, which varied from 7 to 13 years in length depending on the length of the evaluation period.

3) Determining \( \Delta R_{E_0}, \Delta R_e \), and \( \Delta R_e \) by approximating the integral path as a series of line segments

A curve can be approximated as a series of line segments. For a short period, the integral path \( L \) can be considered as linear, which implies a temporally invariant change rate. For a long period in which the change rate may vary over time, \( L \) can be fitted using a number of line segments. Given a reference period and an evaluation period comprising \( N \) subperiods, the catchment state was assumed to evolve from \((P_0, E_{00}, n_0), \ldots, (P_i, E_{0i}, n_i), \ldots, (P_N, E_{0N}, n_N)\), where the subscript "0" denotes the reference period, and "\( i \)" and "\( N \)" denote the i\( \text{th} \) and the final subperiods of the evaluation period, respectively. I used a series of line segments \( L_1, L_2, \ldots, L_N \) to approximate the integral path \( L \), where \( L_i \) connects points \((P_{i-1}, E_{0,i-1}, n_{i-1})\) with \((P_i, E_{0i}, n_i)\). Then \( \Delta R_{E_0}, \Delta R_e \), and \( \Delta R_e \) were evaluated as the sum of the integrals along each of the line segments, which were calculated using Eq. (6).

2.4 Total-differential, decomposition and complementary methods

To evaluate the LI method, I compared it with the existing methods, including the decomposition method, the total differential method, and the complementary method. The total differential method approximated \( \Delta R \) as \( dR \):
\[ \Delta R \approx dR = \frac{\partial R}{\partial P} \Delta P + \frac{\partial R}{\partial E_0} \Delta E_0 + \frac{\partial R}{\partial \Delta n} \Delta n = \lambda_P \Delta P + \lambda_{E_0} \Delta E_0 + \lambda_{\Delta n} \Delta n \]  

where \( \lambda_P = \frac{\partial R}{\partial P} \), \( \lambda_{E_0} = \frac{\partial R}{\partial E_0} \), and \( \lambda_{\Delta n} = \frac{\partial R}{\partial \Delta n} \), representing the sensitivity coefficient of \( R \) with respect to \( P \), \( E_0 \), and \( n \), respectively. Within the total differential method, \( \Delta R_P = \lambda_P \Delta P \), \( \Delta R_{E_0} = \lambda_{E_0} \Delta E_0 \), and \( \Delta R_{\Delta n} = \lambda_{\Delta n} \Delta n \). I used the forward approximation, i.e., substituting the observed mean annual values of the reference period into Eq. (2), to estimate \( \lambda_P \), \( \lambda_{E_0} \), and \( \lambda_{\Delta n} \), as is standard in most studies (Roderick and Farquhar, 2011; Yang and Yang, 2011; Sun et al., 2014).

The decomposition method (Wang and Hejazi, 2011) calculated \( \Delta R_e \) as follows:

\[ \Delta R_e = R_2 - R_1 = (P_2 - E_2) - (P_1 - E_1) = E_2 - E_1 \]  

where \( R_2, P_2, \) and \( E_2 \) represents the mean annual runoff, precipitation and evapotranspiration of the evaluation period, respectively; \( R_1, P_1, \) and \( E_1 \) represent the mean annual runoff and evapotranspiration, respectively, given the climate conditions of the evaluation period and the catchment conditions of the reference period (Fig. 2). Both \( E_2 \) and \( E_1 \) were calculated by Eq. (1), but using \( n \) values of the evaluation period and the reference period respectively.

The complementary method (Zhou et al., 2016) uses a linear combination of the complementary relationship for runoff to determine \( \Delta R_P, \Delta R_{E_0}, \) and \( \Delta R_{\Delta n} \):

\[
\Delta R = a \left[ \left( \frac{\partial R}{\partial P} \right) \Delta P + \left( \frac{\partial R}{\partial E_0} \right) \Delta E_0 + \left( \frac{\partial R}{\partial \Delta n} \right) \Delta n \right] + (1-a) \left[ \left( \frac{\partial R}{\partial P} \right)_1 \Delta P + \left( \frac{\partial R}{\partial E_0} \right)_1 \Delta E_0 + \left( \frac{\partial R}{\partial \Delta n} \right)_1 \Delta n \right]
\]

where the subscript 1 and 2 denotes the reference and the evaluation periods, respectively. \( a \) is a weighting factor and varies from 0 to 1. As suggested by Zhou et al. (2016), I set \( a = 0.5 \). Equation (9) thus gave an estimation of \( \Delta R_P, \Delta R_{E_0}, \) and \( \Delta R_{\Delta n} \) as follows:

\[
\Delta R_P = 0.5 \Delta P \left[ \left( \frac{\partial R}{\partial P} \right)_1 + \left( \frac{\partial R}{\partial P} \right)_2 \right]
\]

\[
\Delta R_{E_0} = 0.5 \Delta E_0 \left[ \left( \frac{\partial R}{\partial E_0} \right)_1 + \left( \frac{\partial R}{\partial E_0} \right)_2 \right]
\]

\[
\Delta R_{\Delta n} = 0.5 \Delta \left( \frac{\partial R}{\partial \Delta n} \right) (P_1, P_2) + 0.5 \Delta \left( \frac{\partial R}{\partial E_0} \right) (E_{01}, E_{02})
\]

2.5 Data

I collected runoff and climate data from 24-19 selected catchments evaluated in previous studies (Table 1). The change-point years given in these studies were directly used to determine the reference and evaluation periods for the LI method. As mentioned above, the LI method further divides the evaluation period into a number of subperiods. For the sake of comparison, the final subperiod of the evaluation period was used as the evaluation period for the decomposition, the total differential and the complementary methods (It can be equally considered that all of the four methods used the final subperiod as the evaluation period, but the LI method cares about the intermediate period between the reference and the evaluation periods and the other methods do not). NineEight of the 21 catchments 19
Table 1

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the intermediate states would result in an imprecise partition, as was illustrated in Fig. 1 using a univariate function, and even a reverse trend estimation (see \( \Delta R_{\text{nc}} \) for Catchment NO. 1 in Table 3).

Superior to the total differential method, the sum of \( \Delta R_{\text{r}}, \Delta R_{\text{nc}}, \) and \( \Delta R_{\text{t}} \) always equaled to \( \Delta R \) for the LI method. Examination of the subperiods revealed that \( \partial R/\partial \overline{n} \) was more temporally variable than \( \partial R/\partial P \) and \( \partial R/\partial E_{\text{t}} \) (discussed below). For this reason, \( \Delta R_{\text{r}} \) showed considerable discrepancies between the two methods, but \( \Delta R_{\text{nc}} \) as well as \( \Delta R_{\text{nc}} \) matched closely between the two methods (Fig. 5).

As with the LI method, the complementary method produced \( \Delta R_{\text{r}}, \Delta R_{\text{nc}}, \) and \( \Delta R \) that exactly summed up to \( \Delta R \). Although its resultant \( \Delta R_{\text{r}}, \Delta R_{\text{nc}}, \) and \( \Delta R_{\text{nc}} \) values were all in accordance with the LI method (Fig. 6), the LI method often yielded values beyond the bounds given by the complementary method (Fig. 7); this is because the maximum or minimum sensitivities do not necessarily occur at the initial or terminal states.

3.2 Spatio-temporal variability of the path-averaged sensitivities

\( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) imply the average runoff change induced by a unit change in \( P, E_{\text{t}}, \) and \( n \), respectively (Appendix D). Their spatio-temporal variability is relevant to the prediction of the runoff change. To evaluate their temporal variabilities, I calculated \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) for each subperiod of the evaluation period and assessed their deviation from those for the whole evaluation period. As shown in Fig. 8, the deviation was rather limited for \( \overline{\Delta R_{\text{r}}} \) (averaged 8.6%), and \( \overline{\Delta R_{\text{nc}}} \) (averaged 13%), but was considerable for \( \overline{\Delta R_{\text{nc}}} \) (averaged 41%). Hence, it seems quite safe to predict the future climate effects on runoff using the earlier \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) values, but care must be taken when using earlier \( \overline{\Delta R_{\text{nc}}} \) to predict future catchment effect.

Different from the temporal variability, \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) all varied by up to several times or even ten folds between the studied catchments (Table 4). Strong correlations were observed between \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) and between \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( n \) (Fig. 9). Fig. 10-8 shows that Eq. (2) reproduced \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) very well taking the long-term means of \( P, E_{\text{t}} \), and \( n \) as inputs, a fact that the dependent variable approached its average if the independent variables were set to be their averages. This finding is of relevance to the spatial prediction of \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \).

Runoff data and, in turn, the parameter \( n \) in the MCY equation are often unavailable. It is thus desirable to make predictions of \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) and \( \overline{\Delta R_{\text{nc}}} \) in the absence of the parameter \( n \). I developed three strategies as follows: 1) using Eq. (2) and assuming \( n = 2 \) as \( n \) is typically in a small range from 1.5 to 3.6 (Roderick and Farquhar, 2011); 2) using \( P \) and \( E_{\text{tc}} \) to establish regression models; 3) using the aridity index to establish regressions as the index appeared to be more strongly correlated with both \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) than \( P \) and \( E_{\text{t}} \) (Fig. 9). As shown in Fig. 11, the three strategies show similar performance although the second one seems to perform better. All the strategies gave acceptable predictions of \( \overline{\Delta R_{\text{r}}}, \overline{\Delta R_{\text{nc}}}, \) but poor results for \( \overline{\Delta R_{\text{nc}}} \), as it was primarily controlled by \( n \) (Fig. 9). Thus, more sophisticated approaches are needed to predict the future catchment effect on runoff in the absence of runoff observations.
4 Discussion

The LI method highlights the role of the evolutionary path in determining the resultant partition. Yet, it seems that no studies have accounted for the path issue while evaluating the relative influences of drivers. The limit of the LI method is high data requirement for obtaining the evolutionary path. When the path data are unavailable, the complementary method can be considered as an alternative. The complementary method is free of residuals; moreover, it employs both data of the reference and the evaluation periods, thereby generally yielding sensitivities closer to the path-averaged results than the total differentiae method.

While using the Budyko models, a reasonable time scale is relevant to meet the assumption that changes in catchment water storage are small relative to the magnitude of fluxes of $P$, $R$ and $E$ (Donohue et al., 2007; Roderick and Farquhar, 2011). A seven-year time scale was used in the present study, as most studies have suggested that a time period of 5-10 years (Zhang et al., 2001; Zhang et al., 2016; Wu et al., 2017a; Wu et al., 2017b; Li et al., 2017) or even one year (Roderick and Farquhar, 2011; Sivapalan et al., 2011; Carmona et al., 2014; Ning et al., 2017) is reasonable. Nevertheless, some studies argued that the time period should be longer than ten years (Li et al., 2016; Dey and Mishra, 2017). If this is the case, the high temporal variation of $E_f$, shown in Fig. 8 might be caused by water storage changes, rather than actual changes in the catchment properties. This uncertainty should be addressed. Using the Gravity Recovery and Climate Experiment (GRACE) satellite gravimetry, Zhao et al. (2011) detected the water storage variations for three largest river basins of China, namely, the Yellow, Yangtze, and Zhujiang. The Yellow River mostly drains an arid and semi-arid region ($P$, 450 $10^{-3}m$; $R$, 70 $10^{-3}m$; $E$, 380 $10^{-3}m$), and the Yangtze ($P$, 110 $10^{-3} m$; $R$, 550 $10^{-3}m$; $E$, 550 $10^{-3}m$) and the Zhujiang river basins ($P$, 1400 $10^{-3}m$; $R$, 780 $10^{-3}m$; $E$, 620 $10^{-3}m$) are humid. The amplitude of the water storage variations between years were 7.372, and 65 $10^{-3}m$ for the three rivers respectively, at one magnitude order smaller than the fluxes of $P$, $R$ and $E$. Although the observations cannot be directly extrapolated to other regions, the possibility seems remote that the use of a 7-year aggregated time strongly violates the assumption of the steady state condition.

The mutual independence between the drivers is crucial for a valid partition. In the present study, although annual $P$ and $E_0$ exhibited significant correlation for most catchments ($p<0.05$), the aggregated $P$, $E_0$ and $n$ over a 7-year period showed minimal correlation (mostly $p>0.1$). The interdependence between the drivers can considerably confound the resultant partitions of the LI method and other existing methods.

The LI method redefines the widely used concept of sensitivity at a point as the path-averaged sensitivity. Mathematically, the LI method is unrelated to a functional form and hence applies to communities other than just hydrology. For example, identifying the carbon emission budgets (an allowable amount of anthropogenic CO$_2$ emission consistent with a limiting warming target), is crucial for global efforts to mitigate climate change. The LI method suggested that the emission budgets depends on both the emission magnitude and pathway (timing of emissions), which is in line with a recent study by Gasser et al. (2018). Hence, an optimal pathway would facilitate an elevated carbon budget unless the carbon-climate system behaves in a linear fashion.

This study presented the LI method using time-series data, but it applies equally to the case of spatial series of data. Given a model that relates fluvial or aeolian sediment load to the influencing
factors (e.g. rainfall and topography), for example, the LI method can be used to separate their contributions to the sediment-load change along a river or in the along-wind direction.

5 Conclusions

Based on the line integral, I created a mathematically precise method to partition the synergistic effects of several factors that cumulatively drive a system to change from a state to the other. The method is relevant for quantitative assessments of the relative roles of the factors on the change in the system state. I applied the LI method to partition the effects of climatic and catchment conditions on runoff within the Budyko framework. The method reveals that in addition to the change magnitude, the change pathways of climatic and catchment conditions also play a role in controlling their impacts on runoff. Instead of using the runoff sensitivity at a point, the LI method uses the path-averaged sensitivity, thereby ensuring a mathematically precise partition. I further examined the spatio-temporal variability of the path-averaged sensitivity. Time-wise, the runoff sensitivity to climate is stable but that to catchment properties is highly variable, suggesting that predicting future climate effects using earlier observations is reliable but care must be taken when predicting future catchment effects. Space-wise (between catchments) the runoff sensitivity both to climatic and catchment conditions was highly variable, but it can be accurately depicted by long-term means of the climatic and catchment conditions.

As a mathematically accurate scheme, the LI method has the potential to be a generic attribution approach in the environmental sciences.

Data availability

The data used in this study are freely available by contacting the authors.

Author contribution

MZ designed the study, analyzed the data and wrote the manuscript.

Competing interests

The authors declare that they have no conflict of interest.

Appendix A: Mathematical proof of \( \Delta z = \int_L f_1(x, y)\,dx + \int_L f_2(x, y)\,dy \)

We define that the curve \( L \) in Fig. 3 is given by a parametric equation: \( x = x(t), \, y = y(t), \, t \in \left[t_0, t_N\right] \), then \( \Delta z = z_N - z_0 = f\left[x(t_0), \, y(t_0)\right] - f\left[x(t_0), \, y(t_0)\right] \). Substituting the parametric equations, we obtain:

The right-hand side of the equation is:

\[
\int_{t_0}^{t_N} f_1(x(t), y(t))\,dx(t) + \int_{t_0}^{t_N} f_2(x(t), y(t))\,dy(t)
\]
Let \( g(t) = f(x(t), y(t)) \), and after using the chain rule to differentiate \( g \) with respect to \( t \), we obtain:

\[
g'(t) = \frac{\partial g}{\partial x} \frac{dx}{dt} + \frac{\partial g}{\partial y} \frac{dy}{dt} = f_x(x(t), y(t))x'(t) + f_y(x(t), y(t))y'(t)
\]

(A2)

Thus, \( g'(t) \) is just the integrand in Eq. (A1), and Eq. (A1) can then be rewritten as:

\[
\int_{t_0}^{t_1} g'(t) \, dt = \left[ g(t) \right]_{t_0}^{t_1} = g(t_1) - g(t_0)
\]

Appendix B: The sum of \( \int_L f_1(x, y) \, dx \) and \( \int_L f_2(x, y) \, dy \) is path-independent

Theorem: Given an open simply-connected region \( G \) (i.e., no holes in \( G \)) and two functions \( P(x, y) \) and \( Q(x, y) \) that have continuous first-order derivatives, if and only if \( \partial P / \partial y = \partial Q / \partial x \) throughout \( G \), then

\[
\int_L P(x, y) \, dx + \int_L Q(x, y) \, dy \text{ is path independent, i.e., it depends solely on the starting and ending point of } L.
\]

We have \( \partial f_1 / \partial y = \partial^2 z / \partial x \partial y \) and \( \partial f_2 / \partial x = \partial^2 z / \partial y \partial x \). As \( \partial^2 z / \partial x \partial y = \partial^2 z / \partial y \partial x \), we can state that \( \partial f_1 / \partial y = \partial f_2 / \partial x \), meeting the above condition and proving that \( \int_L f_1(x, y) \, dx + \int_L f_2(x, y) \, dy \) is path independent. The statement was further exemplified using a fictitious example in Appendix C.

Appendix C: A fictitious example to show how the LI method works

Runoff \( (R, 10^{-3} \text{ m yr}^{-1}) \) at a site is assumed to increase from 120 to 195 \( 10^{-3} \text{ m yr}^{-1} \) with \( \Delta R = 75 \, 10^{-3} \text{ m yr}^{-1} \); meanwhile, precipitation \( (P, 10^{-3} \text{ m yr}^{-1}) \) varies from 600 to 650 \( 10^{-3} \text{ m yr}^{-1} \) \( (\Delta P = 75 \, 10^{-3} \text{ m yr}^{-1}) \) and the runoff coefficient \( (C_R, \text{ dimensionless}) \) varies from 0.2 to 0.3 \( (\Delta C_R = 0.1) \). The goal is to partition \( \Delta R \) into the effects of the precipitation \( (\Delta R_P) \) and runoff coefficient \( (\Delta R_{C_R}) \), provided that \( P \) and \( C_R \) are independent. We have a function \( R = PC_R \) and its partial derivatives \( \partial R / \partial P = C_R \) and \( \partial R / \partial C_R = P \). Given a path \( L \) along which \( P \) and \( C_R \) change and using Eq. (3), the LI method evaluates \( \Delta R_P \) and \( \Delta R_{C_R} \) as:

\[
\Delta R_{C_R} = \int_L \partial R / \partial C_R dC_R = \int_L P dC_R \quad \text{and} \quad \Delta R_P = \int_L \partial R / \partial P dP = \int_L C_R dP \quad \text{(C1)}
\]

The result differs depending on \( L \) but the sum of \( \Delta R_P \) and \( \Delta R_{C_R} \) uniformly equals \( \Delta R \). This dynamic is demonstrated using Fig. 3, in which we considered that the \( x \)-axis represents \( C_R \) and the \( y \)-axis \( P \). Point A denotes the initial state \( (C_R = 0.2, P = 600) \) and point C the terminal state \( (C_R = 0.3, P = 650) \). I calculated \( \Delta R_{C_R} \) and \( \Delta R_P \) along three fictitious paths as follows:

1) \( L = AC \). Line segment AC has equation \( P = 500 C_R + 500, 0.2 \leq C_R \leq 0.3 \). Let’s take \( C_R \) as the parameter and write the equation in the parametric form as \( P = 500 C_R + 500, C_R = C_R, 0.2 \leq C_R \leq 0.3 \). By substituting the equation into Eq. (C1), we have:

\[
\Delta R_{C_R} = \int_{AC} P dC_R = \int_{0.2}^{0.3} (500 C_R + 500) dC_R = 62.5
\]
\[ \Delta R_P = \int_{AC} C \, \text{d}P = \int_{AC} C \, \text{d}(500C_x + 500) = 500 \int_{0.2}^{0.3} C \, \text{d}C_R = 12.5 \]

2) \( L = AB + BC \). To evaluate on the broken line, we can evaluate separately on \( AB \) and \( BC \) and then sum them up. The equation for \( AB \) is \( P = 600, 0.2 \leq C_x \leq 0.3 \), while for \( BC \) is \( C_R = 0.3, 600 \leq P \leq 650 \). Notes that a constant \( C_R \) or \( P \) implies that \( dC_R = 0 \) or \( dP = 0 \). Eq. (C1) then becomes:

\[ \Delta R_{CA} = \int_{AB+BC} P \, \text{d}C_R = \int_{AB} P \, \text{d}C_R + \int_{BC} P \, \text{d}C_R = \int_{0.2}^{0.3} 600 \, dC_R + 0 = 60 \]

\[ \Delta R_P = \int_{AB+BC} C \, \text{d}P = \int_{AB} C \, \text{d}P + \int_{BC} C \, \text{d}P = 0 + \int_{650}^{600} 0.3 \, dP = 15 \]

3) \( L = AD + DC \). The equation for \( AD \) is \( C_R = 0.2, 600 \leq P \leq 650 \) and is \( P = 650, 0.2 \leq C_x \leq 0.3 \) for \( DC \).

\[ \Delta R_P \text{ and } \Delta R_{CA} \text{ are evaluated as:} \]

\[ \Delta R_{CA} = \int_{AD+DC} P \, \text{d}C_R = \int_{AD} P \, \text{d}C_R + \int_{DC} P \, \text{d}C_R = \int_{0.2}^{0.3} 650 \, dC_R = 65 \]

\[ \Delta R_P = \int_{AD+DC} C \, \text{d}P = \int_{AD} C \, \text{d}P + \int_{DC} C \, \text{d}P = \int_{650}^{600} 0.2 \, dP + 0 = 10 \]

As expected, the sum of \( \Delta R_P \) and \( \Delta R_{CA} \) persistently equals \( \Delta R \) although \( \Delta R_P \) and \( \Delta R_{CA} \) varies with \( L \).

**Appendix D: Mathematical proof of the path-averaged sensitivity**

If the interval \([x_0, x_1]\) in Fig. 3 is partitioned into \( N \) distinct bins of the same width \( \Delta x_i = \Delta x/N \). Eq. (3a) can then be rewritten as:

\[ \Delta Z = \int_L f(x, y) \, dx = \lim_{\Delta x \to 0} \sum_{i=1}^{N-1} f(x_i, y) \Delta x_i = \lim_{\Delta x \to 0} \frac{\sum_{i=1}^{N-1} f(x_i, y)}{N} \Delta x = \frac{\sum_{i=1}^{N-1} f(x_i, y)}{N} \Delta x \]

where \( \bar{x}_i = \lim_{\Delta x \to 0} \frac{\sum_{i=1}^{N-1} f(x_i, y)}{N} \), denoting the average of \( f(x, y) \) along the curve \( L \). Likewise, we have

\[ \Delta Z = \bar{x}_i \Delta y, \text{ where } \bar{x}_i \text{ denotes the average of } f(x, y) \text{ along the curve } L. \text{ As a result:} \]

\[ \Delta Z = \bar{x} \Delta x + \bar{y} \Delta y \quad \text{(D1)} \]

The result can readily be extended to a function of three variables. Applying the mathematic derivation determined above to the MCY equation results in a precise form of Eq. (7):

\[ \Delta R = \Delta R_P + \Delta R_{CA} + \Delta R_e = \bar{x} \Delta P + \bar{L} \Delta E_0 + \bar{\lambda} \Delta n, \quad \text{(D2)} \]

where \( \Delta R_P = \bar{x} \Delta P + \Delta R_{CA} = \bar{x} \Delta E_0 + \Delta R_e, \Delta R_e = \bar{L} \Delta n, \), and \( \bar{x}, \bar{L}, \bar{\lambda} \) and \( \bar{x} \) denote the arithmetic mean of \( \partial R/\partial P \), \( \partial R/\partial E_0 \), and \( \partial R/\partial n \) along a path of climate and catchment changes, respectively. Because \( \bar{x}, \bar{L}, \bar{\lambda}, \bar{x} \) also imply the runoff change due to a unit change in \( P, E_0 \) and \( n \), respectively.
Appendix E: Path-averaged sensitivity in one-dimensional cases

Given a one-dimensional function $z=f(x)$ and its derivative $f'(x)$. We assumed that $f'(x)$ averages $\overline{x}$ over the range $(x,x+\Delta x)$, i.e. $\overline{x} = \lim_{\tau \to 0} \frac{\sum_{i=0}^{N} f'(x_i)}{N}$. According to the mean value theorem for integrals, $\overline{x} = \int_{x}^{x+\Delta x} f'(x)dx / \Delta x$. In terms of the Newton-Leibniz formula,

$$\int_{x}^{x+\Delta x} f'(x)dx=f(x+\Delta x)−f(x)=\Delta z.$$ Thus, we obtain: $\overline{x} = \Delta z / \Delta x$.

Acknowledgments

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References


Sun, Y., Tian, F., Yang, L., and H. Hu: Exploring the spatial variability of contributions from climate variation and change in catchment properties to streamflow decrease in a mesoscale basin by three different methods. Journal of Hydrology, 508(2), 170-180,


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<th>Area (10^3 m^2)</th>
<th>R</th>
<th>P</th>
<th>E_0</th>
<th>n</th>
<th>AI</th>
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<th>Evaluation Period</th>
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\[R, P, \text{ and } E_0 \text{ represent the mean annual runoff, precipitation and potential evaporation, all in } 10^3 \text{ m yr}^{-1}, n \text{ (dimensionless) is the parameter representing catchment properties in the MCY equation. } AI \text{ is the dimensionless aridity index } (AI = E_0/P). \text{ Data of Catchments } 1-14 \text{ were derived from Zhang et al. (2010). Data of Catchments } 15-18 \text{ were from Sun et al. (2014). Data of Catchments } 19-21 \text{ were from Zheng et al. (2009), Jiang et al. (2015), and Gao et al. (2016), respectively. I used the change points given in the literatures to divide the observation period into the reference and evaluation periods. The LI method further divides the evaluation period into a number of subperiods. The column “The }\]
final Subperiod” denotes the final subperiod, which was used as the evaluation period for the total differential method, the decomposition method and the complementary method. The bold and italic rows denote that the column “Evaluation Period” is the same as the column “The final Subperiod”.


Table 2. Comparisons of R (mm yr⁻¹), P (mm yr⁻¹), E₀ (mm yr⁻¹), and n (dimensionless) between the reference and the evaluation periods

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The subscript “1” denotes the reference period and "2" denotes the evaluation period. \( \Delta X = X_2 - X_1 \) (\( X \) as a substitute for \( R, P, E_0, \) and \( n \)).
Table 3. Effects of precipitation ($\Delta R_p$, $10^{-3}$m yr$^{-1}$), potential evapotranspiration ($\Delta R_e$, $10^{-3}$m yr$^{-1}$), and catchment changes ($\Delta R_c$, $10^{-3}$m yr$^{-1}$) on the mean annual runoff determined from the four evaluated methods.

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*The bold and italic numbers denote that the evaluation period comprises a single subperiod.*
Table 4. Comparisons of the path-averaged sensitivities with point sensitivities of runoff. If the evaluation period comprised only one subperiod, \( \frac{P_{\text{tr}}}{P_{\text{tr}}} \) and \( \frac{E_{\text{tr}}}{E_{\text{tr}}} \) were calculated as:

\[
\frac{\Delta P_{\text{tr}}}{P_{\text{tr}}} = \frac{\Delta E_{\text{tr}}}{E_{\text{tr}}}
\]

(a, b) \( P_{\text{tr}} \), \( E_{\text{tr}} \), and \( n_{\text{tr}} \) (dimensionless) represent the path-averaged sensitivities of runoff to precipitation, potential evaporation, and catchment properties (see Appendix D).
\( \bar{P}_\bar{x} = \bar{P} / \bar{x} \) and \( \bar{R}_{\bar{x}} = \bar{R} / \bar{x} \). If the evaluation period comprised \( N > 1 \) subperiods, the equations became:
\[
\bar{X} = \frac{\sum_{i=1}^{N} |\bar{X}_i|}{N}, \quad \bar{Y} = \frac{\sum_{i=1}^{N} |\bar{Y}_i|}{N}, \quad \text{and} \quad \bar{Z} = \frac{\sum_{i=1}^{N} |\bar{Z}_i|}{N},
\]
where the subscript \( i \) denotes the \( i \)th subperiod.

\( \bar{P}_\bar{x}, \bar{R}_{\bar{x}} \), and \( \bar{Z} \) represent the point sensitivities of runoff of the total differential method, which was calculated by substituting the observed mean annual values of the reference period into Eq. (2).

Fig. 1. For a non-linear function \( z = f(x) \), the total differential method (a) and the complementary method (b) fail to accurately estimate the effect \( (\Delta z) \) of \( x \) on \( z \) when \( x \) changes by \( \Delta x \), but the LI method (c) does. For a univariate function, the \( z \) change is exclusively driven by \( x \), so that \( \Delta z \), \( \Delta z = \Delta z \) in (c) but not in (a) and (b). \( \bar{\lambda}_x \) in (c) represents the average sensitivity along the curve AC and \( \bar{\lambda}_x = \Delta z / \Delta x \), see Appendix E for details.
Fig. 2. A schematic plot to illustrate the decomposition method. Point A denotes the initial state (the reference period) and Point C denotes the terminal state (the evaluation period). $R_2$ represents the mean annual runoff of the evaluation period, and $R'_2$ the mean annual runoff given the climate conditions of the evaluation period and the catchment conditions of the reference period. See Section 2.4 for details.

Fig. 3. A schematic plot illustrating the LI method.
Fig. 4. Comparisons between the LI method and the decomposition method. (a) Comparison of the estimated contributions to the runoff changes from the catchment changes ($\Delta R_c$); (b) the decomposition method is equivalent to the LI method that assumes a sudden change in catchment properties following climate change. In this case, the integral path of the LI method can be considered as the broken line path ABC–AB+BC in Fig. 3 (x represents climate factors and y catchment properties, i.e. $n$) and

$$\Delta R_c = \int_{AB+BC} \frac{\partial R}{\partial n} dn = \int_{AB} \frac{\partial R}{\partial n} dn + \int_{BC} \frac{\partial R}{\partial n} dn = 0 + \int_{BC} \frac{\partial R}{\partial n} dn = \int_{n_1}^{n_2} f_s(P_s, E_{so}, n) dn,$$

where the subscript "1" denotes the reference period and "2" denotes the final subperiod of the evaluation period.
Fig. 5. Comparisons of the estimated contribution to runoff from the changes in (a) precipitation ($\Delta R_P$), (b) potential evapotranspiration ($\Delta R_{ET}$), and (c) catchment properties ($\Delta R_c$) between the LI method and the total differential method.

Fig. 6. Comparisons of (a) $\Delta R_P$, (b) $\Delta R_{ET}$, and (c) $\Delta R_c$ between the LI method and the complementary method ($a = 0.5$).
Fig. 7. Comparisons of (a) $\Delta R_r$, (b) $\Delta R_{r*}$, and (c) $\Delta R_v$ by the LI method with the upper (---) and lower (---) bounds given by the complementary method. According to Zhou et al. (2016), $\Delta R_r$, $\Delta R_{r*}$, and $\Delta R_v$ reach their bounds when $a$ is 0 or 1.
Fig. 8. Boxplots showing the temporal variability of the path-averaged sensitivities of water yield to precipitation ($\bar{P}$), potential evapotranspiration ($\bar{E}$), and catchment properties ($\bar{N}$). $D$ (%) was calculated as the relative difference between the sensitivity of the whole evaluation period and that of a subperiod. In the calculations, I excluded the catchments that had an evaluation period comprising only one subperiod. The boxes span the inter-quartile range (IQR) and the solid lines are medians. The whiskers represent the data range, excluding statistical outliers, which extend more than 1.5IQR from the box ends.
Fig. 9. $\overline{P}$, $\overline{E_0}$, and $\overline{n}$ in correlation with $P$, $E_0$, $n$, and aridity index.

Fig. 8.10. Performances of Eq. (2) to be used to predict $\overline{P}$, $\overline{E_0}$, and $\overline{n}$. Comparisons of $\overline{P}$, $\overline{E_0}$, and $\overline{n}$ (given in Table 4) with those predicted using Eq. (2) with the long-term mean values of $P$, $E_0$, and $n$ as inputs. $MAE = N^{-1} \sum_{i=1}^{N} |O_i - P_i|$, is the mean absolute error, where $O$ and $P$ are values that actually encountered (given in Table 4S4) and predicted using Eq. (2) respectively, and $N$ is the number of selected catchments.
Fig. 11: Comparisons of $\bar{P}$, $\bar{E}$, and $\bar{N}$ with those predicted by the three strategies. (a)-(c) Predicted by Eq. (2) with a constant $n$ ($n = 2$). (d)-(f) predicted by the regression equations established using $P$ and $E_0$: $\bar{P} = 0.0011P - 0.0006E_0 + 0.21$ ($R^2 = 0.7$), $\bar{E} = 0.0007P - 0.0007E_0 + 0.38$ ($R^2 = 0.87$), and $\bar{N} = -0.302P + 0.372E_0 + 493$ ($R^2 = 0.37$); and (g)-(i) predicted by the regression equations established using only the aridity index, as shown in Fig. 9 (d), (h), and (l). MAE was calculated using the same procedures as in Fig. 10.