

Many thanks for your insightful comments and careful examination of the manuscript. I have studied your comments carefully and tried our best to revise the manuscript. My responses are given as follows. Attached please find the revised version. Thank you and best regards.

1. [The paper describes a mathematical method to attribute a discrete change in runoff to changes in climate and catchment characteristics. The method is directly applicable to common data and yields quite similar results when compared to existing methods. However, it remains open which of these methods is more accurate because there is no data to verify.]

Response: I admitted that I have not provided data to verify the LI method. However, the method is mathematically precise but the other methods are not, so that it is more accurate than other methods.

2. [Still, there are two interesting and valuable aspects of the manuscript: a) The role of the evolution over time b) Reconciling the existing methods and their assumptions on this evolution]

Response: Many thanks for your appreciation.

3. [To consider the path of changes is an important aspect and, as the author illustrates, may thus alter the resultant sensitivity to a change. This is important, since this may allow to better assess the vulnerability of a given catchment to global change. The problem is, that there is usually not sufficient data to constrain the evolution of disturbances.]

Response: I have added a paragraph to discuss the high data requirement associated with the LI method. See line 316-321 for details.

4. [The author uses subperiods of 7 years, where at least the meteorological data provides some constraints. However, the use of shorter periods comes at the cost of potential changes in the catchment water storage, which can then be misinterpreted as changes in catchment characteristics. Figure 6 shows that the temporal variation of the catchment property sensitivity is largest. This might actually be caused by water storage changes, rather than actual changes in the catchment properties. This aspect is not sufficiently discussed in the manuscript.]

Response: I have added a paragraph to justify my use of an aggregated time period of 7 years. See Lines 322-338 for details.

5. [Although I like that the existing methods are discussed in detail, I strongly recommend that the author better visualizes these methods. An attempt is done in Figure 1, but this must be extended and linked to the other methods.]

Response: I have revised Figure 1 as you suggested.

6. [Recommendation: Major Revisions. The relevance/significance of the paper must

be better highlighted. This requires major changes throughout.]

Response: Thanks for your comments. I highlighted the relevance in lines 39-43, 81-92, 94-98, 313-316, and 339-349.

7. [Further comments: Overall, the notation should be more consistent (for example indices) and streamlined]

Response: I have checked the notation throughout the manuscript.

8. [I think that some parts of the paper can be cut. Figure 2b is trivial and can be removed]

Response: A major conclusion of the manuscript is that the decomposition method is a special case of the LI method. Figure 2b lends direct support to the conclusion so that it is not trivial. I am sorry I do not cut it.

9. [It would be better to describe the decomposition method in a conceptual Figure, similar to Fig.1.]

Response: I have revised Figure 1 as you suggested.

10. [The catchments with the largest changes in n have a reference period of only 3 years. This is quite short for a reference period.]

Response: I am sorry that I directly used the data given in Zhou et al (2016). Many thanks for your careful examination, but the data of the catchment NO.10 remains in the manuscript considering the reasons below: 1) the catchment has a high aridity index of 1.5. In dry areas, the carryover of soil water storage between years is relatively small as much of the annual precipitation is evaporated and thus has little effect in altering water storage. For example, a one-year aggregated time period may be appropriate in the semi-arid Loess Plateau (Ning et al., 2017); 2) the carryover of soil water storage would result in an overestimated E , and in turn an overestimated n . The catchment NO.10 had a medium n value (1.7) in the reference period, much smaller than the evaluation period (4.2), so that the effect of the carryover of soil water storage should not be significant in the reference period.

11. [Figure 6: It is unclear what is shown here.]

Response: Figure 6 compares the temporal variability of the sensitivities of water yield to precipitation, potential evapotranspiration, and catchment properties. The boxplot clearly showed that the sensitivities to catchment properties had a much greater temporal variation.

12. [The motivation of the figures 7,8 and 9 is not really clear to me. Please explain or remove]

Response:

Figure 7 shows the correlation of the obtained sensitivities with P , E_0 , n , and aridity index, for purpose to determine the predictors of the sensitivities.

Fig. 8 shows that the path-averaged sensitivities can be well predicted over space if having all data of P , E_0 , and R .

Fig. 9 shows the prediction performance in the absence of runoff data as it frequently occurs in practices.

13. [At Line 311-312 it is argued that the timing of precipitation change is important. I did not see this aspect in the results.]

Response: This sentence is problematic. I have removed it.

Reference

- Zhou, S., B. Yu, L. Zhang, Y. Huang, M. Pan, and G. Wang (2016), A new method to partition climate and catchment effect on the mean annual runoff based on the Budyko complementary relationship. *Water Resources Research*, 52, 7163–7177. <https://doi.org/10.1002/2016WR019046>, 2016.
- Ning, T., Li, Z., and W. Liu: Vegetation dynamics and climate seasonality jointly control the interannual catchment water balance in the Loess Plateau under the Budyko framework, *Hydrology and Earth System Sciences*, 21, 1515-1526. <https://doi.org/10.5194/hess-2016-484>, 2017.

A line integral-based method to partition climate and catchment effects on runoff

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Abstract

It is a common task to partition synergistic impacts of a number of drivers in environmental sciences. However, there is no mathematically precise solution to the partition. Here I presented a line integral-based method, which concerns about the sensitivity to the drivers throughout their evolutionary path so as to ensure a precise partition. The method reveals that the partition depends on both the change magnitude and pathway (timing of change), and not on the magnitude alone unless for a linear system. To illustrate the method, I used the Budyko framework to partition the effects of climatic and catchment conditions on the temporal change in runoff for 21 catchments from Australia and China. The method reduced to the decomposition method when assumed a path along which climate change occurs first followed by an abrupt change in catchment properties. The method re-defines the widely-used concept of sensitivity at a point as the path-averaged sensitivity. The total differential and the complementary methods simply concern about the sensitivity at the initial or/and the terminal state, so that they cannot give precise results. The path-average sensitivity of water yield to climate conditions was found to be stable over time. Space-wise, moreover, it can be readily predicted even in the absence of streamflow observations, whereby facilitates evaluation of future climate effects on streamflow. As a mathematically accurate solution, the method provides a generic tool to conduct the quantitative attribution analyses.

Keywords: Runoff; Climate change; Human activities; Attribution analysis; Budyko

1 Introduction

It is often needed to quantify the relative roles of a few drivers to the observed changes of interest in environmental sciences. In the hydrology community, diagnosing the relative contributions of climate change and human activities to runoff is of great relevance to the researchers and managers as both climate and human activities have pose global-scale impact on hydrologic cycle and water resources (Barnett *et al.*, 2008; Xu *et al.*, 2014; Wang and Hejazi, 2001). Unfortunately, the quantitative

40 attribution analysis of the runoff changes remains a challenge (Wang and Hejazi, 2001; Berghuijs and
41 Woods, 2016; Zhang *et al.*, 2016); this is to a considerable degree due to a lack of a mathematically
42 precise method to decouple synergistic and often confounding impacts of climate change and human
43 activities.

44 Numerous studies have detected the long term variability in runoff and attempted to partition the
45 effects of climate change and human activities by means of various methods (Dey and Mishra, 2017).
46 Among them are the paired-catchments method and the hydrological modeling method. The paired-
47 catchment method is believed to be able to filter the effect of climatic variability and thus isolate the
48 runoff change induced by vegetation changes (Brown *et al.*, 2005). However, the method is
49 capital intensive. Particularly, it generally involves small catchments and is challenged when
50 extrapolating to large catchments (Zhang *et al.*, 2011). The physical-based hydrological models often
51 suffer from limitations including high data requirement, labor-intensive calibration and validation
52 processes, and inherent uncertainty and interdependence in parameter estimations (Binley *et al.*, 1991;
53 Wang *et al.*, 2013; Liang *et al.*, 2015). Interest then turns to the conceptual models over recent years,
54 such as the Budyko-type equations (see Section 2.1).

55 Within the Budyko framework, a large number of studies (Roderick and Farquhar, 2011; Zhang
56 *et al.*, 2016) have used the total differential of runoff (i.e. dR , where R represents runoff) as a proxy for
57 the runoff change (i.e. ΔR) and further evaluated hydrological responses to climate change and human
58 activities (hereafter called the total differential method). However, dR is essentially a first-order
59 approximation of ΔR (Fig. 1(a)). It has been shown that the approximation has caused an error of the
60 climate impact on runoff ranging from 0 to 20 mm (or -118 to 174%) over China (Yang *et al.*, 2014).
61 The total differential method directly used the partial derivatives of runoff as the sensitivities of runoff
62 to climate and catchment conditions. Most studies applied the forward approximation of the runoff
63 change, i.e., using the sensitivities at the initial state while calculation (e.g. Roderick and Farquhar,
64 2011). The elasticity method proposed by Schaake (1990) is also based on the total differential
65 expression (Sankarasubramanian *et al.*, 2001; Zheng *et al.*, 2009). The method uses the “elasticity”
66 concept to assess the climate sensitivity of runoff. The elasticity coefficients, however, have been
67 estimated in an empirical way and is not physically sound (Roderick and Farquhar, 2011; Liang *et al.*,
68 2015).

69 The so-called decomposition method developed by Wang and Hejazi (2011) has also been
70 widely used. The method assumes that climate changes drive a shift along a Budyko curve and then
71 human interferences cause a vertical shift from the Budyko curve to another (Fig. 1(b)). Under this
72 assumption, the method extrapolates the Budyko models calibrated using observations of the reference
73 period, in which human impacts remain minimal, to determine the human-induced changes in runoff
74 occurred during the evaluation period.

75 Recently, Zhou *et al.* (2016) established a Budyko complementary relationship for runoff and
76 applied it to partitioning the climate and catchment effects. Superior to the total differential method, the
77 method culminates with yielding a no-residual partition. Nevertheless, the method depends on a given
78 weighted factor, which is determined in an empirical but not a precise way. Furthermore, Zhou *et al.*
79 (2016) argued that the partition is not unique in the Budyko framework as the path of the climate and
80 catchment changes cannot be uniquely identified.

81 A precise partition remains difficult even given a precise mathematical model. This can be
 82 illustrated by using a precise hydrology model $R = f(x, y)$, where x and y climate factors and catchment
 83 characteristics respectively. We assumed that R changes by ΔR when x changes by Δx and y by Δy , *i.e.*
 84 $\Delta R = f(x + \Delta x, y + \Delta y) - f(x, y)$. To determine the effect of x on ΔR , *i.e.* ΔR_x , a common practice is to
 85 assume that y remains constant when x changes by Δx . We thus get: $\Delta R_x = f(x + \Delta x, y) - f(x, y)$.
 86 Similarly, we can get: $\Delta R_y = f(x, y + \Delta y) - f(x, y)$. Although the derivation seems quite reasonable, it
 87 is problematic as the sum of ΔR_x and ΔR_y is not equal to ΔR . Further examination shows that a
 88 variable's effect on R seems to differ depending on the changing path. For example,
 89 $\Delta R_x = f(x + \Delta x, y) - f(x, y)$ and $\Delta R_y = f(x + \Delta x, y + \Delta y) - f(x + \Delta x, y)$ if x changes first and y
 90 subsequently (Note that the partition is precise with the sum of ΔR_x and ΔR_y equaling ΔR now). If y
 91 changes first and x subsequently, the partition then becomes: $\Delta R_x = f(x + \Delta x, y + \Delta y) - f(x, y + \Delta y)$
 92 and $\Delta R_y = f(x, y + \Delta y) - f(x, y)$. In case of x and y changing simultaneously, unfortunately,
 93 current literature seems not to provide a mathematically precise solution.

94 The aims of this work are to propose a mathematically precise method to conduct quantitative
 95 attribution to drivers. The method is based on the line integer (called the LI method hereafter) and takes
 96 account of the sensitivity throughout the evolutionary path of the drivers rather than at a point as the
 97 total differential does. In this way, the method revises the widely-used concept of sensitivity at a point
 98 as the path-averaged sensitivity. To present and evaluate the method, I decomposed the relative
 99 influences of climate and catchment conditions on runoff within the Budyko framework using data from
 100 21 catchments from Australia and China. I also examined the spatio-temporal variability of the path-
 101 averaged sensitivities of runoff to climatic and catchment conditions and assessed their spatio-temporal
 102 predictability.
 103

104 2 Methodology

105 2.1 The Budyko Framework and the MCY equation

106 Budyko (1974) argued that the mean annual evapotranspiration (E) is largely determined by
 107 water and energy balance of a catchment. Using precipitation (P) and potential evapotranspiration (E_0)
 108 as proxies for water and energy availabilities respectively, the Budyko framework
 109 relates evapotranspiration losses to the aridity index defined as the ratio of E_0 over P . The Budyko
 110 framework has gained wide acceptance in the hydrology community (Berghuijs and Woods, 2016;
 111 Sposito, 2017). Over past decades, a number of equations have been developed to describe the
 112 framework. Among them, the Mezentsev-Choudhury-Yang's equation (Mezentsev, 1955; Choudhury,
 113 1999; Yang *et al.*, 2008) (Called the MCY equation hereafter) has been widely accepted and was used
 114 here:

$$115 \quad \frac{E}{P} = \frac{E_0/P}{(1 + (E_0/P)^n)^{1/n}} \quad (1)$$

116 where $n \in (0, \infty)$ is an integration constant that is dimensionless, and represents catchment properties.
 117 Eq. (3) requires a relative long time scale whereby the water storage of a catchment is negligible and the
 118 water balance equation reduces to be $R = P - E$. Here I adopted a “tuned” n value that can get exact
 119 agreement between the calculated E by Eq. (1) and that actually encountered ($= P - R$).

120 The partial differentials of R with respect to P , E_0 , and n are given as:

$$121 \quad \frac{\partial R}{\partial P} = R_P(P, E_0, n) = 1 - \frac{E_0^{n+1}}{(P^n + E_0^n)^{1/n}} \quad (2a)$$

$$122 \quad \frac{\partial R}{\partial E_0} = R_{E_0}(P, E_0, n) = - \frac{P^{n+1}}{(P^n + E_0^n)^{1/n}} \quad (2b)$$

$$123 \quad \frac{\partial R}{\partial n} = R_n(P, E_0, n) = \frac{-E_0 P n^{-1}}{(P^n + E_0^n)^{1/n}} \left[\frac{\ln(P^n + E_0^n)}{n} - \frac{P^n \ln P + E_0^n \ln E_0}{P^n + E_0^n} \right] \quad (2c)$$

124 2.2 The theory of the line integral-based method

125 To present the LI method, we start by considering an example of a two-variable function $z = f(x,$
 126 $y)$, which has continuous partial derivatives $\partial z / \partial x = f_x(x, y)$ and $\partial z / \partial y = f_y(x, y)$. Suppose that x and y
 127 varies along a smooth curve L (e.g. AC in Fig. 1(c)) from the initial state (x_0, y_0) to the terminal state $(x_N,$
 128 $y_N)$, and z co-varies from z_0 to z_N . Let $\Delta z = z_N - z_0$, $\Delta x = x_N - x_0$, and $\Delta y = y_N - y_0$. Our goal is to seek
 129 for a mathematical solution to quantify the effects of Δx and Δy on Δz , i.e. Δz_x and Δz_y . Δz_x and Δz_y
 130 should be subject to the constraint $\Delta z_x + \Delta z_y = \Delta z$.

131 As shown in Fig. 1(c), points $M_1(x_1, y_1), \dots, M_{N-1}(x_{N-1}, y_{N-1})$ partition L into N distinct segments.
 132 Let $\Delta x_i = x_{i+1} - x_i$, $\Delta y_i = y_{i+1} - y_i$, and $\Delta z_i = z_{i+1} - z_i$. For each segment, Δz_i can be approximated as the
 133 total differential dz_i : $\Delta z_i \approx dz_i = f_x(x_i, y_i)\Delta x_i + f_y(x_i, y_i)\Delta y_i$. We then have:

$$134 \quad \Delta z = \sum_{i=1}^N \Delta z_i \approx \sum_{i=1}^N f_x(x_i, y_i)\Delta x_i + \sum_{i=1}^N f_y(x_i, y_i)\Delta y_i. \text{ We thus obtain an approximation of } \Delta z_x \text{ and } \Delta z_y :$$

$$135 \quad \Delta z_x \approx \sum_{i=1}^N f_x(x_i, y_i)\Delta x_i \text{ and } \Delta z_y \approx \sum_{i=1}^N f_y(x_i, y_i)\Delta y_i. \text{ Define } \tau \text{ as the maximum length among the } N \text{ segments.}$$

136 The smaller the value of τ , the closer to Δz_i the value of dz_i , and then the better the approximations are.
 137 The approximations becomes exact in the limit $\tau \rightarrow 0$. Taking the limit $\tau \rightarrow 0$ then turns sum into
 138 integrals and gives a precise expression (it is an informal derivation and please see Appendix A for a

139 formal one): $\Delta z = \lim_{\tau \rightarrow 0} \sum_{i=1}^N f_x(x_i, y_i)\Delta x_i + \lim_{\tau \rightarrow 0} \sum_{i=1}^N f_y(x_i, y_i)\Delta y_i = \int_L f_x(x, y)dx + \int_L f_y(x, y)dy$, where

$$140 \quad \int_L f_x(x, y)dx = \lim_{\tau \rightarrow 0} \sum_{i=1}^N f_x(x_i, y_i)\Delta x_i \text{ and } \int_L f_y(x, y)dy = \lim_{\tau \rightarrow 0} \sum_{i=1}^N f_y(x_i, y_i)\Delta y_i \text{ denote the line integral of } f_x \text{ and } f_y$$

141 along L (termed integral path) with respect to x and y , respectively. $\int_L f_x(x, y)dx$ and $\int_L f_y(x, y)dy$ exist
 142 provided that f_x and f_y are continuous along L . We thus obtain a precise evaluation of Δz_x and Δz_y :

$$143 \quad \Delta z_x = \int_L f_x(x, y)dx \quad (3a)$$

$$\Delta z_y = \int_L f_y(x, y) dy . \quad (3b)$$

Unlike the total differential method, the sum of Δz_x and Δz_y persistently equals Δz (Appendix B). If $f(x, y)$ is linear, then f_x and f_y are constant. Define $C_x = f_x(x, y)$ and $C_y = f_y(x, y)$, we have $\Delta z_x = C_x \Delta x$ and $\Delta z_y = C_y \Delta y$. Δz_x and Δz_y are thus independent of L . If $f(x, y)$ is non-linear, however, both Δz_x and Δz_y varies with L , as was exemplified in Appendix C. Hence, the initial and the terminal states, together with the path connecting them, determine the resultant partition unless $f(x, y)$ is linear.

The mathematical derivation above applies to a three-variable function as well. By doing the line integrals for the MCY equation, we obtain the desired results:

$$\Delta R_P = \int_L \frac{\partial R}{\partial P} dP \quad (4a)$$

$$\Delta R_{E_0} = \int_L \frac{\partial R}{\partial E_0} dE_0 \quad (4b)$$

$$\Delta R_n = \int_L \frac{\partial R}{\partial n} dn \quad (4c)$$

where ΔR_P , ΔR_{E_0} , and ΔR_n denotes the effects on runoff change of P , E_0 , and n , respectively. The sum of ΔR_P and ΔR_{E_0} represents the effect of climate change, and ΔR_n are often related to human activities although it probably includes the effects of other factors, such as climate seasonality (Roderick and Farquhar, 2011; Berghuijs and Woods, 2016). L denotes a three-dimensional curve along which climate and catchment changes have occurred. I approximated L as a union of a series of line segments. ΔR_P , ΔR_{E_0} , and ΔR_n were finally figured out by summing up the integrals along each of the line segments (see Section 2.3).

2.3 Using the LI method to determine ΔR_P , ΔR_{E_0} , and ΔR_n within the Budyko Framework

1) Determining ΔR_P , ΔR_{E_0} , and ΔR_n assuming a linear integral path

Given two consecutive periods and assumed that the catchment state has evolved from (P_1, E_{01}, n_1) to (P_2, E_{02}, n_2) along a straight line L . Let $\Delta P = P_2 - P_1$, $\Delta E_0 = E_{02} - E_{01}$, and $\Delta n = n_2 - n_1$, then the line L is given by parametric equations: $P = \Delta P t + P_1$, $E_0 = \Delta E_0 t + E_{01}$, $n = \Delta n t + n_1$, $t \in [0, 1]$. Given the equations, Eq. (2) becomes a one-variable function of t , i.e., $\partial R / \partial P = R_P(t)$, $\partial R / \partial E_0 = R_{E_0}(t)$, and $\partial R / \partial n = R_n(t)$. Then, ΔR_P , ΔR_{E_0} , and ΔR_n can be evaluated as:

$$\Delta R_P = \int_L \frac{\partial R}{\partial P} dP = \int_0^1 R_P(t) d(\Delta P t + P_1) = \Delta P \int_0^1 R_P(t) dt \quad (5a)$$

$$\Delta R_{E_0} = \int_L \frac{\partial R}{\partial E_0} dE_0 = \int_0^1 R_{E_0}(t) d(\Delta E_0 t + E_{01}) = \Delta E_0 \int_0^1 R_{E_0}(t) dt \quad (5b)$$

$$\Delta R_n = \int_L \frac{\partial R}{\partial n} dn = \int_0^1 R_n(t) d(\Delta n t + n_1) = \Delta n \int_0^1 R_n(t) dt \quad (5c)$$

Unfortunately, I cannot figure out the antiderivatives of $R_P(t)$, $R_{E_0}(t)$, and $R_n(t)$ and have to make approximate calculations. As the discrete equivalent of integration is summation, we can approximate integration as summation. I divided the $t \in [0, 1]$ interval into 1000 subintervals of the same width, thereby setting dt identically equal to 0.001, and then then calculated $R_P(t) dt$, $R_{E_0}(t) dt$, and $R_n(t) dt$ for

176 each subinterval. Let $t_i = 0.001i, i \in [0, 999]$ and is integer-valued, $\Delta R_P, \Delta R_{E_0}$, and ΔR_n was approximated
 177 as:

$$178 \quad \Delta R_P \approx 0.001\Delta P \sum_{i=0}^{999} R_P(t_i) \quad (6a)$$

$$179 \quad \Delta R_{E_0} \approx 0.001\Delta E_0 \sum_{i=0}^{999} R_{E_0}(t_i) \quad (6b)$$

$$180 \quad \Delta R_n \approx 0.001\Delta n \sum_{i=0}^{999} R_n(t_i) \quad (6c)$$

181 2) Dividing the evaluation period into a number of subperiods

182 I first determine a change point and divide the whole observation period into the reference and
 183 evaluation periods. To determine the integral path, the evaluation period is further divided into a
 184 number of subperiods. The Budyko framework assumes a steady state condition of a catchment and
 185 therefore requires no change in soil water storage. Over a time period of 5-10 years, it is reasonable to
 186 assume that changes in soil water storage are sufficiently small (Zhang *et al.*, 2001). Here I divided the
 187 evaluation period into a number of 7-year subperiods with the exception for the last one, which varied
 188 from 7 to 13 years in length depending on the length of the evaluation period.

189 3) Determining $\Delta R_P, \Delta R_{E_0}$, and ΔR_n by approximating the integral path as a series of line segments

190 As did in Fig. 1(c), a curve can be approximated as a series of line segments. For a short period,
 191 the integral path L can be considered as linear, which implies a temporally invariant change rate. For a
 192 long period, in which the change rate usually varies over time, L can be fitted using a number of line
 193 segments. Given a reference period and an evaluation period comprising N subperiods, I assumed that
 194 the catchment state evolved from $(P_0, E_{00}, n_0), \dots, (P_i, E_{0i}, n_i), \dots$, to (P_N, E_{0N}, n_N) , where the subscript
 195 "0" denotes the reference period, and "i" and "N" denotes the i th and the last subperiods of the
 196 evaluation period, respectively. I used a series of line segments L_1, L_2, \dots, L_N to approximate the
 197 integral path L , where L_1 connects (P_0, E_{00}, n_0) with (P_1, E_{01}, n_1) , L_i connects points $(P_{i-1}, E_{0,i-1}, n_{i-1})$ with
 198 (P_i, E_{0i}, n_i) , and the initial point of L_{i+1} is the terminal point of L_i and. Then $\Delta R_P, \Delta R_{E_0}$, and ΔR_n
 199 are evaluated as the sum of the integrals along each of the line segments, which was calculated using Eq.
 200 (6).

201 2.4 The total-differential, decomposition and complementary methods

202 To evaluate the LI method, I compared it with the decomposition method, the total differential
 203 method, and the complementary method. The total differential method approximated ΔR as dR (Fig.
 204 1(a)):

$$205 \quad \Delta R \approx dR = \frac{\partial R}{\partial P} \Delta P + \frac{\partial R}{\partial E_0} \Delta E_0 + \frac{\partial R}{\partial n} \Delta n = \lambda_P \Delta P + \lambda_{E_0} \Delta E_0 + \lambda_n \Delta n \quad (7)$$

206 where $\lambda_P = \partial R / \partial P$, $\lambda_{E_0} = \partial R / \partial E_0$, and $\lambda_n = \partial R / \partial n$, representing the sensitivity coefficient of R with
 207 respect to P, E_0 , and n , respectively. Within the total differential method, $\Delta R_P = \lambda_P \Delta P$, $\Delta R_{E_0} = \lambda_{E_0} \Delta E_0$, and
 208 $\Delta R_n = \lambda_n \Delta n$. I used the forward approximation, *i.e.* substituting the observed mean annual values of the
 209 reference period into Eq. (2), to estimate λ_P, λ_{E_0} , and λ_n , as did in most studies (Roderick and Farquhar,
 210 2011; Yang and Yang, 2011; Sun *et al.*, 2014).

The decomposition method (Wang and Hejazi, 2011) calculated ΔR_n as follows:

$$\Delta R_n = R_2 - R'_2 = (P_2 - E_2) - (P_2 - E'_2) = E'_2 - E_2 \quad (8)$$

where R_2 , P_2 , and E_2 represents the mean annual runoff, precipitation and evapotranspiration of the evaluation period; and R'_2 and E'_2 represents the mean annual runoff and evapotranspiration respectively, given the climate conditions of the evaluation period and the catchment conditions of the reference period. Both E_2 and E'_2 were calculated by Eq. (1), but using n values of the evaluation period and the reference period respectively.

The complementary method (Zhou *et al.*, 2016) uses a linear combination of the complementary relationship for runoff to determine ΔR_P , ΔR_{E_0} , and ΔR_n :

$$\begin{aligned} \Delta R = a & \left[\left(\frac{\partial R}{\partial P} \right)_1 \Delta P + \left(\frac{\partial R}{\partial E_0} \right)_1 \Delta E_0 + P_2 \Delta \left(\frac{\partial R}{\partial P} \right) + E_{0,2} \Delta \left(\frac{\partial R}{\partial E_0} \right) \right] \\ & + (1-a) \left[\left(\frac{\partial R}{\partial P} \right)_2 \Delta P + \left(\frac{\partial R}{\partial E_0} \right)_2 \Delta E_0 + P_1 \Delta \left(\frac{\partial R}{\partial P} \right) + E_{0,1} \Delta \left(\frac{\partial R}{\partial E_0} \right) \right] \end{aligned} \quad (9)$$

where the subscript 1 and 2 denotes the reference and the evaluation periods, respectively. a is a weighting factor and varies from 0 to 1. As suggested by Zhou *et al.* (2016), I set $a = 0.5$. Equation (9) thus gave an estimation of ΔR_P , ΔR_{E_0} , and ΔR_n as follows:

$$\Delta R_P = 0.5 \Delta P \left[\left(\frac{\partial R}{\partial P} \right)_1 + \left(\frac{\partial R}{\partial P} \right)_2 \right] \quad (10a)$$

$$\Delta R_{E_0} = 0.5 \Delta E_0 \left[\left(\frac{\partial R}{\partial E_0} \right)_1 + \left(\frac{\partial R}{\partial E_0} \right)_2 \right] \quad (10b)$$

$$\Delta R_n = 0.5 \Delta \left(\frac{\partial R}{\partial P} \right) (P_1 + P_2) + 0.5 \Delta \left(\frac{\partial R}{\partial E_0} \right) (E_{0,1} + E_{0,2}) \quad (10c)$$

2.5 Data

I collected data of runoff and climate of 21 selected catchments from previous studies (Table 1). The change-point years gave in the studies was directly used to determine the reference and evaluation periods for the LI method. As mentioned above, the LI method further divides the evaluation period into a number of subperiods. For the sake of comparison, the last subperiod of the evaluation period was used as the evaluation period for the decomposition, the total differential and the complementary methods (It can be equally considered that all methods used the last subperiod as the evaluation period, but the LI method cares about the intermediate period between the reference and the evaluation periods and the others do not). Nine of the 21 catchments had a reference period comprising only one subperiod (Table 1), and the others had two to seven ones.

The 21 selected catchments were located in diverse climates and landscapes. Among them, 14 are from Australia and 7 from China (Table 1). The catchments spanned from tropical to subtropical and temperate and from humid to semi-humid and semi-arid regions, with mean annual rainfall varying from 506 to 1014 mm and potential evaporation from 768 to 1169 mm. The index of dryness ranges between 0.86 and 1.91. The catchment areas vary by five orders of magnitude from 1.95 to 121,972 with a median 606 km². The key data includes annual runoff, precipitation, and potential evaporation.

243 The record length varied between 15 and 75 with a median of 35 years. Among the 21 catchments, the
 244 changes from the reference to the evaluation period ranged between -271 and 79 mm yr^{-1} for
 245 precipitation, and -35 and 41 mm yr^{-1} for potential evaporation (Table 2). The coeval change in the
 246 parameter n of the MCY equation ranged between -0.2 to 2.53 . All of the catchments experienced both
 247 climate change and land cover change over the observation period. The mean annual streamflow
 248 reduced for all of them, by from 0.43 to 229 with a median 38 mm yr^{-1} . More details of data and the
 249 catchments can be found in Zhang *et al.* (2011), Sun *et al.* (2014), Zhang *et al.* (2010), Zheng *et al.*
 250 (2009), Jiang *et al.* (2015), and Gao *et al.* (2016).

252 3 Results

253 3.1 Comparisons with existing methods

254 The LI method first partitions the whole observation period into the reference and evaluation
 255 periods, then further divides the latter into a number of subperiods and evaluates the contributions to
 256 runoff from climate and catchment changes for each subperiod, and finally adds up the derived
 257 contributions. Table 3 lists all of the resultant values of ΔR_P , ΔR_{E_0} , and ΔR_n of the LI method, together
 258 with the three other methods.

259 Fig. 2(a) compares the resultant ΔR_n of the LI method and the decomposition method. Although
 260 they are quite similar, the discrepancies between them can be up to $>20 \text{ mm yr}^{-1}$. The decomposition
 261 method assumes that climate change occurs first and then human interferences cause a sudden change in
 262 catchment properties (Fig. 1(b)). Such a fictitious path is identical to the broken line AB+BC in Fig.
 263 1(c), provided that x represents climate factors and y catchment properties. As a result, the
 264 decomposition method can be considered as a special case of the LI method when adopting the broken
 265 line AB+BC in Fig. 1(c) as the integral path, as was demonstrated clearly in Fig. 2(b).

266 The total differentiae method is predicated on an approximate equation, *i.e.* Eq. (7). The LI
 267 method reveals that the precise form of the equation is $\Delta R = \overline{\lambda_P} \Delta P + \overline{\lambda_{E_0}} \Delta E_0 + \overline{\lambda_n} \Delta n$ (*i.e.* Eq. (D2) in
 268 Appendix D), where $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ (Table 4) denote the path-averaged sensitivity of R to P , E_0 , and n ,
 269 respectively. All points along the path have the same weight in determining $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$. To determine
 270 them, the total differential method utilizes only the initial state and the complementary method utilizes
 271 the initial and the terminal states. Neglecting the intermediate states between the initial and the terminal
 272 ones possibly results in a reverse trend estimation (see ΔR_{E_0} for Catchment NO. 1 in Table 3). Although
 273 the elasticity method exploits information contained over the entire observation period (e.g. Zheng *et al.*,
 274 2009; Wang *et al.*, 2013), the resultant descriptive statistics of climate elasticity may not be robust
 275 (Roderick and Farquhar, 2011; Liang *et al.*, 2015).

276 Superior to the total differential method, the sum of ΔR_P , ΔR_{E_0} , and ΔR_n always equaled to ΔR
 277 for the LI method. Examination of the subperiods revealed that $\partial R / \partial n$ was more temporally variable
 278 than $\partial R / \partial P$ and $\partial R / \partial E_0$ (discussed below). For this reason, ΔR_n showed considerable discrepancies
 279 between the two methods but ΔR_P as well as ΔR_{E_0} matched well between the two methods (Fig. 3).

280 As with the LI method, the complementary method produced ΔR_P , ΔR_{E_0} , and ΔR_n that exactly add
281 up to ΔR . Although its resultant ΔR_P , ΔR_{E_0} , and ΔR_n were all in good agreement with the LI method
282 (Fig. 4), the LI method often yielded values beyond the bounds given by the complementary method
283 (Fig. 5); this is because the initial and terminal states are not equivalent to the maximum and minimum
284 values over the integral path.

285 3.2 The spatio-temporal variability of the path-averaged sensitivities

286 $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ implies the average runoff change induced by a unit change in P , E_0 and n ,
287 respectively (Appendix D). Their spatio-temporal variability is relevant to the prediction of the runoff
288 change. To evaluate their temporal variabilities, I calculated $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ for each subperiod of the
289 evaluation period and assessed their deviation from those for the whole evaluation period. As shown in
290 Fig. 6, the deviation was rather limited for $\overline{\lambda_P}$ (averaged 8.6%) and $\overline{\lambda_{E_0}}$ (averaged 13%), but was
291 considerable for $\overline{\lambda_n}$ (averaged 41%). Hence, it seems quite safe to predict the future climate effects on
292 runoff using earlier $\overline{\lambda_P}$ and $\overline{\lambda_{E_0}}$, but care must be taken when using earlier $\overline{\lambda_n}$ to predict future catchment
293 effect on runoff.

294 Different from the temporal variability, $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ all varied greatly, by up to several or
295 even ten folds, between the studied catchments (Table 4). It was found that there were good correlations
296 between $\overline{\lambda_P}$ and P , between $\overline{\lambda_{E_0}}$ and P , and between $\overline{\lambda_n}$ and n (Fig. 7). Fig. 8 shows that Eq. (2)
297 reproduced $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ very well taking the long-term means of P , E_0 , and n as inputs, a fact that the
298 dependent variable approached its average if setting the independent variables to be their averages. The
299 finding is of relevance to the spatial prediction of $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$; moreover, it would greatly facilitate
300 the prediction of future climate effect on runoff as $\overline{\lambda_P}$ and $\overline{\lambda_{E_0}}$ was rather stable over time as previously
301 mentioned.

302 Runoff data and in turn, the parameter n in the MCY equation, are often unavailable. It is thus
303 desirable to make predictions of $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ in the absence of the parameter n . I developed three
304 strategies as follows: 1) using Eq. (2) and assuming $n = 2$ as n is typically in a small range from 1.5 to
305 2.6 (Roderick and Farquhar, 2011); 2) using P and E_0 to establish regression models; 3) using the aridity
306 index to establish regressions as it appeared to be more correlated with both $\overline{\lambda_P}$ and $\overline{\lambda_{E_0}}$ than P and E_0
307 (Fig. 7). As shown in Fig. 9, the three strategies have similar performance although the second one
308 seems to perform better. All of the strategies gave acceptable predictions of $\overline{\lambda_P}$ and $\overline{\lambda_{E_0}}$, but rather poor
309 results for $\overline{\lambda_n}$ as it was primarily controlled by n (Fig. 7). It was thus needed to seek more sophisticated
310 approaches to predict the future catchment effect on runoff in the absence of runoff observations.
311

312 4 Discussion

313 The LI method re-defines the widely-used concept of sensitivity at a point as the path-averaged
314 sensitivity. The method highlights the role of the evolutionary path in determining the resultant partition.
315 Yet, it seems that no studies have taken into account the path issue while evaluating the relative

316 influences of drivers. Compared with the existing methods, the limit of the LI method is high data
317 requirement for obtaining the evolutionary path. When the data are unavailable, the complementary
318 method can be considered as an alternative. First, the complementary method offer results
319 free of residuals; in addition, it employs both data of the reference and the evaluation periods to
320 determine the sensitivities, thereby generally yielding values closer to the path-averaged sensitivities
321 than the total differentiae method.

322 While using the Budyko models, a reasonable time scale is relevant to meet the assumption that
323 changes in catchment water storage are small relative to the magnitude of fluxes of P , R and E
324 (Donohue et al., 2007; Roderick and Farquhar, 2011). The present study selected seven years as most
325 studies have suggested a time period of 5-10 years (Zhang et al., 2001; Zhang et al., 2016; Wu et al.,
326 2017a; Wu et al., 2017b; Li et al., 2017) or even one year (Roderick and Farquhar, 2011; Sivapalan et
327 al., 2011; Carmona et al., 2014; Ning et al., 2017). Nevertheless, some studies asserted that the time
328 period should be longer than ten years (Li et al., 2016; Dey and Mishra, 2017). If this is the case, the
329 high temporal variation of $\bar{\lambda}_n$ shown in Fig. 6 might be caused by water storage changes, rather than
330 actual changes in the catchment properties. The uncertainty should be addressed. Using the Gravity
331 Recovery and Climate Experiment (GRACE) satellite gravimetry, Zhao et al.(2011) detected the water
332 storage variations for three largest river basins of China, namely, Yellow, Yangtze, and Zhujiang. The
333 Yellow River mostly drains an arid and semiarid region (P , 450mm; R , 70 mm; E , 380mm), and the
334 Yangtze (P , 110mm; R , 550 mm; E , 550mm) and the Zhujiang river basins (P , 1400mm; R , 780 mm; E ,
335 620mm) are humid. The amplitude of the water storage variations between years were 7, 37.2 and 65
336 mm for the three rivers respectively, one magnitude order smaller than fluxes of P , R and E . Although
337 the observations cannot be directly extrapolated to other regions, the possibility seems remote that the
338 use of a 7-year aggregated time strongly violates the assumption of the steady state condition.

339 Mathematically, the LI method is unrelated to a functional form and applies to communities
340 other than just hydrology. For example, identifying the carbon emission budgets (an allowable
341 amount of anthropogenic CO₂ emission consistent with a limiting warming target), is crucial for global
342 efforts to mitigate climate change. The LI method suggested that the emission budgets depends on both
343 the emission magnitude and pathway (timing of emissions), in line with a recent study by Gasser et al.
344 (2018). Hence, an optimal pathway would bring about an elevated carbon budget unless the carbon-
345 climate system behaves in a linear fashion. This study presented the LI method using time-series data,
346 but it applies equally to the case of spatial series of data. Given a model that relates fluvial or aeolian
347 sediment load to the influencing factors (e.g. rainfall and topography), for example, the LI method can
348 be used to separate their contributions to the sediment-load change along a river or in the along-
349 wind direction

351 5 Conclusions

352 Based on the line integral, I found a mathematically precise solution to partition the effects of a
353 number of independent variables on the change in the dependent variable. I then applied the method to
354 partition the effects on runoff of climatic and catchment conditions within the Budyko framework. The
355 method reveals that in addition to the change magnitude, the change pathways of climatic and

356 catchment conditions also exert control on their impacts on runoff. Instead of using the runoff
 357 sensitivity at a point, the LI method uses the path-averaged sensitivity, thereby ensuring a
 358 mathematically precise partition. I further examined the spatiotemporal variability of the path-averaged
 359 sensitivity. Time-wise the runoff sensitivity is stable to climate but highly variable to catchment
 360 properties, suggesting that it is reliable to predict future climate effects using earlier observations but
 361 care must be taken when predicting the catchment effects. Space-wise (between catchments) the runoff
 362 sensitivity was highly variable both to climatic and catchment conditions, but it can be well depicted by
 363 the long-term means of the climatic and catchment conditions. As a mathematically accurate scheme,
 364 the LI method has the potential to be a generic attribution approach in the environmental sciences.
 365

366 **Data availability**

367 The data used in this study are freely available by contacting the authors.
 368

369 **Author contribution**

370 MZ designed the study, analyzing the data and wrote the manuscript.
 371

372 **Competing interests**

373 The authors declare that they have no conflict of interest.
 374

375 **Appendix A: Derivation of equation** $\Delta z = \int_L f_x(x, y)dx + \int_L f_y(x, y)dy$

376 We define that the curve L in Fig. 1(c) is given by a parametric equation: $x = x(t)$, $y = y(t)$,
 377 $t \in [t_0, t_N]$, then $\Delta z = z_N - z_0 = f[x(t_N), y(t_N)] - f[x(t_0), y(t_0)]$. Substituting the parametric equations, we
 378 get:

379 The right-hand side of the equation = $\int_{t_0}^{t_N} f_x[x(t), y(t)]dx(t) + \int_{t_0}^{t_N} f_y[x(t), y(t)]dy(t)$
 380 = $\int_{t_0}^{t_N} \{f_x[x(t), y(t)]x'(t) + f_y[x(t), y(t)]y'(t)\} dt$ (A1)

381 Let $g(t) = f[x(t), y(t)]$, and after using the chain rule to differentiate g with respect to t , we obtain:

382 $g'(t) = \frac{\partial g}{\partial x} \frac{dx}{dt} + \frac{\partial g}{\partial y} \frac{dy}{dt} = f_x[x(t), y(t)]x'(t) + f_y[x(t), y(t)]y'(t)$ (A2)

383 It shows that $g'(t)$ is just the integrand in Eq. (A1), Eq. (A1) can then be rewritten as:

384 The right-hand side of the equation = $\int_{t_0}^{t_N} g'(t) dt = [g(t)]_{t_0}^{t_N} = g(t_N) - g(t_0)$
 385 = $f[x(t_N), y(t_N)] - f[x(t_0), y(t_0)]$ = The left-hand side of the equation

386 **Appendix B: The sum of $\int_L f_x(x, y)dx$ and $\int_L f_y(x, y)dy$ is path independent**

387 **Theorem:** Given an open simply-connected region G (i.e., no holes in G) and two functions $P(x, y)$
 388 and $Q(x, y)$ that have continuous first-order derivatives, if and only if $\partial P / \partial y = \partial Q / \partial x$ throughout G ,
 389 then $\int_L P(x, y)dx + \int_L Q(x, y)dy$ is path independent, i.e., it depends solely on the starting and ending
 390 point of L .

391 We have $\partial f_x / \partial y = \partial^2 z / \partial x \partial y$ and $\partial f_y / \partial x = \partial^2 z / \partial y \partial x$. As $\partial^2 z / \partial x \partial y = \partial^2 z / \partial y \partial x$, we can state that
 392 $\partial f_x / \partial y = \partial f_y / \partial x$, meeting the above condition and proving that $\int_L f_x(x, y)dx + \int_L f_y(x, y)dy$ is path
 393 independent. The statement was further exemplified using a fictitious example in Appendix C.

394 **Appendix C: A fictitious example to show how the LI method works**

395 It is assumed that runoff (R , mm yr⁻¹) at a site increases from 120 to 195 mm yr⁻¹ with $\Delta R = 75$ mm
 396 yr⁻¹; meanwhile, precipitation (P , mm yr⁻¹) varies from 600 to 650 mm yr⁻¹ ($\Delta P = 75$ mm yr⁻¹) and
 397 runoff coefficient (C_R , dimensionless) from 0.2 to 0.3 ($\Delta C_R = 0.1$). The goal is to partition ΔR into the
 398 effects of precipitation (ΔR_P) and runoff coefficient (ΔR_{C_R}) provided that P and C_R are independent.
 399 We have a function $R = PC_R$ and its partial derivatives $\partial R / \partial P = C_R$ and $\partial R / \partial C_R = P$. Given a path L
 400 along which P and C_R change and using Eq. (3), the LI method evaluates ΔR_P and ΔR_{C_R} as:

401
$$\Delta R_{C_R} = \int_L \partial R / \partial C_R dC_R = \int_L P dC_R \text{ and } \Delta R_P = \int_L \partial R / \partial P dP = \int_L C_R dP \quad (C1)$$

402 The result differs depending on L but the sum of ΔR_P and ΔR_{C_R} uniformly equals ΔR . It will be
 403 demonstrated using Fig. 1(c), in which the x -axis represents C_R and the y -axis P . Point A denotes the
 404 initial state ($C_R = 0.2, P = 600$) and point C the terminal state ($C_R = 0.3, P = 650$). I calculated ΔR_P and
 405 ΔR_{C_R} along three fictitious paths as follows:

406 1) $L=AC$. Line segment AC has equation $P = 500C_R + 500, 0.2 \leq C_R \leq 0.3$. Let's take C_R as the
 407 parameter and write the equation in the parametric form as $P = 500C_R + 500, C_R = C_R, 0.2 \leq C_R \leq 0.3$.
 408 By substituting the equation into Eq. (C1), we have:

409
$$\Delta R_{C_R} = \int_{AC} P dC_R = \int_{0.2}^{0.3} (500C_R + 500) dC_R = 62.5$$

 410
$$\Delta R_P = \int_{AC} C_R dP = \int_{AC} C_R d(500C_R + 500) = 500 \int_{0.2}^{0.3} C_R dC_R = 12.5$$

411 2) $L=AB+BC$. To evaluate on the broken line, we can evaluate separately on AB and BC and then sum
 412 them up. The equation for AB is $P = 600, 0.2 \leq C_R \leq 0.3$, and is $C_R = 0.3, 600 \leq P \leq 650$ for BC.
 413 Notes that a constant C_R or P implies that $dC_R = 0$ or $dP = 0$. Eq. (C1) then becomes:

414
$$\Delta R_{C_R} = \int_{AB+BC} P dC_R = \int_{AB} P dC_R + \int_{BC} P dC_R = \int_{0.2}^{0.3} 600 dC_R + 0 = 60$$

 415
$$\Delta R_P = \int_{AB+BC} C_R dP = \int_{AB} C_R dP + \int_{BC} C_R dP = 0 + \int_{600}^{650} 0.3 dP = 15$$

416 3) $L=AD+DC$. The equation for AD is $C_R = 0.2$, $600 \leq P \leq 650$ and is $P = 650$, $0.2 \leq C_R \leq 0.3$ for
 417 DC. ΔR_P and ΔR_{C_R} are evaluated as:

$$418 \quad \Delta R_{C_R} = \int_{AD+DC} PdC_R = \int_{AD} PdC_R + \int_{DC} PdC_R = 0 + \int_{0.2}^{0.3} 650dC_R = 65$$

$$419 \quad \Delta R_P = \int_{AD+DC} C_RdP = \int_{AD} C_RdP + \int_{DC} C_RdP = \int_{600}^{650} 0.2dP + 0 = 10$$

420 As we expect, the sum of ΔR_P and ΔR_{C_R} persistently equals ΔR although ΔR_P and ΔR_{C_R} varies with L .
 421

422 Appendix D: Derivation of $\Delta R = \overline{\lambda_P}\Delta P + \overline{\lambda_{E_0}}\Delta E_0 + \overline{\lambda_n}\Delta n$

423 If we partition the interval $[x_0, x_N]$ in Fig. 1(c) into N distinct bins of the same width $\Delta x_i = \Delta x/N$.
 424 Eq. (3a) can then be rewritten as:

$$425 \quad \Delta Z_x = \int_L f_x(x, y)dx = \lim_{\tau \rightarrow 0} \sum_{i=0}^{N-1} f_x(x_i, y_i)\Delta x_i = \lim_{\tau \rightarrow 0} N\Delta x_i \frac{\sum_{i=0}^{N-1} f_x(x_i, y_i)}{N} = \Delta x \lim_{\tau \rightarrow 0} \frac{\sum_{i=0}^{N-1} f_x(x_i, y_i)}{N} = \overline{\lambda_x}\Delta x$$

426 where $\overline{\lambda_x} = \lim_{\tau \rightarrow 0} \frac{\sum_{i=0}^N f_x(x_i, y_i)}{N}$, denoting the average of $f_x(x, y)$ along the curve L . Likewise, we have

427 $\Delta Z_y = \overline{\lambda_y}\Delta y$, where $\overline{\lambda_y}$ denotes the average of $f_y(x, y)$ along the curve L . As a result, we have:

$$428 \quad \Delta Z = \overline{\lambda_x}\Delta x + \overline{\lambda_y}\Delta y \quad (D1)$$

429 The result can readily be extended to a function of three variables. Applying the mathematic
 430 derivation above to the MCY Equation results in a precise form of Eq. (7):

$$431 \quad \Delta R = \Delta R_P + \Delta R_{E_0} + \Delta R_n = \overline{\lambda_P}\Delta P + \overline{\lambda_{E_0}}\Delta E_0 + \overline{\lambda_n}\Delta n, \quad (D2)$$

432 where $\Delta R_P = \overline{\lambda_P}\Delta P$, $\Delta R_{E_0} = \overline{\lambda_{E_0}}\Delta E_0$, $\Delta R_n = \overline{\lambda_n}\Delta n$, and $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ denote the arithmetic mean of $\partial R/\partial P$,
 433 $\partial R/\partial E_0$, and $\partial R/\partial n$ along a path of climate and catchment changes, respectively. Because $\overline{\lambda_P} = \Delta R_P / \Delta P$,
 434 $\overline{\lambda_{E_0}} = \Delta R_{E_0} / \Delta E_0$, and $\overline{\lambda_n} = \Delta R_n / \Delta n$, $\overline{\lambda_P}$, $\overline{\lambda_{E_0}}$ and $\overline{\lambda_n}$ also implies the runoff change due to a unit change in
 435 P , E_0 and n , respectively.
 436

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441

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Table 1. Summary of the long-term hydrometeorological characteristics of the selected catchments^a

Catchment No. ^b	Area (km ²)	R	P	E_0	n	AI	Reference Period	Evaluation Period	The Last Subperiod
1	391	218	1014	935	3.5	0.92	1933-1955	1956-2008	1998-2008
2	16.64	32.9	634	1087	3.16	1.71	1979-1984	1985-2008	1999-2008
3	559	183	787	780	2.68	0.99	1960-1978	1979-2000	1993-2000
4	606	73	729	998	3.07	1.37	1971-1995	1996-2009	2003-2009
5	760	77.9	689	997	2.66	1.45	1970-1995	1996-2009	2003-2009
6	502	57.2	730	988	3.59	1.35	1974-1995	1996-2008	1996-2008
7	673	431	1013	953	1.34	0.94	1947-1955	1956-2008	1998-2008
8	390	139	840	1021	2.61	1.22	1966-1980	1981-2005	1995-2005
9	1130	20.7	633	1077	3.79	1.7	1972-1982	1983-2007	1997-2007
10	3.2	37.5	631	954	3.49	1.51	1989-1991	1992-2009	1999-2009
11	1.95	111	767	901	3.06	1.18	1990-1992	1993-2005	1993-2005
12	89	272	963	826	2.82	0.86	1958-1965	1966-1999	1987-1999
13	243	38.5	735	1010	4.27	1.37	1989-1995	1996-2007	1996-2007
14	56.35	65.8	744	1007	3.35	1.35	1989-1995	1996-2008	1996-2008
15	14484	385	893	1022	1.11	1.14	1970-1989	1990-2000	1990-2000
16	38625	461	985	1087	1.03	1.1	1970-1989	1990-2000	1990-2000
17	59115	388	897	1161	1.02	1.29	1970-1989	1990-2000	1990-2000
18	95217	371	881	1169	1.03	1.33	1970-1989	1990-2000	1990-2000
19	121,972	171	507	768	1.17	1.52	1960-1990	1991-2000	1991-2000
20	106,500	60.5	535	905	2.25	1.69	1960-1970	1971-2009	1999-2009
21	5891	34.4	506	964	2.54	1.91	1952-1996	1997-2011	2004-2011

^a R , P , and E_0 represents mean annual runoff, precipitation and potential evaporation, all in mm yr⁻¹. n (dimensionless) is the parameter representing catchment properties in the MCY equation. AI is dimensionless aridity index ($AI = E_0/P$). Data of Catchments 1-14 were derived from Zhang *et al.* (2010). Data of Catchments 15-18 were from Sun *et al.* (2014). Data of Catchments 19-21 were from Zheng *et al.* (2009), Jiang *et al.* (2015), and Gao *et al.* (2016), respectively. I used the change points given in the literatures to divide the observation period into the reference and elevation periods. The LI method further divides the evaluation period into a number of subperiods. The column “The Last Subperiod” denotes the last one, which was used as the evaluation period for the total differential method, the decomposition method and the complementary method. The bold and italic rows denote that the column “Evaluation Period” is the same as the column “The Last Subperiod”.

^bCatchments 1-14 are in Australia and the others in China. 1: Adjungbilly CK; 2: Batalling Ck; 3: Bombala River; 4: Crawford River; 5: Darlot Ck; 6: Eumeralla River; 7: Goobarragandra CK; 8: Jingellic CK; 9: Mosquito CK; 10: Pine Ck; 11: Red Hill; 12: Traralgon Ck; 13: Upper Denmark River; 14: Yate Flat Ck; 15: Yangxian station, Hang River; 16: Ankang station, Hang River; 17: Baihe station, Hang River; 18: Danjiangkou station, Hang River; 19: Headwaters of the Yellow River Basin; 20: Wei River; 21: Yan River.

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Table 2. Comparisons of R (mm yr⁻¹), P (mm yr⁻¹), E_0 (mm yr⁻¹), and n (dimensionless) between the reference and the evaluation periods^a

Catchment No.	R_1	R_2	P_1	P_2	E_{01}	E_{02}	n_1	n_2	ΔR	ΔP	ΔE_0	Δn
1	223	216	959	1038	950	928	2.7	4.1	-7.2	79.2	-21	1.4
2	40.6	31	655	629	1087	1087	3	3.2	-9.7	-27	0	0.2
3	249	127	847	736	780	780	2.3	3.2	-122	-112	0.4	0.9
4	90.6	41.5	753	685	1002	989	2.9	3.7	-49	-67	-13	0.8
5	94.9	46.3	718	633	1000	992	2.5	3	-49	-85	-9	0.5
6	70.8	34.3	756	687	989	987	3.4	4.1	-36	-69	-2	0.6
7	575	406	1123	995	931	957	1.1	1.4	-169	-128	25	0.3
8	139	139	871	821	1043	1008	2.7	2.5	-0.4	-50	-35	0
9	24.1	19.2	659	621	1100	1067	3.7	3.8	-4.9	-37	-33	0.1
10	116	24.3	588	638	927	958	1.7	4.2	-92	50.4	31	2.5
11	297	68	986	716	884	905	2.3	3.6	-229	-271	22	1.3
12	301	265	992	956	820	828	2.7	2.8	-36	-36	7.4	0.1
13	48.5	32.6	752	725	991	1021	4.2	4.4	-16	-28	30	0.2
14	90.4	52.6	753	739	991	1015	2.9	3.7	-38	-14	24	0.8
15	435	295	948	795	1008	1047	1.1	1.2	-139	-153	38	0.1
16	520	353	1035	894	1074	1109	1	1.2	-167	-141	35	0.2
17	441	291	939	820	1149	1182	1	1.2	-151	-119	33	0.2
18	412	296	913	821	1163	1179	1	1.1	-116	-92	15	0.2
19	180	144	512	491	774	751	1.1	1.3	-36	-21	-23	0.2
20	90.2	52.1	585	520	895	908	2.1	2.3	-38	-65	13	0.2
21	37.7	24.6	521	462	954	995	2.6	2.5	-13	-59	41	0

^aThe subscript "1" denotes the reference period and "2" denotes the evaluation period. $\Delta X = X_2 - X_1$ (X as a substitute for R , P , E_0 , and n).

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Table 3. Effects of precipitation (ΔR_P , mm yr⁻¹), potential evapotranspiration (ΔR_{E_0} , mm yr⁻¹), and catchment changes (ΔR_n , mm yr⁻¹) on the mean annual runoff resulting from the four methods

Catchment NO. ^a	LI Method			Decomposition Method	Total Differential Method			Complementary Method		
	ΔR_P	ΔR_{E_0}	ΔR_n	ΔR_n	ΔR_P	ΔR_{E_0}	ΔR_n	ΔR_P	ΔR_{E_0}	ΔR_n
1	-70.9	-8.99	-24.3	-44.6	-67	4.82	-62	-60.7	4.34	-47.3
2	-6.49	0.95	-9.74	-9.65	-7.2	1.3	-13	-6.23	1.13	-10.2
3	-89	25.9	-140	-128	-104	26.6	-483	-88	25.7	-140
4	-18.1	2.09	-35.4	-36.3	-18	2.37	-58	-14.8	1.99	-38.5
5	-27.9	1.14	-21.3	-18.6	-34	1.18	-27	-28.1	0.97	-20.9
6	-19.9	0.29	-16.7	-14.9	-24	0.36	-22	-19.9	0.29	-16.7
7	-211	-7.19	-101	-90.9	-236	-6.9	-134	-211	-6.21	-102
8	-32.2	12.3	-14.4	-12.6	-35	12.6	-15	-32.9	11.9	-13.3
9	-11.8	3.02	-9.96	-8.45	-13	0.85	-20	-8.76	0.56	-10.5
10	19.47	-5.61	-119	-96.5	0.91	-10	-291	0.56	-6.53	-99.1
11	-150	-7.46	-71.8	-60.7	-188	-9.4	-113	-144	-7.04	-78.3
12	-9.88	-3.99	-79.2	-82	-11	-0.5	-154	-10.8	-0.57	-81.6
13	-6.98	-4.36	-4.54	-4.21	-8	-5.1	-5.2	-7	-4.38	-4.51
14	-4.84	-4.42	-28.7	-27.9	-5.6	-5	-37	-4.85	-4.4	-28.6
15	-104	-8.56	-24.8	-23	-110	-9.4	-27	-103	-8.52	-25.1
16	-99.3	-7.99	-58.8	-56	-105	-8.3	-68	-99	-7.92	-59.1
17	-78.8	-6.26	-63.9	-61	-84	-6.5	-76	-78.6	-6.2	-64.2
18	-60.1	-2.79	-53.5	-52	-64	-2.9	-62	-60	-2.77	-53.6
19	-11.9	3.89	-27.6	-27	-12	3.81	-31	-11.9	3.85	-27.5
20	-27.5	-2.46	-18.5	-17	-31	-4.4	-26	-25.5	-3.47	-19.5
21	-10.4	-3.47	-2.11	-3.4	-9.9	-4.8	-4.8	-8.27	-3.86	-3.82

^aThe bold and italic numbers denote that the evaluation period comprises a single subperiod.

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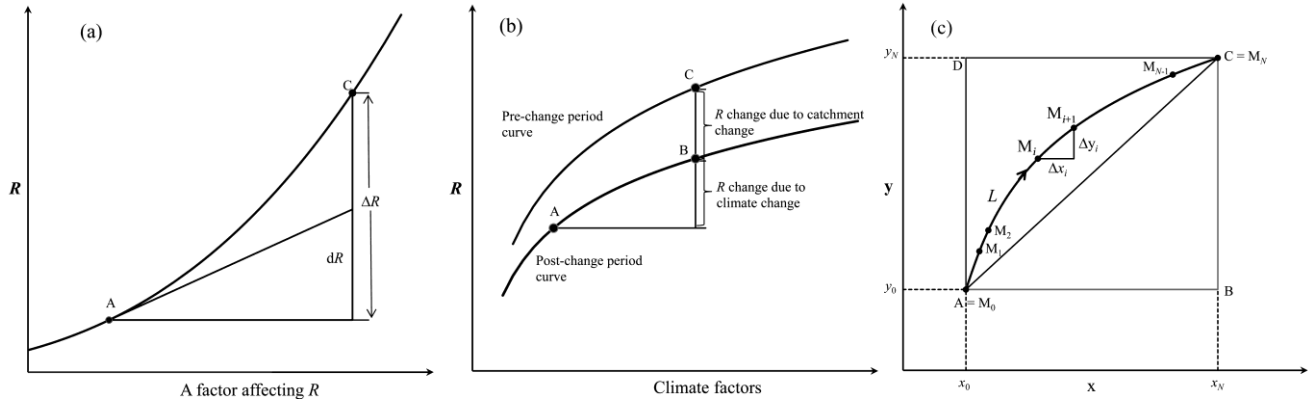
Table 4. Comparisons of the path-averaged with the point sensitivities of runoff^{a, b}

Catchment NO.	$\overline{\lambda}_P$	$\overline{\lambda}_{E_0}$	$\overline{\lambda}_n$	λ_{Pf}	λ_{E_0f}	λ_{nf}	λ_{Pb}	λ_{E_0b}	λ_{nb}
1	0.68	-0.55	-17	0.621	-0.39	-71.8	0.497	-0.32	-39.7
2	0.2	-0.08	-27.3	0.227	-0.1	-30.9	0.168	-0.07	-19.6
3	0.58	-0.36	-26.7	0.68	-0.42	-79	0.473	-0.39	-6.29
4	0.3	-0.16	-30.5	0.39	-0.2	-50.1	0.248	-0.14	-21
5	0.33	-0.14	-43.1	0.394	-0.19	-59.4	0.264	-0.12	-33.2
6	0.29	-0.16	-26.5	0.352	-0.2	-34.9	0.228	-0.12	-19.1
7	0.71	-0.32	-223	0.781	-0.33	-299	0.615	-0.26	-157
8	0.49	-0.26	-77.9	0.478	-0.27	-64.9	0.429	-0.24	-50.7
9	0.16	-0.07	-11.8	0.161	-0.07	-17.6	0.052	-0.02	-4.31
10	0.27	-0.12	-40.9	0.45	-0.16	-99.9	0.101	-0.05	-7.8
11	0.55	-0.35	-56.1	0.695	-0.44	-88.2	0.367	-0.22	-30.7
12	0.72	-0.45	-57.3	0.74	-0.53	-61.1	0.775	-0.67	-16.7
13	0.25	-0.15	-19.8	0.29	-0.17	-22.5	0.219	-0.12	-17.1
14	0.34	-0.18	-37.2	0.393	-0.21	-48.6	0.291	-0.16	-27.8
15	0.68	-0.22	-275	0.719	-0.25	-303	0.635	-0.2	-246
16	0.7	-0.23	-326	0.745	-0.24	-378	0.659	-0.21	-279
17	0.66	-0.19	-320	0.708	-0.2	-378	0.609	-0.18	-267
18	0.65	-0.19	-315	0.692	-0.19	-363	0.614	-0.18	-270
19	0.58	-0.17	-153	0.602	-0.17	-175	0.552	-0.17	-134
20	0.32	-0.12	-50.1	0.402	-0.16	-69.6	0.255	-0.1	-37.7
21	0.2	-0.06	-29.2	0.234	-0.09	-34	0.157	-0.05	-22.6

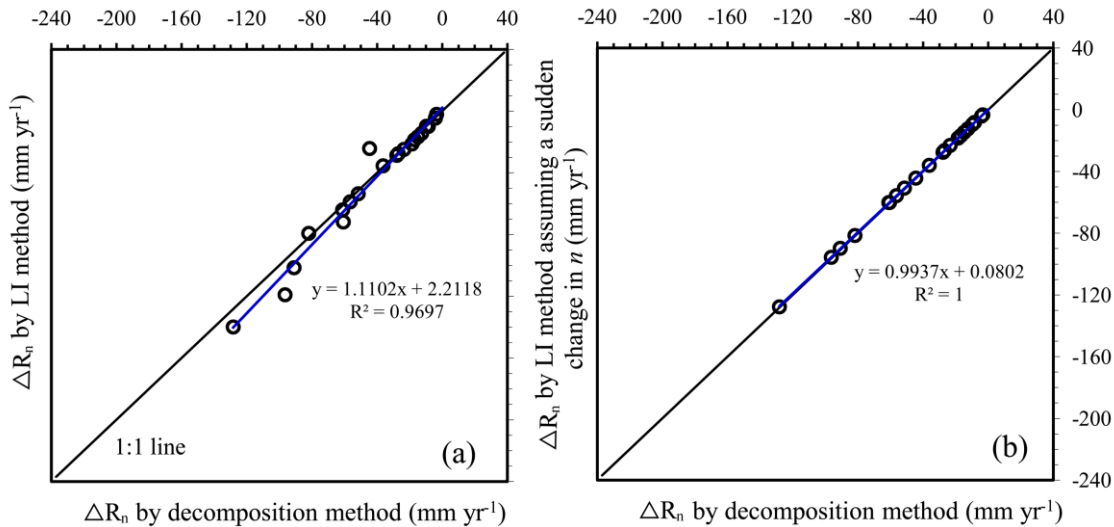
637 ^a $\overline{\lambda}_P$ (mm mm⁻¹), $\overline{\lambda}_{E_0}$ (mm mm⁻¹), and $\overline{\lambda}_n$ (dimensionless) represent the path-averaged sensitivities of
638 runoff to precipitation, potential evaporation, and catchment properties (see Appendix D). If the
639 evaluation period comprises only one subperiod, $\overline{\lambda}_P$, $\overline{\lambda}_{E_0}$ and $\overline{\lambda}_n$ was calculated as: $\overline{\lambda}_P = \Delta R_P / \Delta P$,
640 $\overline{\lambda}_{E_0} = \Delta R_{E_0} / \Delta E_0$, and $\overline{\lambda}_n = \Delta R_n / \Delta n$. If the evaluation period comprises $N > 1$ subperiods, the equations become:
641 $\overline{\lambda}_P = \sum_{i=1}^N |\Delta R_{Pi}| / \sum_{i=1}^N |\Delta P_i|$, $\overline{\lambda}_{E_0} = - \sum_{i=1}^N |\Delta R_{E_0i}| / \sum_{i=1}^N |\Delta E_{0i}|$, and $\overline{\lambda}_n = - \sum_{i=1}^N |\Delta R_{ni}| / \sum_{i=1}^N |\Delta n_i|$, where the subscript i denotes the i th
642 subperiod.

643 ^b λ_P , λ_{E_0} , and λ_n represent the point sensitivities of runoff. The subscript “ f ” represents a forward
644 approximation, i.e. substituting the observed mean annual values of the reference period into Eq. (2) to
645 calculate the sensitivities, while the subscript “ b ” represents a backward approximation, i.e. substituting
646 the observed mean annual values of the evaluation period into Eq. (2).
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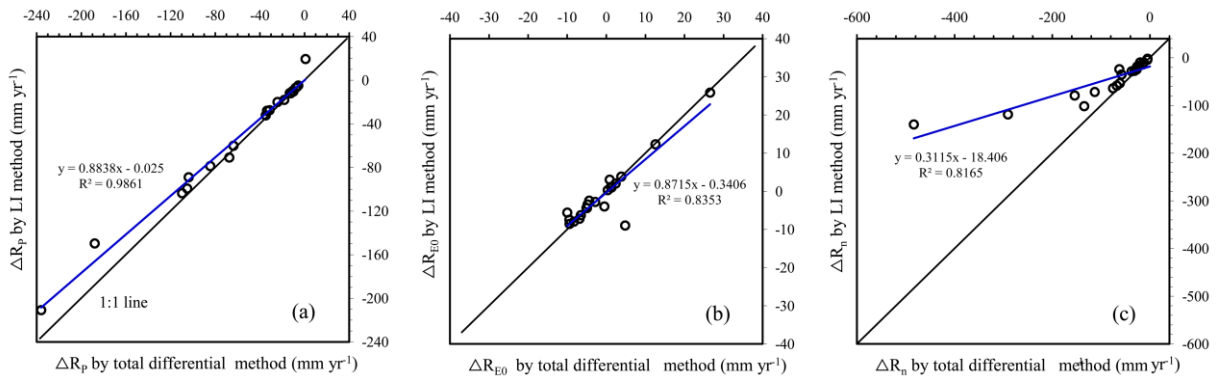
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653 **Fig. 1.** A schematic plot to illustrate (a) the total differential method, (b) the decomposition method,
654 and (c) the LI method. Point A denotes the initial state and Point C the terminal state.
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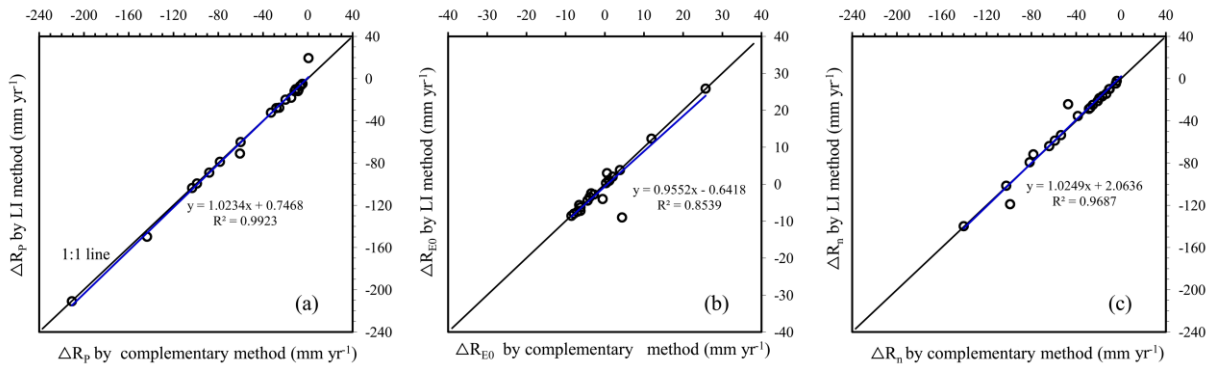


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657 **Fig. 2.** Comparison between the LI method and the decomposition method. (a) Comparison of the
658 estimated contribution to the runoff change from catchment change (ΔR_n); (b) the decomposition
659 method is equivalent to the LI method that assumes a sudden change in catchment properties following
660 climate change. In this case, the integral path of the LI method is the broken line AB+BC in Fig. 1(c) (x
661 represents climate factors and y catchment properties, *i.e.* n) and
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$$\Delta R_n = \int_{AB+BC} \frac{\partial R}{\partial n} dn = \int_{AB} \frac{\partial R}{\partial n} dn + \int_{BC} \frac{\partial R}{\partial n} dn = 0 + \int_{BC} \frac{\partial R}{\partial n} dn = \int_{n_1}^{n_2} f_n(P_2, E_{02}, n) dn$$
, where the subscript "1"
663 denotes the reference period and "2" denotes the last subperiod of the evaluation period.
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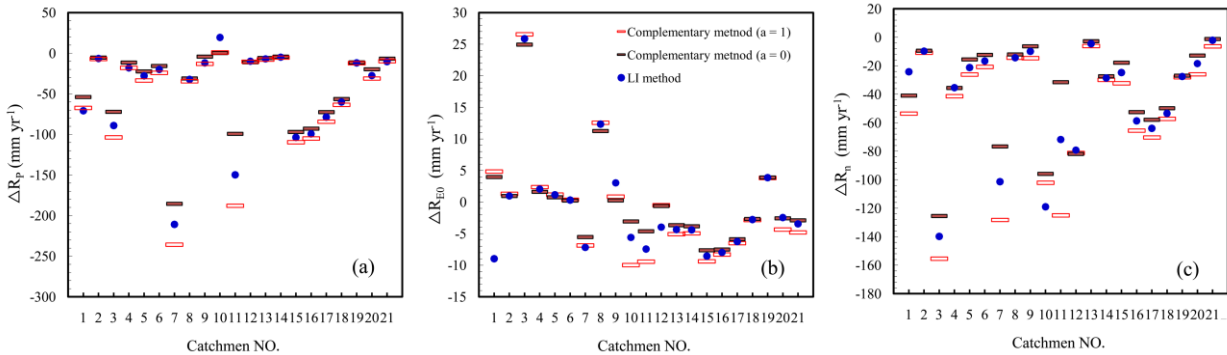
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Fig. 3. Comparison of the estimated contribution to runoff from the changes in (a) precipitation (ΔR_P), (b) potential evapotranspiration (ΔR_{E_0}), and (c) catchment properties (ΔR_n) between the LI method and the total differential method.



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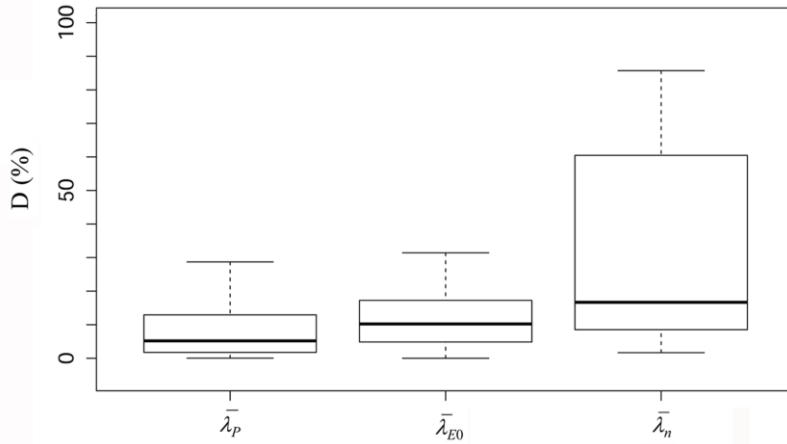
Fig. 4. Comparison of (a) ΔR_P , (b) ΔR_{E_0} , and (c) ΔR_n between the LI method and the complementary method ($a = 0.5$).



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Fig. 5. Comparison of (a) ΔR_P , (b) ΔR_{E_0} , and (c) ΔR_n by the LI method with the upper ($a=1$) and lower ($a=0$) bounds given by the complementary method. According to Zhou *et al.* (2016), the upper and lower bounds of ΔR_P , ΔR_{E_0} , and ΔR_n are reached when a is 0 or 1.

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Fig. 6. Boxplots showing the temporal variability of the path-averaged sensitivities of water yield to precipitation ($\bar{\lambda}_P$), potential evapotranspiration ($\bar{\lambda}_{E0}$), and catchment properties ($\bar{\lambda}_n$). D (%) was calculated as the relative difference between the sensitivity of the whole evaluation period and that of a subperiod. In the calculations, I excluded the catchments whose evaluation periods were not long enough to comprise two or more subperiods. Box spans the inter-quartile range (IQR) and solid lines are medians. Whiskers represent data range, excluding statistical outliers, which extend more than 1.5IQR from the box ends.

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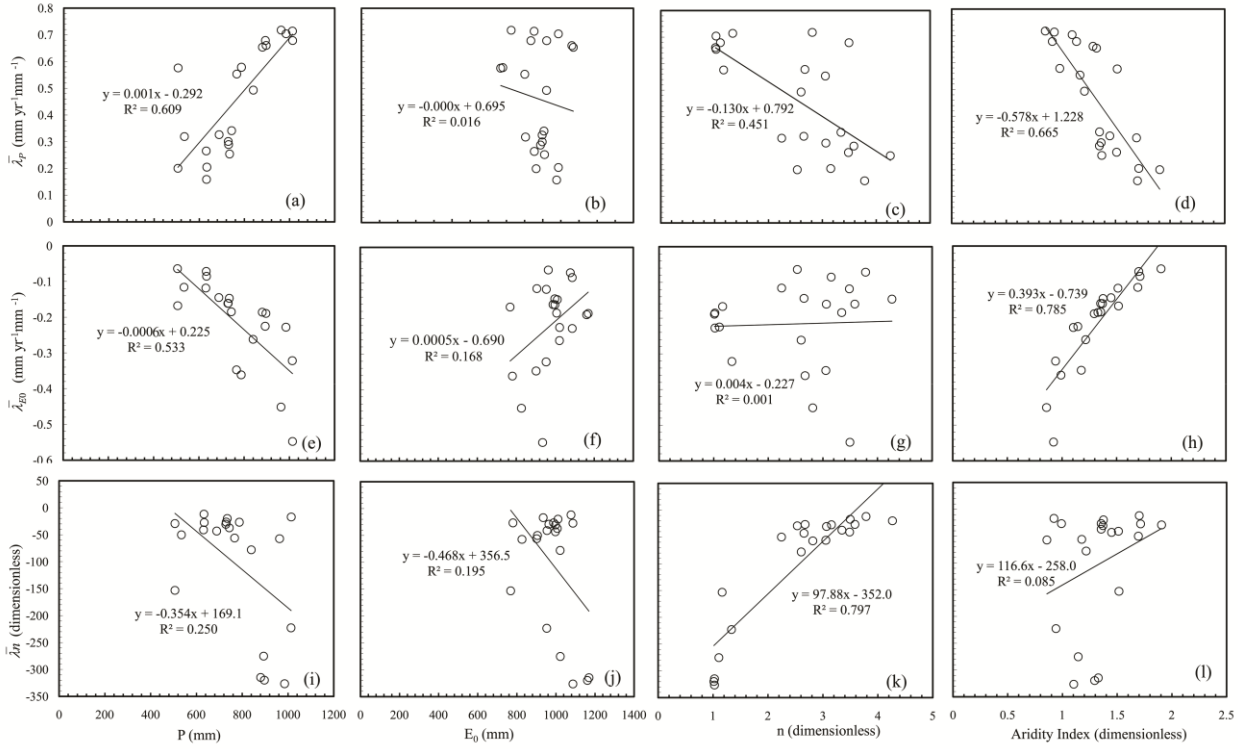


Fig. 7. $\bar{\lambda}_P$, $\bar{\lambda}_{E_0}$ and $\bar{\lambda}_n$ in correlation with P , E_0 , n , and aridity index.

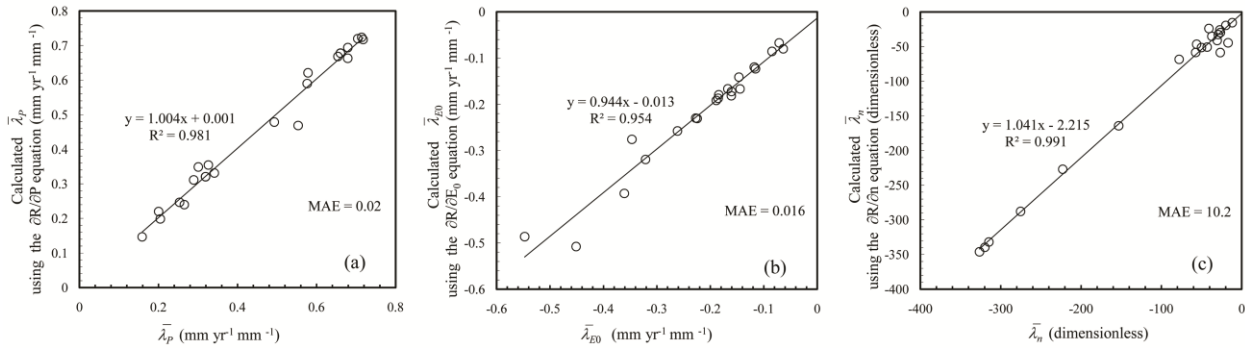
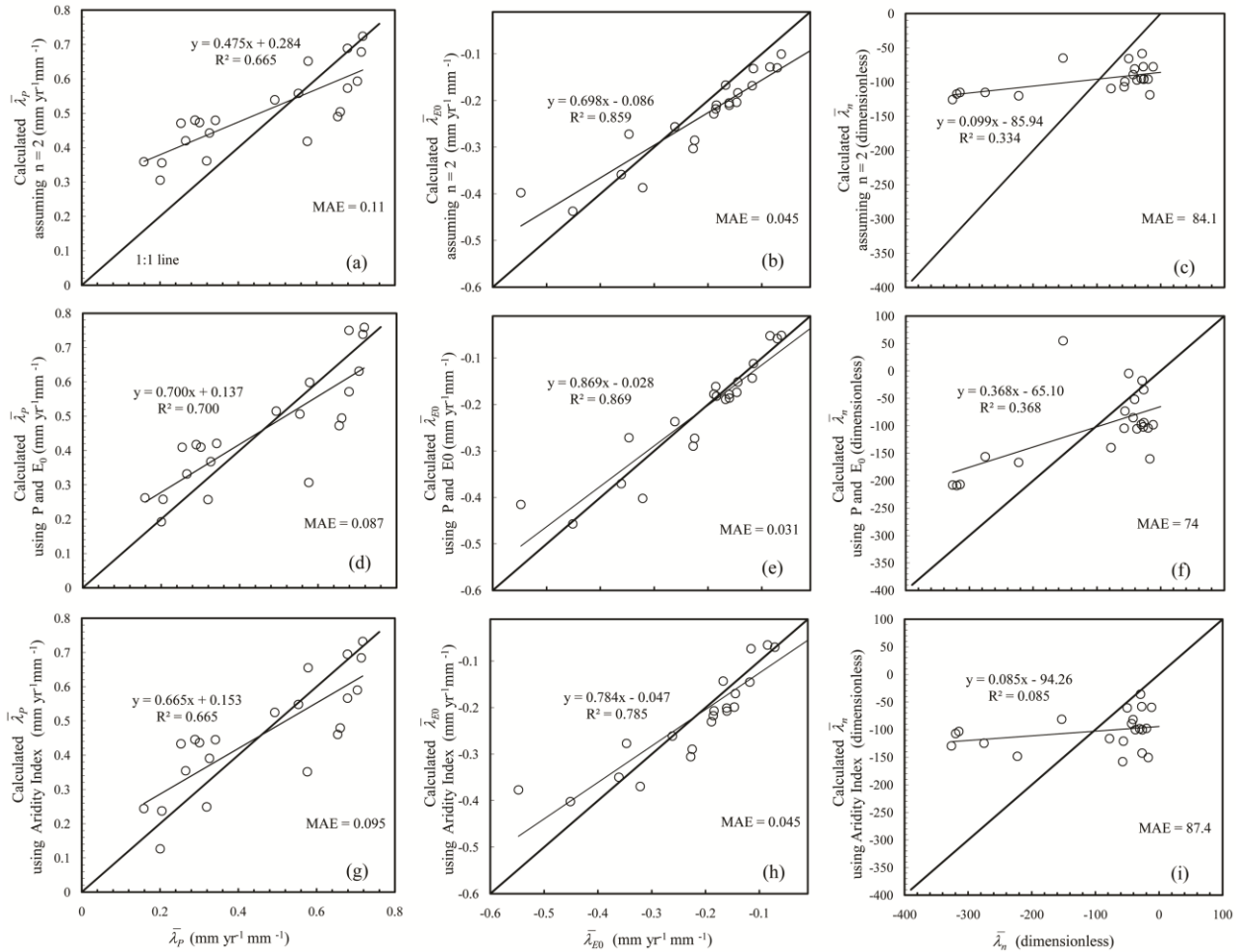


Fig. 8. Comparisons of $\bar{\lambda}_P$, $\bar{\lambda}_{E_0}$ and $\bar{\lambda}_n$ (given in Table 4) with those predicted using Eq. (2) with the long-term mean values of P , E_0 , and n as inputs. $MAE = N^{-1} \sum_{i=1}^N |O_i - P_i|$, is the mean absolute error, where O and P are values that actually encountered (given in Table 4) and predicted using Eq. (2) respectively, and N is the number of selected catchments.



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Fig. 9. Comparisons of $\bar{\lambda}_P$, $\bar{\lambda}_{E0}$ and $\bar{\lambda}_n$ with those predicted by the three strategies. (a)-(c) by Eq. (2) with a constant n ($n = 2$), (d)-(f) by the regression equations established using P and E_0 : $\bar{\lambda}_P = 0.0011P - 0.0006E_0 + 0.21$ ($R^2 = 0.7$), $\bar{\lambda}_{E0} = 0.0007P - 0.0007E_0 - 0.38$ ($R^2 = 0.87$), and $\bar{\lambda}_n = -0.302P - 0.372E_0 + 493$ ($R^2 = 0.37$), and (g)-(i) by the regression equations established using only the aridity index, as shown in Fig. 7 (d), (h) and (l). MAE was calculated as for Fig. 8.

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