Dear Editor,

We are grateful to the three anonymous referees for their constructive review, encouraging comments and useful suggestions that helped us to improve the quality of the manuscript. Following their comments, we improved the manuscript. In the following lines please find our replies (AR) **in bold fonts** to the reviewers' comments (RC), reported in *italic*. Changes to the manuscript are quoted and reported in **"*bold italic*"** font. Finally, as a first general remark suggests by the Referees #2 and #3, a careful proofreading for English language was performed on the revised version of the manuscript from a professional native English speaker. Here, you can find the revisited text also in comments to Referees.

Kind regards,

Vincenzo Totaro

Andrea Gioia

Vito Iacobellis

Referee #1

RC. The first is about the assumptions made in this work as their significance with respect to natural phenomena; e.g. the non-stationary model accounts only for the variability of mean in time (which should be explicitly shown for the sake of clarity by reporting the theoretical expressions of the three first order moments a functions of the parameters), yet in nature the non-stationary behavior could imply also a variability in terms of the second order moment. Further, natural time series often depict dependence in time, which significantly affects the power of statistical methods for non-stationarity detection, as also recognized by the Authors themselves. I generally suggest the Authors to improve the discussion on practical limitations of those tests and of the conditions analyzed in their work, yet this is only a personal suggestion to improve the completeness of the discussion.

AC. Authors wish to thank the reviewer for these useful suggestions. We introduced in section 2.4 the theoretical expressions by Muraleedharan et al. (2010) of the three first order moments as functions of the parameters. We enlarged the discussion regarding general limits of statistical methods for detection of non-stationarity in both the introduction and the conclusive sections.

In particular we specify, also dealing with the final remark from Referee #2, that our purpose here is to show that, in some cases, even a weak linear trend in the mean suffices to reduce power to unacceptable values, then, we decided to limit the investigation to variability of mean in time. Nevertheless, when dealing with natural series, a number of potential other sources of uncertainty, including variability in time of the second order moment, should be considered as we have remarked in the conclusions of the revised manuscript.

As main changes to the manuscript, in the introduction the following lines were introduced:

"T*he use of null hypothesis significance tests for trend detection has raised concerns and severe criticisms in a wide range of scientific fields since many years (e.g. Cohen, 1994), as outlined by Vogel et al. (2013). Serinaldi et al. (2018) provided an extensive critical review focusing on logical flaws and misinterpretations often related to their misuse.*"

and later,

"*Nevertheless, as claimed by different authors (Milly et al. 2015, Beven, 2016, among others) the importance of power in earth system sciences fields has been largely overlooked in years while a strong attention is always given to the level of significance (i.e. type I error). As pointed out by Vogel et al. (2013) "a type II error in the context of an infrastructure decision implies under-preparedness, which is often an error much more costly to society than the type I error (overpreparedness)".*

For changes to conclusions please see response to final remark from Referee #2.

RC. Second, I would like to see a deeper comparison with previous literature works on the same topic; e.g. the Authors mention in the conclusion section the paper from Serinaldi et al. (2018), without giving further details. I believe that the comparison with previous literature results could help strengthen the general discussion presented in the conclusion section

AC. **We accepted this suggestion, we decided to move the comparison with previous literature in the introduction. This includes a more specific reference to the work from Serinaldi et al (2018) and also a more general discussion about the use of statistical power for trend detection in hydrology (see response to point 5) of Referee #3 for changes to manuscript).**
Moreover, in the conclusion section, we introduced some final remarks about the lack of ergodicity to be considered when dealing with nonstationary stochastic process (see response to final remark of Referee #2 for changes to manuscript).

RC. Finally, the Authors should spend some efforts to improve the readability of the figures, e.g. by making the lines thick, by increasing the character size etc.
AC. Suggestion accepted.

Referee #2

RC. The title of the manuscript is not informative of its content. I suggest "Monte Carlo investigation of the power of parametric and non-parametric tests for trend detection in annual maxima series", or something similar.

AC. We thank the reviewer for this comment and for the suggested title we slightly changed it "Numerical investigation on the power of parametric and non-parametric tests for trend detection in annual maxima series"

RC. Lines 29, 147 and other parts of the manuscript: The statement attributed to Salas (1993) (see line 29) is theoretically incorrect. Stationarity is an attribute of stochastic processes (i.e. models) not of time-series (i.e. their realizations). More precisely, a stochastic process is said to be stationary, if and only if:

$$X_t \overset{\mathrm{d}}{=} X_{t+\tau},\ \forall\ t, \tau$$

where $\overset{\mathrm{d}}{=}$ *denotes equality in all finite-dimensional CDF's* $F_{X,n}$, *n = 1, 2,.... For example,* $F_{X,1}(x; t) = F_{X,1}(x)$, $F_{X,2}(x_1, x_2; t_1, t_2) = F_{X,2}(x_1, x_2; |t_2-t_1|)$, $F_{X,3}(x_1, x_2, x_3; t_1, t_2, t_3) = F_{X,3}(x_1, x_2, x_3; |t_2-t_1|,|t_3-t_1|)$ *and so on.*

In the above context: a) lack of trends shifts and periodicities in a timeseries does not necessarily mean that the parent process is stationary, b) the wording in the manuscript should be properly modified to avoid use of the terms: "stationary timeseries" and "non-stationary timeseries"

AC. We thank the reviewer for this important comment. The statement which is recognized as "theoretically incorrect" is a literal quote from Chapter 18 of Handbook of Hydrology (Maidment, 1993). As mentioned by Koutsoyiannis and Montanari (2015), in their paper section "Semantic and historical review" about the concept of stationarity, this is not the only case. Also the Kendall and Stuart's book make reference to "stationary series", for not counting the large number of papers that may arise from such a keyword search in a database such as Scopus. While we recognize the huge importance of semantic consistency in the scientific literature, we observe that in our paper there is little chance for misconception if considering that we are working in the framework of a Monte Carlo experiment by using time series generated from theoretical models. Nevertheless, in order to avoid any possible confusion, we accepted the reviewer's suggestion by removing the quoted sentence in the introduction (line 29) and introducing a more formal definition of stationarity. Then, we have checked throughout the manuscript for the use of "stationary (and nonstationary) time series". We found, besides the line 147 that was indicated by the reviewer, only one other "suspicious case" at line 264. In lines 147 we rephrased "if the time series is non-stationary, […]. Vice versa if the time series is stationary" into "*if the time series arises from a non-stationary process, […]. Vice versa if the process is stationary*"; Line 264 "the generation of stationary series …" into "*the generation of series from a stationary model…*".

RC. The results presented in Figures 7-13 need to be discussed in more detail.

AC. We have introduced some more description of such results, nevertheless, following comment 15) from Referee #3, we have also reduced the number of figures and subplots, limiting their display to representative selected cases.

The following comment was added to the revised manuscript

"Subplots show that the presence of a strong trend coefficient may produce significant loss in the estimator efficiency probably due to deviation from normal distribution of the sample estimates also for long samples. This suggests the need of more robust estimation procedures which provides higher efficiency for estimates of ε and σ in case of strong observed trend. It should be highlighted that efficiency in parameter estimation increases with sample size for $\varepsilon = [0, 0.4]$, while it decreases for both ε and σ, in the case $\varepsilon = [-0.4]$, where the trend of the location parameter implies a shift in time of the distribution upper bound."

See response to comment 15) from Referee #3 regarding the reduction of figures and subplots.

RC. As a final remark, I think that in the concluding section, the Authors should at least comment on an important aspect related to the presented analysis: When inferring the properties of a stochastic process from data, one needs to analyze the available time-series assuming ergodicity.

Since a non-stationary process is (by definition) non-ergodic, the stationarity assumption is central to any type of time-series analysis. Hence, non-stationary modeling of physical processes based on data (i.e. a single realization of a stochastic process) is theoretically inconsistent. That said, I believe that the findings of the Authors regarding uncertainty aspects of parametric and non-parametric tests in detecting non-stationarities, significantly underestimate those emerging when real world data is used.

AC. Also this comment is particularly welcome because it provide us with the possibility to share our general perspectives on issues concerning real data analysis. Ergodicity, in fact, is not only an important theoretical property of stationary stochastic processes but it also affects practical inferential tasks. Then, we added a final remark about different sources of uncertainty and perspectives about data usage and exogenous information exploitation to be used in environmental change modeling.

The following lines were added in conclusions:

"*As a final remark, concerning real data analysis, in our numerical experiment we showed that, in some cases, a weak linear trend in the mean suffices to reduce power to unacceptable values. Yet we explored the simplest nonstationary working hypothesis by introducing a deterministic linear dependence on time of the location parameter of the parent distribution. Obviously, when making inference from real observed data other sources of uncertainty may affect statistical inference (trend, heteroscedasticity, persistence, nonlinearity, etc), and moreover, if considering a nonstationary process with underlying deterministic dynamics, the process turns out to be non-ergodic, implying that statistic inference from sampled series is not representative of the process's ensemble properties (Koutsoyiannis and Montanari, 2015).*

As a consequence, while considering a nonstationary stochastic process as produced by a combination of a deterministic function and a stationary stochastic process, other sources of information and deductive arguments should be exploited in order to identify the physical mechanism underlying such relationships. Even in such a case observed time series have a crucial role in order to calibrate and validate deterministic modeling or, in other words, for confirming or disproving the model hypotheses.

In the field of frequency analysis of extreme hydrological events, considering the high spatial variability of sample length, trend coefficient, scale and shape parameters, etc, physically based probability distributions could be further developed and exploited for selection and assessment of the parent distribution in the context of non-stationarity and change detection. Physically based probability distributions we refer to are: (i) those arising from stochastic compound processes introduced by Todorovic and Zelenhasic (1970), which include also the GEV (see Madsen et al., 1997) and the TCEV (Rossi et al., 1984), and (ii) the theoretically derived distributions following Eagleson (1972) whose parameters are provided by clear physical meaning and are usually estimated with support of exogenous information in regional methods (e.g. Gioia et al., 2008; Iacobellis et al., 2011; see also for a more extensive overview Rosbjerg et al., 2013).

Hence, we believe that "learning from data" (Sivapalan, 2003), will remain in future years a key task for hydrologists facing the challenge of consistently identifying both deterministic and stochastic components of change (Montanari et al., 2013). This involves crucial and interdisciplinary research to develop suitable methodological frameworks for enhancing physical knowledge and data exploitation, in order to reduce the overall uncertainty of prediction in a changing environment."

Referee #3

Specific comments will be addressed in the following lines according to the same numbering provided by the reviewer:

RC 1. Lines 113-116. Description of Sen slope estimation and equation (2) should be revised: If N is the number of univocal (non-repeated) couples and j is an index for the j-th couple (x_i, x_k), why should be j > k? Maybe the authors mean i > k? Please check and better specify the role of j index. Remove also "Sorting in ascending order ....", declaring that the median values is the final estimate is enough and the reader understand.

AC. Suggestion accepted.

RC 2. Lines 179-188 + Appendix. These lines + Appendix should be removed. All the analyses in the manuscript are based on ML estimates, thus there is no reason to keep a description of PWM and L-moments.

AC. Suggestion accepted, the appendix and the expressions of L-moments were removed and replaced by theoretical expressions of moments as from first comment from Referee #1.

RC 3. Section 2.5. It should be written that stationarity is assumed as null hypothesis (e.g. in line 207 and 210)

AC. Suggestion accepted, we modified lines 205 and 211 by adding, respectively, that "*the null hypothesis of stationarity is false*" and "*the null hypothesis of stationarity (which) is true*".

RC 4. Section 2.5, lines 201 and 209. Just a curiosity: why experiments are conducted with a different number of samples (2000 vs 10000)?

AC. We used N = 10000 generations to be sure that the effect of sample variability is negligible when estimating the real level of significance and the corresponding threshold of acceptance. For the evaluation of power (i.e. in all generations with values of $\zeta_1 \neq 0$) we found N = 2000 is a good compromise between quality of results and reasonable computational time. To this purpose we performed sample checks with N = 10000 obtaining negligible differences. We did not report such details for the sake of simplicity and also because power values obtained with generations with $\zeta_1 \neq 0$ are only shown in graphical form and as such they are not even distinguishable from those shown. Moreover, N = 2000 is the same number of generations used by Yue et al. (2002a). In the revised manuscript we added this short explanation about this point in section 2.5:

"*We used a reduced number of generations (N = 2000) for the evaluation of power as good compromise between quality of results and computational time and, also in analogy with Yue et al. (2002a).*"

Regarding the number of generations N, see also answers to points 7), 8) and 14).

RC 5. Section 2.5, line 198. Authors can provide a reference for the sentence "1-to-4 trade-off between α and β is accepted"?

AC. Such question is everything but trivial, we can provide a reference about this (Cohen, 1994) and we introduced it in the revised manuscript, nevertheless such paper does not come from hydrological literature but from Psychology and is by far the most cited paper in Scopus about "statistical power". Reflecting about this point has stimulated a discussion about the apparent lack of references of this type in the earth system sciences that we have added in the conclusion section. The following lines were added in the conclusion section:

"Considering the feasibility of numerical evaluation of power, allowed by the parametric approach, we observe that, while the awareness of the crucial role of type II error is growing in latest years in the hydrological literature, a common debate would deserve more development about which power values should be considered acceptable. Such an issue is much more enhanced in other scientific fields where the experimental design is traditionally required to estimate the appropriate sample size to adequately support results and conclusions. In psychological research, Cohen (1994) proposed 0.8 as a conventional value of power, to be used with level of significance 0.05 thus leading to a ratio 4 to 1 between the risk of type II and type 1 error. The conventional value proposed by Cohen (1994) has been taken as reference by thousands of papers in social and behavioural sciences. In pharmacological and medical research, depending on real implications and nature of the type II error, conventional values of power may be as high as 0.999. This is the value suggested by Liebher (1990), when testing a treatment for patients' blood pressure. He stated, while "guarding against cookbook application of statistical methods", that "it should also be noted that, at times, type II error may be more important to an investigator then type I error".

We believe that, selecting between stationarity and non-stationarity models for extreme hydrological event prediction, a fair comparison between the null and the alternative hypotheses as $\alpha = \beta = 0.05$ should be taken, which provides power = 0.95. In our discussion we considered 0.8 as minimum threshold for acceptable power values."

RC 6. Lines 214-219. I understand the choice of GEV parameters and it is reasonable to my knowledge of rainfall maxima in Mediterranean climate. I was wandering whether it can be more informative to present results and figures in a more general way, e.g. as a function of the relative trend ζ_1/ζ_0 (which has dimension 1/time)?

AC. This comment is particularly important and stimulating. Nevertheless, such generalization is out of the purpose of this paper and would require more extensive investigation. In facts, presenting results in terms of the ratio ζ_1/ζ_0 would make sense if results of the analyses were the same for different couples of ζ_1 and ζ_0 values producing the same ratio. Actually, this is not the case when σ is fixed. We believe that invariant properties of the frequency distribution of rainfall or floods annual maximum values could be exploited for a generalization of these analyses, but this would involve consideration of scaling features of different order moments. This could be a quite interesting future development of this study, involving also time dependence of the scale parameter. Not any change has been done to the manuscript.

RC 7. Line 236. "a multi-peak . . ..": are you sure that it is not a sampling effect? Increasing the number of simulations is the result the same?

AC. In this figure we show some distributions of sampled errors in terms of difference between the power obtained with AIC and the power obtained with AIC_c. The curves show that the entire range of sampled errors provides negligible values compared to the expected power values. The different peaks in one curve (L = 30) are observed because we merged sample errors obtained for different values of *σ* characterized by a small different (random) bias. We added such considerations in the revised paper:

"*Aim of this figure is to show that the difference between the power obtained with AIC and the power obtained with AIC_c is negligible. Anyway, different peaks in one curve (L = 30) can be explained by merging sample errors obtained for different values of σ.*"

RC 8. Lines 267-273 and Table 1. Similar to previous comment: are you sure that variability observed for different σ (but keeping constant the other constraints) are not a sampling effect? Increasing the number of simulations is the result the same?

AC. Results shown in Table 1 are, to some extent, affected by a sampling effect. We had numerically checked the sample variability of the actual level of significance, which is quite smaller than the difference between the designed (0.05) and the actual value of the significance level for the LR test. We didn't report such analysis for the sake of simplicity nevertheless we used a very high number of generations (N = 10000) to produce these values (see also response to comment #4). On the other hand, we agree that there is not such evidence with specific reference to different σ values, while ε and L mostly affect results and accordingly we revised the manuscript rephrasing the sentence as:

"*Such effect is exalted when the parent distribution is upper bounded (ε=-0.4) and for shorter series (L = 30).*"

RC 9. Line 301. I would specify here that series are stationary.

AC. Suggestion accepted

RC 10. Section 3.4 (and maybe other parts of the manuscript, figures and tables). GEV parameters are always estimated by ML, thus I suggest to avoid the use of the prefix "ML-" before the symbol of the parameter (e.g. in Line 310 and 313). This would made more clear text, figure and tables.

AC. We would rather maintain the use of the prefix "ML-" in order to distinguish between values (ML-ε, ML-σ, ML-ζ_1) estimated from the series and the theoretical values (ε, σ, ζ_1) used in the parent distribution. Not any change was done.

RC 11. Line 316. "Figs. 6 and 9": usually figures should be ordered as they are cited.

AC. Suggestion accepted by revising the text by removing such reference to fig. 9 which is introduced later.

RC 12. Figures. Please consider the opportunity to use larger fonts for labels, they are not readable here, and most probably Figure will be reduced in the final formatting.

AC. Suggestion accepted

RC 13. Figure 6. Use the same range of scales (e.g. 0-0.6) in the y-axis for a fair comparison.
AC. Suggestion accepted

RC 14. Figure 9. Again: are you sure that fluctuations are not due to sampling effects? See e.g. the subplot in the right part of the Figure 9.
AC. We checked such results with different sets of random generations also increasing N up to 10000. Qualitatively results do not change. "*Randomness of results for $L = 30$ and $\sigma = 15, 20$ is probably due to a reduced efficiency of the algorithm that maximizes the log-Likelihood function, for heavy tailed distributions.*" Such consideration was added in the revised manuscript (Fig. 9 is now Fig. 8, because of the following comment).

RC 15. Figures 7, 8, 10, 11, 12, 13 are not much informative. Please show only a selection of the most representative case. An additional option is to move these figures as supplementary material.
AC. Suggestion accepted. Results shown in figs. 7-8-10-11-12-13 are now shown for representative selected cases in figures 7-9-10.

Authors' supplement to response at comments from Referees: references.

Following the general comment and suggestions from Referee #1, involving also the final remark of Referee #2, and comment 5) from Referee # 3, asking for more insights about the implications of this work within the general framework of real data analysis, we have introduced in the revised manuscript a number of references hereafter reported with indication of the position in the revised manuscript.

Section 1 Introduction, see response to 1st comment from Referee #1:

Beven, K.: Facets of uncertainty: Epistemic uncertainty, non-stationarity, likelihood, hypothesis testing, and communication, Hydrol. Sci. J., doi:10.1080/02626667.2015.1031761, 2016.

Cohen, J.: The earth is round (p < .05), American Psychol., 49, 997–1003, 1994.

Milly, P. C. D., Betancourt, J., Falkenmark, M., Hirsch, R. M., Kundzewicz, Z. W., Lettenmaier, D. P., Stouffer, R. J., Dettinger, M. D. and Krysanova, V.: On Critiques of "stationarity is Dead: Whither Water Management?" Water Resour. Res., doi:10.1002/2015WR017408, 2015.

Vogel, R. M., Rosner, A. and Kirshen, P. H.: Brief communication: Likelihood of societal preparedness for global change: Trend detection, Nat. Hazards Earth Syst. Sci., doi:10.5194/nhess-13-1773-2013, 2013.

Section 2.4 The GEV parent distribution, see response to 1st comment from Referee #1:

Muraleedharan, G., Guedes Soares, C. and Lucas, C.: Characteristic and moment generating functions of generalised extreme value distribution (GEV), in Sea Level Rise, Coastal Engineering, Shorelines and Tides., 2011.

Section 4 Conclusions, see response to 2nd comment from Referee #1 and final remark from Referee #2:

Eagleson, P. S.: Dynamics of flood frequency, Water Resour. Res., doi:10.1029/WR008i004p00878, 1972.

Gioia, A., Iacobellis, V., Manfreda, S. and Fiorentino, M.: Runoff thresholds in derived flood frequency distributions, Hydrol. Earth Syst. Sci., doi:10.5194/hess-12-1295-2008, 2008.

Iacobellis, V., Gioia, A., Manfreda, S. and Fiorentino, M.: Flood quantiles estimation based on theoretically derived distributions: Regional analysis in Southern Italy, Nat. Hazards Earth Syst. Sci., doi:10.5194/nhess-11-673-2011, 2011.

Madsen, H., Rasmussen, P. and Rosbjerg, D.: Comparison of annual maximum series and partial duration series for modelling exteme hydrological events: 1. At sit modelling, Water Res. Res., 1997.

Montanari, A., Young, G., Savenije, H.H.G., Hughes, D., Wagener, T., Ren, L.L., Koutsoyiannis, D., Cudennec, C., Toth, E., Grimaldi, S., Blöschl, G., Sivapalan, M., Beven, K., Gupta, H., Hipsey, M., Schaefli, B., Arheimer, B., Boegh, E., Schymanski, S.J., Di Baldassarre, G., Yu, B., Hubert, P., Huang, Y., Schumann, A., Post, D., Srinivasan, V., Harman, C., Thompson, S., Rogger, M., Viglione, A., McMillan, H., Characklis, G., Pang, Z., and Belyaev, V., “Panta Rhei—Everything Flows”: Change in hydrology and society—The IAHS Scientific Decade 2013–2022. Hydrological Sciences Journal. 58 (6) 1256–1275, 2013.

Rosbjerg, D., Blöschl, G., Burn, D., Castellarin, A., Croke, B., Di Baldassarre, G., V. Iacobellis, T. R. Kjeldsen, G. Kuczera, R. Merz, A. Montanari, D. Morris, T. B. M. J. Ouarda, L. Ren, M. Rogger, J. L. Salinas, E. Toth, and Viglione, A.: Prediction of floods in ungauged basins. In G. Blöschl, M. Sivapalan, T. Wagener, A. Viglione, & H. Savenije (Eds.), Runoff Prediction in Ungauged Basins: Synthesis across Processes, Places and Scales (pp. 189-226). Cambridge: Cambridge University Press. doi:10.1017/CBO9781139235761.012, 2013.

Rossi, F., Fiorentino, M. and Versace, P.: Two-Component Extreme Value Distribution for Flood Frequency Analysis, Water Resour. Res., doi:10.1029/WR020i007p00847, 1984.

Sivapalan, M., Prediction in Ungauged Basins: A Grand Challenge for Theoretical Hydrology, Hydrol. Process. 17, 3163–3170, 2003.

Todorovic, P. and Zelenhasic, E.: A Stochastic Model for Flood Analysis, Water Resour. Res., doi:10.1029/WR006i006p01641, 1970.

Section 4 Conclusions, see response to 2nd comment from Referee #1 and comment 5) from Referee #3:

Cohen, J., A power primer, Psychological Bulletin, Vol 112(1), Jul 1992, 155-159, 1992.

Lieber, R. L.: Statistical significance and statistical power in hypothesis testing, J. Orthop. Res., doi:10.1002/jor.1100080221, 1990.

Dear Editor

please, find below a list of all relevant changes to the original version of the manuscript. For the sake of clarity, we report rows numbers of the original manuscript, stating the corresponding comment of Referee which justified modification. We use an ***Italic bold*** font when reporting changes in the manuscript.

Sincerely

Vincenzo Totaro
Andrea Gioia
Vito Iacobellis

List of relevant changes to submitted manuscript:

- Lines 1-2: accepting suggestion n. 1 of Referee #2, we changed title of the manuscript in "***Numerical investigation on the power of parametric and non-parametric tests for trend detection in annual maximum series***".

- Line 27: some short historical notes on the concept of stationarity is introduced (suggestion n. 2 of Referee #2).
Kolmogorov in 1931 introduced the concept of stationarity of a probability distribution, formally defined by Khintchine in 1934, as depicted in the historical review provided by Koutsoyiannis and Montanari (2015)

- Line 48: in order to improve the part about practical limitations of tests used into a non-stationary framework, we introduced a discussion about the mutual importance of type I and type II errors in statistical tests (see comment n.1 of Referee #1).
On the other hand, the use of null hypothesis significance tests for trend detection has raised concerns and severe criticisms in a wide range of scientific fields since many years (e.g. Cohen, 1994), as outlined by Vogel et al. (2013). Serinaldi et al. (2018) provided an extensive critical review focusing on logical flaws and misinterpretations often related to their misuse.
In general, the use of statistical tests involves different errors, such as type I (rejecting the null hypothesis when it is true) and type II (accepting the null hypothesis when it is false). The latter is related to the test power, i.e. the probability of rejecting the null hypothesis when it is false but, as recognized by a few authors (e.g. Milly et al. 2015; Beven, 2016), in years the importance of power has been largely overlooked in earth system science fields. A strong attention has always been given to the level of significance (i.e. type I error) though, as pointed out by Vogel et al. (2013), "a type II error in the context of an infrastructure decision implies under-preparedness, which is often an error much more costly to society than the type I error (over-preparedness)".
Moreover, as already proven by Yue et al. (2002a), the power of the Mann-Kendall test, despite its non-parametric structure, actually shows a strong dependence on the type and parametrization of the parent distribution

- Lines 179-187: we replaced the discussion on L-moments with the expression of the three first order moments of GEV distribution, as suggested by comment n.1 of Referee #1.
According to Muraleedharan et al. (2010), the first three moments of GEV distribution are:

$$\boldsymbol{Mean = \zeta + \frac{\sigma}{\varepsilon}(g_1 - 1)} \qquad \boldsymbol{\varepsilon \neq 0, \varepsilon < 1} \qquad \textit{(10)}$$

$$\boldsymbol{Variance = \frac{\sigma^2}{\varepsilon^2}(g_2 - g_1^2)} \qquad \boldsymbol{\varepsilon \neq 0, \varepsilon < \frac{1}{2}} \qquad \textit{(11)}$$

$$\boldsymbol{Skewness = sgn(\varepsilon) \cdot \frac{g_3 - 3g_2 g_1 + 2g_1^3}{(g_2 - g_1^2)^{3/2}}} \qquad \boldsymbol{\varepsilon \neq 0, \varepsilon < \frac{1}{3}} \qquad \textit{(12)}$$

where $g_k = \Gamma(1 - k\varepsilon)$, with $k \in \mathbb{Z}^+$ and $\Gamma(\cdot)$ is the Gamma function. It is worth noting that, following Eqs. (10), (11) and (12), the trend in the position parameter only affect the Mean while Variance and Skewness remain constant.

- Line 198: we added a sentence for better specifying values of tests' level of significance and power we consider acceptable for our purposes.
  Then, in our experiment we assumed always significance level 0.05, and, for the following description of results and discussion we considered a power level less than 0.8 as too low and hence unacceptable. In the conclusions section we report further considerations about this choice.

- Line 212: We justified the use of the number N = 2000 as selected number of simulations (in the light of comment n.4 of Referee #3).
  We used a reduced number of generations (N = 2000) for the evaluation of power as good compromise between quality of results and computational time and, also in analogy with Yue et al. (2002a).

- Line 237: We added a sentence (according to comment n.7 of Referee #3) for commenting the use of Fig. 2.
  The purpose of this figure is to show that the difference between the power obtained with AIC and the power obtained with AIC_c is negligible. Different peaks in one curve (L = 30) can be explained by the merge of sample errors obtained for different values of σ.

- Line 327: we added a sentence (according to comment n.14 of Referee #3) for interpreting of fluctuations in Fig. 9 (which become Fig. 8 in revised manuscript).
  Randomness of results for $L = 30$ and $\sigma = [15, 20]$ is probably due to a reduced efficiency of the algorithm that maximizes the log-Likelihood function, for heavy tailed distributions.

- Line 334: Further comments to figures are introduced (according to point n.3 of Referee #2).
  It should be highlighted that efficiency in parameter estimation increases with sample size for $\varepsilon = [0, 0.4]$, while it decreases for both ε and σ, in the case $\varepsilon = [-0.4]$, where the trend of the location parameter implies a shift in time of the distribution upper bound.

- Line 338: Moving from comment n. 5 of Referee #3, we decided to introduce a discussion about the importance of dealing with type II error in statistical test used in Earth sciences.

Considering the feasibility of numerical evaluation of power, allowed by the parametric approach, we observe that, while the awareness of the crucial role of type II error is growing in latest years in the hydrological literature, a common debate would deserve more development about which power values should be considered acceptable. Such an issue is much more enhanced in other scientific fields where the experimental design is traditionally required to estimate the appropriate sample size to adequately support results and conclusions. In psychological research, Cohen (1992) proposed 0.8 as a conventional value of power, to be used with level of significance 0.05 thus leading to a ratio 4 to 1 between the risk of type II and type 1 error. The conventional value proposed by Cohen (1992) has been taken as reference by thousands of papers in social and behavioural sciences. In pharmacological and medical research, depending on real implications and nature of the type II error, conventional values of power may be as high as 0.999. This is the value suggested by Liebher (1990), when testing a treatment for patients' blood pressure. He stated, while "guarding against cookbook application of statistical methods", that "it should also be noted that, at times, type II error may be more important to an investigator then type I error".

We believe that, selecting between stationarity and non-stationarity models for extreme hydrological event prediction, a fair comparison between the null and the alternative hypotheses as $\alpha = \beta = 0.05$ should be taken, which provides power = 0.95. In our discussion we considered 0.8 as minimum threshold for acceptable power values.

– Line 392: In order to contemplate the final remark of Referee n. 2, we decided to introduce a wide discussion about ergodicity and the different sources of uncertainty that can affect real data analysis, with some considerations about possible future developments.

As a final remark, concerning real data analysis, in our numerical experiment we showed that, in some cases, a weak linear trend in the mean suffices to reduce power to unacceptable values. Yet we explored the simplest nonstationary working hypothesis by introducing a deterministic linear dependence on time of the location parameter of the parent distribution. Obviously, when making inference from real observed data other sources of uncertainty may affect statistical inference (trend, heteroscedasticity, persistence, nonlinearity, etc), and moreover, if considering a nonstationary process with underlying deterministic dynamics, the process turns out to be non-ergodic, implying that statistic inference from sampled series is not representative of the process's ensemble properties (Koutsoyiannis and Montanari, 2015).

As a consequence, while considering a nonstationary stochastic process as produced by a combination of a deterministic function and a stationary stochastic process, other sources of information and deductive arguments should be exploited in order to identify the physical mechanism underlying such relationships. Even in such a case observed time series have a crucial role in order to calibrate and validate deterministic modeling or, in other words, for confirming or disproving the model hypotheses.

In the field of frequency analysis of extreme hydrological events, considering the high spatial variability of sample length, trend coefficient, scale and shape parameters, etc, physically based probability distributions could be further developed and exploited for selection and assessment of the parent distribution in the context of non-stationarity and change detection. Physically based probability distributions we refer to are: (i) those arising from stochastic compound processes introduced by Todorovic and Zelenhasic (1970), which include also the GEV (see Madsen et al., 1997) and the TCEV (Rossi et al., 1984), and (ii) the theoretically derived distributions following Eagleson (1972) whose parameters are provided by clear physical meaning and are usually

estimated with support of exogenous information in regional methods (e.g. Gioia et al., 2008; Iacobellis et al., 2011; see also for a more extensive overview Rosbjerg et al., 2013).
Hence, we believe that "learning from data" (Sivapalan, 2003), will remain in future years a key task for hydrologists facing the challenge of consistently identifying both deterministic and stochastic components of change (Montanari et al., 2013). This involves crucial and interdisciplinary research to develop suitable methodological frameworks for enhancing physical knowledge and data exploitation, in order to reduce the overall uncertainty of prediction in a changing environment.

- Lines 396-415: Accepting suggestions n.1 of Referee #1 and n. 2 of referee #3, we removed Appendix A.

- References: we introduced new references added in the modified version of manuscript.

- All figures: accepting suggestions of Referees #1 and #3, we improved readability of all figures.

- Figures 7-8: we condensed Figures 7-8 into a single Fig. 7 in the final version of the manuscript.

- Fig. 9 become Fig. 8:

- Figures 10-11: we condensed Figures 10-11 into a single Fig. 9 in the final version of the manuscript.

- Figures 12-13: we condensed Figures 12-13 into a single Fig. 10 in the final version of the manuscript.

Please, note that changes of Figures were made for fulfilling comments n. 3 of Referee #2 and n. 15 of Referee #3.

Numerical investigation on the ~~Power~~ power of parametric and non-parametric tests for trend detection in annual maximum series

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Abstract. The need of fitting time series characterized by the presence of trend or change points has generated in latest years an increased interest in the investigation of non-stationary probability distributions. Considering that the available hydrological time series can be recognized as the observable part of a stochastic process with a definite probability distribution, two main topics can be tackled in this context: the first one is related to the definition of an objective criterion for choosing whether the stationary hypothesis can be adopted, while the second one regards the effects of non-stationarity on the estimation of distribution parameters and quantiles for assigned return period and flood risk evaluation. Although the time series trend or change points are usually detected by non-parametric ~~can be recognized using classical~~ tests available in literature (e.g. Mann-Kendal or CUSUM test), ~~for design purpose it is still required~~ the correct selection of the stationary or non-stationary probability distribution is still required for design purposes. By this light, the focus is shifted toward model selection criteria which implies the use of parametric methods with all related issues i~~o~~n parameters estimation. The aim of this study is to compare the performance of parametric and non-parametric methods for trend detection analysing their power and focusing on the use of traditional model selection tools (e.g. Akaike Information Criterion and Likelihood Ratio test) within this context. Power and efficiency of parameter estimation, including the trend coefficient, were investigated through Monte Carlo simulations using Generalized Extreme Value distribution as parent with selected parameter sets.

1 Introduction

Long and medium-term prediction of extreme hydrological events under non-stationary conditions, is one of the major challenges of our times. Streamflow, as well as temporal rainfall and many other hydrological phenomena, can be considered stochastic processes (Chow, 1964), i.e. families of random variables with an assigned probability distribution ~~(Koutsoyiannis and Montanari, 2015)~~, while time series are the observable part of this process. One of the main goals of the extreme events frequency analysis ~~of extreme events~~ is the estimation of distribution quantiles related to a certain non-exceedance probability. They are usually obtained after fitting a probabilistic model to observed data. Kolmogorov in 1931 introduced the concept of stationarity of a probability distribution, formally defined by Khintchine in 1934, as depicted in the historical review provided by Koutsoyiannis and Montanari (2015).

Quae cum ita sint, detecting the existence of time-dependence in a stochastic process~~, has to~~ should be considered a necessary task in the statistical analysis of recorded time series.

~~According to Salas (1993) "a hydrological time series is stationary if is free of trends, shifts, or periodicity (cyclicity)".~~ Starting from this statement several considerations can be done in updating some important hydrological concepts while assuming that non-exceedance probability varies with times or other covariates. For example, return period may be reformulated in two different ways, Expected Waiting Time (EWT, Olsen et al., 1998) or Expected Number of Events (ENE, Parey et al., 2007, 2010) which lead to a different evaluation of quantiles ~~into~~within a non-stationary approach. As proved by Cooley (2013~~01~~), EWT and ENE are differently affected by non-stationarity, possibly producing ambiguity in engineering design practice (Du et al., 2015; Read and Vogel, 2015). Salas and Obeysekera (2014) provided a detailed report about relationships between stationary and non-stationary EWT values within a parametric approach for the assessment of non-stationary conditions. In such a framework, a strong relevance is given to statistical tools for detecting changes in non-normally distributed time series (Kundewicz and Robson, 2004).

So far~~On the other hand~~, the vast majority of research ~~undertaken~~ about climate change and detection of non-stationary conditions has been ~~so far~~ developed through non-parametric approaches. One of the most used non-parametric measures of trend is~~ the~~ Sen's slope (Gocic and Trajkovic, 2013). Also, a wide gamma of non-parametric tests for detecting non-stationarity ~~in time series~~ is available (e.g. Kundewicz and Robson, 2004~~-~~). Statistical tests include Mann-Kendall (*MK;* Mann, 1945; Kendall, 1975) and Spearman (Lehmann, 1975) for detecting trends, Pettitt (Pettitt, 1979~~8~~) and CUSUM (Smadi and Zghoul, 2006) for change point detection. All of these tests are based on a specific null hypothesis and have to be performed for an assigned significance level. Non-parametric tests are usually preferred to parametric ones because they are distribution-free and do not require knowledge of the parent distribution. In the frequency analysis of extreme events they are also suggested being less sensitive to the presence of outliers with respect to parametric tests (Wang et al., 2005).

On the other hand, the use of null hypothesis significance tests for trend detection has raised concerns and severe criticisms in a wide range of scientific fields since many years (e.g. Cohen, 1994), as outlined by Vogel et al. (2013). Serinaldi et al. (2018) provided an extensive critical review focusing on logical flaws and misinterpretations often related to their misuse.

In general, the use of statistical tests involves different errors, such as type I (rejecting the null hypothesis when it is true) and type II (accepting the null hypothesis when it is false). The latter is related to the test power, i.e. the probability of rejecting the null hypothesis when it is false but, as recognized by a few authors (e.g. Milly et al. 2015; Beven, 2016), in years the importance of power has been largely overlooked in earth system science fields. A strong attention has always been given to the level of significance (i.e. type I error) though, as pointed out by Vogel et al. (2013), "*a type II error in the context of an infrastructure decision implies under-preparedness, which is often an error much more costly to society than the type I error (over-preparedness)*".

Moreover, as already proven by Yue et al. (2002a), the power of the Mann-Kendall test, despite its non-parametric structure, actually shows a strong dependence on the type and parametrization of the parent distribution.~~An important characteristic is the power of such tests, i.e. the probability of rejecting the null hypothesis when it is false. It is worth noting that, as already~~

~~proven by numerical experiments by Yue et al. (2002a), the power of the Mann-Kendall test, despite its non-parametric structure, actually shows a strong dependence on the type and parametrization of the parent distribution.~~

In a parametric approach, the estimation of quantiles of the extreme events distribution requires the search for the underlying distribution and for time-dependant hydrological variables, providing the identification of a model, which can be stationary or not (Montanari and Koutsoyiannis, 2014). In other words, it is necessary to define if variables are iid (independent identically distributed) or i/nid (independent/non identically distributed) and, accordingly, to select between a stationary or not-stationary distribution model (Serinaldi and Ki~~r~~lsby, 2015).

In this perspective, the detection of non-stationarity may exploit, besides traditional statistical tests, well known properties of model selection tools. Even in this case several measures and criteria are available for selecting a best-fit model, among these we find Akaike Information Criterion (AIC, Akaike, 1974), Bayesian Information Criterion (BIC, Schwarz, 1978) and Likelihood Ratio test (LR, Coles, 2001), the latter is suitable when dealing with nested models.

The purpose of this paper is to provide further insights on the use of parametric and non-parametric approaches in the framework of extreme events frequency analysis ~~of extreme events~~ under non-stationary conditions. The comparison between those different approaches is not straightforward. Non-parametric tests do not require the knowledge of the parent distribution and their properties strongly rely on the choice of a null hypothesis. Parametric methods for model selection, on the other hand, require the selection of the parent distribution and the estimation of its parameters, but are not necessarily associated with a specific null hypothesis. Nevertheless, in both cases the evaluation of the rejection threshold is usually based on a statistical measure of trend that, under the null hypothesis of stationarity, follows a specific distribution (e.g. gaussianity of the Kendall statistic for the MK non-parametric test; χ^2 distribution of deviance statistic for the LR parametric test).

Considering *pros* and *cons* of different approaches, we believe that specific remarks should be made about the use of parametric or non-parametric methods for the analysis of extreme event series. For this purpose, we set up a numerical experiment to compare performances of: 1 the MK as a non-parametric test for trend detection, 2 the LR parametric test for model selection, 3 the AIC_R parametric test as defined in section 2.3~~4~~. In particular, the AIC_R is a measure for model selection, based on the AIC, whose distribution was numerically evaluated, under the null hypothesis of a stationary process, for comparison purposes with other tests.

We aim to provide (i) a comparison of test power between MK, LR and AIC_R, (ii) a sensitivity analysis of test power to parameters of a known parent distribution used to generate sample data, (iii) an analysis of the influence of sample size on test power and significance level.

We conducted the analysis using Monte Carlo techniques, by generating samples from parent populations assuming one of the most popular extreme event distributions, the Generalized Extreme Value (Jenkinson, 1955), with linear (and without any) trend in the position parameter. From generated samples we numerically evaluated the power and significance level of tests for trend detection, using MK, LR and AIC_R. For the latter we also checked the option of using the modified version AIC_c, suggested by Sugiura (1978) for smaller samples.

Considering that parametric methods involve the estimation of the parent distribution parameters, we also analysed the efficiency of the Maximum Likelihood (ML) estimator used in trend assessment by comparing the sample variability of the ML estimate of trend with the non-parametric Sen's slope. We also scoped the sample variability of GEV parameters in the stationary and non-stationary cases.

2 Methodological framework

This section is divided into five parts. Subsections 2.1, 2.2 and 2.3 report main characteristics of respectively, *MK*, *LR* and AIC_R based test. In the fourth subsection the probabilistic model used for generations, based on the use of the GEV distribution, is described in the stationary and non-stationary cases. Subsection 2.5 outlines the procedure for numerical evaluation of tests' power and significance level.

2.1 The Mann-Kendall test

Hydrological time series are often composed by non-normally independent realizations of phenomena, and this characteristic makes the use of non-parametric trend tests very attractive (Kundzewicz and Robson, 2004). Mann-Kendall test is a widely used rank-based tool for detecting monotonic, and not necessarily linear, trends. Given a random variable z, and assigned a sample of L independent data $\boldsymbol{z} = (z_1, \dots, z_L)$, the Kendall S statistic (Kendall, 1975) can be defined as:

$$S = \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} sgn(z_j - z_i), \quad (1)$$

with *sgn* sign function.

The null hypothesis of this test is the absence of any statistically significant trend in the sample, while it is contemplated by the alternative hypothesis. Yilmaz and Perera (2014) reported that serial dependence can lead to a more frequent rejection~~ng~~ of null hypothesis. For $L \geq 8$, Mann (1945) reported how Eq. (1) is ~~an~~ approximatively a normal~~ly~~ ~~distributed~~ variable with zero mean and variance that, in the presence of t_m *m*-length ties, can be expressed as:

$$V = \frac{L(L-1)(2L+5) - \sum_{m=1}^{n} t_m m(m-1)(2m+5)}{18}.$$

In practice, Mann-Kendall test is performed using the Z statistic

$$Z = \begin{cases} \frac{S-1}{\sqrt{V(S)}} & S > 0 \\ 0 & S = 0 \\ \frac{S+1}{\sqrt{V(S)}} & S < 0 \end{cases},$$

which follows a standard normal distribution. With this approach, it is simple to evaluate the p-value and compare it with an assigned level of significance or, equivalently, to calculate~~evaluate~~ the Z_α threshold value to be compared with Z, where Z_α is the $(1-\alpha)$ quantile of a standard normal distribution.

Yue et al. (2002b) observed that autocorrelation in time series can influence the ability of *MK* test ~~in~~ to detect~~ing~~ trends. ~~For~~ To avoid~~ing~~ this problem, a correct approach in trend analysis should contemplate a preliminary check for autocorrelation and, if necessary, the application of pre-whitening procedures.

A non-parametric tool for a reliable estimation of a trend in a time series of length L and with N~~N~~ pairs of data is the Sen's slope estimator (Sen, 1968):

$$\delta_j = \frac{z_i - z_k}{i - k}, \qquad j = 1, \dots, N \tag{2}$$

being i~~j~~ $> k$. Then, ~~Sorting in ascending order the δ_j's,~~ Sen's slope estimator can be defined as the~~ir~~ median of δ_j~~δ~~.

2.2 Likelihood Ratio Test

The Likelihood Ratio statistical test allows to compare two candidate models. Like its name suggests, it is based on the evaluation of the likelihood function of different models.

The LR test has been used ~~different~~ multiple times (Tramblay et al., 2013; Cheng et al., 2014; Yilmaz et al., 2014) ~~for~~ to select~~ing~~ between stationary and non-stationary models in the context of nested models. Given a stationary model characterized by a parameter set $\boldsymbol{\theta}_{st}$ and a non-stationary model, with parameter set $\boldsymbol{\theta}_{ns}$, if $\ell(\hat{\boldsymbol{\theta}}_{st})$ and $\ell(\hat{\boldsymbol{\theta}}_{ns})$ are their respective maximized log-likelihoods, the Likelihood Ratio test can be defined through the deviance statistic

$$D = 2\left[\ell(\hat{\boldsymbol{\theta}}_{ns}) - \ell(\hat{\boldsymbol{\theta}}_{st})\right], \tag{3}$$

D is approximately, for large L, χ_m^2 distributed, with $m = dim(\boldsymbol{\theta}_{ns}) - dim(\boldsymbol{\theta}_{st})$ degrees of freedom. The null hypothesis of stationarity is rejected if $D > C_\alpha$, where C_α is the $(1-\alpha)$ quantile of χ_m^2 distribution (Coles, 2001).

Besides the analysis of power, we also checked (in subsection 3.3) the approximation $D \sim \chi_m^2$ as a function of the sample size L for the evaluation of the level of significance.

2.3 Akaike Information Criterion Ratio test

Information criteria are useful tools for model selection. It is reasonable to retain that Akaike Information Criterion (*AIC*; Akaike, 1974) is the most famous among them. Based on the Kullback-Leibler discrepanc~~ye~~ measure, if $\boldsymbol{\theta}$ is the parameter set of a k-dimensional model ($k = dim(\boldsymbol{\theta})$), AIC is defined as:

$$AIC = -2\ell(\hat{\boldsymbol{\theta}}) + 2k. \tag{4}$$

The model that best fits data has the lowest value of *AIC* between candidates. It is useful to observe that the term proportional to the number of model parameters allows to account for the increase~~d~~ of the estimator variance as the number of model parameters increases. ~~due to a larger parametrization and embodies the principle of parsimony.~~

Sugiura (1978) observed that *AIC* can lead to misleading results for small samples; he proposed a new measure for AIC:

$$AIC_c = -2\ell(\hat{\theta}) + \frac{2k(k+1)}{L-k-1} \qquad (5)$$

where a second-order bias correction is introduced. Burnham e Anderson (2004) suggested to use this version only when $L/k_{max} < 40$, being k_{max} the maximum number of parameters between the compared models. However, for larger L, AIC_c converges to AIC. For a quantitative comparison between AIC and AIC_c in the extreme value stationary model selection framework see also Laio et al. (2009).

In order to select between stationary and nonstationary candidate models, we use the ratio

$$AIC_R = \frac{AIC_{ns}}{AIC_{st}}. \qquad (6)$$

where the subscripts indicate the AIC value obtained for a stationary (*st*) and a non-stationary (*ns*) model, both fitted with maximum likelihood to the same data series.

Considering that the better fitting model has lower AIC, if the time series arises from a is non-stationary process, the AIC_R should be less than 1. Vice versa if the time seriesprocess is stationary.

In order to provide a rigorous comparison between the use of MK, LR and AIC_R, we evaluated the $AIC_{R,\alpha}$ threshold value corresponding to significance level α by numerical experiments.

More in detail, we adopted the following procedure:

1. N = 10000 samples are generated from a stationary GEV parent distribution, with known parameters;
2. for each of these samples the AIC_R is evaluated, by fitting the stationary and non-stationary GEV models described in section 2.4, thus providing its empirical distribution (see pdf in fig. 1);
3. exploiting the empirical distribution of AIC_R the threshold associated with a significance level of $\alpha = 0.05$ is numerically evaluated: this value, $AIC_{R,\alpha}$, represents the threshold for rejecting the null hypothesis of stationarity (which in these generations is true) in 5% of the synthetic samples.

This procedure was applied both for AIC and AIC_c. The experiment was repeated for a few selected sets of the GEV parameters, including different trend values, and different sample lengths, as detailed in section 3.

2.4 The GEV parent distribution

The cumulative Generalized Extreme Value (GEV) distribution (Jenkinson, 1955) can be expressed as:

$$F(z, \boldsymbol{\theta_{st}}) = \begin{cases} exp\left\{-\left[1+\varepsilon\left(\frac{z-\zeta}{\sigma}\right)\right]^{-1/\varepsilon}\right\} & \varepsilon \neq 0 \\ exp\left\{-exp\left[-\left(\frac{z-\zeta}{\sigma}\right)\right]\right\} & \varepsilon = 0 \end{cases} \quad \sigma > 0 \qquad (7)$$

where $\zeta, \sigma, \varepsilon$ are respectively known as the position, scale and shape parameter, $\boldsymbol{\theta_{st}} = [\zeta, \sigma, \varepsilon]$, is a general and comprehensive way to express the parameter set in the stationary case. The flexibility of GEV in contemplating Gumbel, Fréchet and Weibull distributions (for $\varepsilon = 0$, $\varepsilon > 0$ and $\varepsilon < 0$ respectively) makes it eligible for a more general discussion about non-stationarity implications.

Traditional extreme value distributions can be used into a nonstationary framework, modelling their parameters as function of time or other covariates (Coles, 2001) producing $\boldsymbol{\theta}_{st} \longrightarrow \boldsymbol{\theta}_{ns} = [\zeta_t, \sigma_t, \varepsilon_t]$.

In this study, only a deterministic linear dependence on trend with time t ofin the position parameter ζ has been introduced, leading Eq. (7) to be expressed as:

$$F(z, \boldsymbol{\theta}_{ns}) = \begin{cases} exp\left\{-\left[1 + \varepsilon\left(\frac{z-\zeta_t}{\sigma}\right)\right]^{-1/\varepsilon}\right\} & \varepsilon \neq 0 \\ exp\left\{-exp\left[-\left(\frac{z-\zeta_t}{\sigma}\right)\right]\right\} & \varepsilon = 0 \end{cases} \quad \sigma > 0 \tag{8}$$

with

$$\zeta_t = \zeta_0 + \zeta_1 t, \tag{9}$$

and $\boldsymbol{\theta}_{ns} = [\zeta_0, \zeta_1, \sigma, \varepsilon]$.

It is important to note that Eq. (8) is a more general way to define the GEV and has the property of degenerating into Eq. (7) for $\zeta_1 = 0$: in other words Eq. (7) represents a nested model of Eq. (8) which confirms the suitability of the Likelihood Ratio test for model selection.

The estimation of GEV parameters is often performed by means of the L-moments (Hosking, 1990), linear combinations of PWM (Hosking et al., 1985). Given a time series of values **z**, sorting it in ascending order, sample PWM can be expressed using the following relationships:

$$\beta_0 = \frac{1}{L}\sum_{j=1}^{L} z_j$$

$$\beta_1 = \frac{1}{L}\sum_{j=1}^{L} \frac{(j-1)}{(L-1)} z_j$$

Between PWM and L-moments the following relationships hold:

$$\lambda_1 = \beta_0 = \frac{1}{L}\sum_{j=1}^{L} z_j$$

$$\lambda_2 = 2\beta_1 - \beta_0 = 2\frac{1}{L}\sum_{j=1}^{L} \frac{(j-1)}{(L-1)} x_j - \frac{1}{L}\sum_{j=1}^{L} z_j$$

We observe that, imposing trend in the mean has reflections only in λ_1, and does not affect λ_2. The analytical proof based on sample relationships is provided in Appendix.

According to Muraleedharan et al. (2010), the first three moments of GEV distribution are:

$$\text{Mean} = \zeta + \frac{\sigma}{\varepsilon}(g_1 - 1) \qquad \varepsilon \neq 0, \varepsilon < 1 \qquad (10)$$

$$Variance = \frac{\sigma^2}{\varepsilon^2}(g_2 - g_1^2) \qquad \varepsilon \neq 0, \varepsilon < \frac{1}{2} \qquad (11)$$

$$Skewness = sgn(\varepsilon) \cdot \frac{g_3 - 3g_2 g_1 + 2g_1^3}{(g_2 - g_1^2)^{3/2}} \qquad \varepsilon \neq 0, \varepsilon < \frac{1}{3} \qquad (12)$$

where $g_k = \Gamma(1 - k\varepsilon)$, with $k \in \mathbb{Z}^+$ and $\Gamma(\cdot)$ is the Gamma function. It is worth noting that, following Eqs. (10), (11) and (12), the trend in the position parameter only affect the Mean, while Variance and Skewness remain constant.

In this work we used the maximum likelihood method (ML) to estimate GEV parameters from sample data. ML allows to treat ζ_1 as an independent parameter, as well as ζ_0, σ and ε. To this purpose we exploited the R package extRemes (Gilleland and Katz, 2016).

2.5 Numerical evaluation of test power and significance level

The power of a test is related to the second type error and is the probability of correctly rejecting the null hypothesis when it is false. In particular, defining α (level of significance) the probability of a Type I error and β the probability of a Type II error, we have *power* = 1 – β. The maximum value of power is 1, which correspond to β = 0, i.e. no probability of Type II error. A fair comparison between the null and the alternative hypotheses would see α = β = 0.05, which provides power = 0.95. In most applications conventional values are α = 0.05 and β = 0.2, meaning that a 1-to-4 trade-off between α and β is accepted. Thus, assuming a significance level 0.05, a power level less than 0.8 should be considered too low. Then, in our experiment we assumed always significance level 0.05, and, for the following description of results and discussion we considered a power level less than 0.8 as too low and hence unacceptable. In the conclusions section we report further considerations about this choice. For each of the tests described in subsections 2.1, 2.2 and 2.3, the power was numerically evaluated according to the following procedure:

1) N = 2000 Monte Carlo synthetic series are generated using the non-stationary GEV in Eqs. (8-9) as parent distribution with fixed parameter set $\boldsymbol{\theta}_{ns} = [\zeta_0, \zeta_1, \sigma, \varepsilon]$ $(\zeta_0, \zeta_1, \sigma, \varepsilon)$ and length L, being $\zeta_1 \neq 0$.

2) The threshold $AIC_{R,\alpha}$ associated with a significance level $\alpha = 0.05$ is numerically evaluated, as described in section 2.3 using the corresponding parameter set $\boldsymbol{\theta}_{st} = [\zeta_0, \sigma, \varepsilon]$~~$(\zeta_0, \sigma, \varepsilon)$~~ of GEV parent distribution.

3) From these synthetic series the power of the test is estimated as:

$$rejection\ rate = \frac{N_{rej}}{N}$$

where N_{rej} is the number of series for whom the null hypothesis is rejected, as in Yue et al. (2002a).

The same procedure, with N = 10000, was used in order to check the actual significance level of test, which is the probability of first type error i.e. the probability of rejecting the null hypothesis of stationarity when it is true. The task was performed by following the above steps from 1 to 3 while replacing $\boldsymbol{\theta}_{ns}$ with $\boldsymbol{\theta}_{st}$ at step 1), in such a case the rejection rate N_{rej}/N represents the actual level of significance α.

We used a reduced number of generations (N = 2000) for the evaluation of power as good compromise between quality of results and computational time and, also in analogy with Yue et al. (2002a).

3 Sensitivity analysis, results and discussion

A comparative evaluation of the tests' performance was carried out for all the GEV parameter sets $\boldsymbol{\theta}_{ns}$ obtained considering three values of ε (-0.4, 0, 0.4) and three values of σ (10, 15, 20). The position parameter was kept always constant and equal to $\zeta_0 = 40$. Then, for any possible couple of σ and ε, we considered ζ_1 ranging from -1 to 1 with step 0.1. Such a range of parameters represents a wide domain in the hydrologically feasible parameters space of annual maximum daily rainfall. Upper bounded ($\varepsilon = -0.4$), EV1 ($\varepsilon = 0$), and heavy tailed ($\varepsilon = +0.4$) cases are included. Moreover, for each of these parameter sets $\boldsymbol{\theta}_{ns}$, N samples of different size (30, 50 and 70) were generated.

For a clear exposition of results, this section is divided into four subsections. In the first one we focus on the opportunity of using AIC or AIC_c for the evaluation of AIC_R, in the second one the comparison of test power and its sensitivity analysis to parent distribution parameters and sample size is shown. In the third one, the evaluation of the level of significance for all tests and in particular the validity of the χ^2 approximation for the D statistic is discussed. In the fourth subsection the numerical investigation on the sample variability of parameters is reported.

3.1 Evaluation of AIC_R, with AIC or AIC_c

Considering the non-stationary GEV four-parameter~~s~~ model, in order to satisfy the relation $L/k_{max} < 40$ suggested by Burnham e Anderson (2004), a time series with a record length not less than 160 should be available. Following this simple reasoning the *AIC* should be considered *de facto* not-applicable to any annual maximum series showing a changing point in the ’70-80s (e.g. Kiely, 1999). In our numerical experiment, the second-order bias correction of Sugiura (1978) should be always used because for the maximum sample length, L = 70, we have L/k_{max} = 70/4 = 17.5 for the non-stationary GEV. Nevertheless, we checked if using AIC or AIC_c is important in such a use of the ratio AIC_R.may affect results. To this purpose we evaluated from synthetic series the percentage differences between the power of AIC_R, evaluated obtained by means of AIC and AIC_c. In Fig. 2 the empirical probability density functions of such percentage differences, grouped according to sample length, are plotted for generations with ε = 0.4 and different values of σ. It is interesting to note that only for L = 70 the error distribution shows a regular and unbiased bell-shaped distribution. Then we observe for L = 50 a small negative bias (about -0.02%), while for L = 30 a bias of -0.08 with a multi-peak and negatively skewed pdf. The latter pdf also has a higher variance than the others. The purpose of this figure is to show that the difference between the power obtained with AIC and the power obtained with AIC_c is negligible. Different peaks in one curve (L = 30) can be explained by the merge of sample errors obtained for different values of σ. Similar results were obtained for all values of ε, providing a general amount of differences always very low always very low differences and allowing to conclude that the use of AIC or AIC_c does not significantly affect the power of AIC_R for the cases examined. This follows the combined effect of the sample size (whose minimum value considered here is 30) and the limited difference in the number of parameters in the selected models. In the following we will refer and show only the plots obtained for the AIC_R in Eq. (6) with AIC evaluated as in Eq. (4).

3.2 Dependence of power on parent distribution parameters and sample size

The effect of parent distribution parameters and sample size on the numerical evaluation of power and significance level of MK, LR and AIC_R for different values of ε, σ and ζ_1 is shown in Fig. 3. The curves represent both significance level which is shown for $\zeta_1 = 0$ (true parent is the stationary GEV) and power for all other values $\zeta_1 \neq 0$ (true parent is the non-stationary GEV). Each subplot in Fig. 3 shows the dependence on the trend coefficient of power and significance level of MK, LR and AIC_R for one set of parameter values and different sample sizes. In all subplots the test power strongly depends on trend coefficient and sample size. This dependence is also affected by parent parameter values. In all cases the power reaches 1 for strong trend and approaches 0.05 (the chosen level of significance) for weak trend (ζ_1 close to 0). In all combinations of the shape and scale parameters, and expecially especially for short samples, for a wide range of trend values the power hasexhibits values well below the conventional value 0.8. The curves’ slope between 0.05 and 1 is sharp for long samples and slow for short samples. It also depends on the parameter set, being such a slope generally slower for higher values of the scale (σ) and shape (ε) parameters of the parent distribution. Significant difference of power between MK, LR and AIC_R is observable when the sample size is smaller and still more when the parent is heavy tailed ($\varepsilon = +0.4$).

In particular, for $\varepsilon = 0$, -0.4 and $L = 50, 70$ it is possible to report a slightly larger power of LR with respect to AIC_R and MK, but values are very close to each other. Interesting is the reciprocal position of MK and AIC_R power curves: in fact, the AIC_R power is always larger than the MK one, except when $\varepsilon = -0.4$, without sensible influence of the scale parameter.

Higher difference is found for heavy tailed parent distribution ($\varepsilon = +0.4$). While *LR* keeps having the largest power, the difference with respect to AIC_R remains small while the *MK*'s power almost collapses to values always smaller than 0.5. Practical consequences of such patterns are very important and are discussed in the concluding~~sion~~ section.

3.3 Sensitivity and evaluation of the actual significance level

We evaluated the threshold values (corresponding to a significance level of 0.05) for accepting/rejecting the null hypothesis of stationarity according to the methodologies recalled in subsections 2.1 and 2.2 for *MK* and *LR* tests and introduced in section 2.3 for AIC_R. Based on such thresholds we exploited the generation of series from a stationary model ~~series~~ ($\zeta_1 = 0$) in order to numerically evaluate the rate of rejection of the null-hypothesis, i.e. the actual significance level of the tests considered in the numerical experiment, following the procedure described in subsection 2.5.

Table 1 shows the numerical values of the actual level of significance, obtained numerically, to be compared to the theoretical value 0.05 for all the considered sets of parameters and sample size. Among the three measures for trend detection the *LR* shows the worst performance. Results in Table 1 show that the rejection rate of the (true) null hypothesis is systematically higher than it should be, and it is also dependent on parent parameter values. Such effect is exalted when the parent distribution is upper bounded ($\varepsilon = -0.4$) and for shorter series ($L = 30$)~~higher values of the scale parameter~~. In practice this implies that when using the *LR* test, as described in subsection 2.2, ~~one actually has a probability of rejecting the true null hypothesis of stationarity quite higher than he knows~~ there is a quite higher probability of rejecting the null hypothesis of stationarity (if it is true) than expected or designed.

On the other hand, the performances of *MK* with respect to the designed level of significance are less biased and independent from the parameter set. Similar good performances are trivially obtained for the AIC_R, whose rejection threshold is numerically evaluated.

The plot in Fig. 4 is displayed in order to focus on the actual value of the level of significance and in particular on the *LR* approximation $D \sim \chi^2_m$ as a function of the sample length *L*~~*n*~~. The difference between theoretical and numerical values of the significance level is represented by the distance between the bottom value of the curve (obtained for $\zeta_1 = 0$, i.e. the stationary GEV model) and the chosen level of significance 0.05 which is represented by the horizontal dotted line. In particular in Fig. 4 results for the parameter set ($\sigma = 15, \varepsilon = -0.4$) show that the actual rate of rejection is always higher than the theoretical one and changes significantly with the sample size, which means that the χ^2_m approximation leads to significant~~ly~~ underestimation~~ng~~ of the rejection threshold of the D statistic. Moreover, it seems that the entire curves of the *LR* power (in red) are upward translated as a consequence of the significance level overestimation, meaning that the *LR* test power is also overestimated because of the approximation $D \sim \chi^2_m$. These results suggest, for the *LR* test, the use of a numerical procedure (as the one introduced for AIC_R in subsection 2.3) for evaluating the D distribution and the rejection threshold.

Other considerations can be made on the use of AIC_R. As explained in subsection 2.3 we empirically evaluated by numerical generations the $AIC_{R,\alpha}$ threshold value with significance level 0.05 for each of the parameter sets and sample sizes considered.

Similar results were obtained using the AIC_c which are not shown for brevity. We found a significative dependence of $AIC_{R,\alpha}$ on the sample size. Fig. 5 shows curves of $AIC_{R,\alpha}$ obtained for each of the parameter sets vs sample size. It is also worth noting that all curves asymptotically trend to 1 as L increases. This property is due to the structure of *AIC* and peculiarity of the nested models used in this paper: while using a sample generated with weak non-stationarity (i.e. when $\zeta_1 \rightarrow 0$ in Eq. (9)) the maximum likelihood of model (7), $\ell(\widehat{\boldsymbol{\theta}}_{st})$, tends to $\ell(\widehat{\boldsymbol{\theta}}_{ns})$ of model (8) leaving only the bias correction in *AIC* to be discriminant for model selection. As a consequence, $AIC_{R,\alpha}$ should~~will~~ be always lower than 1, but, increasing sample size, also both the likelihood terms $-2\ell(\widehat{\boldsymbol{\theta}}_{st})$ and $-2\ell\left(\widehat{\boldsymbol{\theta}_{ns}}\right)$ in Eq. (4) will increase, pushing AIC_R toward the limit 1. On the other hand, Fig. 5 shows that the threshold value $AIC_{R,\alpha}$ is significatively smaller than 1 up to L values well beyond the length usually available in this kind of analysis. Hence the numerical evaluation of the threshold has to be considered as a required task in order to provide an assigned significance level to model selection. On the other hand, the simple adoption of the selection criteria $AIC_R < 1$ (i.e. $AIC_{R,\alpha} = 1$), would correspond to an unknown significance level dependent on the parent distribution and sample size. In order to highlight this point, we evaluated the significance level α corresponding to $AIC_{R,\alpha} = 1$ following the procedure described in subsection 2.5 by generating N = 10000 synthetic series, from a stationary model, for any parameter set and sample length. Results, provided in Tab. 2, show that, in the explored GEV parameter domain, α ranges between 0.16 and 0.26 mainly depending on the sample length and the shape parameter of the parent distribution.

3.4 Sample variability of parent distribution parameters

Results shown above, with regard to performances of parametric and non-parametric tests, are in our opinion quite surprising and important. It is proved~~n~~ that the preference widely accorded to non-parametric tests, being their statistics allegedly independent from the parent distribution, is not well founded. On the other hand, the use of parametric procedures raises the problem of correctly estimating the parent distribution and, for the purpose of this paper, its parameters. Moreover, as being the trend coefficient ζ_1 a parameter of the parent distribution in non-stationary condition, the proposed parametric approach provides a maximum likelihood-based estimation of the same trend coefficient which is hereafter called ML-ζ_1. For a comparison with non-parametric approaches we also evaluated the sample variability of the Sen's slope measure (δ) of the imposed linear trend. In order to provide insights into these issues, from the same sets of generations exploited above, we also analysed the sample variability of the maximum likelihood estimates ML-ε ML-σ, for different parameter sets and sample length.

We evaluated sample variability $s[\cdot]$~~s[.]~~, as the standard deviation of the ML estimates of parameter values obtained from synthetic series. ~~Results are shown in Figs. 6 and 9, for different parameter sets and sample size, vs true ζ_1 values. In Fig 6, on the first subplots row we show s[ML-ζ_1] and on the second row the Sen's slope median s[δ].~~ In the upper panels of Fig 6 we show s[ML-ζ_1] and the lower panels the Sen's slope median $s[\delta]$. The sample variability of the linear trend is in both cases strongly dependent on sample size and independent from the true ζ_1 value in the range examined $[-1,1]$. It reaches high values for short samples and in such cases also its dependence on the scale and shape parent parameters is relevant. The ML estimation of the trend coefficient is always more efficient than Sen's slope and this is observed in particular for heavy tailed distributions. In Fig~~s~~. 7 ~~and 8~~ we show the empirical distributions of the Sen's slope δ and ML-ζ_1 estimates obtained from samples of size $L = 30$ with parent distribution characterized by $\sigma = 15$ and $\varepsilon = [-0.4, 0, 0.4]$, providing ~~a~~ visual information about the range of trend values that may result from a local evaluation. Similar results, characterized by smaller sample variability shown in Fig. 6, are obtained for $L = 50$ and $L = 70$ and are not shown for brevity.

Fig. 8~~9~~ shows the sample variability of ML-ε and ML-σ, which is still independent from the true ζ_1 for values of $\varepsilon = 0$ and 0.4 while for upper bounded GEV distributions ($\varepsilon = -0.4$) it shows a significant increase for higher values of σ and high trend coefficients ($|\zeta_1| > 0.5$). Randomness of results for $L = 30$ and $\sigma = [15,20]$ is probably due to a reduced efficiency of the algorithm that maximizes the log-Likelihood function, for heavy tailed distributions.

In order to better analyse such patterns, for the scale and shape parent parameters we report also the distribution of their empirical ML estimates for different parameter sets vs the true ζ_1 value used in generation. The sample distribution of ML-ε for $\sigma = 15$ is shown in Fig. 9~~10~~ for $L = 30$ and ~~Fig. 11 for~~ $L = 70$. The sample distribution of ML-σ for $\sigma = 15$ is shown in Fig. 10~~2~~ for $L = 30$ and ~~Fig. 13 for~~ $L = 70$. Subplots show that the presence of a strong trend coefficient may produce significant loss in the estimator efficiency probably due to deviation from normal distribution of the sample estimates also for long samples. This suggests the need of more robust estimation procedures which provides higher efficiency for estimates of ε~~ϵ~~ and σ in case of strong observed trend. It should be highlighted that efficiency in parameter estimation increases with sample size for $\varepsilon = [0, 0.4]$, while it decreases for both ε and σ, in the case $\varepsilon = -0.4$, where the trend of the location parameter implies a shift in time of the distribution upper bound.

4 Conclusions

The results shown have important practical implications. The dependence of test power on the parent distribution parameters may significantly affect results of both parametric and non-parametric tests including the widely used Mann-Kendall. Considering the feasibility of numerical evaluation of power, allowed by the parametric approach, we observe that, while the awareness of the crucial role of type II error is growing in latest years in the hydrological literature, a common debate would deserve more development about which power values should be considered acceptable. Such an issue is much more enhanced in other scientific fields where the experimental design is traditionally required to estimate the appropriate sample size to adequately support results and conclusions. In psychological research, Cohen (1992) proposed 0.8 as a conventional value of power, to be used with level of significance 0.05 thus leading to a ratio 4 to 1 between the risk of type II and type 1 error. The conventional value proposed by Cohen (1992) has been taken as reference by thousands of papers in social and behavioural sciences. In pharmacological and medical research, depending on real implications and nature of the type II error, conventional values of power may be as high as 0.999. This is the value suggested by Liebher (1990), when testing a treatment for patients' blood pressure. He stated, while "*guarding against cookbook application of statistical methods*", that "*it should also be noted that, at times, type II error may be more important to an investigator then type I error*".

We believe that, selecting between stationarity and non-stationarity models for extreme hydrological event prediction, a fair comparison between the null and the alternative hypotheses as $\alpha = \beta = 0.05$ should be taken, which provides power = 0.95. In our discussion we considered 0.8 as minimum threshold for acceptable power values.

For all the generation sets and tests conducted, under the null hypothesis of stationarity, the power has values ranging between the chosen significance level (0.05) and 1 for large (and larger) ranges of the trend coefficient. The test power always collapses to very low values for weak (but climatically important) trend values (in the case of annual maximum daily rainfall, ζ_1 equal to 0.2 or 0.3 mm per year, for example). In presence of trend, the power is also affected by the scale and shape parameters of the GEV parent distribution. This observation can be made with reference to samples of all the lengths considered in this paper (from 30 to 70 years of observation) but the use of smaller samples significantly reduces the test power and dramatically extends the range of ζ_1 values for which the power is below the conventional value 0.8. The use of this sample size is not rare considering that significative trends due to anthropic effects are typically investigated in periods following a changing point often observed in the '80s.

These results also imply that in spatial fields where the alternative hypothesis of non-stationarity is true, but the parent's parameters (including the trend coefficient) and the sample length are variable in space, the rate of rejection of the false null-hypothesis may be highly variable from site to site and practically out of control. In other words, in such a case, the probability of recognizing the alternative hypothesis of non-stationarity as true from a single observed sample may unknowingly change (between 0.05 and 1) from place to place. For small samples (as $L = 30$ in our analysis) and heavy tailed distributions, the power is always very low for all the investigated range of the trend coefficient.

Hence, considering the high spatial variability of the parent distribution parameters and the relatively short period of reliable and continuous historical observations usually available, a regional assessment of trend non-stationarity may suffer from the different probability of rejection of the null hypothesis of stationarity (when it is false).

These problems affect, in slightly different measures, both parametric and non-parametric tests. While these considerations are generally applicable to all the tests considered, differences also emerge between them. For heavy tailed parent distributions and smaller samples, the *MK* test power decreases more rapidly than for the other tests considered. Low values of power are already observable for $L = 50$. The *LR* test slightly outperforms the AIC_R for small sample size and higher absolute values of the shape parameter. Nevertheless, the higher value of the *LR* power seems to be overestimated as a consequence of the χ_m^2 approximation for the D statistic distribution (see section 3.3).

Results also suggest that theoretical distribution of the *LR* test-statistic based on the null hypothesis of stationarity may lead to a significative increase of the rejection rate compared to the chosen level of significance i.e. an abnormal rate of rejection of the null hypothesis when it is true. In this case the use of numerical techniques, based on the use of synthetic generations performed by exploiting a known parent distribution, should be preferred.

By the light of these results we conclude that in trend detection on annual maximum series the assessment of the parent distribution and the choice of the null hypothesis should be considered as fundamental preliminary tasks. According to this remark, it is advisable to make use of parametric tests by numerically evaluating both the rejection threshold for the assigned significance level and the power corresponding to alternative hypotheses. This also requires developing robust techniques for ~~individuation of~~ selecting the parent distribution and estimating~~on~~ ~~of~~ its parameters. To this perspective, the use of a parametric measure such as the AIC_R, may take into account ~~of~~ different choices for the parent distribution and, even more importantly, allows one to set the~~a~~ null hypothesis differently from the stationary case, based on a priori information.

The need of robust procedures for assessing the parent distribution and its parameters is also proven by the numerical simulations we conducted. Sample variability of parameters (including the trend coefficient) may increase rapidly for series with L as low as 30 years of annual maxima. Moreover, we observed that, in case of high~~est~~ trends, numerical instability and non-convergence of algorithms may affect the estimation procedure for upper bounded and heavy tailed distributions. Nevertheless, the sample variability of the ML trend estimator was found always smaller than the Sen's slope sample variability. Finally, it is worth noting that also the non-parametric Sen's slope method, applied to synthetic series, showed dependence on the parent distribution parameters with sample variability higher for heavy tailed distributions.

This analysis shed lights onto important eventual flaws in the at-site analysis of climate change provided by non-parametric approaches. Both test power and trend evaluation are affected by the parent distribution as ~~well as they are in~~it is the case also in parametric methods. It is not ~~a case~~by chance, in our opinion, that many technical studies conducted in years around the world, provide inhomogeneous maps of positive/negative trends and large areas of stationarity characterized by weak trends that are considered not statistically significany~~tive~~. ~~Analogous concerns about the use of statistical tests have been expressed by Serinaldi et al. (2018).~~

As already said, an advantage of using parametric tests and numerical evaluation of the test-statistic distribution is given by the possibility of assuming a null hypothesis based on a preliminary assessment of the parent distribution including trend detection by evaluation of non-stationary parameters. This could lead to a regionally homogeneous and controlled assessment of both significance level and power in a fair mutual relationship. With respect to the estimation of parameters of the parent distribution, results suggest that at site analysis may provide highly biased results. More robust procedures are necessary like hierarchic estimation procedures (Fiorentino et al., 1987) providing estimates of ε and σ from detrended series (Strupczewski et al., 2016; Kochanek et al., 2013).

~~Considering the high spatial variability of sample length, trend coefficient, scale and shape parameters we believe that the application of well-known and developed regional methods for selection and assessment of the parent distribution could be easily and profitably exploited in the context of non-stationarity and climate change detection in annual maximum series and will be tackled in future research.~~

As a final remark, concerning real data analysis, in our numerical experiment we showed that, in some cases, a weak linear trend in the mean suffices to reduce power to unacceptable values. Yet we explored the simplest nonstationary working hypothesis by introducing a deterministic linear dependence on time of the location parameter of the parent distribution.

Obviously, when making inference from real observed data other sources of uncertainty may affect statistical inference (trend, heteroscedasticity, persistence, nonlinearity, etc), and moreover, if considering a nonstationary process with underlying deterministic dynamics, the process turns out to be non-ergodic, implying that statistic inference from sampled series is not representative of the process's ensemble properties (Koutsoyiannis and Montanari, 2015).

As a consequence, while considering a nonstationary stochastic process as produced by a combination of a deterministic function and a stationary stochastic process, other sources of information and deductive arguments should be exploited in order to identify the physical mechanism underlying such relationships. Even in such a case observed time series have a crucial role in order to calibrate and validate deterministic modeling or, in other words, for confirming or disproving the model hypotheses.

In the field of frequency analysis of extreme hydrological events, considering the high spatial variability of sample length, trend coefficient, scale and shape parameters, etc, physically based probability distributions could be further developed and exploited for selection and assessment of the parent distribution in the context of non-stationarity and change detection. Physically based probability distributions we refer to are: (i) those arising from stochastic compound processes introduced by Todorovic and Zelenhasic (1970), which include also the GEV (see Madsen et al., 1997) and the TCEV (Rossi et al., 1984), and (ii) the theoretically derived distributions following Eagleson (1972) whose parameters are provided by clear physical meaning and are usually estimated with support of exogenous information in regional methods (e.g. Gioia et al., 2008; Iacobellis et al., 2011; see also for a more extensive overview Rosbjerg et al., 2013).

Hence, we believe that "learning from data" (Sivapalan, 2003), will remain in future years a key task for hydrologists facing the challenge of consistently identifying both deterministic and stochastic components of change (Montanari et al., 2013). This involves crucial and interdisciplinary research to develop suitable methodological frameworks for enhancing physical knowledge and data exploitation, in order to reduce the overall uncertainty of prediction in a changing environment.

Appendix

Let us consider two different but consequent years, t and t + 1, setting $z(t) = z_0 + \alpha t$, for λ_1 there is:

$$\lambda_1(t) = \frac{1}{L}\sum_{j=1}^{L} z_0 + \alpha t = \frac{1}{L}\sum_{j=1}^{L} z_0 + \frac{1}{L}\sum_{j=1}^{L} \alpha t$$

$$\lambda_1(t+1) = \frac{1}{L}\sum_{j=1}^{L} z_0 + \alpha(t+1) = \frac{1}{L}\sum_{j=1}^{L} z_0 + \frac{1}{L}\sum_{j=1}^{L} \alpha(t+1) =$$

$$= \frac{1}{L}\sum_{j=1}^{L} z_0 + \frac{1}{L}\sum_{j=1}^{L} \alpha t + \frac{1}{L}\sum_{j=1}^{L} \alpha$$

Subtracting side by side:

$$\lambda_1(t+1)-\lambda_1(t)=\frac{1}{L}\sum_{j=1}^{L}z_0+\frac{1}{L}\sum_{j=1}^{L}\alpha t+\frac{1}{L}\sum_{j=1}^{L}\alpha-\frac{1}{L}\sum_{j=1}^{L}z_0-\frac{1}{L}\sum_{j=1}^{L}\alpha t=\frac{1}{L}\sum_{j=1}^{L}\alpha$$

Which proves that a trend in the mean value produces a related trend in λ_1.

By using the same approach for λ_2, we observe that:

$$\lambda_2(t)=2\sum_{j=1}^{L-1}\frac{(L-j)(z_j+\alpha t)}{L(L-1)}-\frac{1}{L}\sum_{j=1}^{L}(z_j+\alpha t)$$

$$\lambda_2(t+1)=2\sum_{j=1}^{L-1}\frac{(L-j)[z_j+\alpha(t+1)]}{L(L-1)}-\frac{1}{L}\sum_{j=1}^{L}[z_j+\alpha(t+1)]$$

Subtracting side by side:

$$\lambda_2(t+1)-\lambda_2(t)=2\sum_{j=1}^{L-1}\frac{(L-j)(z_j+\alpha t)}{L(L-1)}-2\sum_{j=1}^{L-1}\frac{(L-j)[z_j+\alpha(t+1)]}{L(L-1)}-\frac{1}{L}\sum_{j=1}^{L}(z_j+\alpha t)+\frac{1}{L}\sum_{j=1}^{L}[z_j+\alpha(t+1)]=$$

$$=2\sum_{j=1}^{L-1}\frac{(L-j)(z_j+\alpha t)}{L(L-1)}-2\sum_{j=1}^{L-1}\frac{(L-j)(z_j+\alpha t+\alpha)}{L(L-1)}-\frac{1}{L}\sum_{j=1}^{L}(z_j+\alpha t)+\frac{1}{L}\sum_{j=1}^{L}(z_j+\alpha t+\alpha)=$$

$$=2\sum_{j=1}^{L-1}\frac{(L-j)(z_j+\alpha t-\alpha-x_j-\alpha t)}{L(L-1)}+\frac{1}{L}\sum_{j=1}^{L}(z_j+\alpha t+\alpha-x_j-\alpha t)=$$

$$=-2\sum_{j=1}^{L-1}\frac{(L-j)\alpha}{L(L-1)}+\frac{1}{L}\sum_{j=1}^{L}\alpha=\frac{-2\alpha}{L(L-1)}\sum_{j=1}^{L-1}(L-j)+\frac{L\alpha}{L}=\frac{-2\alpha}{L(L-1)}\left[\sum_{j=1}^{L-1}L-\sum_{j=1}^{L-1}j\right]+\alpha=$$

$$=\frac{-2\alpha}{L(L-1)}\left[L(L-1)-\frac{L(L-1)}{2}\right]+\alpha=\frac{-2\alpha}{L(L-1)}\frac{L(L-1)}{2}+\alpha=-\alpha+\alpha=0$$

Which proves that a trend in the mean value does not affect λ_2.

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Table 1: Actual level of significance of tests for different sample size, scale and shape parent parameters

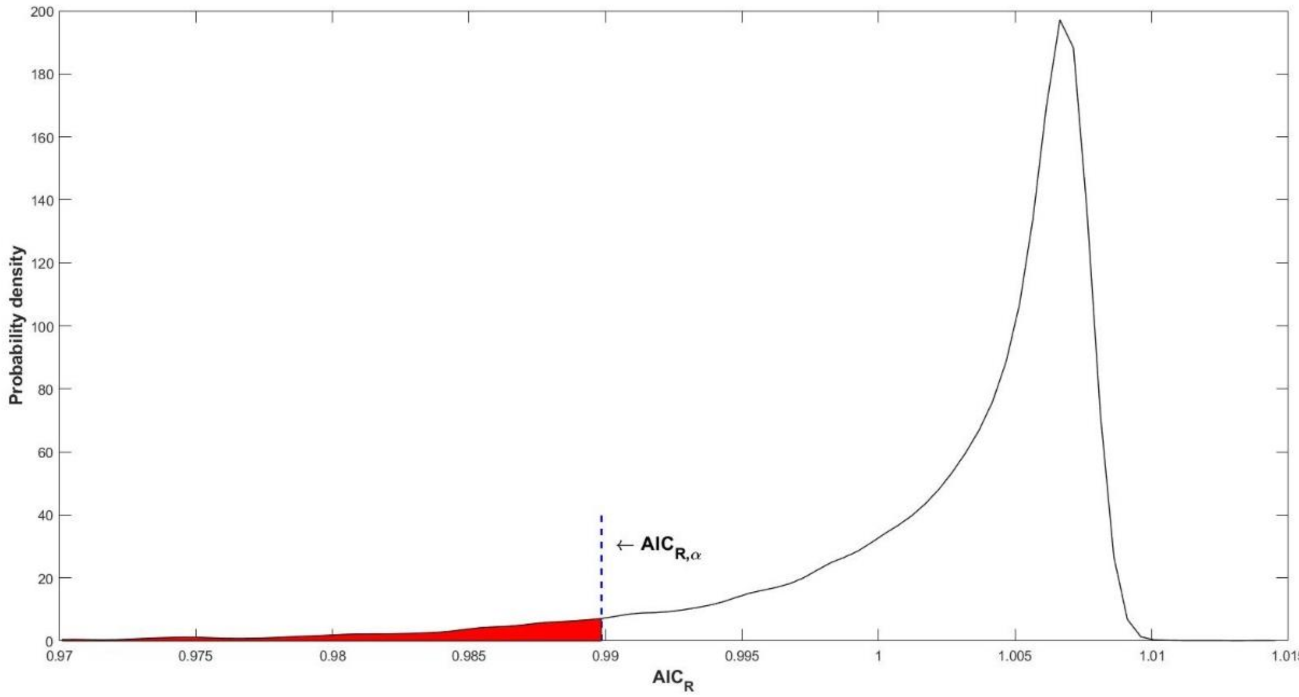
	L = 30								
	$\varepsilon = -0.4$			$\varepsilon = 0$			$\varepsilon = +0.4$		
	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$
MK	0.048	0.047	0.047	0.047	0.050	0.050	0.046	0.049	0.048
AIC_R	0.050	0.046	0.052	0.051	0.052	0.045	0.052	0.054	0.051
LR	0.104	0.103	0.115	0.061	0.064	0.060	0.084	0.081	0.083

	L = 50								
	$\varepsilon = -0.4$			$\varepsilon = 0$			$\varepsilon = +0.4$		
	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$
MK	0.050	0.047	0.046	0.044	0.047	0.050	0.049	0.044	0.048
AIC_R	0.053	0.053	0.046	0.051	0.051	0.057	0.050	0.050	0.053
LR	0.079	0.078	0.074	0.060	0.063	0.063	0.070	0.069	0.070

	L = 70								
	$\varepsilon = -0.4$			$\varepsilon = 0$			$\varepsilon = +0.4$		
	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$
MK	0.050	0.052	0.054	0.052	0.051	0.047	0.049	0.048	0.046
AIC_R	0.047	0.051	0.051	0.058	0.058	0.052	0.050	0.054	0.051
LR	0.069	0.069	0.073	0.063	0.065	0.058	0.062	0.062	0.063

Table 2: Actual level of significance of AIC_R test for $AIC_{R,\alpha} = 1$

	$\varepsilon = -0.4$			$\varepsilon = 0$			$\varepsilon = +0.4$		
	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$
L = 30	0.246	0.254	0.261	0.188	0.191	0.181	0.220	0.221	0.215
L = 50	0.213	0.209	0.206	0.171	0.175	0.170	0.188	0.207	0.195
L = 70	0.192	0.192	0.201	0.168	0.169	0.173	0.184	0.204	0.184


Probability density
AIC_R
← AIC_{R,α}
0
20
40
60
80
100
120
140
160
180
200
0.97
0.975
0.98
0.985
0.99
0.995
1
1.005
1.01
1.015

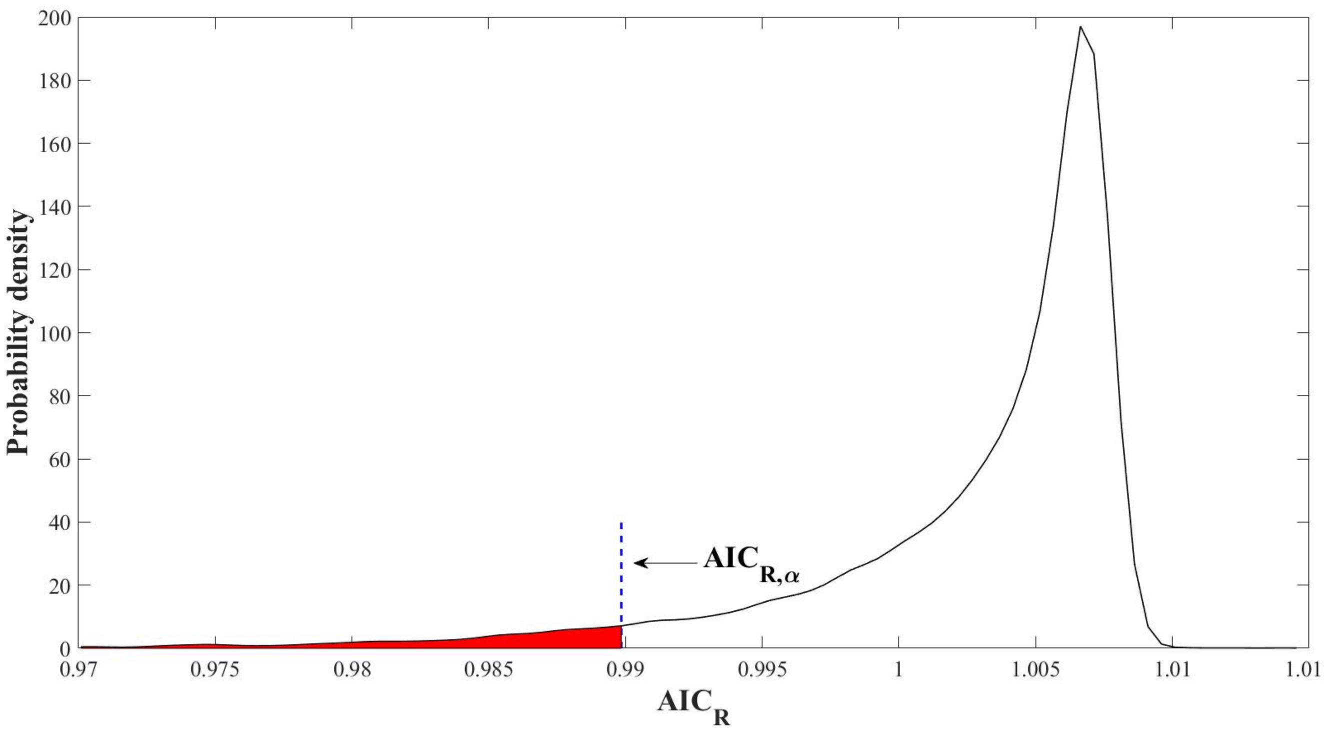
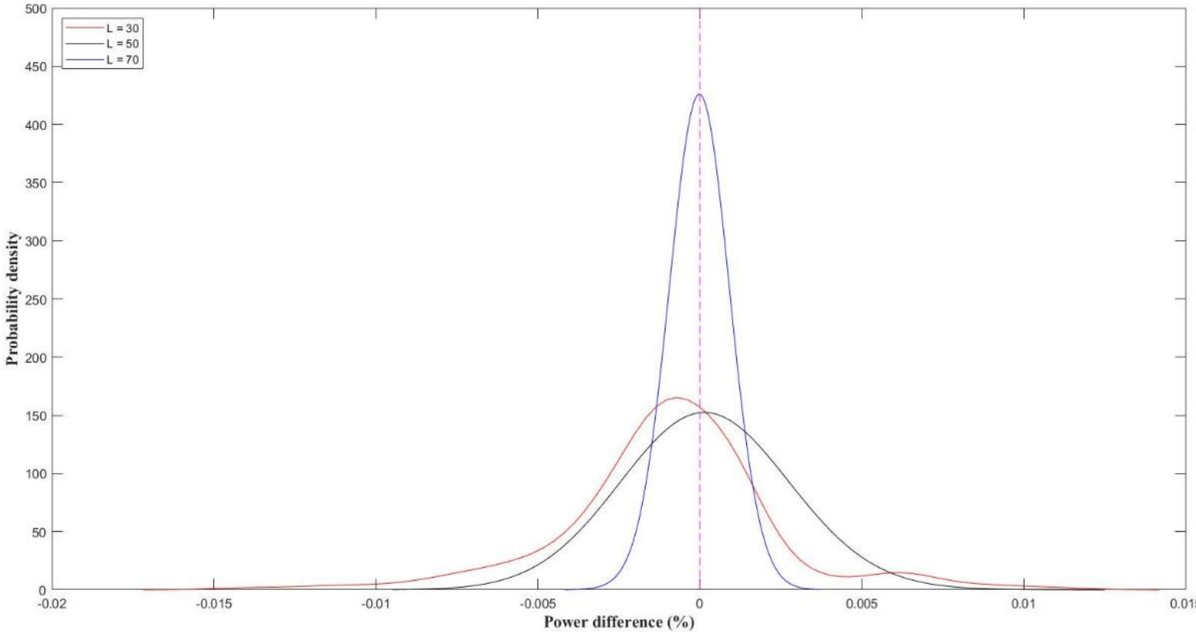

Probability density
AIC_R
AIC_{R,α}
0
20
40
60
80
100
120
140
160
180
200
0.97
0.975
0.98
0.985
0.99
0.995
1
1.005
1.01
1.015

Figure 1: Empirical distribution of AIC_R and rejection threshold $AIC_{R,\alpha}$ of the null hypothesis (stationary GEV parent)



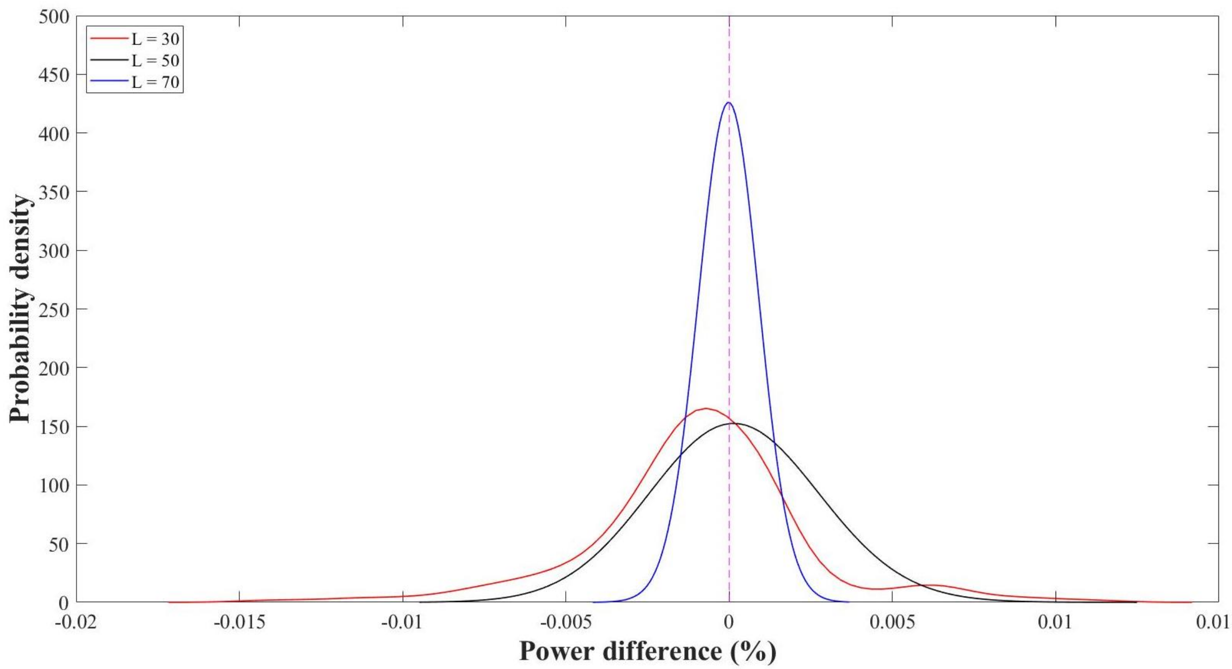
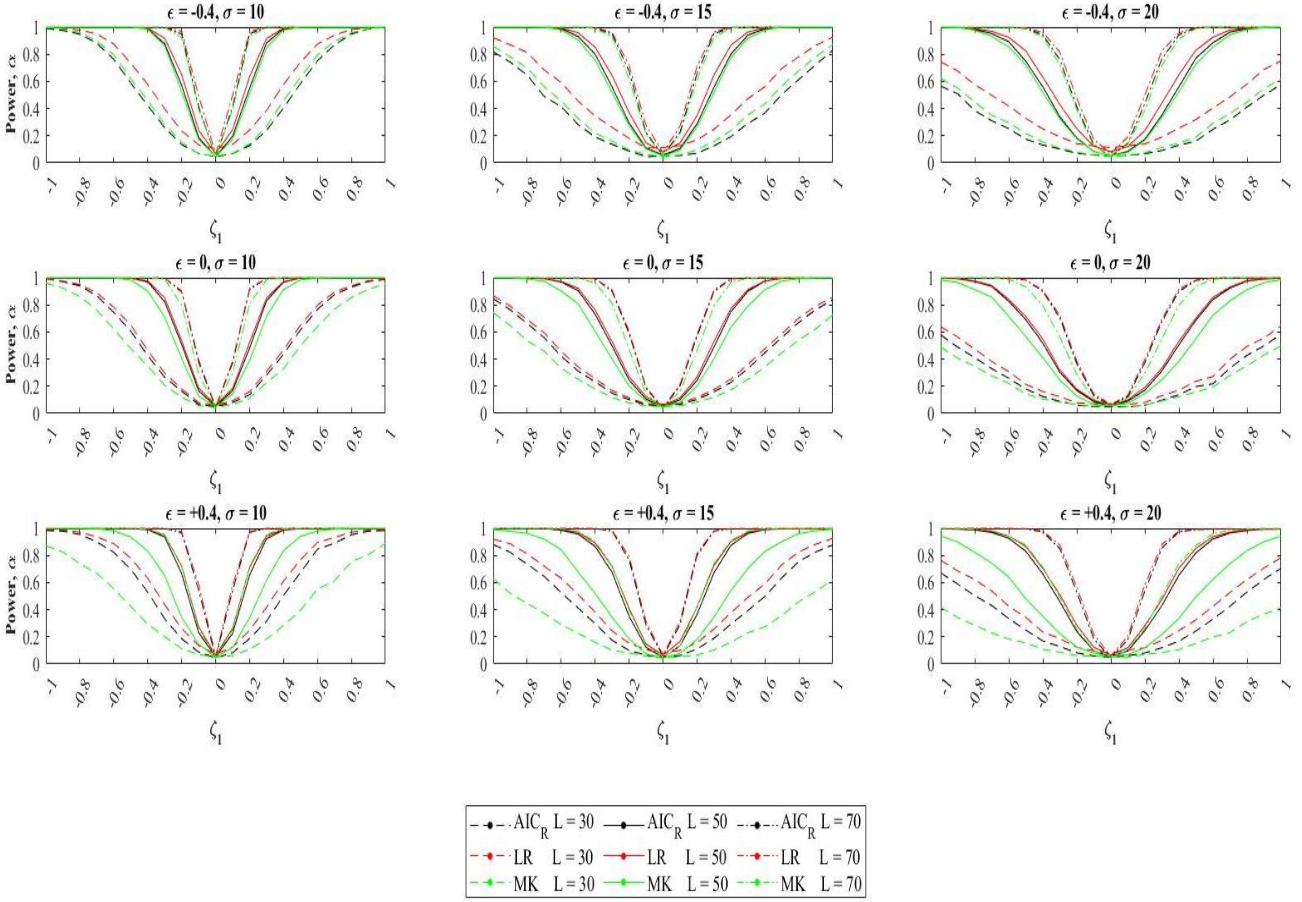


Figure 2: Distributions of the differences between power of AIC_R evaluated with AIC and AIC_c for $\varepsilon = 0.4$


ϵ = -0.4, σ = 10
ϵ = -0.4, σ = 15
ϵ = -0.4, σ = 20
ϵ = 0, σ = 10
ϵ = 0, σ = 15
ϵ = 0, σ = 20
ϵ = +0.4, σ = 10
ϵ = +0.4, σ = 15
ϵ = +0.4, σ = 20
Power, α
ζ1
AIC_R L = 30
AIC_R L = 50
AIC_R L = 70
LR L = 30
LR L = 50
LR L = 70
MK L = 30
MK L = 50
MK L = 70

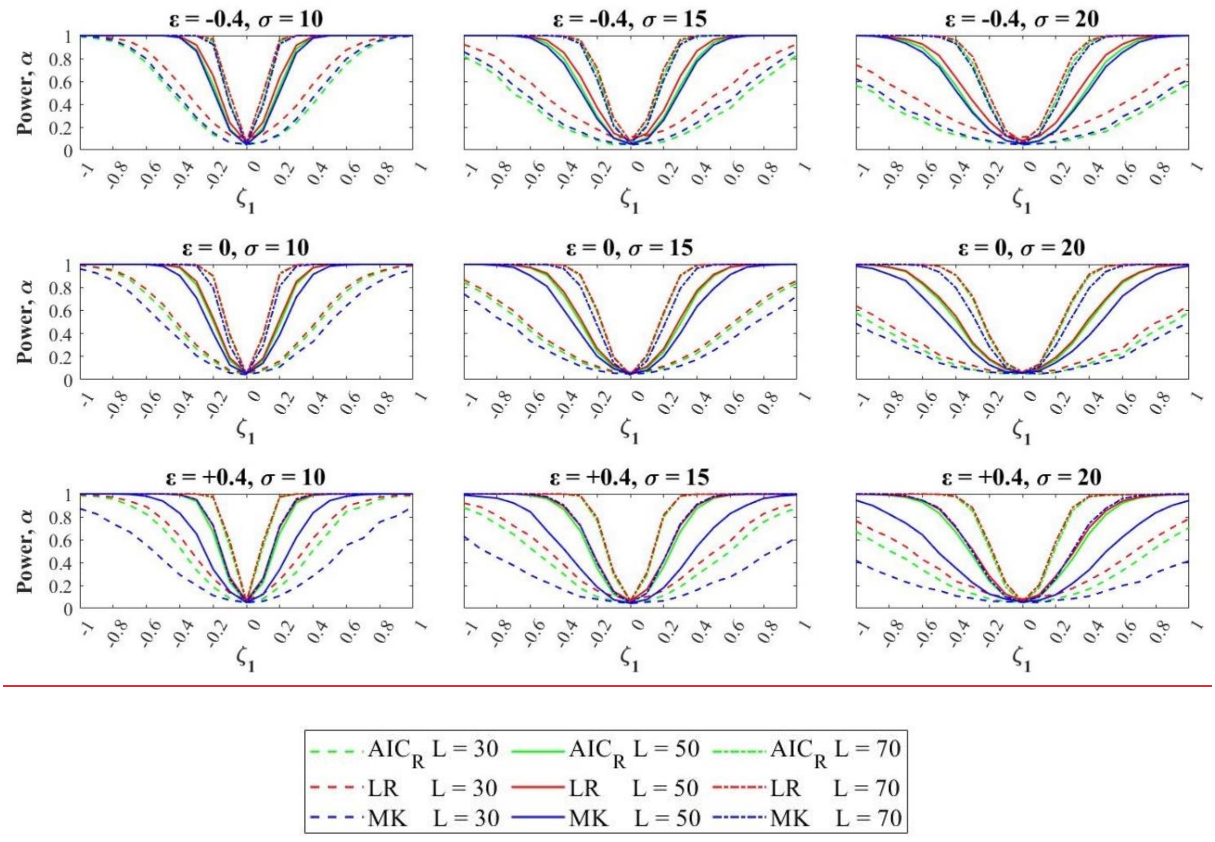
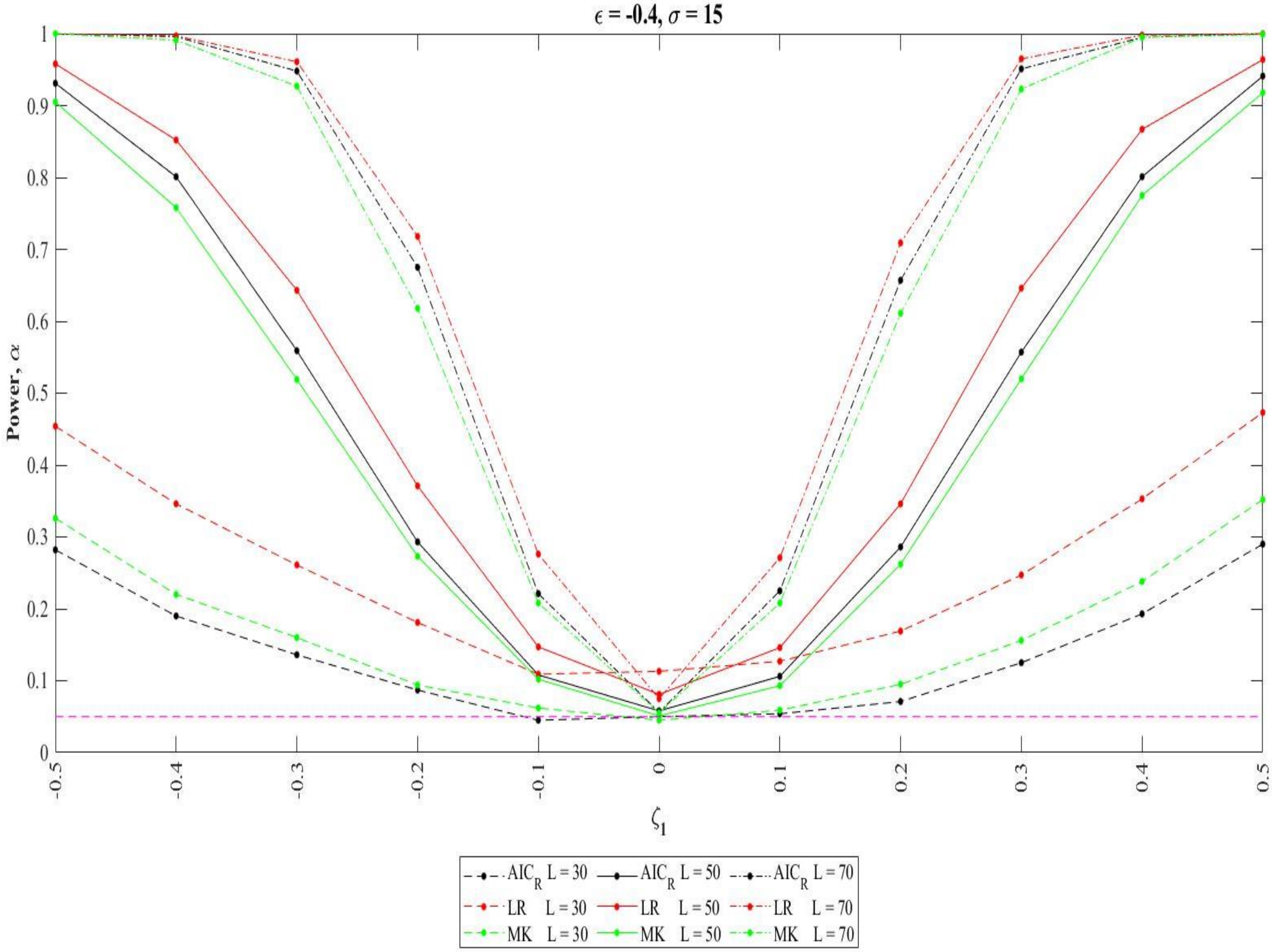


Figure 3: Dependence of test power on trend coefficient, sample size, scale and shape parent parameters.


ϵ = -0.4, σ = 15
Power, α
ζ1
AIC_R L = 30
AIC_R L = 50
AIC_R L = 70
LR L = 30
LR L = 50
LR L = 70
MK L = 30
MK L = 50
MK L = 70

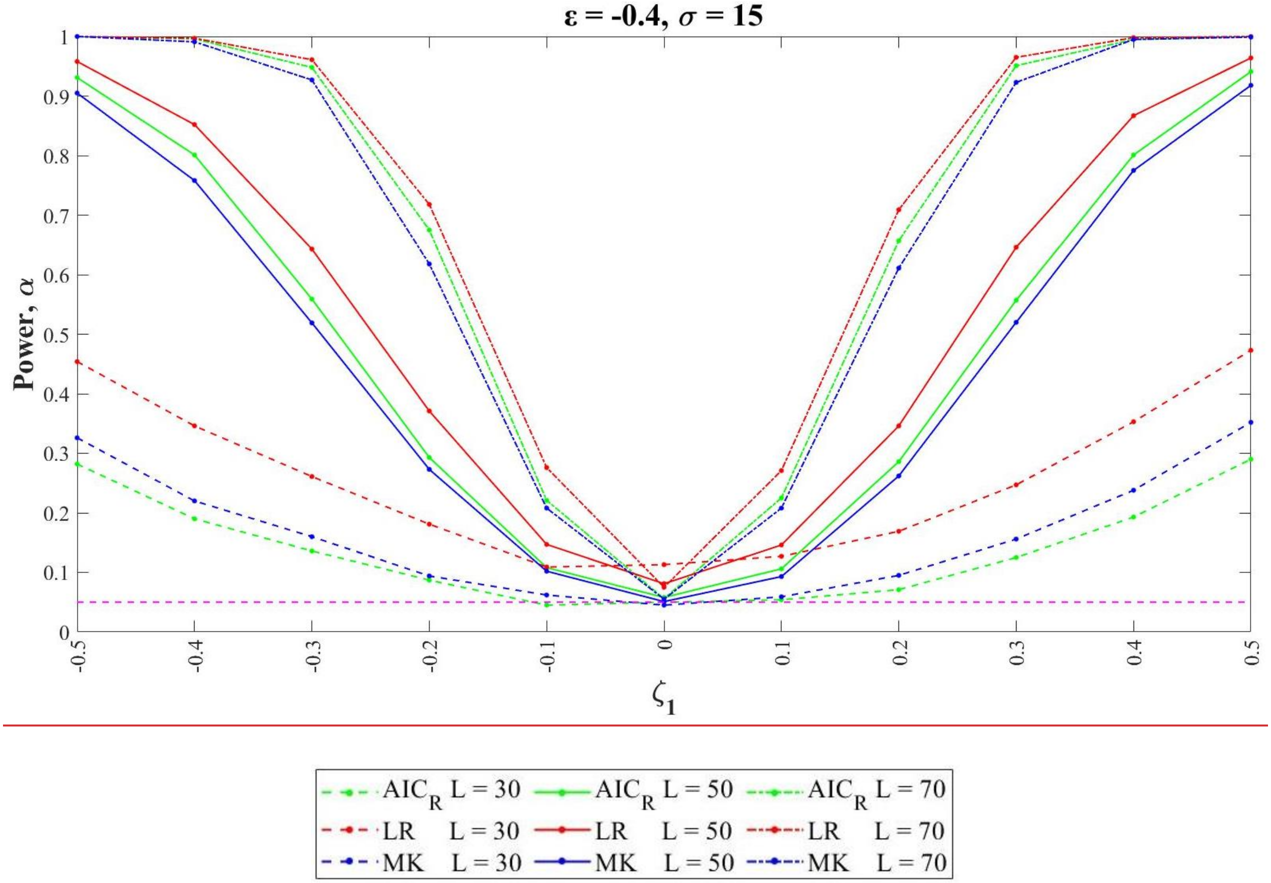
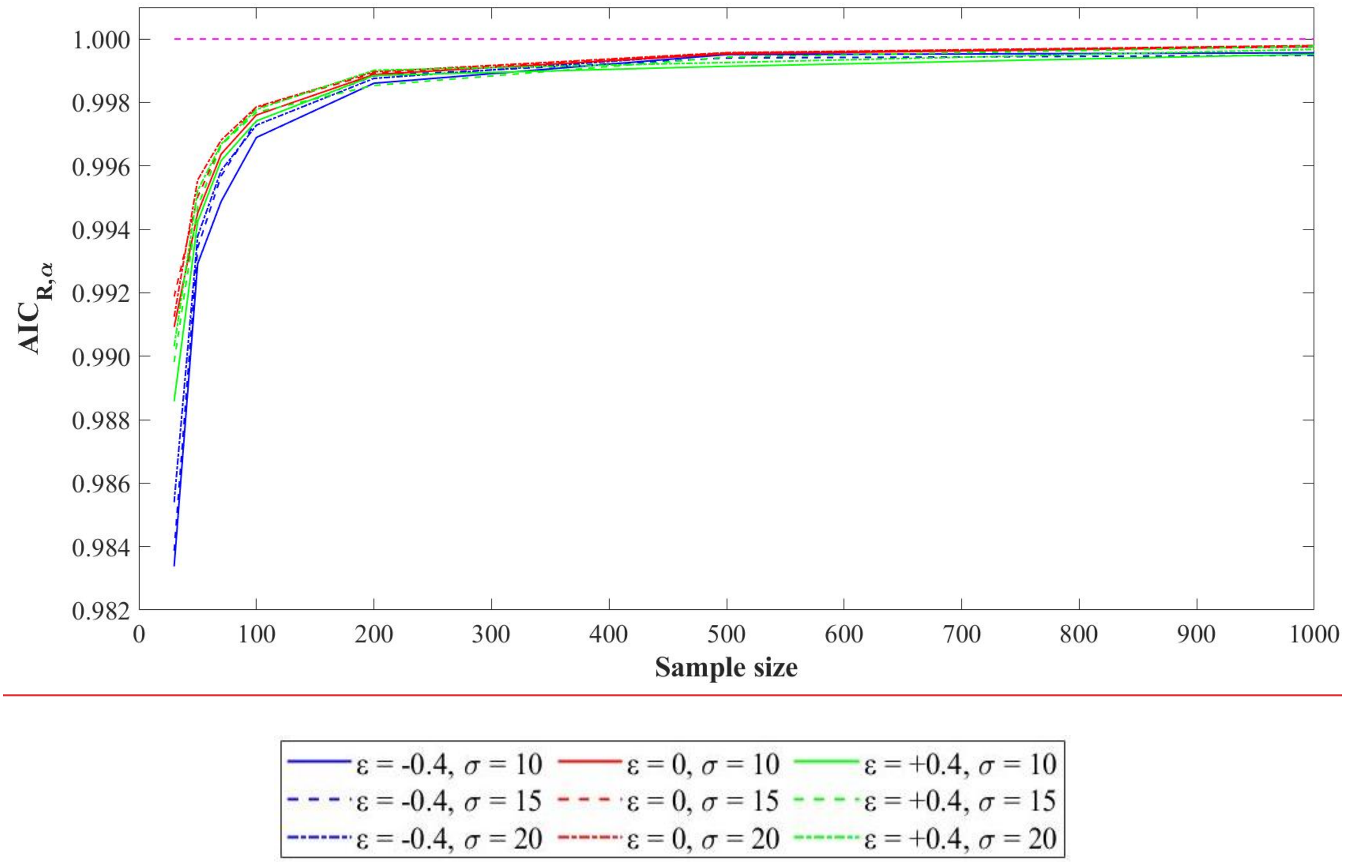


Figure 4: Focus on the actual level of significance reported for $\zeta_1 = 0$.



1.000
0.998
0.996
0.994
0.992
0.990
0.988
0.986
0.984
0.982
$AIC_{R,\alpha}$
0
100
200
300
400
500
600
700
800
900
1000
Sample size
ε = -0.4, σ = 10
ε = 0, σ = 10
ε = +0.4, σ = 10
ε = -0.4, σ = 15
ε = 0, σ = 15
ε = +0.4, σ = 15
ε = -0.4, σ = 20
ε = 0, σ = 20
ε = +0.4, σ = 20

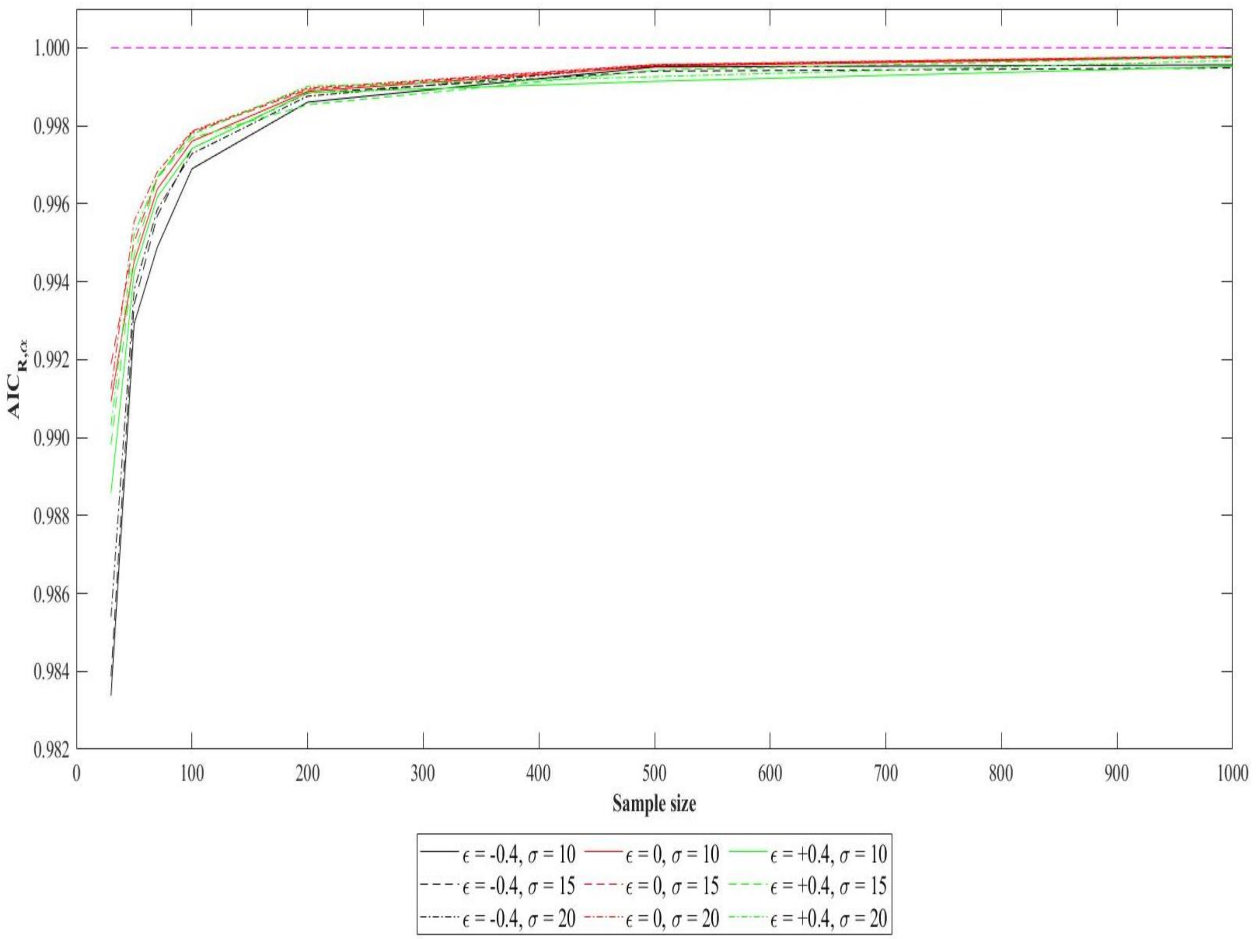
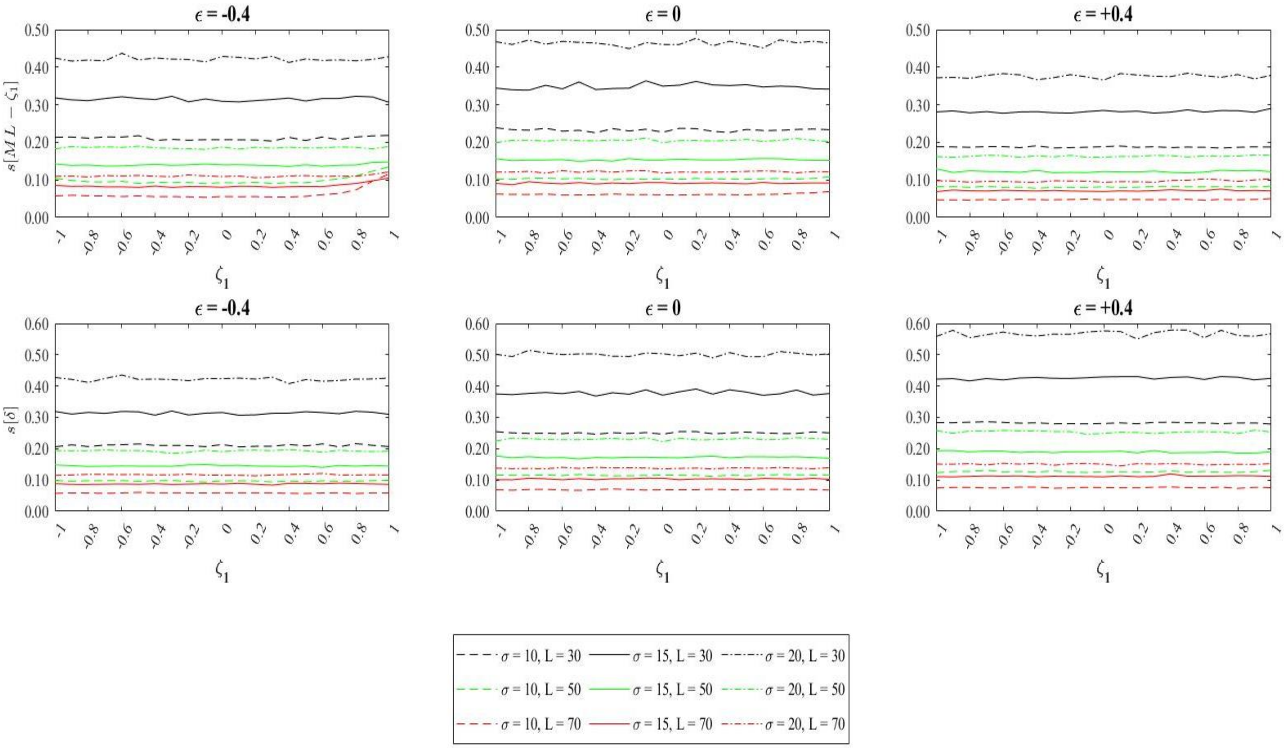


Figure 5: $AIC_{R,\alpha}$ **thresholds for different parameter sets vs sample size**



ϵ = -0.4
ϵ = 0
ϵ = +0.4
s[ML − ζ1]
s[δ]
ζ1
σ = 10, L = 30
σ = 15, L = 30
σ = 20, L = 30
σ = 10, L = 50
σ = 15, L = 50
σ = 20, L = 50
σ = 10, L = 70
σ = 15, L = 70
σ = 20, L = 70

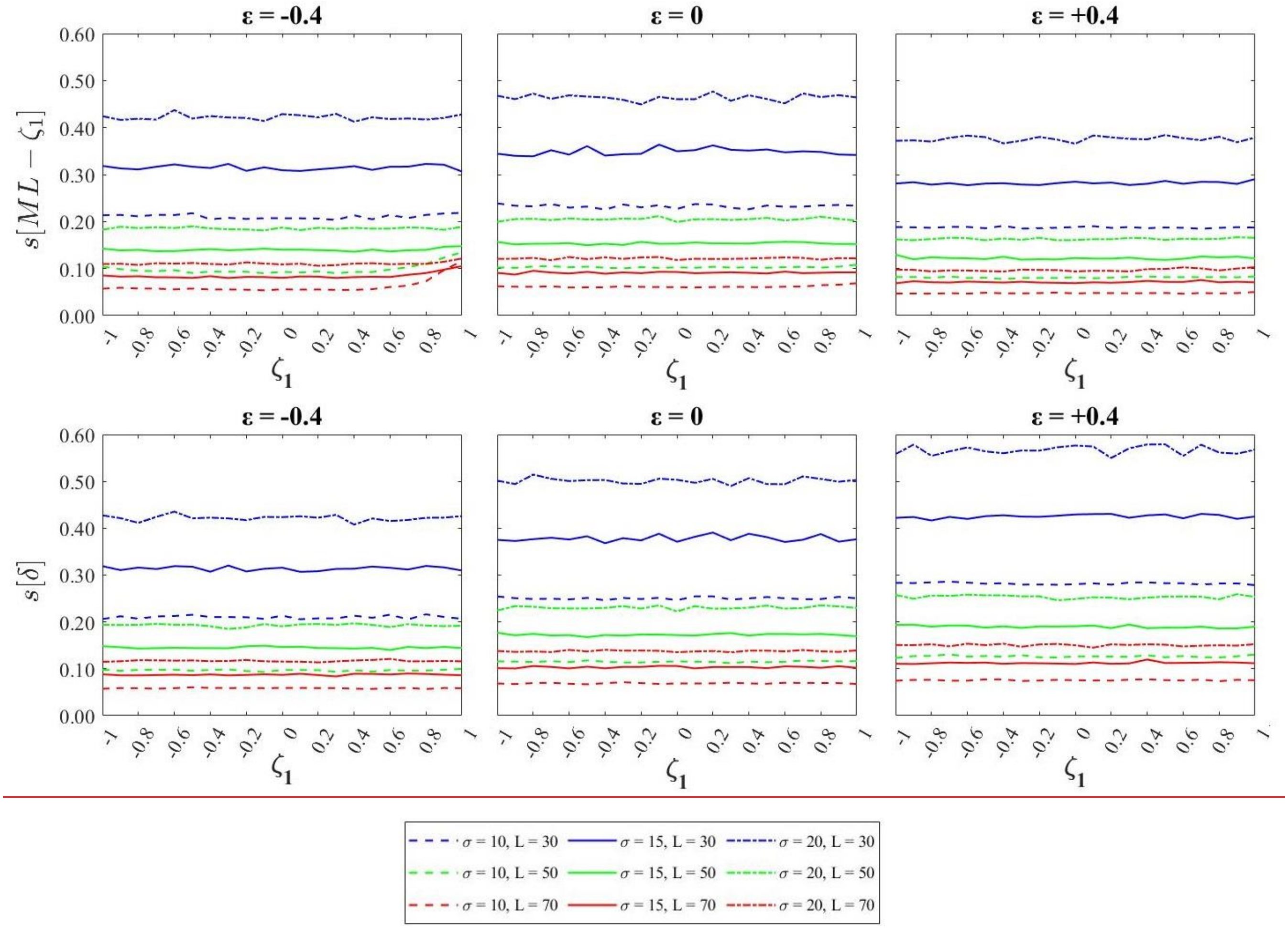
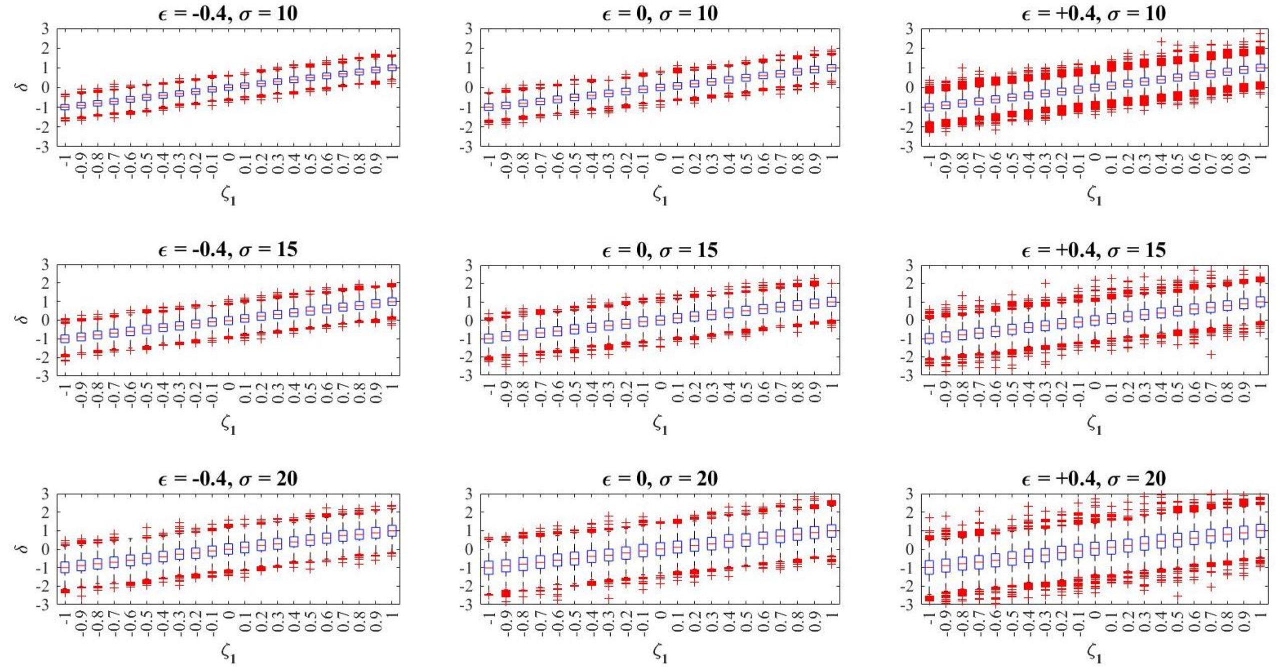
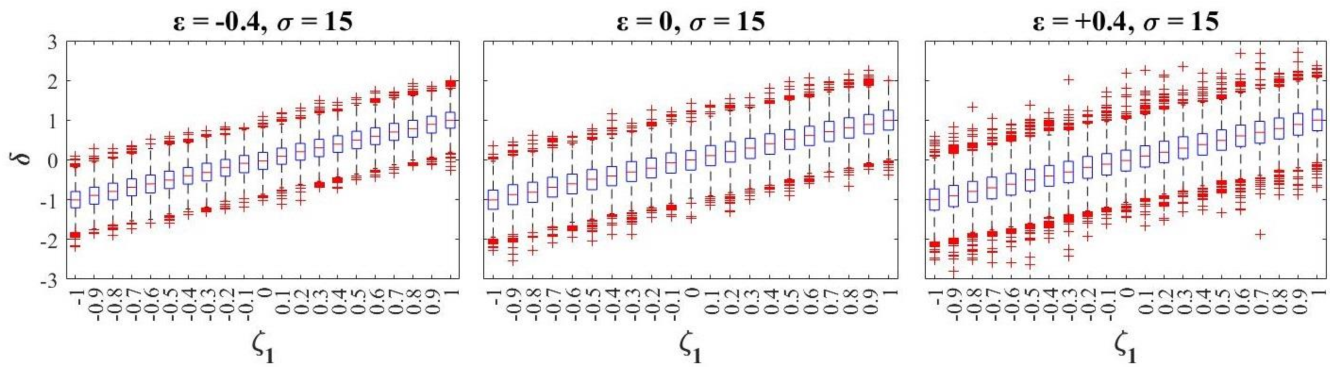


Figure 6: Sample variability of ML-ζ_1 and δ vs trend coefficient ζ_1


ϵ = -0.4, σ = 10
ϵ = 0, σ = 10
ϵ = +0.4, σ = 10
ϵ = -0.4, σ = 15
ϵ = 0, σ = 15
ϵ = +0.4, σ = 15
ϵ = -0.4, σ = 20
ϵ = 0, σ = 20
ϵ = +0.4, σ = 20
δ
ζ1


ε = -0.4, σ = 15
ε = 0, σ = 15
ε = +0.4, σ = 15
δ
ζ1

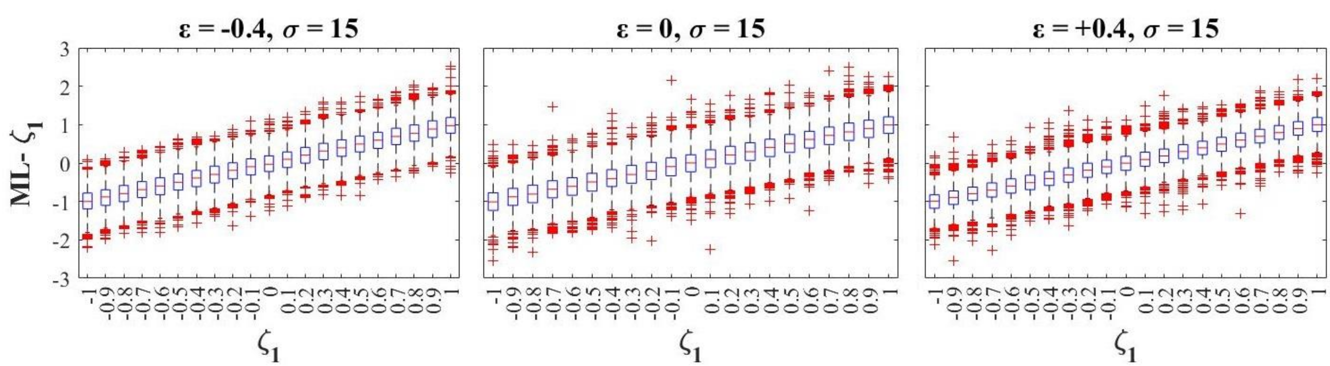
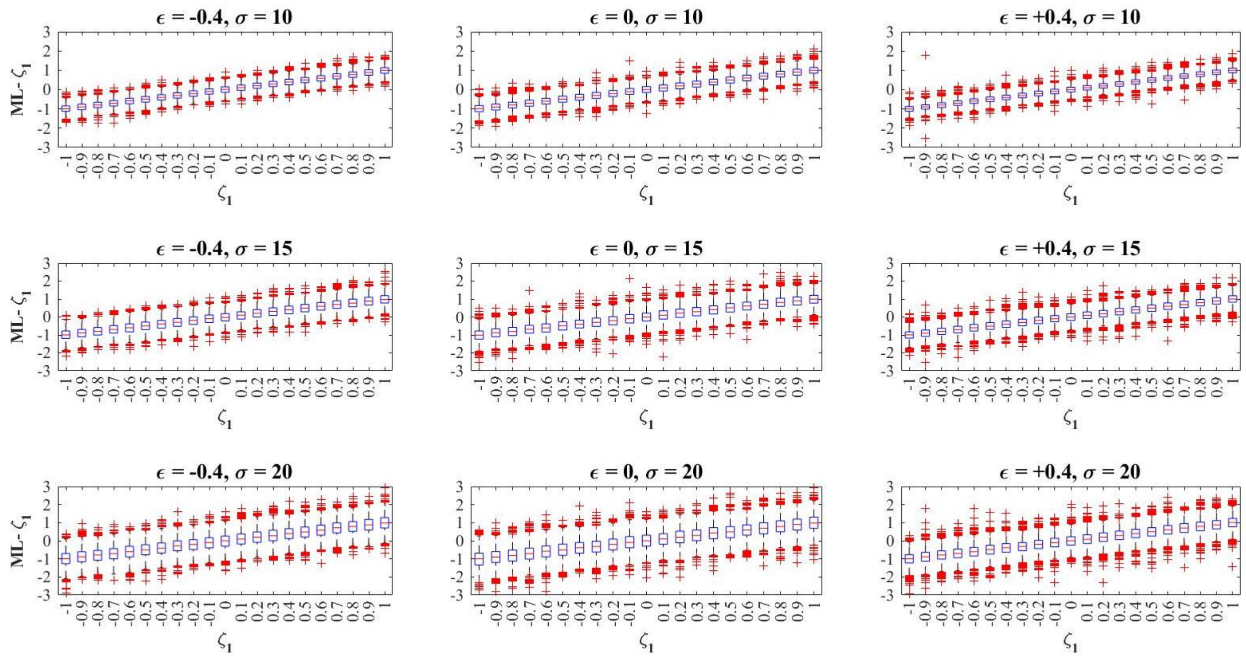
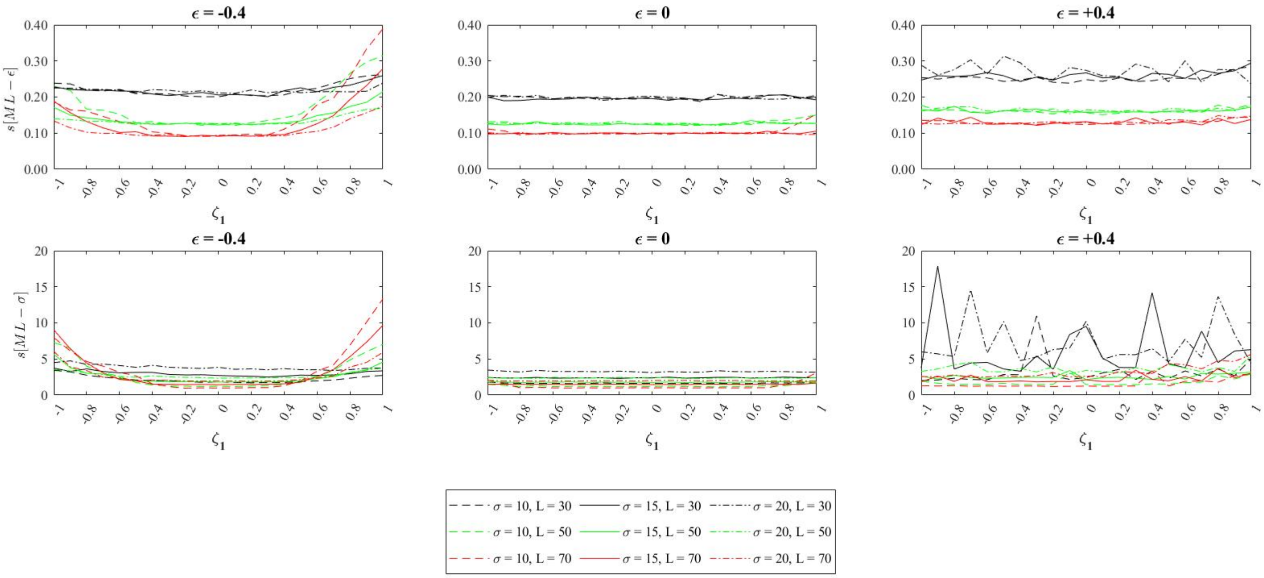
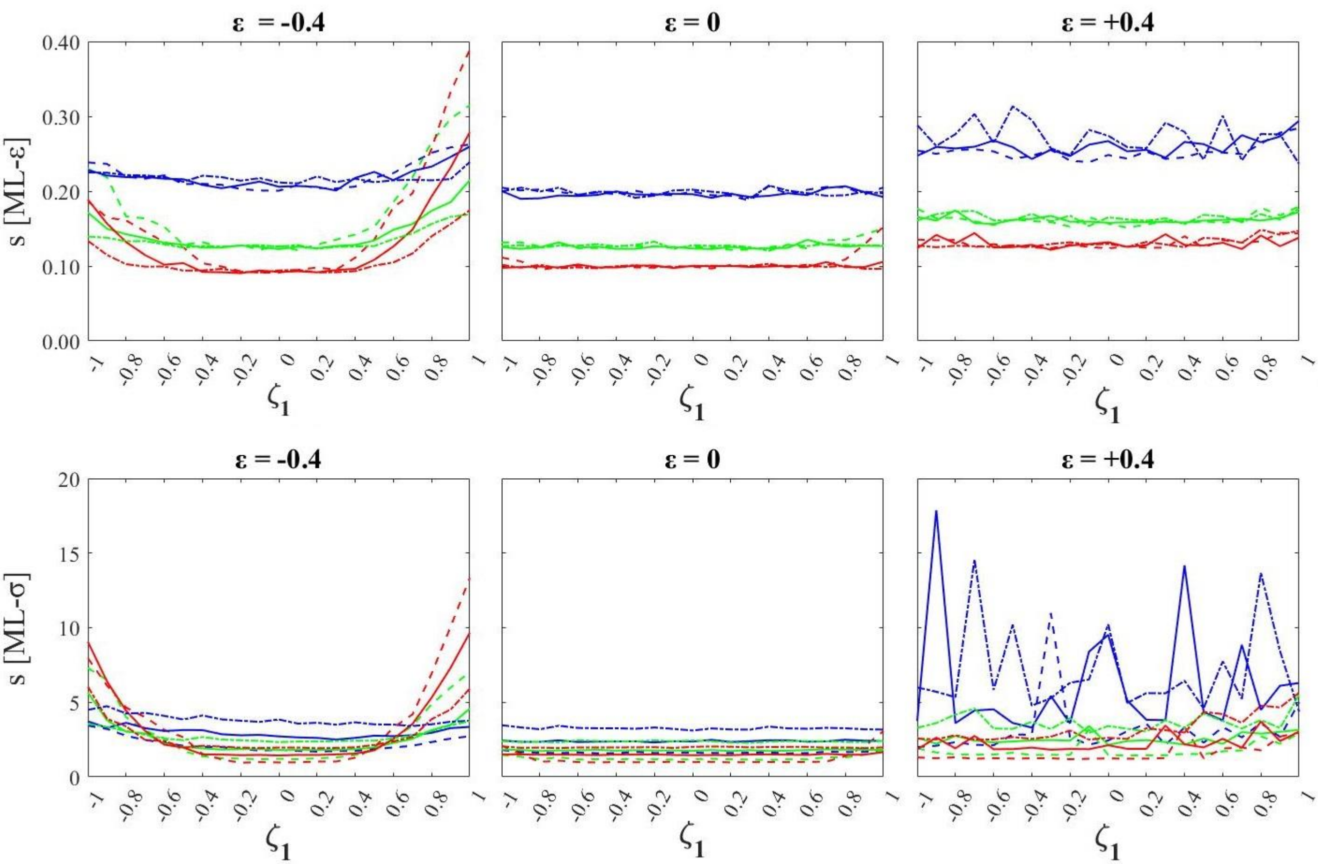

ε = -0.4, σ = 15
ε = 0, σ = 15
ε = +0.4, σ = 15
ML- ζ1
ζ1

Figure 7: Empirical distributions of δ <u>and ML-ζ_1</u> evaluated from samples with $L = 30$ and $\sigma = 15$ vs trend coefficient ζ_1



~~**Figure 8: Empirical distributions of ML-ζ_1 evaluated from samples with $L = 30$ vs trend coefficient ζ_1**~~


ϵ = -0.4
ϵ = 0
ϵ = +0.4
s[ML − ϵ]
s[ML − σ]
ζ1
σ = 10, L = 30
σ = 15, L = 30
σ = 20, L = 30
σ = 10, L = 50
σ = 15, L = 50
σ = 20, L = 50
σ = 10, L = 70
σ = 15, L = 70
σ = 20, L = 70


ε = -0.4
ε = 0
ε = +0.4
s [ML-ε]
s [ML-σ]
ζ1

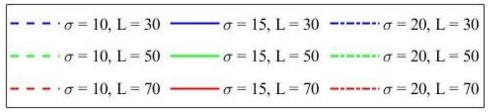
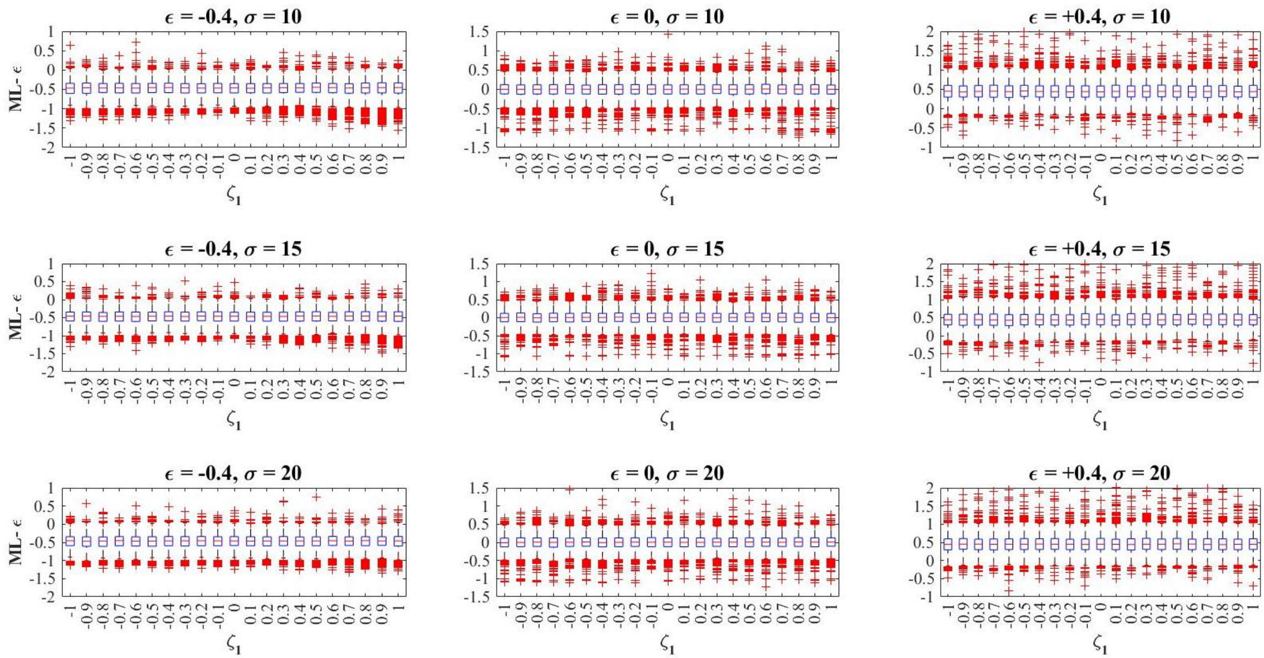


Figure 89: Sample variability of ML-ε and ML-σ vs trend coefficient ζ_1



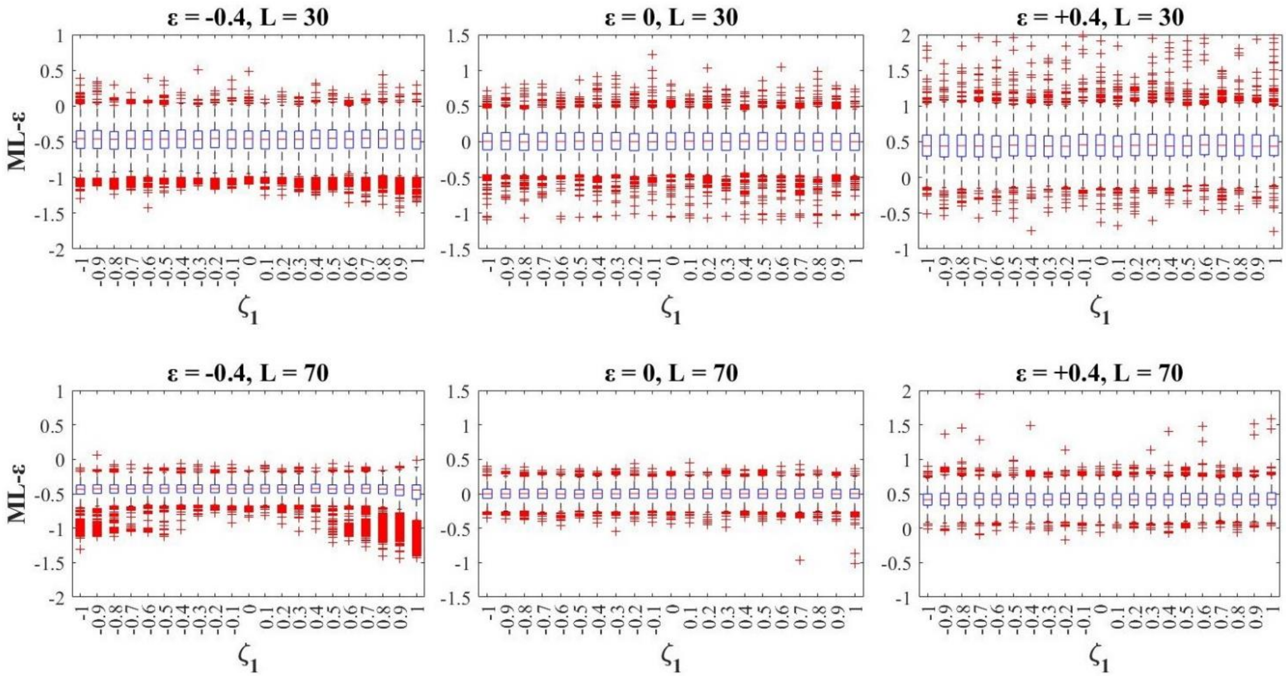
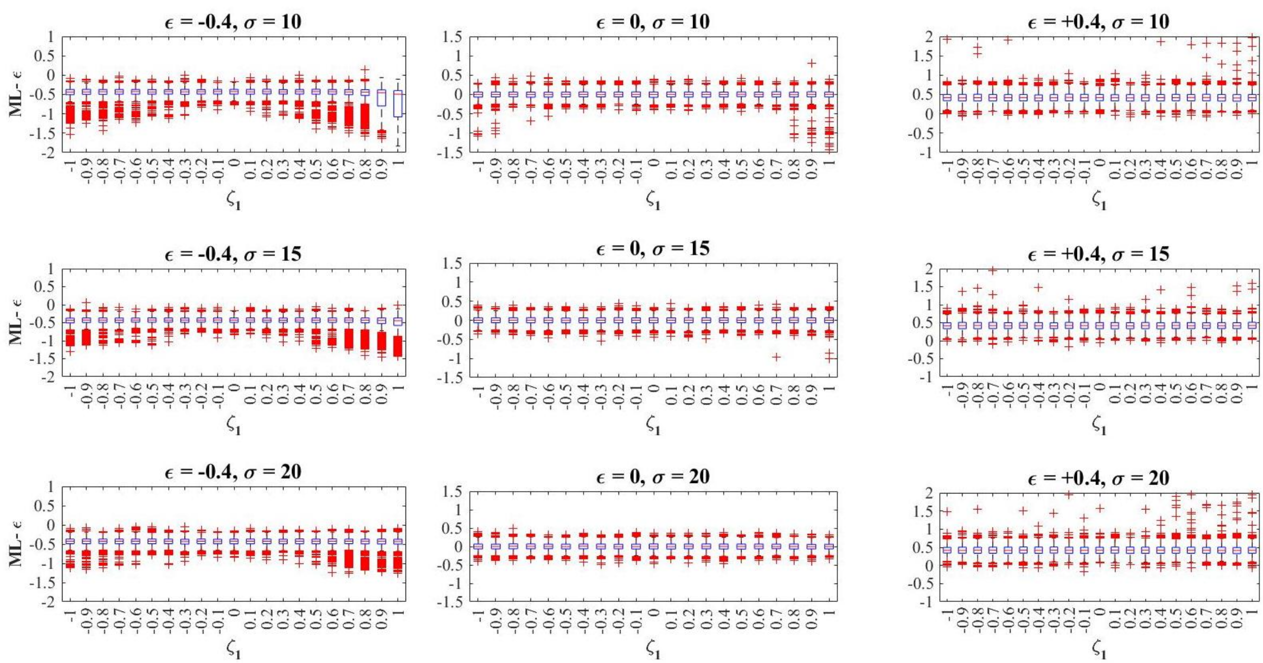


Figure 9~~10~~: Empirical distributions of ML-ε evaluated for $\sigma = 15$ from samples with $L = 30$ and $L = 70$ vs trend coefficient ζ_1



~~**Figure 11: Empirical distributions of ML-ε evaluated from samples with $L = 70$ vs trend coefficient ζ_1**~~

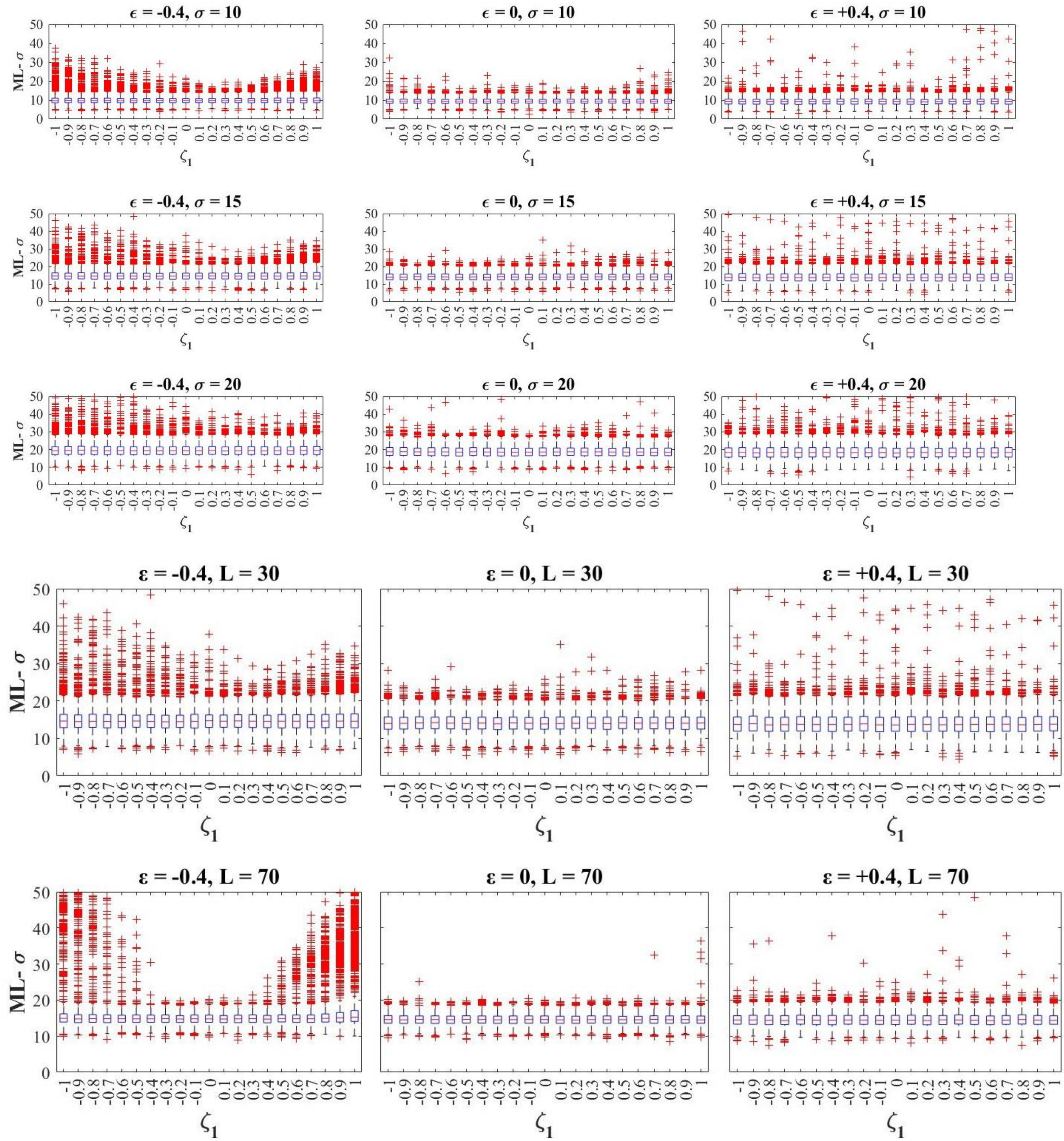


Figure ~~12~~10: Empirical distributions of ML-σ evaluated for $\sigma = 15$ from samples with $L = 30$ and $L = 70$ vs trend coefficient ζ_1

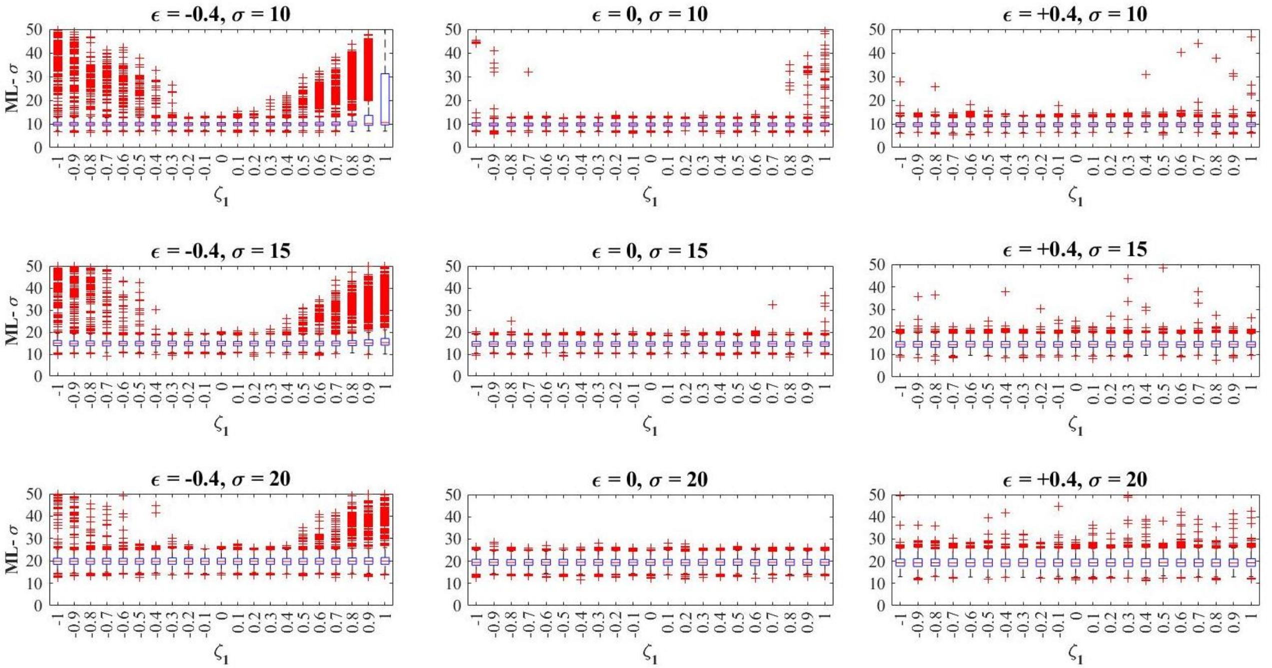


Figure 13: Empirical distribution of ML-σ evaluated from samples with $L = 70$