

Dear Colleague,

Thank you for your review. Please find below a detailed response to the points you raised:

- **Although the topic is relevant to the current literature and well within the scope of the journal, I am struggling to understand what the exact nature of the problem is, and how and why the proposed work represents an improvement of the current knowledge about the research topic.**

In this technical note, our purpose was to present as factually as possible a new mathematical representation for the concentration-discharge relationship. We chose the technical note format because we thought it was more adapted to a paper, which only aimed at discussing a mathematical formulation. We hope that the answers brought below will help you better understand our purpose. In any case, we will use your comments to improve our manuscript.

- **The literature review presented in the introduction section (rather rushed) does not at all bring the reader to the idea that a different data transformation, beyond the log-log transformation, is desirable by the scientific community for the representation of Q-C relationship and for what reason it should be.**

Because this is a technical note, we tried to go straight to the point in the introduction; this is probably where the impression of “rushed” comes from. You are perfectly right to mention that there has been (to our knowledge) no direct critic of the log-log transformation in the literature. However, in papers like that of Moatar (2017), the variety of shapes is a clear indication that the log-log transformation lacks generality (we will add this point in the revised manuscript). In a recent paper of the same group, Minaudo et al. (2019) mention that “*fitting a single linear regression on C-Q plots is sometimes questionable due to large dispersion in C-Q plots (even log transformed)*”. We also believe that the recent advent of high-frequency time series allows better scrutinizing the C-Q relationships (cf. l. 44-46).

- **The motivation provided in the “About the excess of log-log transformation” section (the change of the shape of the Q-C relation) is quite weak. Authors probably reach an interesting point/motivation when they introduce (line 111) the problem of the representation of high flow discharge data concentration that arises for high frequency database but, surprisingly, when they come to the results, they mention the difficulties of the model to reproduce this type of data (line 151). But many more questions come about the scientific idea.**

Because our new mathematical representation comes to address a problem unsolved by the log-log transformation, we thought that we had to discuss the shortcomings of the log-log transformation. We chose a graphical demonstration (Figure 2) to show that data do not “line up” after transformation. We will make this point clearer in the revised manuscript.

- **Why authors choose the Box-Cox transformation? Aren't there alternative? If they do not compare the performance of the log-log transformation with the proposed two-sided power transformation, how can the reader guess it is an improved representation? How is the improvement demonstrated by the authors?**

We used the Box-Cox transformation (the two-sided power transformation) because (i) it has the requested flexibility and converges towards the classical log-log transformation for high n values, (ii) from our point of view it is the simplest alternative to the one-sided power transformation, (iii) it is almost universally known in the field of statistics and time series analysis. We will make this point clearer in the revised manuscript.

We agree that we did not provide an exhaustive numerical evidence of the superiority of the Box-Cox transformation (we only showed how the coefficient of determination is improved in Table 1). Below, we added in Table 1 a column computing the RMSE of prediction:

Table 1: Coefficient of determination (R^2) and RMSE calculated for $n = 1$ (no transformation), $n =$ optimal value for Box-Cox transformation (Figure 4) and $n \rightarrow \infty$ (log-log transformation) for each ion and for EC. Note that while the R^2 is computed between transformed values, the RMSE is computed between untransformed values.

ion	n	R^2	RMSE
Sodium	$n = 1$ (no transformation)	0.54	1.11 mgL ⁻¹
	$n = 3$ (optimal)	0.73	0.97 mgL ⁻¹
	$n \rightarrow \infty$ (log-log)	0.53	1.22 mgL ⁻¹
Sulfate	$n = 1$ (no transformation)	0.32	3.06 SmgL ⁻¹
	$n = 5$ (optimal)	0.81	2.00 SmgL ⁻¹
	$n \rightarrow \infty$ (log-log)	0.77	2.21 SmgL ⁻¹
Chloride	$n = 1$ (no transformation)	0.52	3.34 mgL ⁻¹
	$n = 3$ (optimal)	0.88	1.92 mgL ⁻¹
	$n \rightarrow \infty$ (log-log)	0.69	2.91 mgL ⁻¹
EC	$n = 1$ (no transformation)	0.38	60.02 μ Scm ⁻¹
	$n = 5$ (optimal)	0.79	37.26 μ Scm ⁻¹
	$n \rightarrow \infty$ (log-log)	0.74	41.32 μ Scm ⁻¹

- **1) The introduction section is quite rushed and presents a figure published elsewhere by other authors. Generally figures are not included in the introduction but if needed why do not use authors own data?**

Our aim in the introduction was to illustrate the fact that the log-log transformation could sometimes be well-adapted, but we found no such case in our own dataset. We understand that this can be misleading for the reader. In the revised paper, we will remove this figure and replace it by a citation.

- **2) Figures frequently do not indicate neither the range of variability of the data not the unit of measurements**

We had removed the unit of measurements and the range of variability in order to focus on the shape of the scatterplot. We understand that it can be misleading for the reader and we will replace them in the revised paper.

- **3) The dataset used for the analysis is not clearly presented**

Because a technical note should be very short, we had kept the dataset description as short as possible. We will extend its description in the revised paper.

- **4) Figure 3: to which a and b parameters does it correspond?**

Figure 3 only shows the data (transformed and untransformed), and a and b refer to the fitted model. Their values will depend on the objective function(s) chosen for fitting; this is why we cannot mention them at this point (they will be given in Table 3).

- **5) Figure 4: how can I judge by visual inspection that black dots represent the best performing transformation if I do not know about the empirical relationship (figure 4 presents the model)?**

We mention in lines 85-86 that “The optimal shape could be chosen numerically (see Table 1), but we first followed the 86 advice of Box et al. (2016, p. 331) and did it visually”. Figure 3 provides a graphical illustration, while Table 1 provides the numerical demonstration.

- **6) Table 1: for sulfate and EC (half of the database) the coefficient of determination for $n = 5$ (optimal) and $n = \sqrt{C \cdot e}$ (log-log) is almost the same. What the improvement is?**

See our response above (updated table 1 with RMSE).

- **7) Comparison between observation and model only appear at the very end (figure 6) but no comparison is provided with the log-log transformation (where the improvement is?).**

We now have a numerical comparison (RMSE) in Table 1 and we will modify Figure 6 in the revised paper, showing the comparison between the Box-Cox transformation and the log-log transformation (see below).

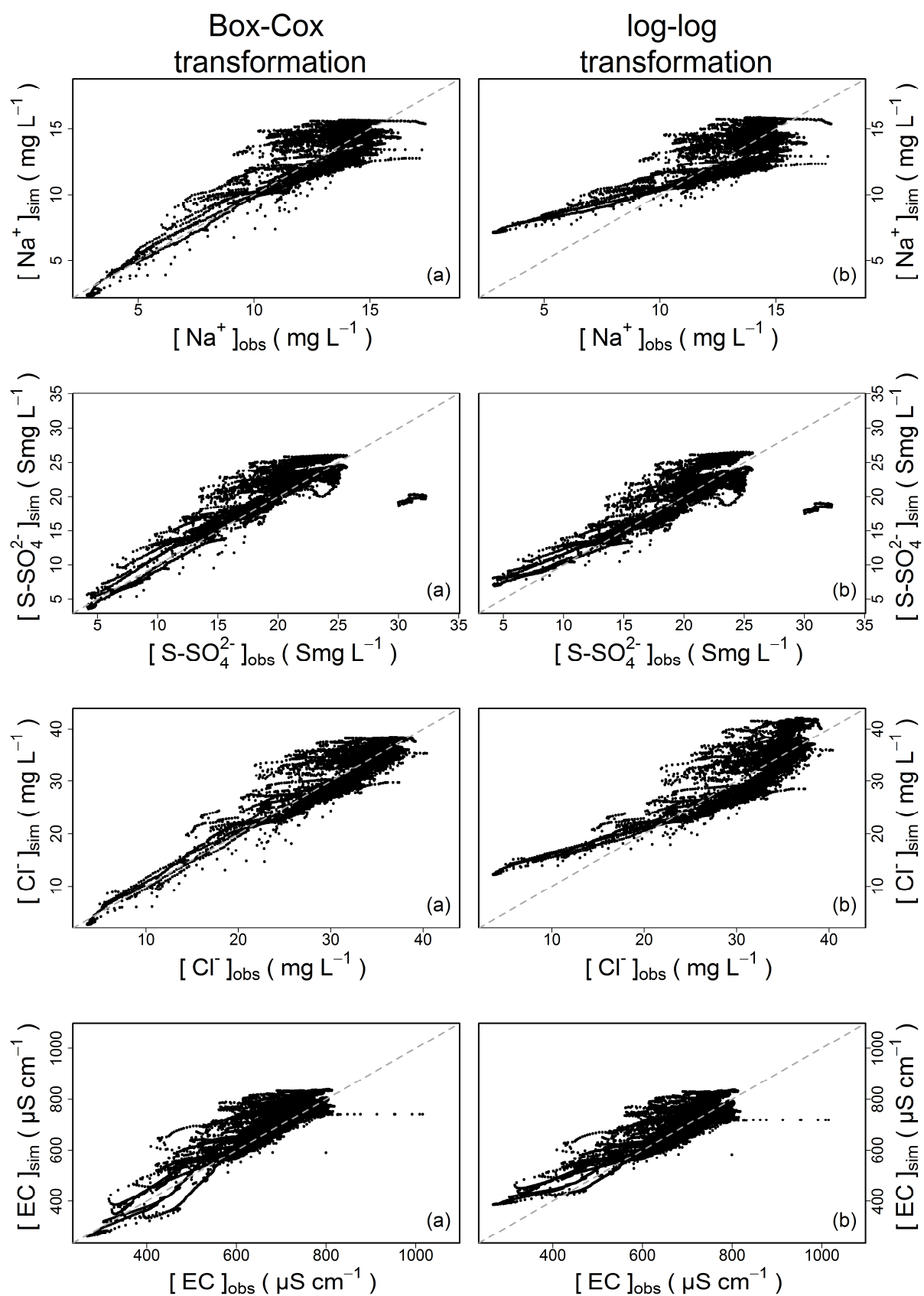


Figure 6: Comparison of observed concentrations with simulated concentrations by: (a) Box-Cox transformation, (b) log-log transformation

References

- Minaudo, C. et al., 2019. Seasonal and event-based concentration-discharge relationships to identify catchment controls on nutrient export regimes. *Advances in Water Resources*, 131: 103379.
- Moatar, F., Abbott, B., Minaudo, C., Curie, F., and Pinay, G.: Elemental properties, hydrology, and 228 biology interact to shape concentration-discharge curves for carbon, nutrients, sediment, and major 229 ions, *Water Resources Research*, 53, 1270-1287, 2017.