

Response to Anonymous Referee #1

The authors aim to define a new Budyko-type relationship (it seems) by using an Ohm's approach. The article has a very strange structure. There is really no justification on the knowledge gap for the research. The literature review is rather "old" and convoluted, mentioning relevant but rather old formulations. Much has happened in the Budyko field after the Zhang, Fu and MCY formulations. Budyko-type empirical models of ET. I couldn't even find in the text what is the Ohm's law or approach that the authors are discussing. In the beginning, it sounded interesting to be able to include moisture recycling into the Budyko framework, by using a Lagrangian particle tracking method, but then, in Page 4 to 6 I just got lost. The manuscript is made up of disconnected parts without a clear thread. You end up with another expression of E in terms of P and E_0 before the discussion, the MCY formulation. Is this suppose to be the main result, that your formulations ends up in the same expression? I think the article goes into too much detail, and loses the big picture. You are giving mathematical explanations and derivations from the beginning, but I don't really understand what is the research question of this manuscript. Which are the objectives? What is the hypothesis? I am not saying that the new formulation is wrong, but rather what is the purpose of it? It might be interesting what you state, but the relevance or how it contributes to the field is not evident.

Response:

We thank the reviewer for taking the time to review our manuscript and for the invaluable comments.

As for the knowledge gap for this research, we think it is that between the mathematical equation and hydrological mechanism and stated it both in the abstract and introduction, i.e. "few hydrological processes were involved in the derivation". To our knowledge, the Fu and MCY formulations have been considered as the analytical ones and have been the most widely used ones, and their equivalence has been proved. Therefore, we focused on the MCY formulation. Previous derivation is filled with mathematical reasoning and the dimensional analysis. There was so few hydrological process involved in previous derivation, except the boundary conditions, that we tried to link it with more hydrological meaning. Although recent studies tried to derive the mean annual water-energy balance based on physical assumptions such as the principle of maximum entropy production and Carnot Limit, as reviewed in our introduction, their results include several not easily accessible and vague parameters, which leads to that they were hardly applied. Therefore, the purpose of our manuscript was stated in the abstract and the introduction: to fill in the gap between the derivation of MCY equation and hydrological processes.

As for the "old and convoluted literature review", we agree with the reviewer that "much has happened in the Budyko field after the Zhang, Fu and MCY formulations", but the Fu and MCY formulations were considered as the analytical ones and have been the most widely used ones, and their equivalence has been proved. In fact, we tried to derive the mean annual water-energy balance base

on the Ohm's balance and obtained a formulation same to the MCY equation. Therefore, we focused on the MCY formulation and gave a detail review on it.

Regarding "mentioning relevant but rather old formulations", this paper aims at the **derivation of mean annual water-energy balance**, so we have to focus on the "old and convoluted" equations before Zhang, Fu and MCY formulations. By tracking those equations, we get much more understanding of the derivation.

As for the Ohm's law, I am sorry that we didn't describe that clearly. We will give more detailed explanation in the revised version. As well known, at a long time scale, the water evaporated into atmosphere will be precipitated on land due to water cycle. In our manuscript, we defined the catchment network for water (vapor) transformation and transportation. **To define the catchment network, we track the water movement using Lagrangian particle tracking method, i.e. we took the water precipitated into the first catchment as research object** and marked it as P_1 ; we focuses on the subsequent transportation and transformation of P_1 and all the catchments that the water enters into was defined as the catchment network. Remarkably, for a special catchment of the catchment network, part of precipitation comes from P_0 and the rest comes from other sources, and we only studied the former.

In Figure 1, Catchments $A_{2,j}$ ($j=1, 2, 3, \dots$) represent all the catchments that the evaporated water from Catchment 1 can fall down with a form of precipitation, where water vapor has been through once of evaporation-precipitation process from P_1 ; while Catchments $A_{3,j}$ ($j=1, 2, 3, \dots$) represent the catchments that the evaporated water from Catchments $A_{2,j}$ ($j=1, 2, 3, \dots$) can fall down with a form of precipitation, where water vapor has been through twice of evaporation-precipitation process from P_1 . Regarding the precipitation in Catchment 2 as the reviewer concerns, some comes from Catchment 1 and the rest comes from other sources; however, according to the Lagrangian particle tracking method, we only focus on the part from Catchment 1. Also, we can establish the balance equation of **only the water from Catchment 1** for Catchment 2, $P_2 = E_2 + R_2$, where P_2 being the precipitation from Catchment 1, E_2 and R_2 being the evaporation and the runoff from the evaporated water from Catchment 1, respectively.

Further remarks:

abstract: "The abstract and conclusions are complicated and dense to read.

Response:

"Thank very much for your comment and we will add some detailed explanations in the revised version.

L. 23 what equation, you have not shown it

Response:

This equation represents the mean annual water-energy balance equation mentioned in L. 22.

Eq. 1 and 2 – you are not defining variables in the equations, and Eq. looks strange.
E in terms of E?

Response:

They were defined in L. 22 and L. 23. E in terms of E is just a mathematical form.

L: 5 and 6 . Which is the dry condition and which the wet condition?

Response:

Respectively, $E \rightarrow P$ as $E0 \rightarrow \infty$ is dry condition, i.e., evaporation approaches precipitation when potential evaporation is large enough, and $E \rightarrow E0$ as $P \rightarrow \infty$ is wet condition, i.e., evaporation approaches potential evaporation when precipitation is large enough.

L. 9 an analytical equation of what?

Response:

An analytical equation of the Budyko hypothesis. We will revise the expression to make it more clear.

L. 10 a derivative of the mean water energy balance? that is not variable?

Response:

I am sorry for the inaccurate expression, and it should be a derivative of E with respect to precipitation. We will revise it.

L.11 What is n, what is m? Specially n results being important, since it is shown in the last equation but you never say what it means.

Response:

n is a parameter introduced to reflect the characteristics of underlying surface by Bagrov (1953) and *m* is a variable defined as $m=(n+1)/n$ by Mezentsev (1955) to integrate the derivative. We will revise the expression in the resubmitted version.

L: 20 but isn't Fus equation also an implicit function? Some figures could greatly help, showing the Budyko space.

Response:

Yes. Fu's equation is an implicit function of $E_0 - E$ and P (or $P - E$ and E_0), stated in L.14. Thank you for your suggestion, and we will add figures to show the difference between the two equations.

L. 13 I have never her of the Carrot Limit.

Response:

I am sorry for our carelessness. It's a typo. It should be "Carnot Limit". I am sorry for our carelessness and will revise it in the revised version.

In Eq. 9, what is the physical meaning of conductance?

Response:

I am sorry for our carelessness, and it should be flux.

L. 2 what is the Ohms approach? You have not talked about it or introduced it?

Response:

*As for the Ohm's law, I am sorry that we didn't describe that clearly. The Ohm's law can be referred as Equation (11), i.e. the flux can be expressed as the function of the wate potential difference and the resistance of the water vapor movement or transportation process. We will give more detailed explanation in the revised version. As well known, at a long time scale, the water evaporated into atmosphere will be precipitated on land due to water cycle. In our manuscript, we defined the catchment network for water (vapor) transformation and transportation. **To define the catchment network, we track the water movement using Lagrangian particle tracking method, i.e. we took the water precipitated into the first catchment as research object** and marked it as P_1 ; we focuses on the subsequent transportation and transformation of P_1 and all the catchments that the water enters into*

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The structure of the manuscript is really strange. Where are the results?

Response:

I am sorry for the unclear structure. This manuscript aimed to derive the mean annual water-energy balance equation, so the results are Section 2. And we will revise the expression in the resubmitted version.