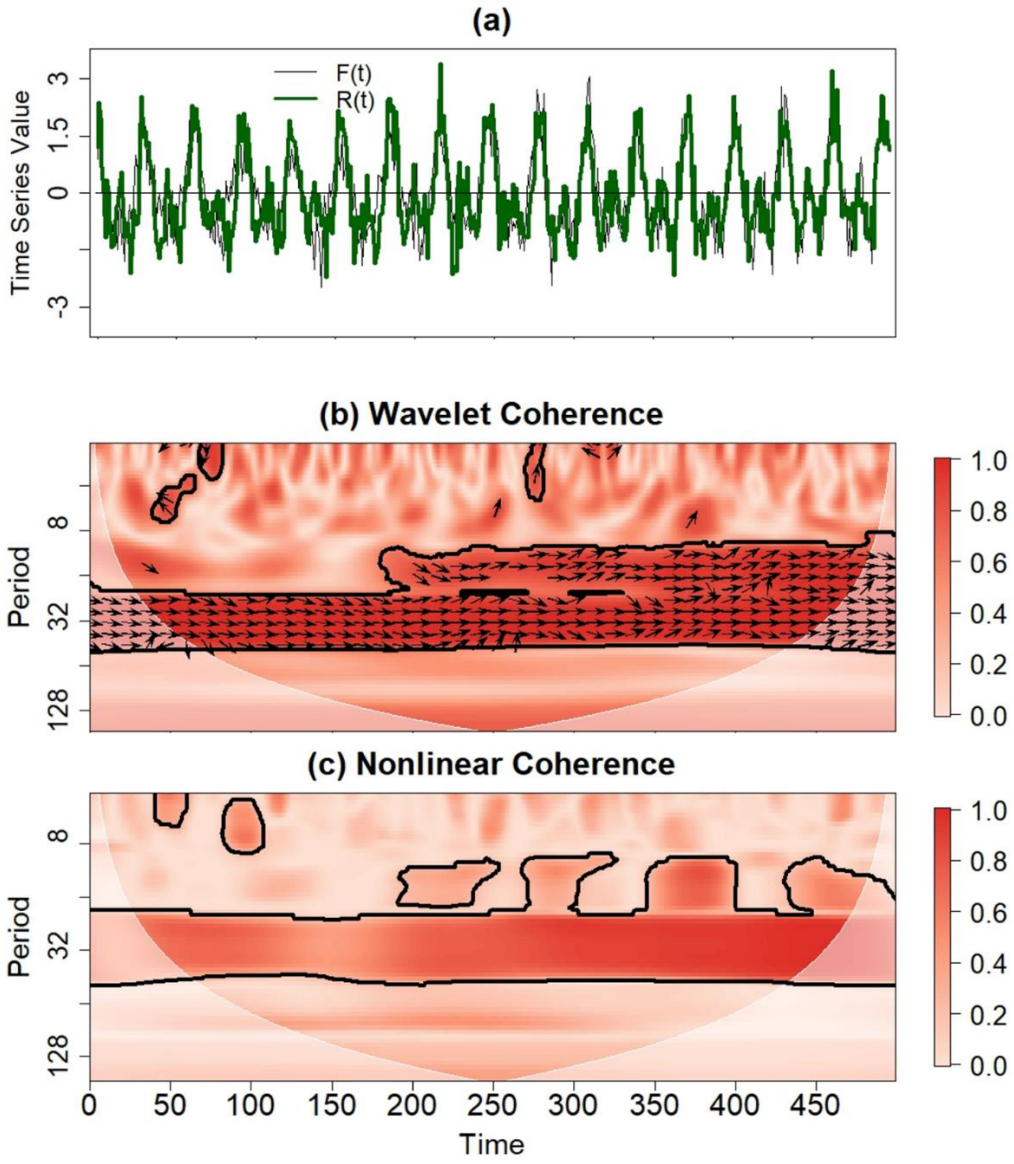

Figure S1. The forcing times series, *F*(*t*), from the main text and the response *R*(t) = cos($\frac{2\pi}{p_1}$ *t* + ϕ)+ cos($\frac{2\pi}{p_3}$ t) + $W_R(t)$ where p_1 = 32, $p_3 = p_1/2$ = 16. (b) Linear coherence and (c) nonlinear coherence between *F*(*t*) and *R*(*t*). Contours enclose regions of 5% cumulative area-wise significance and the light-shaded region represents the cone of influence. Although linear coherence suggests that *F(t*) and *R*(*t*) are not related at the period p_3 = 16 from *t* = 0 to almost *t* = 200, nonlinear coherence shows that they are related, which should be the case by construction because both *R*(*t*) and *F*(*t*) have cosine functions with period of 16. The time series *F*(*t*) and *R*(*t*) must be related at p_3 = 16 otherwise the relative bi-phase would fluctuate randomly according to Eq. (16), resulting in statistically insignificant nonlinear wavelet coherence at p_3 = 32. In other words, $\phi_n^X(32) - \phi_n^Y(32)$ and $\phi_n^X(16) - \phi_n^Y(16)$ do not fluctuate randomly so that *K* is not a random function of time.

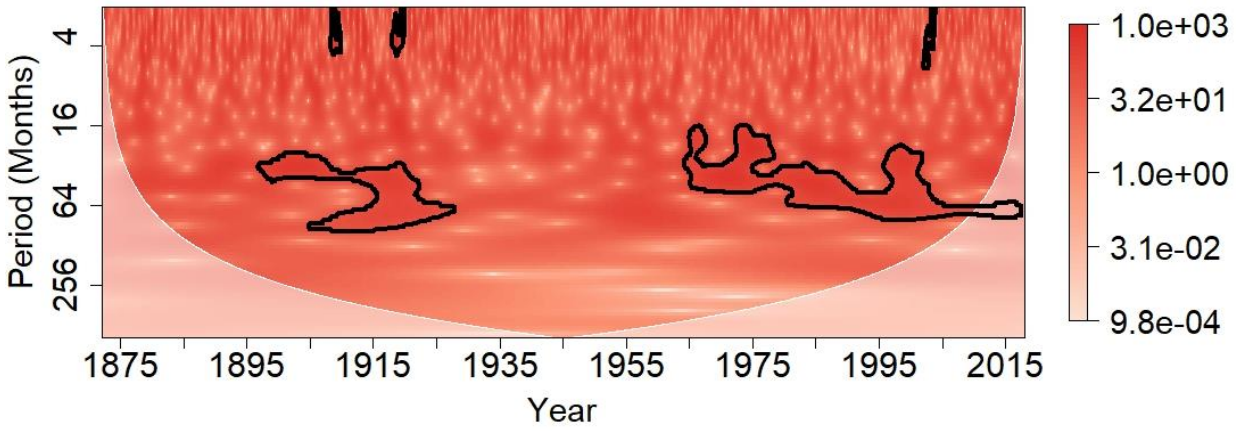

Figure S2. Wavelet power spectrum of Nino 1+2 – AIR.

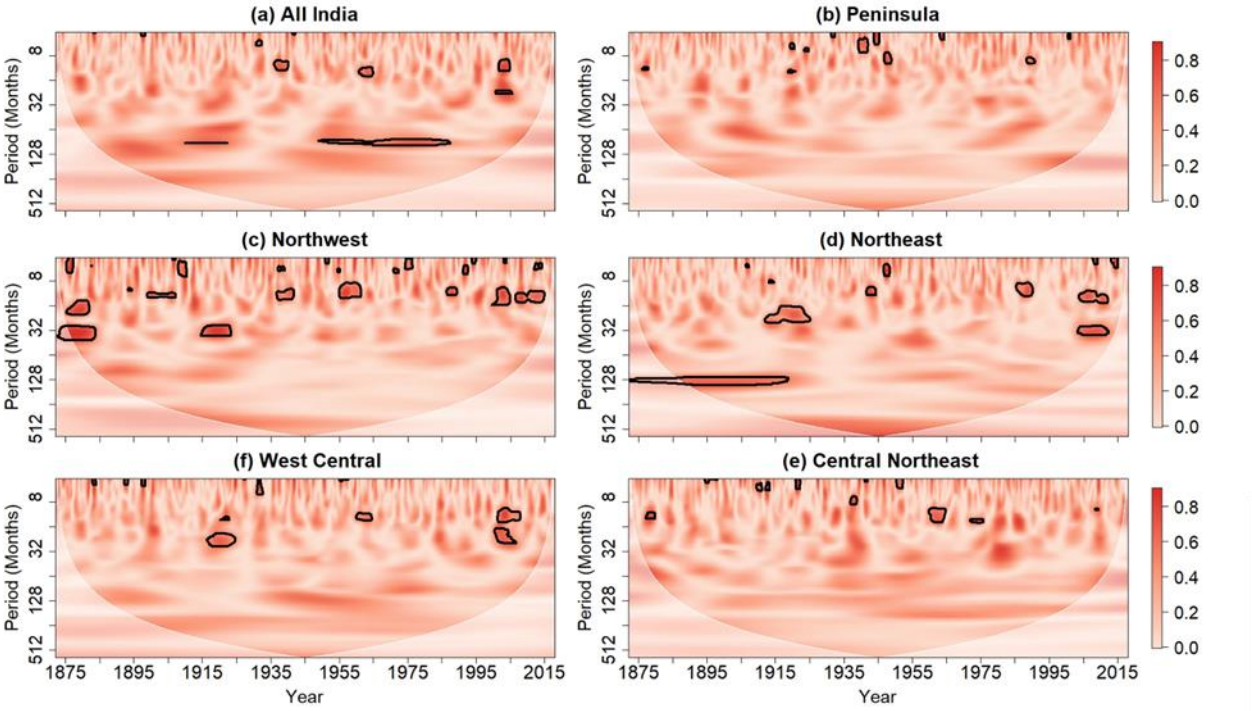

Figure S3. Local diagonal slice of auto-bicoherence spectra corresponding to the (a) All-India, (b) Peninsula, (c) Northwest, (d) Northeast, (e) West Central, and (f) Central Northeast time series. Contours enclose regions of 5% cumulative arc-wise significance. Light shading represents the cone of influence.

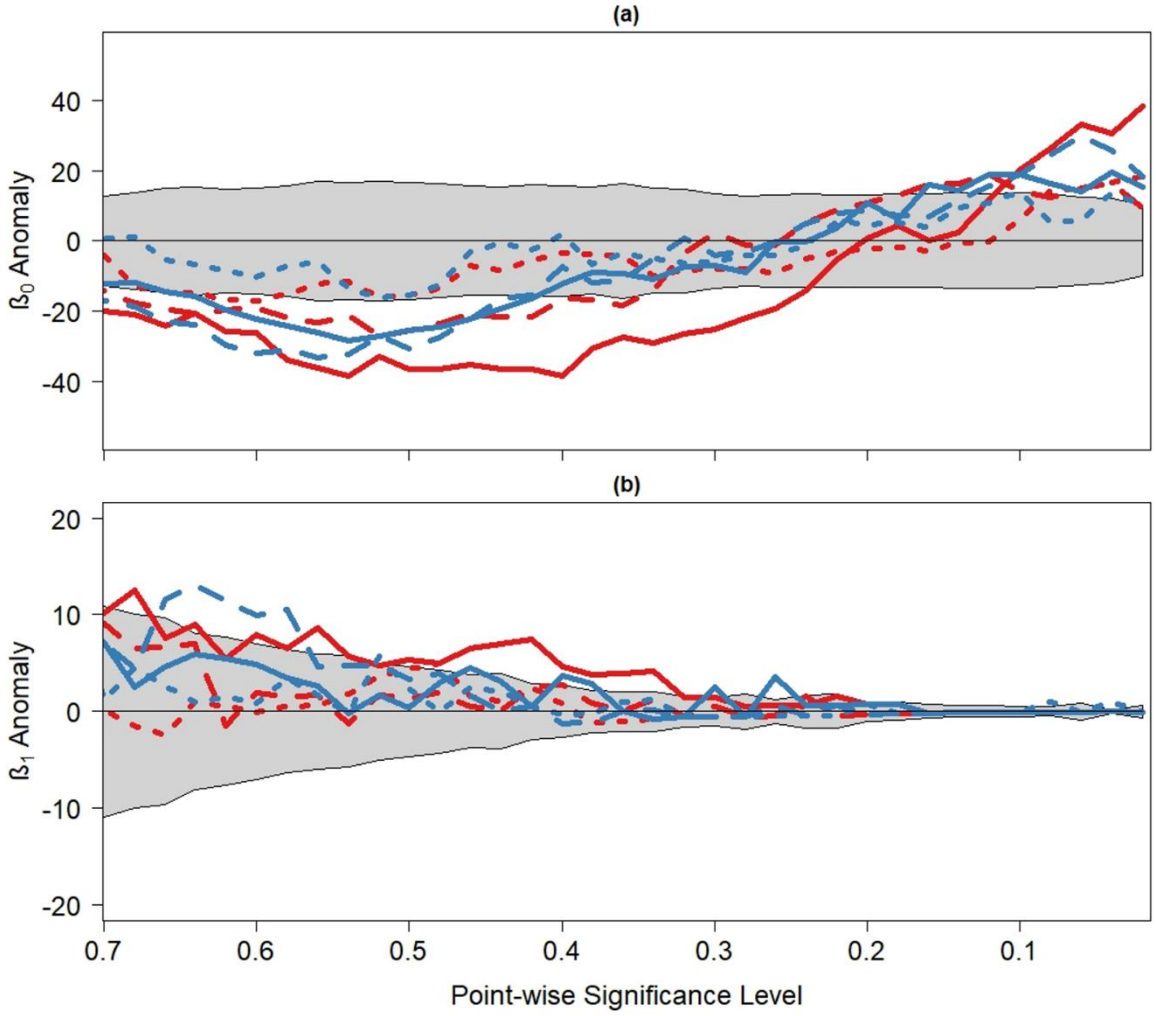

Figure S4. The (a) 0- and (b) 1-dimensional persistent homology profiles (Schulte, 2018) corresponding to the local auto-bicoherence spectra of the India rainfall time series. The grey shaded region is the test non-rejection region, the region in which the null hypothesis of red noise cannot be rejected.

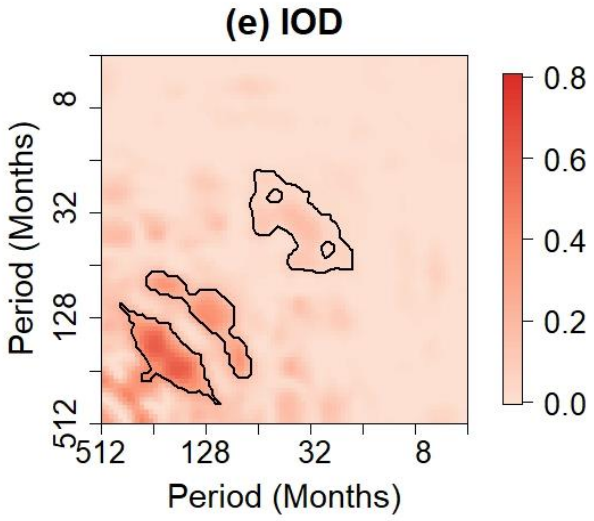

Figure S5. Global auto-bicoherence spectrum of the Indian Ocean dipole index. Contours enclose regions of 5% cumulative area-wise significance.

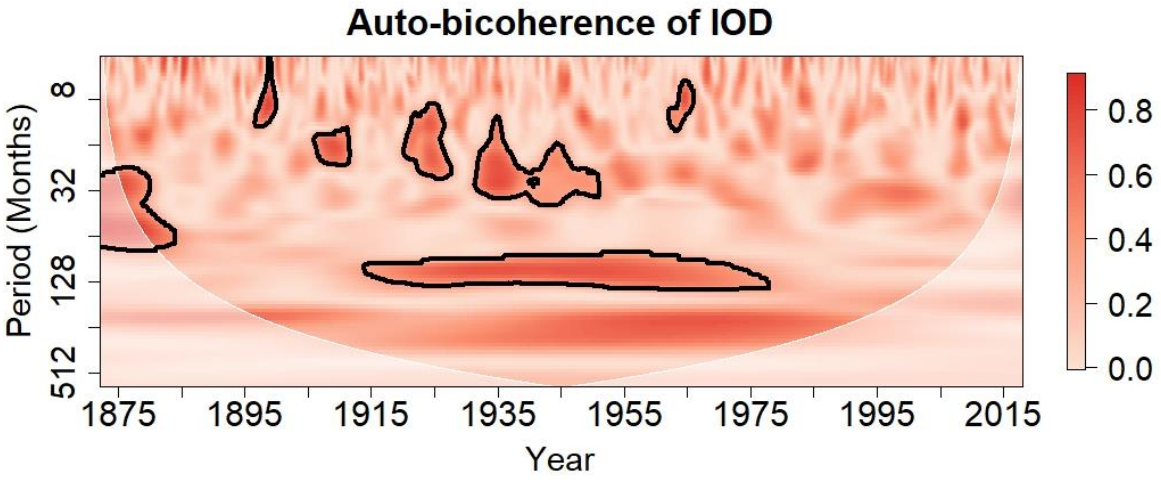

Figure S6. Local auto-bicoherence spectrum of the Indian Ocean dipole index. Contours enclose regions of 5% cumulative area-wise significance.

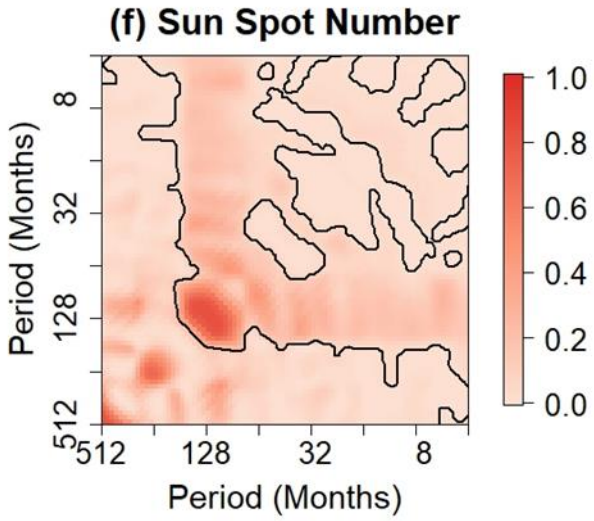

Figure S7. Global auto-bicoherence spectrum of the Sun spot index. Contours enclose regions of 5% cumulative area-wise significance.

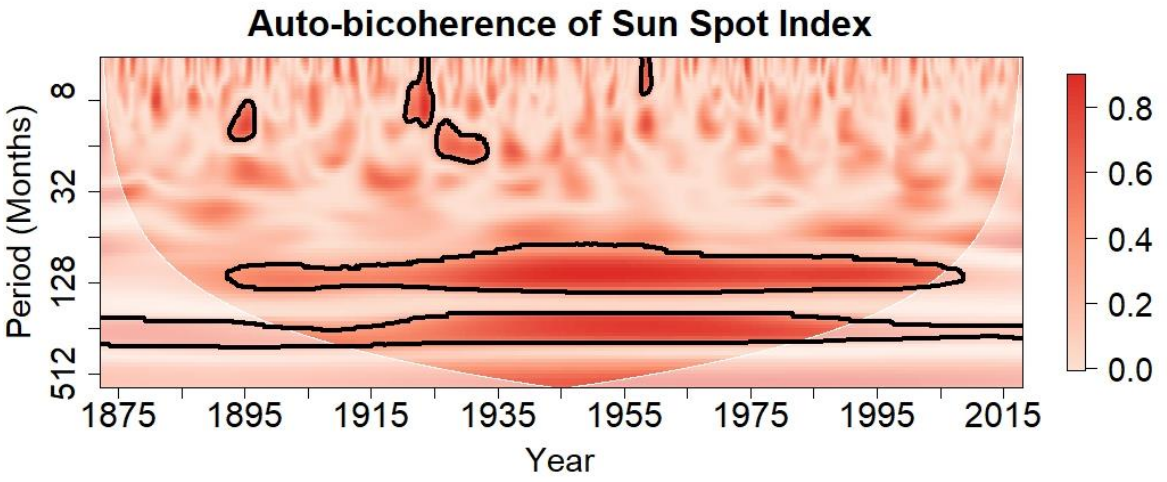

Figure S8. Local auto-bicoherence spectrum of the sun spot index. Contours enclose regions of 5% cumulative area-wise significance.