## General Reply

I thank all three reviewers for their constructive comments on the manuscript, which helped to improve it significantly. Please find detailed replies to all comments on the following pages. These replies were uploaded very fast to encourage a discussion in the open review process. However, after the open review phase is closed now an additional focus was given to increase the readability of the manuscript. The following steps were carried out:

(i) The readability was increased by adding a flowchart (Fig. 2) illustrating all applied methods, a notation where to find their description in the manuscript and the resulting data sets.

(ii) Additional information are provided at the beginning of sections including method descriptions for an easier navigation.

(iii) Although I have rejected it before, I followed now the suggestion of two reviewers to leave out the comparison between MRA and MMD. Only the MMD approach is still included in the manuscript.

With these changes the readability of the manuscript should have been increased and the navigation should be (hopefully) easier.

**Review by anonymous Referee #1**

The objective of the paper is to compare different versions of a rainfall disaggregation model that aims to produce high resolution times series (10min) from daily time series of precipitation. The versions is applied on a set of 24 recording stations. The main challenge for the author is to reproduce well the autocorrelation that was observed in the measurements.

This issue is obviously of high interest for the specific configuration of hydrological design in rapid response catchments, especially in urban areas. The manuscript however suffers from a number of limitations. The model is very rough and some basic assumptions of it do likely not hold. This make the model likely poorly relevant. On the other hand, other disaggregation models have been proposed in past years and the present work should at least include some in their comparison (it just compares variants of the present model but those are not really convincing). The description of the model / results is often rough and requires improvement. I could not understand what is done with some model variants and with the “resampling” process.

I would thus suggest rejection with the recommendation for a resubmission after clarification / improvements. For the Editor, it is not comfortable to have a numbering of lines which is reset to zero at every page. A unique numbering would make the review more convenient.

I thank the reviewer for the effort and the time spend on this manuscript. The reviewer points out the importance of the study for urban hydrological applications. His/her major concerns are an insufficient model description and missing comparisons of the results with other references. For both concerns detailed examples are provided by the reviewer. A point-by-point reply can be found below. All page and line numbers refer to the original submission.

Specific comments:

1. The objective of the paper is to compare different versions of a rainfall disaggregation model that aims to produce high resolution times series (10min) from daily time series of precipitation. The versions is applied on a set of 24 recording stations.

This comment is copied from the general part of the review for clarification. The disaggregated time series have a final temporal resolution of 5 min, not 10 min. Although the observed time series have a temporal resolution of 5 min (see e.g. P4L20, P8L3-5, P13L15-16,…), it seems the information was missing at other parts of the manuscript. Hence, the information was added in the abstract (P1L9-10):

"In this paper two cascade model modifications are analysed regarding their ability to improve the autocorrelation **in disaggregated time series with 5 minute resolution**."

and in the introduction of the method section (P5L5):

"The overall aim of this investigation is the improvement of the autocorrelation $r_{t1,t2}$ of the generated time series **with a temporal resolution of 5 minutes.**"

2. In the introduction, the authors mention that cascade models underestimate autocorrelation. This is not always true. See the comparative study of Hingray and Ben Haha (2005). They present results obtained for different disaggregation models including the so-called pattern-based microcanonical model. The reproduction of the autocorrelation with this model is almost perfect. This model should likely be included in the present work for comparison.

The reference of Hingray and Ben Haha (2005) was not included in the review of autocorrelation results so far. However, it was not the intention to "hide" results which do not fit to the motivation for the current study. Indeed, it is pointed out that under- and overestimations can occur by the application of a micro-canonical cascade model (P2L29-P3L4) due to several reasons (P3L5-10). Hingray and Ben Haha use for their investigation the time series of only one rain gauge and over a short range of disaggregation levels. Despite the low representation using only one rain gauge, by the few disaggregation levels it can be expected to achieve good results in terms of autocorrelation. The disaggregation starts with observed hourly values (so on this level the autocorrelation is "perfect", because it results from observations) and ends after four disaggregation steps (start: 1 h, 30 min, 15 min, 7.5 min…even less, since afterwards the time series is aggregated back to 10 min time steps). To keep the "perfect" autocorrelation over four disaggregation steps is much easier than over 7 disaggregation steps (start: 1d, 8 h, 4 h, 2 h, 1 h, 30 min, 15 min, 7.5 min [5 min]) as done in the current study. It can be questioned if the autocorrelation would be as good represented by a disaggregation of daily values down to 5 min values. Also, the results are shown in Hingray and Ben Haha only for lag-1 autocorrelation. As shown in the current study, although a good fit can be achieved for lag-1, strong underestimations can occur for other lags (see e.g. Fig. 6, Method C). Hence, I have implemented the reference into the literature review, but do not attempt to apply the disaggregation model additionally for the sake of comparison (especially in accordance with comment 8 of the reviewer, that the manuscript is already complex).

The following sentences were implemented after P3L4:

"A good representation of the lag-1 autocorrelation was achieved by Hingray and Ben Haha (2005) with two micro-canonical cascade models. However, since only four disaggregation steps were applied (from hourly to 7.5 min time steps) it remains unclear, if the good representation would have been achieved for more disaggregation steps."

3. Other models mentioned in the introduction perform also relatively well for AC reproduction. The best ones should be at least included in the present comparative work.

I double-checked the references cited in the introduction regarding autocorrelation results. As far as I can see, the references not discussed do not include autocorrelation results (e.g. Molnar and Burlando, 2005, Licznar et al. 2011, 2015). The autocorrelation results of Paschalis et al. (2014) are not discussed, since they result from combination of different rainfall generators, thus the error cannot be assigned exactly to the micro-canonical cascade model or the others. However, for the majority of rain gauges and rainfall generator combinations, the lag-1 autocorrelation is also underestimated. Since the reviewer mentioned no reference explicitly, it remains unclear for me which results should be discussed additionally.

A comparison with other rainfall generators is way beyond the scope of the current study, which aims at improving the general model structure. The basic idea behind the preceding cascade model and the resampling approach can be transferred to other micro-canonical cascade models, so this study is considered more as a methodological development than a best-choice model comparison.

4. The author state in their introduction (p.1 – ln 21/22) that "Since time series with 1280 minutes do not exist as observations, these studies [the studies related to the other models] are rather theoretical than practical from an engineering point of view." It seems to be the reason why the author disregarded the related models. This statement does obviously not hold. All the suggested models can be of high practical interest even from an engineering point of view. You just have to push the disaggregation process at the right temporal level (as the author does it actually in the present disaggregation process > disaggregation to 2.5min + reagregation to 5min).

The reviewer is right, a disaggregation could have started also on daily values applying the same disaggregation model until the disaggregated time series have a temporal resolution below the desired resolution and then a kind of transformation can be applied. However, this was not done in the mentioned studies. As shown in Müller and Haberlandt (2015) the kind of transformation has a significant influence on the disaggregation results, e.g. over- or underestimation of the rainfall intensities and especially on extreme rainfall values. Hence, a fair comparison is not possible. However, it was not the intention to disregard these disaggregation models. Hence I added the following subsequent sentence:

„Of course, by the application of an additional transformation process a desired temporal resolution can be achieved, whereby the transformation process affects the characteristics of the disaggregated time series."

5. Relevance of the present model: Variant A : in the first disaggregation step, the daily amount is distributed uniformly on the wet times steps (the wet 8hrs time step can be 1, 2 or 3). This is obviously not realistic at all. The model should relax this strong assumption which obviously cannot be validated from observations (or if it can, this has to be shown)

The reviewer is right, the uniform distribution over the as wet identified 8 h-time steps is a strong assumption. Although the number of wet 8 h-time steps is realistic, the resulting rainfall amounts are maybe less realistic. There are two reasons for this assumption.

First, the number of wet 8 h-time steps tells us something about the genesis of the rainfall event. If it is only one wet time step, it is likely a convective event and hence the whole daily rainfall amount is put into this single 8 h-time step. If the rainfall event lasts longer than 8 h, it is likely a stratiform event, so a long-lasting and less intense event (with often more or less uniform rainfall occurrence). The uniform distribution can be found only on the 8 h-level, on finer temporal resolutions intensities vary due to the b=2-splitting so the resulting final time series does not show any uniform rainfall intensity distribution anymore.

Second, the assumption with the uniform distribution demands only a few parameters. For method A, two parameters are required (P(0/0/1) and P(0/0.5/0.5)). The only other tested approach with a b=3-splitting in the first disaggregation step was introduced by Lisniak et al. (2013), who use for example 8 distribution functions for a splitting with 3 wet 8 h intervals. To keep the cascade model parameter parsimonious, the uniform distribution has been chosen. Also, in previous publications (Müller and Haberlandt, 2015, 2018, Müller-Thomy et al., 2018) with this assumption good representation of rainfall characteristics have been achieved.

6. The amount generated at the 7.5min time step are distributed uniformily on 2.5min time steps and then aggregated back to 5min. The uniform assumption is again really strong. Why don't you do the disaggregation to a finer resolution (3.75 min) and then aggregate back (using the same disaggregation model than the one you used for the previous time step) ?

The idea of the reviewer is to apply a different transformation to achieve a final temporal resolution of 5 min. In a previous publication Müller and Haberlandt (2018) tested transformations at 3 different temporal resolutions, starting with 15 min, 7.5 min and 3.75 min, for the same data set. The transformation starting with 7.5 min led to the overall best results.

An alternative would be to continue with the disaggregation to a very fine temporal resolution (few seconds) and then aggregating the time steps to 5 min, so that there is no need for any transformation. However, since a bounded cascade model is applied, the parameters for this so-called 'fine-graining' process have to be estimated from observations with the same temporal resolution, which are not available for the current study.

7. Variant C : Clarify. I can not understand how it works. The scheme of Figure 2 is very unclear. I do not understand at all.

For a better understanding, the disaggregation level and temporal resolutions have been added to Fig. 2 and colours have been removed. Unfortunately, without a certain comment what is unclear for the reviewer (the position definition, the time step indices, the increase in number of position classes or what exactly in the figure remains unclear) the manuscript can hardly be improved further in this point.

8. Avoiding time steps with too small rainfall intensities. Two approaches are considered to tackle this issue. This makes the paper rather complex. The results obtained with both approaches differ not a lot. I would suggest to keep only one of both (The one that mimic the measurement device would be likely relevant).
The reviewer suggests to leave the MRA approach out to reduce the complexity of the manuscript. However, both approaches represent possible solutions and none of them has been tested before to the authors knowledge. To reduce the complexity, only the MMD approach has been applied for the resampling investigations. I prefer to keep both approaches in the current study, especially since reviewer 2 identifies this issue as the more fundamental one. The comparison of the two approaches is also useful for other researchers, because they know that the outcome is very similar and don't have to carry out a study on their own.

8. Resampling. What do you do with the resampling step? Please clarify.
The introduction of subsection 3.3 has been extended to clarify, what is done in the resampling algorithm (which is explained more in detail afterwards). The following sentence has been added after P10L7:
„In a resampling process, two elements (here: relative diurnal cycles of the disaggregated time series) are swapped to improve an objective function (here: minimizing the deviation of the autocorrelation function of the disaggregated time series from the observed time series)."

9. What is the archive of observed structure you use ? Please clarify. Give perhaps a graphical scheme for illustration.
For the resampling, there is no archive of observed structures. The structure of the disaggregated time series results solely from the disaggregation process. After the disaggregation, two relative diurnal cycles of the disaggregated time series are swapped with the aim to improve the autocorrelation function (see point 1 of the resampling scheme, P12L2-3).

10. P10 ln 12. Which structure ? what are the volume classes ?
The reviewer refers to the sentence: "The structure of position and volume classes in the disaggregated time series generated by the cascade model should be conserved." Indeed, the sequence of position (e.g. starting, enclosed, ending, isolated, see Fig. 3) and volume classes combination (lower and upper volume class) of a day defines the structure of the disaggregated time series.
Both type of classes are introduced on P7L15-18.

11. P10 ln 16. "Restriction b) is fulfilled by swapping only relative diurnal cycles as time series elements, which does not affect the daily rainfall amount." I do not understand. Clarify.
One advantage of the micro-canonical cascade model is that the daily rainfall amounts are conserved exactly. If for the resampling absolute diurnal cycles are swapped, daily totals will differ from the original coarse time series, which has been disaggregated. To avoid this issue, no absolute values, only relative diurnal cycles are swapped. To achieve a "relative" diurnal cycle, for each 5 min time step its fraction of the total daily rainfall amount is determined. This sequence of fractions are swapped and subsequently multiplied with the rainfall amount of the other day chosen for the swap. Hence, the daily rainfall amount remains constant for each day.

12. Why should you swap structure from one day to another ?
The aim of the resampling is to improve the autocorrelation function (P10L7). Structures are not swapped, only relative diurnal cycles. The structure of the time series is kept.

13. what about the configuration where the rainfall event lasts more than one day ? do you swap 2-days structures ? if no, why not ?
If rainfall lasts for more than one day, this is taken into account by the position classes. For example, in three consecutive days there is {no rain; rain; rain}, the rainfall of the second day (starting position) can be most likely found at the end of the

day after the disaggregation process due to the position dependency of the parameters. This is the content of Table 3, 4 and 5 and explained at P14L3-11. To answer the question, the relative diurnal cycle of only one day is swapped, but the long-lasting event should not be destroyed by the swap.

14. Definition of an event : P4. Ln 23. Why should you define "events". The separation of events is always rather subjective and all results would depend on the separation rules. Here, you consider that "An event is hereby defined as a wet period enclosed by at least one time step without rainfall before and after the wet period." What is the time step there ? This definition seems to be not really relevant if it is 10mn or 1hour. A number of events present intensities interruption. We cannot consider that a break of one or 2 hours makes different events.
The reviewer questions the event definition in the manuscript. Indeed, the event definition is always subjective and from the authors opinion there is no right or wrong for this case. The reviewer argues that 5 min dryness are not long enough for a separation, but also 2 hours are not long enough in the reviewers opinion. A plausible event definition would differ between events, since it depends on the kind of event (for a convective event with a duration of few minutes or hours the separation time would be much shorter than for stratiform events, which last over several days). For this kind of validation much more information than the rainfall time series would be required, e.g. circulation patterns or air pressure time series. But this would be way beyond the scope of the current study. Since there is no "true" separation time" as the reviewer points out herself/hisself, I prefer to keep the 5 min time step for separation as an objective criteria. For clarification, the sentence has been updated to:
„The definition for a single event is according to Dunkerley (2008); having a minimum of one dry **5 min** time step before and after the rainfall occurrence."

<u>Minor comments</u>:

P8 : clarify : is model B non preceding model ? model C : preceding model ?
Method C is the only cascade model modification which is considered as "preceding" cascade model. For clarification purposes, the following sentence has be added after P8L18:
"Since only method C is referred to as preceding cascade model, method A and B can be considered as non-preceding cascade models."

P9 ln 2 : what is the "so-called non preceding" ??
Please see the reply to your former comment.

p7 ln 20 Clarify : how works the bounded cascade
The differentiation into "bounded" and "unbounded" relates to the range of temporal scales used for the estimation of the cascade model parameters. In an unbounded cascade model it is assumed, that the parameters are similar over the applied range of temporal scales due to a mon-fractal scaling behaviour and hence the same parameter set can be applied for all disaggregation steps (e.g. from daily values to hourly time steps). In a bounded cascade model, the parameter differ between the disaggregation levels (e.g. the parameter set applied for the disaggregation from 8 h to 4 h is different than from 10 min to 5 min) to take into account the multi-fractal scaling behaviour, so for each disaggregation step a particular parameter set is applied (which is 'bound' to the certain disaggregation step). The explanation on P7L20:
"For each step of the disaggregation process a particular parameter set is applied, which is 'bound' to the specific transition of temporal resolution."
has been extended to:
„For each step of the disaggregation process a particular parameter set is applied, which is 'bound' to the specific transition of temporal resolution. The need for particular parameter sets for each disaggregation step arise from the wide temporal range (from daily to 5 min time steps) and hence the underlying processes covered causing multi-fractal scaling behaviour."
to clarify this issue.

Equation 2 and equation 4 : what is the sum of probabilities ?

The sum for each line of the equations is ‘1’ as mentioned at P6L1 for Eq. 2:
“The sum of the weights is equal to 1 in each split, so the rainfall amount is conserved exactly throughout the disaggregation process.“
For clarification purposes, this information has been repeated for Eq. 4:
“Again, all probabilities (P(0/1), P(1/0) and P(x/(1-x))) sum up to 1. “

How many parameters have to be estimated for each model ? A table is required.
I agree with the reviewer, a new table (Table 2) has been implemented, which lists the parameter amount for each branching type and cascade model variant exactly.

Can you precise what is an event based and a continuous based evaluation ?
For an event-based evaluation first events are separated and then the characteristics of these events are determined. For a continuous-based evaluation the whole time series is considered, event-independent. The two sentences written bold has been added for explanation (at P13L1-5):
„**For an event-based evaluation, first the rainfall events are identified and then the characteristics of these events are determined.** Event-based rainfall characteristics include wet and dry spell duration as well as wet spell amount. An event is hereby defined as a wet period enclosed by at least one time step without rainfall before and after the wet period.

**For a continuous-based evaluation, the whole time series is considered, without differentiation into single events.** As continuous time series characteristics, the average intensity, the fraction of dry intervals and the autocorrelation are analysed.“

P 11. Ln 13-15. This statement has to be justified
The statement is justified by the citation of Müller and Haberlandt (2018). Additional explanations as e.g. deviations of flooding volume or combined sewer overflow volume would require an additional brief description of the investigated sewage system to plausibilize these values (independent if absolute or relative deviations would be shown). This would move the scope of the paragraph to a wrong direction. Hence, no additional information has been added for justification.

P11. Ln 18-19. “The return period Tn=1.5 years is assumed to be representative for typical return periods for dimensioning purposes in urban hydrology (Tn={1, 2, 5, 10 years},”. This can not be possible T15 can not be representative of other T.
The reviewer points out a sentence, which is indeed misleading. Of course, the absolute values of Tn=1.5 years are different from other return periods. The intention was to express, that an under- or overestimation of observed values for Tn=1.5 yrs is similar for these return periods, since they are close to each other from a statistical point of view. Of course, for a return period of Tn=100 years additional analysis would have to be carried out. The sentence was rephrased to:
“It is assumed that the results regarding under- or overestimation for the return period $T_n$=1.5 years are representative for typical return periods for dimensioning purposes in urban hydrology ($T_n$={1, 2, 5, 10 years}, DWA-A 531 (2012)).“

p11. Ln 23 : the amount of diurnal cycle ? what is this ?
The word “amount” was wrong here. It was changed to “number”.

p12. Ln 23 : for the sake of completeness. I do not see why this is completeness there
Without the parameter values it would be only a method description, not exactly repeatable. With the absolute values a repetition is possible, because all information are provided. However, the sentence has been changed to:
“~~For the sake of completeness the~~ The following setup was chosen for the resampling: $T_{a,start}=1*10^{-4}$, $dt$=0.99, $K$=500, $M$=200 and $thr_{O,auto}=1*10^{-9}$.”

p13 : what is partial duration series ??? is it a standard terminology ?
Yes, at least in Germany it is a standard terminology. However, the subsequent sentence includes the information that it is similar to the peak-over-threshold approach:
„Partial duration series are similar to the peak-over-threshold approach, whereby the threshold is defined in order to select 3 extreme rainfall events on average per year.“

p13 : ln 22 : what is “the single out of all n realisations “?
The sentence has been rephrased to:
“with *i* as control variable of all realisations *n*:“

Table 1 : what are AC values (to be given in the table)
The autocorrelation values for lag-1 have been added to Table 1.

Table 2 : can not understand what is presented there
The caption of the table was changed to “Nomenclature of dataset abbreviations used in this study“. The intention of the table is to provide an overview, which combinations of data sets were investigated in the study.

Table 3 and 4 and 5 : which model ?
The first column of each table (“position-independent”) belongs to method A, while all other columns belong to method B (“position-dependent).
The sentence:
„The resulting probabilities are shown in Table 3, 4 and 5.“ (P14L3)
has been extended to:
„The resulting probabilities are shown in Table 3, 4 and 5 (columns with position-dependent entries) in comparison to the position-independent probabilities estimated for method A (first column in each table).

what should be the sum of probabilities ?
The sum of probabilities has to be 1 as defined on P7L1-2: “The sum of the weights is equal to 1 in each split, so the rainfall amount is conserved exactly throughout the disaggregation process.“ For example, the sum of the probabilities for one wet interval, position-independent (Tab. 3, first column, first entry) P=40 %, for two wet intervals, position-independent (Tab. 4) P=35 % and for three wet intervals, position-independent (Table 5) P=25 % is ∑=100 %. Small deviations from 100 % can occur, since the mean for 24 stations is shown and hence rounding errors can occur.

Table 4 and 5 : they have the same caption !!!
The reviewer is right, for Table 5 it should have been “…three wet 8 hour interval…” instead of “…two wet 8 hour interval…”. This has been corrected.

Table 6 / 7 : what is the variability between stations ?
In Table 6 and 7 the relative errors for rainfall characteristics and extreme values are presented. These error variability differs between the investigated methods. However, I think an information about the standard deviation of the relative error or something similar is not useful for the reader and hence it was not added to the tables..

<u>Reference</u>:
Hingray, B., Ben Haha, M. (2005). Statistical performances of various deterministic and stochastic models for rainfall series disaggregation. Atm. Res. 77:152-175.
**Müller-Thomy, H.**, Wallner, M., Förster, K. (2018): Rainfall disaggregation for hydrological modeling: Is there a need for spatial consistence?, Hydrology and Earth System Sciences, 22, 5259-5280

**Review by Elena Volpi, Referee #2**

**1 Summary**

The manuscript deals with micro-canonical Multiplicative Random Cascade (MRC) application for rainfall disaggregation (starting from the daily scale) at sub-hourly scales. Specifically, the Author investigates the effectiveness of some modifications to MRC model proposed by Müller and Haberlandt (2018) in correctly reproducing the autocorrelation function of observed rainfall sub-hourly time series and the occurrence of small rainfall values. This topic is of interest for many hydrological applications; however, the manuscript presents some weaknesses that should be addressed before the paper can be considered for publication in HESS. My comments are reported in the following; I hope they will be helpful for manuscript improvement.

With appreciation,
Elena Volpi

I thank Elena Volpi for her useful and constructive comments and all her time, she spend on the manuscript. A point-by-point reply can be found below. All page and line numbers refer to the original submission.

**2 General comment**

1. The first concern is related to the motivation of this work. In the Abstract and Introduction Section two main issues that limit the applicability of "classical" MRC models for rainfall disaggregation are mentioned, namely the underestimation of the autocorrelation function of the fine scale process and the presence in the disaggregated series of very small rainfall values, that are not observed in measured data. The first problem does not seem to be of general interest, i.e. it does not characterize all the MRC models, but only the reference model here (that proposed by Müller and Haberlandt, 2018); the second problem is related to the properties of the observed data, which are characterized by a finite resolution so that values smaller than this resolution cannot be recorded. This second problem seems to affect significantly the estimation of the autocorrelation function, as also demonstrated by the results of this work, so that the simulated autocorrelation function does not correspond to the observed one. Hence, the second issue seems to be the fundamental one to improve the effectiveness of the reference model in reproducing the characteristics of the observed data.
From the literature review it is obvious that the reproduction of the autocorrelation function is a general problem of the micro-canonical cascade models, whereby under- and overestimations can occur. The introduced position definitions and the resampling approach were proven to solve the underestimation and it is assumed, that an overestimation would be reduced by both approaches as well. Both approaches are transferable to other cascade models and hence represent general improvements/solutions, which are not restricted to the micro-canonical cascade model applied in this study.
Of course, I agree with the reviewer that the generation of too small rainfall intensities is an important issue, especially since all rainfall characteristic comparisons are biased if the time steps with hydrologic irrelevant rainfall intensities are taken into account. Solving this second issue provides the basement for solving the first issue.

2. Further, only at the end of Introduction Section another fundamental problem characterizing MRC model is mentioned, that is the stationarity of the disaggregated process. The Author cites the paper from Lombardo et al. (2012) where the stationarity issue of MRC model is discussed and an alternative model is proposed that is proved to be stationarity. The Author drew inspiration from the latter model, yet not to guarantee stationarity but only to improve the reference model performance in terms of autocorrelation function. This is the main issue here; based on my opinion, stationarity should be addressed before improving the accuracy of the simulated autocorrelation function.
Indeed, the problem of stationarity is not solved in this manuscript, but this was also not the intention. The inspiration from Lombardo et al. (2012) for the current study refers "only" to the determination of the most important time steps to generate highly correlated time series. However, the analysed position definitions as well as the resampling approach can be transferred to other cascade models, so also later for stationary models.

3. Finally, I personally believe that the manuscript is rather difficult to follow because most of the mathematic behind the models is not explained (see the specific and technical comments that follow). As an example, it is not clear how many parameters rule the model behavior (in its different modified versions) and how these parameters can be estimated starting from the observed data, even if it seems that in this case the Author does not assume an universal generator (hence a simple-/multi-scaling behavior), am I correct?

The reviewer points out missing information about the method and its implication. A new subsection was added (Section 3.1.4, including a model parameter comparison) as well as a paragraph for the parameter estimation. All detailed comments from the reviewer have been addressed and can be found below. The rainfall generator is described by Eq. 2 (Eq. 4 for method B and C) for b=3 and Eq. 3. for b=2.

**3 Specific and technical comments**

4. Lines 14-17, page 2. The difference between canonical and micro-canonical is that between downscaling and disaggregation, as pointed out in Koutsoyiannis and Langousis (2011). (Koutsoyiannis, D. and Langousis, A. (2011). Precipitation. In Treatise on water science, Edited by: Wilderer, P. and Uhlenbrook, S. Vol. 2, 27–78. Oxford: Academic Press)

The additional information was implemented in the manuscript:

„Based on Koutsoyiannis and Langousis (2011), the canonical version of the cascade model (conservation of rainfall amount on average during the disaggregation, e.g. Molnar and Burlando, 2005, Paschalis et al., 2012) represents a downscaling technique, while the micro-canonical version (exact rainfall amount conservation for each time step, e.g. Olsson, 1998, Güntner et al., 2001, Licznar et al., 2011, 2015) represents a disaggregation technique."

5. Lines 14-17, page 2. This seems to be a minor problem, which is in general solved by disaggregating at a finer time scale and then aggregating at the desired one. It might have some implications in parameter estimation, depending on the structure of the generator. Is this the case?

I assume the comment refers to P2L17-22. The implication of putting a1440 min rainfall amount (1 d) into 1280 min and start the disaggregation is that you will end up with a day which is 160 min (~2.5 h) too short. So either you assume that the missing time steps have a rainfall amount of 0 mm (the question is then: Where to put them?) or you apply any transformation to the daily time series before the disaggregation. Both approaches will affect the resulting rainfall time series characteristics. The following sentence was added:

„Of course, by the application of an additional transformation process a desired temporal resolution can be achieved, whereby the transformation process affects the characteristics of the disaggregated time series."

6. Lines 17-19, page 3. Among the problems that the Author cites that justify the modifications of the MRC proposed in the manuscript, there is the non-stationarity issue. However, this is not mentioned in the abstract, but only at the end of the Introduction Section, and nowhere else in the manuscript; in other words process stationarity is not considered a problem here (see also the general comment).

Of course, non-stationarity is a problem, but it is not the motivation for the analysis carried out in this study. Indeed, mentioning the problem in the introduction was misleading and this sentence has been removed (see also the reply to your comment 20).

7. Lines 19-20, page 3. The description of the work by Lombardo et al. is not clear and, based on my opinion, what emerges does not correspond to the work done in the cited papers. The Author should improve his synthetic explanation; further, note that the method proposed by Lombardo et al. is not based on a MRC, but on an "additive" cascade.

The reviewer is right, the disaggregation models differ. The related parts of the text have been rephrased, so that only the information of Lombardo et al. is briefly cited, which are the most worth time steps to consider for the generation of highly correlated time series under the burden of computational efforts. A new paragraph has been added to the section "5.3 Study limitation" for clarification:

“Fourth, although method C is based on a finding in Lombardo et al. (2012, 2017), the disaggregation method differs from the additive cascade model in Lombardo et al. (2012, 2017). Hence, the by Lombardo et al. identified problem of non-stationarity of the disaggregation is not solved by the introduced cascade model variants and remains an open challenge for further studies.“
Please see also the reply to your comments 6 and 20.

8. Lines 26-28, page 3. I’m not sure I fully understood the issue of small values that are generated by the random cascade. Are the small values too small with respect to those characterizing observed rainfall time series at the target temporal resolution? If the reference truth is the observed high-resolution time series, is the Author arguing that the reference truth is not correct? This issue, which constitutes one of the most important motivations of this study, should be better explained to the potential readers (see also the general comment).
I thank the reviewer for pointing out this vague lines. This part of the introduction has been rephrased to:
“Koutsoyiannis et al. (2003) argue that it is unclear, if the values generated by the cascade model are too small in comparison to the observed minimum rainfall intensities or if the resolution of the measurement device is not fine enough to observe the very small rainfall intensities generated by the cascade model. From a practical point of view, these low-intensity time steps are not important, but they have an impact on the autocorrelation function. To enable comparisons between the autocorrelation of observed and disaggregated rainfall time series two methods are analysed in this study which ensure a minimum rainfall intensity in the disaggregated time series.“

9. Lines 13-14, page 5 and 1-2, page 6. The sentence is not clear.
The reviewer is right, the sentences were more misleading than helpful. Both sentences were removed from the manuscript.

10. Line 4, page 6. “direction”?
The sentence has been reduced to:
“The autocorrelation function is based on two elements: the covariance $s_{t1,t2}$ of the original and the shifted time series ($t_1$ and $t_2$), that describes the ~~direction of the~~ relation of both time series, and the standard deviations of both time series, $s_{t1}$ and $s_{t2}$, for the standardization of the covariance.“

11. Line 11, page 6. What does it mean that the aim is achieving a “minimum rainfall intensity”?
The text has been rephrased to:
“…to achieve the same minimum rainfall intensity as in the observed time series, to enable comparisons between the autocorrelation in observed and disaggregated rainfall time series.“

12. Line 12, page 6. Resampling as a subsequent step after disaggregation could be define as a post-processing technique/strategy.
The term “subsequent step” was replaced by “post-processing strategy” throughout the manuscript.

3. Line 24, page 6. “no” instead of “on”?
I thank the reviewer for this error spot, it was corrected.

14. Eq. (2). Are the possible outcomes for the three “events” at the disaggregation level 2 all mutually exclusive? In such a case, should the sum of the corresponding probabilities be equal to 1? If this is not the case, the disaggregation scheme at level 2 should be better explained.
Indeed, in Eq. (2) the occurrence of rainfall amounts in the three 8 h time steps is independent from each other. In a first step, based on the probabilities P(0/0/1), P(0/½/½) and P(1/3 / 1/3 / 1/3) (which sum up to $\sum P=1$), the number of wet 8 h time steps is identified. The choice of the wet intervals among all three possible intervals.

15. Line 4, page 7. What does it mean that the (probability?) parameters depend on the volume? How? Even if this is explained in a previous paper, it should be briefly recalled here for the sake of clarity.

The following explanation has been added:
"Müller and Haberlandt (2015) have shown that for days with high rainfall amounts (above a quantile $q_{0.998}$) the probability for two and especially three wet 8 h time steps is much higher than for lower daily rainfall amounts. Without a consideration of this volume-dependency of the parameters, the probability is too high that high daily rainfall amounts are put into one 8 h time step, which will lead to an overestimation of extreme rainfall values."

16. Lines 15-20, page 7 and 1-2, page 8. How many parameters characterize this modified version of the model? Which is the rule of dependence for starting, enclosed, etc. elements in the disaggregated series? The explanation is incomplete, or at least too vague.
Regarding the parameter number, Table 3 has been added to the manuscript. The explanation has been extended by the following sentences to describe the parameter estimation procedure more concise:
"To summarize the previous explanation regarding parameter estimation: for each temporal scale two fine time steps are aggregated (or three finer time steps for $b$=3, respectively) to one coarser time step, whereby the position and the volume class of the coarser time step determines to which position-volume class-combination the current splitting belongs. The cascade model parameters are then estimated over all splittings of a position-volume class combination."

17. Section 3.2.1. Method B adds additional probability parameters, letting them vary with the "position". It is unclear how these multiplicative parameters are determined to reproduce the statistical characteristics of the process across scales based on the structure of the cascade (i.e. b=3 for level one and b=2 for all the remaining levels up to the desired temporal resolution). See also previous comment.

I assume that this comment refers to section 3.1.2. Additional to the added explanation (see my reply to your previous comment) a summarizing figure (Fig. 3) has been added, including all position definitions applied in this study.

18. Figure 2 could be improved to help for reader understanding the difference between method B and C (but also starting, ending etc. elements of the cascade). Further, it should be explicitly mention if blue and white denote wet and dry states.
We thank the reviewer for her suggestion. We added the temporal scales to Fig. 2. Since a new figure (Fig. 3) has been added to the manuscript, which illustrates in detail the different possible position classes, the colours have been removed from Fig. 2.

19. Line 11, page 9. "isolated" instead of the second "ending"?
The reviewer is right, "ending" was replaced by "isolated".

20. There is a substantial difference between the approach proposed here and that used in Lombardo et al. (2012, 2017). Here the Author introduces a "conditional" probability that determines the probability of a wet or dry state, while in Lombardo et al. the information of previously disaggregated elements at the same time-resolution is used to feed a linear interpolation model estimating the disaggregated value of the subsequent element (see eq. (7) in Lombardo et al., 2017).
The reviewer is right, only the identification of the time steps „most worth to consider" has been overtaken from Lombardo et al. (2017), while the disaggregation method itself differs. The related part of the manuscript (P3L17-20) has been rephrased to:
„The second, more complex modification follows an idea of Lombardo et al. (2012, 2017). Lombardo et al. analysed which time steps are most worth to consider to generate highly correlated time series under the burden of computational efforts. Their conclusion is adapted in this investigation."

21. Line 7 page 10. The underestimation of the autocorrelation function characterizing the starting model is not a characteristic of all multiplicative random cascade models, as stated by the Authors in the literature review. Is there a motivation for this? Is it possible to generalize the problem to a specific MRC model (i.e. generator) structure?

I'm not sure if the text index refers to the comment (this reply is to the comment itself).

First, the deviation of the autocorrelation in the generated time series from the observed time series refers to micro-canonical cascade models, not cascade models in general. This information has been added where possible in the introduction.

Second, under- and overestimations are reported by the references in the introduction. But indeed, the previous results from the basic model for this study leads to underestimations of the autocorrelation.

However, it is assumed that with the new position definitions and the resampling approach under- and overestimations of the autocorrelation function can be avoided and improved subsequently, respectively.

22. Line 17, page 14. If the reference model works well in terms of autocorrelation, that is the main issue here, why two different modifications are proposed?

In the referred sentence only the results for lag-1 autocorrelation are discussed, how it is done very often for rainfall characteristics. However, as shown later in e.g. Fig. 7 and related paragraphs, the analysis of lag-1 is not enough, the whole autocorrelation function has to be analysed, or at least more lags as done in this study for lag-6 and lag-36.

23. I do understand that the dataset is not so rich, but I expected to find two separated datasets, one for "calibration" and one for "validation" of the models. The Author should justify this choice.

The maximum time series length is 20 years. For the parameter-intensive method C an artificial shortening of the time series should be avoided. Also, for later "real-life" applications, the whole high-resolution time series is used for parameter estimation for the disaggregation of the daily time series. Hence, a shortening of the time series could lead to a worsening of the disaggregation results, which would not occur otherwise. The following explanation was added in the validation section 3.1.4:

„A split-sampling into calibration and validation period was not carried out to keep the time series as long as possible for the parameter estimation (see also the discussion in Section 3.1.4)."

**Review by anonymous referee, Referee #3**

**1 Summary**

The paper deals with the improvement of a given disaggregation model using microcanonical cascade. In general the topic is relevant for the community. The paper is interesting, but cannot be published in its current state, and requires major modifications.

Reviewer #3 is gratefully acknowledged for her/his efforts and the time spend on the manuscript. I think the comments of the reviewer refer to the original submitted version, not to the „current" version. The current version was uploaded later and includes already the reviews of Elena Volpi (reviewer #2) and reviewer #1 and covers some of the issues and concerns raised by the reviewer. Nevertheless, please find below a detailed point-by-point reply under the assumption, that the review was based on the original submitted manuscript. All page and line numbers refer to the original submission.

**2. General comments:**

- The paper is quite hard to read with many models being compared. Explanations for the slight variations between the various models are sometimes hard to follow. There is a lack of mathematical details in the presentations of the various models.

The reviewer points out several issues about the method description and the complexity of the manuscript. These issues are repeated as detailed comments, hence please find there a detailed reply.

- Only comparison between variations of a given model are provided. Comparisons with other type of cascade models should at least be discussed.

I agree with the reviewers suggestion that the manuscript would benefit from comparisons of the disaggregation results with results from other cascade models. However, most references compare only the value for the lag-1 autocorrelation, which is not representative (e.g. as shown for method C in Fig. 5 with an overestimation of the lag-1 autocorrelation, while in Fig. 6 a clear underestimation for the majority of lags can be identified). Hence, a comparison is hardly possible, especially because a comparison based only on lag-1 values could be misleading.

Also, the autocorrelation depends on the rainfall processes and genesis in the study area, which enables comparison of absolute values from other studies. A fair comparison could only be carried out with the direct application of another cascade model, but this would be contrary to the reviewers comment on the already very large complexity of the manuscript with too much model version comparisons.

- There are numerous parameters to be estimated per model (not even very clear which number according to the model choice). It is not clear whether a calibration period and a validation period were used.

To provide a better overview an additional figure and a table were added to the manuscript. Fig. 3 provides an explanation of position definitions for method A, B and C in dependence of the branching number. Table 2 lists the cascade model parameters for methods A, B and C in dependence of the applied branching number. From Table 2 it is visible, that for method C a high number of parameters is required. Since the maximum length of the observed time series is 20 years, no split-sampling has been applied, to ensure a best-possible parameter estimation. Also, for later "real-life" applications, the whole high-resolution time series is used for parameter estimation for the disaggregation of the daily time series. Hence, a shortening of the time series could lead to a worsening of the disaggregation results, which would not occur otherwise. The following explanation was added in the validation section 3.1.4:

„A split-sampling into calibration and validation period was not carried out to keep the time series as long as possible for the parameter estimation (see also the discussion in Section 3.1.4)."

**3. Detailed comments:**

1) Introduction

- p.2 l.21 : “since time series with 1280 minutes do not exist as observation do not exist”. I do not understand this statement and this does not seem a real issue. Anyway, if needed, you can disaggregate at a higher resolution and up-scale to the desired one.

The aim of real-life applications is to disaggregate daily values to achieve high-resolution rainfall time series. The implication of putting a1440 min rainfall amount (1 d) into 1280 min and start the disaggregation is that you will end up with a day which is 160 min (~2.5 h) too short. So either you assume that the missing time steps have a rainfall amount of 0 mm (the question is then: Where to put them?) or you apply any transformation to the daily time series before the disaggregation. Both approaches will affect the resulting rainfall time series characteristics. The following sentence was added for clarification:

„Of course, by the application of an additional transformation process a desired temporal resolution can be achieved, whereby the transformation process affects the characteristics of the disaggregated time series.“

2) Rainfall data

- p. 3 l.28-29 : “from a practical. . . have an impact on the autocorrelation function”. Why not trying the compute the autocorrelation using higher moments to limit the influence of smaller values ?

The reviewer suggests another way of computing the autocorrelation. Indeed, by doing so the influence of the too small intensities would be limited, but it solves not the problem itself. Also a threshold could be introduced to “ignore” the lower values. However, especially for method C very long wet spells with too small rainfall intensities are generated. This underestimation of the average intensity (Fig. 5) poses a serious problem for later applications. Hence, the introduction of MRA and MMD to avoid too small rainfall intensities is also done for practical applications, not only for a more representative computation of the autocorrelation.

- p.4 l.10 : “how can a minimum rainfall intensity be ensured during the disaggregation process?”. It is not very clear to me the need for this, since as pointed out by the author and references cited, it might very well be simply due to the rain gauge measurement limitations. It might be worth testing a time series obtained with a disdrometer which enables better representation of small values of rainfall.

Indeed, a disdrometer time series would gain more insights in this relation of observed and disaggregated rainfall intensities, but is not available to me for this study area. However, the long wet spells with very low rainfall intensities generated by method C have to be avoided (please see also my reply to your previous comment). This is the motivation for the introduction of MRA and MMD (see P14L25-32).

3) Methods

- p.5 l.8 : “actual” - - > “Actually”

This has been corrected.

- Eq. 1 : it should be added how lambda is related to t1 and t2 (I guess lambda = t2-t1)

This has been added in the paragraph before Eq. 1.

- p. 6 l.24 : “on” - - > “no” ?

This has been corrected.

- Eq. 2 : shouldn’t it be P(0/0/1)/3 in the first line since there are three possibilities (100,010,001) for the same P(0/0/1) ? This remark is also valid for all the other probabilities except P(1/3,1/3,1/3)

The reviewer is right, the way Eq. 2 was written was misleading. Eq. 2 has been changed to:

$$W_1, W_2, W_3 = \begin{cases} \{1, 0, 0;\ 0, 1, 0\ or\ 0, 0, 1\} & with\ P(0/0/1) \\ \left\{\frac{1}{2}, \frac{1}{2}, 0;\ \frac{1}{2}, 0, \frac{1}{2}\ or\ 0, \frac{1}{2}, \frac{1}{2}\right\} & with\ P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} & with\ P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) \end{cases}$$

- p.7 l.11 : "an empirical function", please be more specific (see also general comment on the lack of mathematical details).

The description of the empirical distribution function was extended to: „An empirical distribution function is used to represent *f(x)*, with a maximum of 14 equidistant bins (based on the number of available splittings, see Storm (1988, p. 86))."

- p.7 l.15-18 : a summary table or scheme would be helpful.

Fig. 3 has been added to illustrate all position definitions.

- It remains weird to have different branching number and probabilities weights for the first cascade steps which seems to be in contradiction with the underlying scaling properties.

The different branching numbers are a compromise between scaling theory and practical applications. Indeed, it can be questioned if scaling properties are met by b=3 and especially by the uniform distribution in this first disaggregation step. Although the number of wet 8 h-time steps is realistic, the resulting rainfall amounts are maybe less realistic. There are two reasons for this assumption.

First, the number of wet 8 h-time steps tells us something about the genesis of the rainfall event. If it is only one wet time step, it is likely a convective event and hence the whole daily rainfall amount is put into this single 8 h-time step. If the rainfall event lasts longer than 8 h, it is likely a stratiform event, so a long-lasting and less intense event (with often more or less uniform rainfall occurrence). The uniform distribution can be found only on the 8 h-level, on finer temporal resolutions intensities vary due to the b=2-splitting so the resulting final time series does not show any uniform rainfall intensity distribution anymore.

Second, the assumption with the uniform distribution demands only a few parameters. For method A, two parameters are required (P(0/0/1) and P(0/0.5/0.5)). The only other tested approach with a b=3-splitting in the first disaggregation step was introduced by Lisniak et al. (2013), who use for example 8 distribution functions for a splitting with 3 wet 8 h intervals. To keep the cascade model parameter parsimonious, the uniform distribution has been chosen. Also, in previous publications (Müller and Haberlandt, 2015, 2018, Müller-Thomy et al., 2018, Müller-Thomy and Sikorska-Senoner, 2019) with this assumption good representation of rainfall characteristics have been achieved.

- Section 3.1.3 : I found the paragraph quite hard to read. may be a more precise scheme could be helpful. It should be mentioned that it adds a lot of parameters. In general, a summary table with the number of parameters according to the model would be helpful.

The existing scheme in Fig. 2 was improved and Table 3 including all parameters in dependence of the applied branching number was added as well as section 3.1.4.

- Section 3.2 : why presenting two different models (especially given that they provide rather similar results) ? It adds complexity to a paper with already a lot of comparison. I would keep only the MMD which is the more realist I believe.

This suggestion was also made by reviewer #1. He suggested as well to leave out the MRA approach to reduce the complexity of the manuscript. However, both approaches represent possible solutions and none of them has been tested before to the authors knowledge. To reduce the complexity, only the MMD approach has been applied for the resampling investigations. I prefer to keep both approaches in the current study, especially since reviewer 2 identifies this issue as the more fundamental one. The comparison of the two approaches is also useful for other researchers, because they know that the outcome is very similar and don't have to carry out a study on their own.

- Section 3.3 : The process with Ir and more generally the swapping seems rather ad hoc. It seems that the underlying physical meaning of cascade process is lost. I think that this issue should at least be discussed.
Maybe the swapping seems ad hoc because it is not related to cascade models in general. However, in a previous publication Müller and Haberlandt (2018, Fig. 12) have shown, that the scaling behaviour of the disaggregated time series is not changed by the resampling process. The following sentence has been added:
„As proven by Müller and Haberlandt (2018), the resampling does not affect the scaling behaviour of the disaggregated time series, because the total rainfall amount as well as the number of wet time steps are kept.“

- p. 13 l. 18 : "30 realisations". Why such a small number, it seems that much more could have been performed.
The disaggregation is a random process. In a prior study it was analysed, which number of realisations is required to cover the stochastic behaviour of the disaggregation based on the rainfall characteristics of the disaggregated time series analysed in this study. It was found that 30 realisations are sufficient. Also, as pointed out as last point in section "5.3 Study limitations", the simulated annealing is an optimization process that demands a high computational effort. Hence, 30 realisations are kept for the study.

4) Results
- Table 3 and 4 are really hard to follow. I think a scheme representing the various cases could be really helpful.
A scheme of all possible positions is provided in Fig. 3 for a better interpretation of both tables.

- Why the average rainfall intensity changes is such a micro-canonical cascade ?
I'm not sure if I understand the question correctly. The average rainfall intensity depends only on the number of wet time steps generated by the cascade model, since the total rainfall amount remains the same for the whole daily time series. Hence an overestimation of wet time steps leads to an underestimation of the average intensity and vice versa.

- p. 19 l. 5-10 : may be a graph showing the sensitivity of the results to Ir would be be needed.
The reviewer is right, this would be useful. However, a 2D-graph could only include the results for one lag and one extreme event (of a certain duration and return period). Since the problem is more complex, a simplification as a 2D-graph was not added to the manuscript.

5) Discussion - p. 21 l. 14-15 : "identified similarities. . . used for simplification", please clarify and be more explicit.
The reviewer is right, due to the high number of parameters every possibility to reduce their number should be listed in detail. The following sentence has been added:
„These similarities are e.g. P(0/1) for starting and P(1/0) for ending positions (and vice versa) as well as P(0/1) and P(1/0) for both, enclosed positions and isolated positions.

References:

Müller-Thomy, H., Sikorska-Senoner, A. (2019): Does the complexity in temporal precipitation disaggregation matter for a lumped hydrological model?, Hydrological Sciences Journal, accepted

Storm, R.: Wahrscheinlichkeitsrechnung, mathematische Statistik und statistische Qualitätskontrolle, VEB Fachbuchverlag, Leipzig, 9th edition, 360 pages, 1988.

# Temporal rainfall disaggregation using a micro-canonical cascade model: Possibilities to improve the autocorrelation

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**Abstract.** In urban hydrology rainfall time series of high resolution in time are crucial. Such time series with sufficient length can be generated through the disaggregation of daily data with a micro-canonical cascade model. A well-known problem of time series generated so is the underestimation of the autocorrelation. In this paper two cascade model modifications are analysed regarding their ability to improve the autocorrelation in disaggregated time series with 5 minute resolution. Both modifications are based on a state-of-the-art reference cascade model. In the first modification, a position-dependency is introduced in the first disaggregation step. In the second modification the position of a wet time step is redefined in addition. Both modifications led to an improvement of the autocorrelation, especially the position redefinition. Simultaneously, two approaches are investigated to avoid the generation of time steps with too small rainfall intensities, the conservation of a minimum rainfall amount during the disaggregation process itself and the mimicry of a measurement device after the disaggregation process. The mimicry approach shows slight better results for the autocorrelation and hence was kept for a ~~subsequent~~ resampling investigation using Simulated Annealing as a post-processing strategy. For the resampling, a special focus was given to the conservation of the extreme rainfall values. Therefore, a universal extreme event definition was introduced to define extreme events a priori without knowing their occurrence in time or magnitude. The resampling algorithm is capable of improving the autocorrelation, independent of the previously applied cascade model variant. Also, the improvement of the autocorrelation by the resampling was higher than by the choice of the cascade model modification. The best overall representation of the autocorrelation was achieved by method C in combination with the resampling algorithm. The study was carried out for 24 rain gauges in Lower Saxony, Germany.

## 1. Introduction

For many applications in hydrology high resolution rainfall time series are crucial (see the review of Cristiano et al. (2017)) to match the scale of the underlying processes (Blöschl and Sivapalan, 1995). Schilling (1991) concludes that for urban hydrology, in particular for overland flow, a temporal resolution of 5 minutes is acceptable. Berne et al. (2004) points out that the required temporal resolution depends on the catchment size, and recommend for urban catchments with area sizes of about 1000 ha a temporal resolution of 6 minutes and for 10 ha or smaller a temporal resolution of 1 min. Unfortunately, lengths of time series with such a high temporal resolution are insufficient for most applications. However, for the non-recording stations (registration of daily values) the time series lengths are usually sufficient, but the temporal resolution is not fine enough to cope with the dynamics in urban hydrology (Ochoa-Rodriguez et al., 2015) or for erosion processes (Jebari et al, 2012).

A possible solution for this data scarcity is rainfall disaggregation. Information of short, high-resolution time series are used to disaggregate coarser time series. The disaggregation results in high-resolution time series with sufficient lengths as well as a higher network density in most cases. Several methods exist for the temporal disaggregation, e.g. method of fragments (Wójcik and Buishand, 2003, Westra et al., 2012, Breinl et al., 2015~~, 2019~~, Breinl and Di Baldassarre~~e~~, 2019), rectangular pulse models (Koutsoyiannis and Onof, 2001) and cascade models. Cascade models are well-known disaggregation models for the generation of high-resolution rainfall time series and were developed originally in the field of turbulence theory (Mandelbrot, 1974). Based on Koutsoyiannis and Langousis (2011), the ~~The temporal rainfall disaggregation models can be divided between~~ canonical version of the cascade model (conservation of rainfall amount on average during the disaggregation, e.g. Molnar and Burlando, 2005, Paschalis et al., 2012) represents a downscaling technique, ~~and~~ while the micro-canonical ~~cascade models~~ version (exact rainfall amount conservation for each time step, e.g. Olsson, 1998, Güntner et al., 2001, Licznar et al., 2011, 2015) represents a disaggregation technique. However, the majority of investigations with cascade models focus on the disaggregation of quasi-daily time series (with time step durations of 1280 minutes instead of 1440 minutes, e.g. Licznar et al., 2011, 2015, Molnar and Burlando, 2005, ~~2008,~~ Paschalis et al., 2014) down to 10 minute or 5 minute time series, which has the advantage of using the same branching number of $b=2$ throughout the disaggregation process, that determines the number of finer time steps (here: two) with equal duration from one coarser time step. Since time series with 1280 minutes do not exist as observations, these studies are rather theoretical than practical from an engineering point of view. Of course, by the application of an additional transformation process a desired temporal resolution can be achieved, whereby the transformation process affects the characteristics of the disaggregated time series. ~~Hence~~To overcome this issue, Müller and Haberlandt (2018) developed a micro-canonical cascade model, which enables the rainfall disaggregation from daily values to 5 minutes. Müller and Haberlandt evaluated the disaggregated rainfall time series in terms of rainfall characteristics and showed good performances regarding continuous (average intensity, fraction of dry intervals) and event-based rainfall characteristics (wet and dry spell duration, wet spell amount) as well as extreme values. An additional validation with an urban hydrological model led to comparable results for event-based combined sewer overflow volume as well as manhole flooding volume when forced with observed and disaggregated rainfall time series, respectively.

However, Müller and Haberlandt (2018) also show that the autocorrelation of the disaggregated time series is underestimated. This is critical, because the autocorrelation describes the memory of a process. So for continuous

applications especially deviations can be expected whether e.g. an urban hydrological model is forced with observed or disaggregated rainfall time series. The underestimation of the autocorrelation in the generated time series has been identified before when micro-canonical cascade models were used for the disaggregation by e.g. Olsson (1998), Güntner et al. (2001) and Paschalis et al. (2012, 2014). Lisniak et al. (2013) divided the study period into a calibration and validation period. While for the calibration period the autocorrelation was underestimated, a good representation was achieved for the validation period. Rupp et al. (2009) and Pohle et al. (2018) analysed four and three different kinds of cascade models, respectively. Depending on the choice of the model, under- and overestimations of the autocorrelation function were identified. A good representation of the lag-1 autocorrelation was achieved by Hingray and Ben Haha (2005) with two micro-canonical cascade models. However, since only four disaggregation steps were applied (from hourly to 7.5 min time steps) it remains unclear, if the good representation would have been achieved for more disaggregation steps.
Summarizing the previous findings, ~~the~~ an adequate reproduction of the autocorrelation function ~~cannot be reproduced adequately~~ by the multiplicative micro-canonical cascade model can be difficult. The reasons for over- and underestimation differ depending on the choice of the cascade model. For example, in Olsson (1998) and Müller and Haberlandt (2018) the underestimation is caused by the generation of dry time steps inside rainfall events, causing shorter wet spells in the disaggregated time series in comparison with the observed time series. In Pohle et al. (2018) an overestimation of the autocorrelation is identified for a cascade based on Menabde and Sivaplan (2000), which disables the generation of dry finer time steps from one wet coarser time step.

In this study, I investigate modifications of the cascade model itself, but also a post-processing strategy ~~subsequent methods~~ after the disaggregation procedure to improve the representation of the autocorrelation in the disaggregated time series. The basis for all investigations in this study is the multiplicative random cascade model as proposed by Müller and Haberlandt (2018). According to Marshak et al. (1994) it is a bounded cascade model with a single parameter set for each disaggregation level. The parameters are estimated by the aggregation of observed, high-resolution time series (Carsteanu and Foufoula-Georgiou, 1996). The modifications are based on the introduction of position-dependencies with two different degrees of complexity. The first, less complex modification includes taking into account the position of the wet day in the time series. The second, more complex modification follows an idea of Lombardo et al. (2012, 2017)~~, who found that the stochastic process of the cascade model is non-stationary~~. Lombardo et al. analysed which time steps are most worth to consider to generate highly correlated time series under the burden of computational efforts. Their conclusion is adapted in this investigation~~transform the disaggregation into a stationary process by changing the time steps taking into account the position definition in all disaggregation steps~~. Both modifications are expected to improve the autocorrelation function and lead with the basis model to three different cascade model variants in this study.

Simultaneously, a second general issue of the cascade model ~~should be~~ is solved: the generation of time steps with very small rainfall intensities. Molnar and Burlando (2005) identified a fraction of rainfall intensities lower than the measurement resolution of the investigated time series of 48 % for 10 min time series, starting with the disaggregation from quasi-daily values. Müller and Haberlandt (2015) found for a disaggregation from daily to hourly values a fraction of underestimated rainfall intensities of 35 %. Koutsoyiannis et al. (2003) argue that it is ~~Hence it remains~~ unclear, if the values generated by the cascade model are too small in comparison to the observed minimum intensities or if the resolution of the measurement device is not fine enough to observe the very small rainfall intensities generated by the cascade model~~(Koutsoyiannis et al., 2003)~~. From a practical point of view, these low-intensity time steps are not important, but they have an impact on the autocorrelation function. To enable comparisons between the autocorrelation of observed and disaggregated rainfall time series ~~Therefore two methods are analysed~~ a novel method is applied in this study ~~which~~ to ensure a minimum rainfall intensity in the disaggregated time series.

In addition to the modifications of the cascade model itself, a ~~subsequent~~ resampling algorithm as post-processing strategy is analysed to improve the autocorrelation. A similar approach has been investigated by Bárdossy (1998), who used a simulated annealing algorithm to resample time series generated with a Markov chain Monte Carlo method. Bárdossy investigated temporal resolutions of 1 hour and 5 minutes, the autocorrelation function could be reproduced well for both. For this investigation, the proposed resampling algorithm of Müller and Haberlandt (2015) will be modified to include the autocorrelation function in the optimization process.

As a summary from the introduction, the main research question of this study is:

~~(i)~~ How can the autocorrelation in the disaggregated time series be improved?

~~Along with this question, a second research question is addressed:~~

~~(ii) How can a minimum rainfall intensity be ensured during the disaggregation process?~~

## 2. Rainfall data

For the investigation 24 stations in and around Lower Saxony, Northern Germany, are used (see Fig. 1). The same data set has been used before by e.g. Callau and Haberlandt (2017) for rainfall generation.

There are three dominating topographical regions with a coastal area around the North Sea, followed by the flatland around the Lüneburger Heide and the Harz middle mountains with altitudes up to 1141 m a.s.l. (from North to South). Due to the climate classification after Köppen-Geiger (Peel et al., 2007) the study area can be divided into a temperate climate in the north and a cold climate in the mountainous region. Both climates exhibit no dry seasons, but hot summers. For the Harz mountains, average annual precipitation amounts greater than 1400 mm can be identified.

In Fig. 1, the 24 recording stations operated by the German Weather Service (DWD) with long term time series ranging from 9 to 20 years and a temporal resolution of 5 minutes are shown. The validation of the cascade model modifications is based on these 24 stations with a focus on the autocorrelation, but also on overall characteristics (average intensity, fraction of dry hours), event characteristics (dry spell duration, wet spell duration, wet spell amount) as well as extreme values. The definition for a single event is according to Dunkerley (2008); having a minimum of one dry time step before and after the rainfall occurrence. For the definition of a dry time step the accuracy of the measuring instrument is not applied here, instead a threshold of 0 mm/5 min rainfall intensity is used. This enables comparisons between observed and disaggregated time series, which are not limited to the measuring accuracy and hence also includes smaller values (Molnar and Burlando, 2005). The rainfall time series characteristics along with further information of the rain gauges are provided in Table 1.

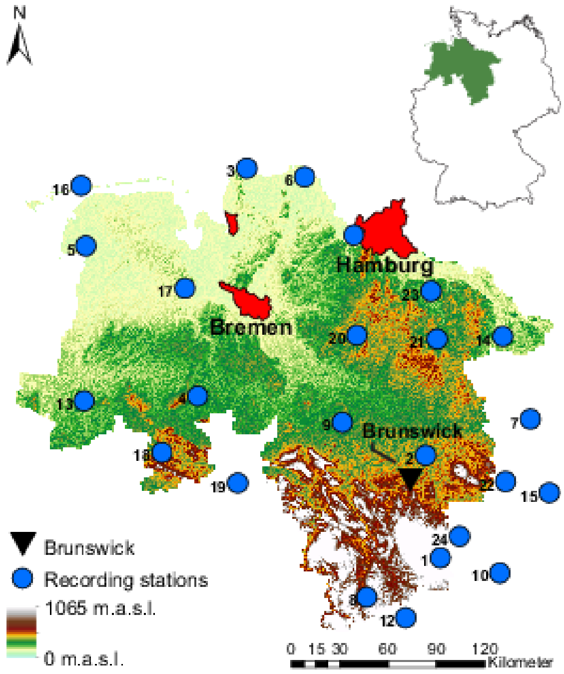



**Fig. 1: Study area of Lower Saxony (and its location in Germany) with 24 recording stations.**

### 3. Methods

The overall aim of this investigation is the improvement of the autocorrelation $r_{t1,t2}$ of the generated time series with a temporal resolution of 5 minutes. The autocorrelation function (Eq. 1) describes the memory of a process (here: rainfall) by the comparison of time series with itself in the future (shifted time series), whereby the future is represented by a certain number of $\lambda$ future time steps in the time series (lags). For rainfall time series Pearson's autocorrelation and Spearman's rank autocorrelation are applied in literature (e.g. Pohle et al., 2018). Actually, the Pearson's autocorrelation can only be applied if the data population is normally distributed, while Spearman's rank autocorrelation demands no assumption about the distribution of the data. However, using Pearson's autocorrelation has two advantages: i) it enables comparisons with results from the literature, since it is applied more often than Spearman's rank autocorrelation, and ii) in terms of the later introduced resampling algorithm (see Sect. 3.2) for Pearson's autocorrelation no rank analysis of the whole time series has to be performed, since it can be calculated straight forwardly from the absolute values of the time series, which essentially fastens the optimization. ~~If for the calculation of Pearson's autocorrelation the ranks of rainfall intensities are used instead of their absolute values, the result is identical with Spearman's rank autocorrelation, which means an optimization of one criteria leads to an optimization of the other criteria as well.~~ Hence, the Pearson's autocorrelation is applied throughout this study.

The autocorrelation function is based on two elements: the covariance $s_{t1,t2}$ of the original and the shifted time series ($t_1$ and $t_2$, shifted by the lag $\lambda$ with $\lambda = t_2 - t_1$), that describes the ~~direction of the~~ relation of both time series, and the standard deviations of both time series, $s_{t1}$ and $s_{t2}$, for the standardization of the covariance. While $t_1$ consists of $n$ time steps, the rainfall amount at a single time step $i$ is represented by $x$.

$$r_{t1,t2} = \frac{s_{t1,t2}}{s_{t1} s_{t2}} = \frac{\sum_{i=1}^{n-\lambda}(x_i - \overline{x})(x_{i+\lambda} - \overline{x})}{\sqrt{\sum_{i=1}^{n-\lambda}(x_i - \bar{x})^2 \sum_{i=1}^{n-\lambda}(x_{i+\lambda} - \bar{x})^2}} \tag{1}$$

To improve the autocorrelation of the disaggregated time series, several methods are introduced. A flowchart of the applied methods is presented in Fig. 2. ~~Hence, t~~The method chapter is divided into ~~four~~ three subsections, which will be briefly described. Section 3.1 includes the model description of the cascade model and two modifications to improve the representation of the autocorrelation in the disaggregated time series. These three cascade model variants are compared at the end of Section 3.1. ~~Three variants of the cascade model are described which can be combined with two modifications to achieve a minimum rainfall intensity, which are introduced in Sect. 3.2.~~ In Sect. 3.2~~3~~ a resampling algorithm to increase autocorrelation as a ~~subsequent step~~ post-processing strategy after the disaggregation process is explained. The evaluation strategy for the disaggregated time series based on rainfall characteristics is explained in Sect. 3.3~~4~~.

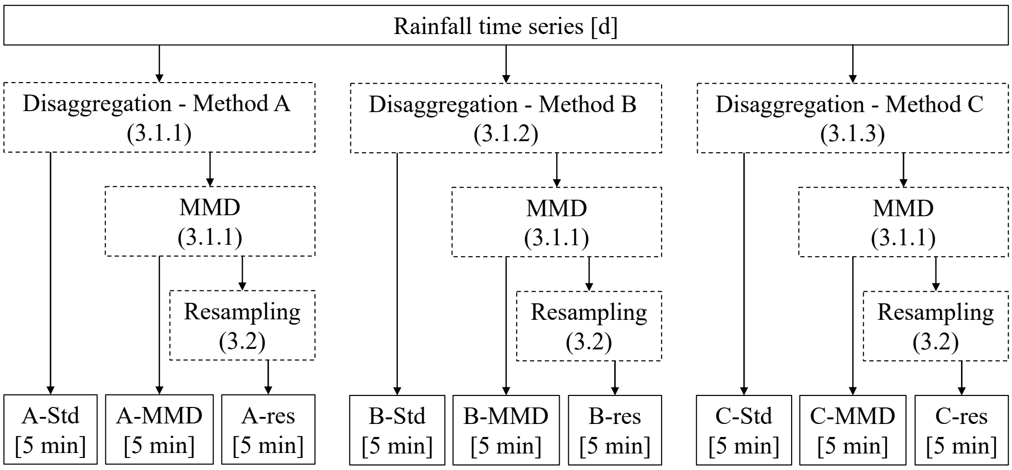



**Fig. 2: Overview of applied methods (dashed rectangles) and the resulting data sets (bottom line). In the brackets behind the applied methods the subsection with the method description is referenced.**

### 3.1 Disaggregation model

### 3.1.1 General scheme (Method A)

The principle of the micro-canonical, bounded cascade model applied in this investigation is illustrated for the first two disaggregation steps in Fig. 3~~2~~ (top) and was introduced by Müller and Haberlandt (2018). A coarse time step is disaggregated into $b$ finer time steps, with $b$ named the branching number.

Starting with daily values, $b=3$ is applied and three time steps with 8 h duration are generated (similar to Lisniak et al., 2013). The daily rainfall amount can occur in only one (0/0/1-splitting), in two ($0/\frac{1}{2}/\frac{1}{2}$) or in all three of the finer time steps, whereby the rainfall amount is distributed uniformly on the wet time steps (as it can be seen from the numbers in brackets that identify the fractions of the daily rainfall amount). The required parameters for this splitting are the probabilities $P$ for one ($P(0/0/1)$), two ($P(0/\frac{1}{2}/\frac{1}{2})$) and three ($P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3})$) wet 8 h-intervals in a day. The parameters $P(0/0/1)$ and $P(0/\frac{1}{2}/\frac{1}{2})$ have no~~on~~ influence on the position of the wet boxes, only on the number. The position is assigned randomly. The probability for $P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3})$ can be determined by $P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) = 1-P(0/0/1)-P(0/\frac{1}{2}/\frac{1}{2})$. The possible splittings and the distribution of the daily rainfall amount on the finer time steps are summarized in the cascade generator for $b=3$ (see Eq. 2). By multiplying the rainfall volume $V$ of the coarser time step with the so-called multiplicative weights $W_1$, $W_2$ and $W_3$ the rainfall amounts of the finer time steps are derived. ~~the rainfall~~ The sum of the weights is equal to 1 in each split, so the rainfall amount is conserved exactly throughout the disaggregation process.

$$W_1, W_2, W_3 = \begin{cases} \{1,0,0;\ 0,1,0 \ or\ 0,0,1\} & with\ P(0/0/1) \\ \{\frac{1}{2},\frac{1}{2},0;\ \frac{1}{2},0,\frac{1}{2}\ or\ 0,\frac{1}{2},\frac{1}{2}\} & with\ P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{3},\frac{1}{3},\frac{1}{3} & with\ P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) \end{cases} \qquad (2)$$

$$\cancel{W_1, W_2, W_3} = \begin{cases} 1, 0 \text{ and } 0 & \text{with } P(0/0/1) \\ 0, 1 \text{ and } 0 & \text{with } P(0/0/1) \\ 0, 0 \text{ and } 1 & \text{with } P(0/0/1) \\ \frac{1}{2}, \frac{1}{2} \text{ and } 0 & \text{with } P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{2}, 0 \text{ and } \frac{1}{2} & \text{with } P(0/\frac{1}{2}/\frac{1}{2}) \\ 0, \frac{1}{2} \text{ and } \frac{1}{2} & \text{with } P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{3}, \frac{1}{3} \text{ and } \frac{1}{3} & \text{with } P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) \end{cases} \qquad \cancel{(2)}$$

Also, a volume-dependency of the parameter was identified for $b=3$. Müller and Haberlandt (2015) have shown that for days with high rainfall amounts (above a quantile $q_{0.998}$) the probability for two and especially three wet 8 h time steps is much higher than for lower daily rainfall amounts. Without a consideration of this volume-dependency of the parameters, the probability is too high that high daily rainfall amounts are put into one 8 h time step, which will lead to an overestimation of extreme rainfall values. Hence, ~~P~~parameters are estimated for a lower and an upper volume class, with the quantile $q_{0.998}$ of all daily total rainfall amounts as threshold (see Müller and Haberlandt (2015) for more details).

After the first disaggregation step, only $b=2$ is applied. The generated intermediate time series have temporal resolutions of $\Delta t=\{4\ h,\ 2\ h,\ 1\ h,\ 30\ min,\ 15\ min,\ 7.5\ min\}$. The rainfall amount of the coarser time step can be assigned either to the first (1/0-splitting) or to the second (0/1) finer time step only or to both finer time steps (x/(1-x)). Again, all probabilities (P(0/1), P(1/0) and P(x/(1-x))) sum up to 1. For the x/(1-x)-splitting, the relative fraction of the rainfall volume that is assigned to the first of the two finer time steps is considered as a random variable $x$ with $0 < x < 1$. An empirical distribution function is used to represent $f(x)$, with a maximum of 14 equidistant bins (based on the number of available splittings, see Storm (1988, p. 86)). The cascade generator for $b=2$ is given in Eq. 3:

$$W_1, W_2 = \begin{cases} 0 \text{ and } 1 & \text{with } P(0/1) \\ 1 \text{ and } 0 & \text{with } P(1/0) \\ x \text{ and } 1-x & \text{with } P(x/(1\text{-}x));\ 0<x<1 \end{cases} \qquad (3)$$

The parameters for the splitting with $b=2$ depend on both, the position and the volume class of the current time step to disaggregate in the time series (see e.g. Olsson, 1998, Güntner et al., 2001). The position of a time step is defined by the wetness state of the surrounding time steps, so starting (time step before is dry, time step afterwards is wet, dry-wet-wet), enclosed, ending and isolated positions can be distinguished (see Fig. 4 for an illustration). For each position two volume classes are defined, whereby the lower and upper volume class are separated by the mean rainfall volume of each position.

All parameters for *b=2*- and *b=3*-splittings can be estimated by aggregating observed time series with the same temporal resolution (Carsteanu and Foufoula-Georgiou, 1996). As mentioned before, a bounded cascade model is applied (Marshak et al., 1994). For each step of the disaggregation process a particular parameter set is applied, which is 'bound' to the specific transition of temporal resolution. The need for particular parameter sets for each disaggregation step arise from the wide temporal range (from daily to 5 min time steps) and hence the underlying processes covered causing multi-fractal scaling behaviour.

To summarize the previous explanation regarding parameter estimation: for each temporal scale two fine time steps are aggregated (or three finer time steps for *b*=3, respectively) to one coarser time step, whereby the position and the volume class of the coarser time step determines to which position-volume class-combination the current splitting belongs. The cascade model parameters are then estimated over all splittings of a position-volume class combination.

A final temporal resolution of 5 min is achieved via uniform transformation (Müller and Haberlandt, 2018). The rainfall amounts of time steps with $\Delta t$=7.5 min are distributed uniformly on 2.5 min time steps and afterwards aggregated non-overlapping to $\Delta t$=5 min.

As mentioned in Sect. 1, the cascade model tends to generate too many time steps with too low intensities. To overcome the issue~~, two possible solutions are presented in the following section, namely 'minimum rainfall amount' (MRA) and 'mimicry of the measurement device' (MMD). However, if no modification is applied the disaggregation is referred to as standard.~~

~~By MRA, the cascade generator is modified dependent on the rainfall amount of the current coarse time step to disaggregate. This modification affects the disaggregation process in several ways. If the rainfall amount is smaller than twice the minimum rainfall amount *min* (defined by the minimum resolution of the measuring device of each time series) for a *b=2*-splitting, only 1/0- and 0/1-splittings are possible (beginning from this time step to all finer time steps resulting from it). If the rainfall amount of a time step is higher than this threshold, x/(1-x)-splittings are possible, but under the restriction $V \cdot x \geq min$ and $V \cdot (1-x) \geq min$. For disaggregation steps with *b=3*, the cascade generator is affected in the same way with a threshold of $V \geq 3 \cdot min$ to enable all splittings and $V \geq 2 \cdot min$ to enable $(0/\frac{1}{2}/\frac{1}{2})$-splittings, respectively.~~

~~By MMD, the cascade generator itself is not modified. After the disaggregation,~~ the behaviour of a measurement device is imitated after the disaggregation. Rainfall amounts smaller than the minimum resolution of the measurement device are summated in the chronological order of the time steps until the sum $S_{thr}$ exceeds this threshold. The former wet time steps with smaller intensities are set to 0 mm, while $S_{thr}$ is moved to the last time step of the summation. Afterwards, $S_{thr}$ is set back to 0 mm. This process is carried out over the whole disaggregated time series and is referred to as 'mimicry of a measurement device' (MMD).

### 3.1.2 Introduction of position-dependency in the uniform splitting (Method B)

For method B, only the first disaggregation step is (uniform splitting) modified from Method A in terms of the introduction of a position dependency. All further disaggregation steps remain identical to Method A and are not changed.

For the disaggregation of daily values into 8 hours the cascade model is applied with a branching number of $b=3$. Although the number of wet 8 hour-intervals depends on estimated probabilities, their position is chosen randomly in Method A. This is assumed to cause deviations of the autocorrelation.

Therefore, in addition to the volume classes, the position of the daily time step in the time series is also taken into account for the parameter estimation in Method B. The same positions are applied for the further disaggregation steps with $b=2$ (starting, enclosed, ending and isolated position, see also Fig. 4). For each position, the probability of possible placements of wet and dry 8 hour-intervals is estimated. The daily rainfall amount is split uniformly between the as-wet-defined 8 hour-intervals. Based on the possible placements, the resulting cascade generator for the first disaggregation step is shown in Eq. 4 and substitutes Eq. 2.

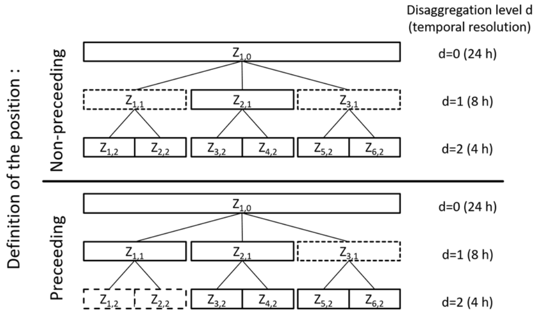
$$W_1, W_2, W_3 = \begin{cases} 1,\ 0 \text{ and } 0 & \text{with } P(1/0/0) \\ 0,\ 1 \text{ and } 0 & \text{with } P(0/1/0) \\ 0,\ 0 \text{ and } 1 & \text{with } P(0/0/1) \\ \frac{1}{2},\ \frac{1}{2} \text{ and } 0 & \text{with } P(\frac{1}{2}/\frac{1}{2}/0) \\ \frac{1}{2},\ 0 \text{ and } \frac{1}{2} & \text{with } P(\frac{1}{2}/0/\frac{1}{2}) \\ 0,\ \frac{1}{2} \text{ and } \frac{1}{2} & \text{with } P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{3},\ \frac{1}{3} \text{ and } \frac{1}{3} & \text{with } P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) \end{cases} \tag{4}$$

### 3.1.3 Introduction of a preceding cascade model (Method C)

In the modification called preceding cascade model, the position-dependency for the whole disaggregation process is extended. Hence, the modifications for Method C affect all disaggregation steps. Since only method C is referred to as preceding cascade model, method A and B can be considered as non-preceding cascade models. Besides the modified position classes definition, the disaggregation process remains identical to Method A.

An example of the position-dependency extension is illustrated in Fig. 3~~2~~ (bottom) and will be used for explanation. The indices $f$ and $g$ of each time step $Z_{f,g}$ represent an index for each time step ($f$=1, 2, .., $n$ with $n$=length of the time series) and each disaggregation level ($g$=1, 2, …, 7), respectively.

For a time step $Z$ ($Z_{2,1}$) the wetness state of the time steps before ($Z_{1,1}$) and afterwards ($Z_{3,1}$) of the same disaggregation level are taken into account for the identification of the position so far (so-called "non-preceding" in Fig. 3~~2~~). Hence, the type of splitting (1/0, 0/1 or x/(1-x)) is chosen independently from the wetness state of two already disaggregated time steps ($Z_{1,2}$ and $Z_{2,2}$) in the next disaggregation level. For the position definition in the preceding cascade model, the information about the wetness state of the two finer, already disaggregated time steps ($Z_{1,2}$ and $Z_{2,2}$) and the following, coarse time step ($Z_{3,1}$) is taken into account (according~~ly~~ to Lombardo et al. (2012, 2017)).



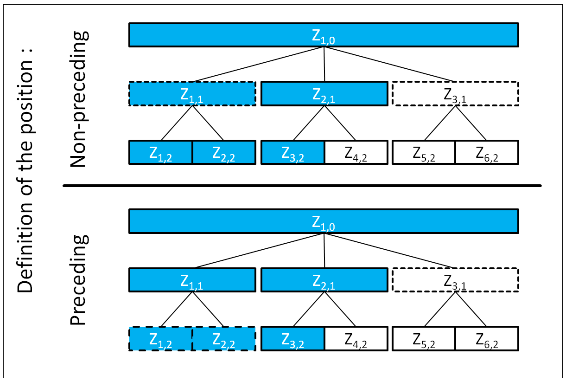




**Fig. 3~~2~~: Scheme for position definition in a non-preceding (upper part) and preceding (lower part) cascade model. The dashed boxes indicate the time steps taken into account for position definition of the time step $Z_{2,1}$.**

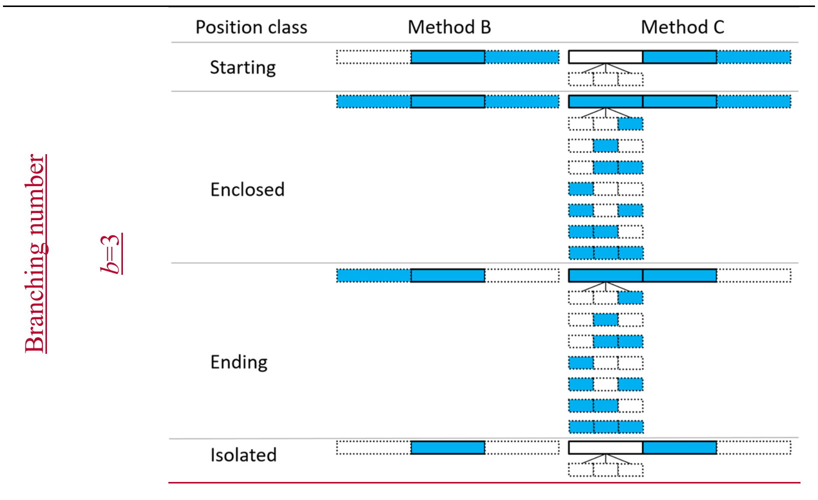
Due to the new definition, the number of positions is extended from four in the non-preceding cascade model positions (starting, ending, enclosed, ~~ending~~isolated) to eight in the preceding cascade model (see also Fig. 4): one starting ({0,0},1,1 with 0=dry and 1=wet and {} indicating the wetness state of the preceding, already disaggregated time steps), three enclosed ({0,1},1,1; {1,0},1,1 and {1,1},1,1), three ending {0,1},1,0; {1,0},1,0 and {1,0},1,0) and one isolated position ({0,0},1,0) for $b$=2.

### 3.1.4 Comparison of cascade model variants

Due to the new introduced position definitions a comparative overview of the different position classes (Fig. 4) and the resulting number of cascade model parameters (Table 2) is provided. For the sake of comparability, the empirical distribution function used for the x/(1-x)-splitting for $b$=2 is considered simplified as an additional parameter, since the complexity of the disaggregation method is higher with $f(x)$ than without. Nevertheless, it remains an empirical distribution function and is not a single parameter value.

From Table 2 it is visible, that the introduction of a position-dependency for $b$ =3 for cascade model B and the refinement of the position definition for cascade model C leads to an increase in cascade model parameters. Especially for method C the number of possible distributions of rainfall amounts in the already disaggregated time step before the current time step to disaggregate (see Fig. 4) leads to a strong increase of model parameters. Since all model parameters are estimated directly from observations (Carsteanu and Foufoula-Georgiou, 1996) as mentioned before, no parameter calibration is required and there is no problem with equifinality.

However, especially for the $b$ =3-splitting for the upper volume class in method C, the number of parameters is critical. Since only days with rainfall amounts higher than the $q_{0.998}$ quantile are taken into account, only a few days exist for the parameter estimation if the observed time series for parameter estimation is not long enough. This will lead to probabilities with $P$=0 for several splittings. While for some splittings $P$=0 seems reasonable from a physical interpretation (e.g. P(½/0/½)=0 is reasonable, since the highest daily rainfall amounts have no dry spell in between with a minimum of 8 h in the observed data set), for other probabilities this can result from the too small population for parameter estimation.



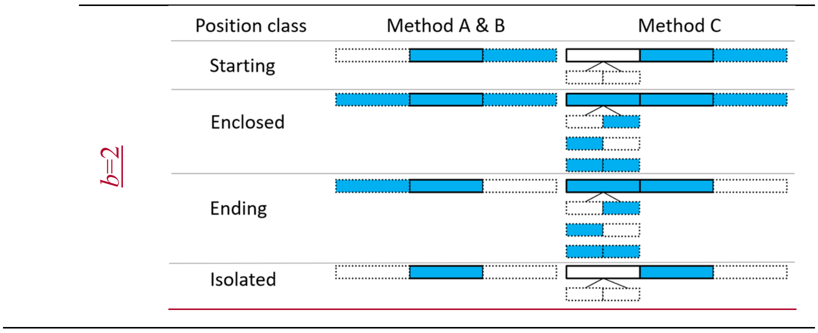



**Fig. 4: Comparison of position classes definition for methods A, B and C. For method A, no position classes differentiation is applied for *b*=3. The dashed boxes indicate the time steps, which are analysed regarding their wetness state for the definition of the position class. Blue boxes indicate a wet time step, white boxes a dry time step.**

~~Due to the new definition, the number of positions is extended from four in the non-preceding cascade model positions (starting, ending, enclosed, ending) to eight in the preceding cascade model: one starting ({0,0},1,1 with 0=dry and 1=wet and {} indicating the wetness state of the preceding, already disaggregated time steps), three enclosed ({0,1},1,1; {1,0},1,1 and {1,1},1,1), three ending {0,1},1,0; {1,0},1,0 and {1,0},1,0) and one isolated position ({0,0},1,0) for *b=2*.~~

## ~~3.2 Assurance of a minimum rainfall intensity~~

~~As mentioned in Sect. 1, the cascade model tends to generate too many time steps with too low intensities. To overcome the issue, two possible solutions are presented in the following section, namely 'minimum rainfall amount' (MRA) and 'mimicry of the measurement device' (MMD). However, if no modification is applied the disaggregation is referred to as standard.~~

~~By MRA, the cascade generator is modified dependent on the rainfall amount of the current coarse time step to disaggregate. This modification affects the disaggregation process in several ways. If the rainfall amount is smaller than twice the minimum rainfall amount *min* (defined by the minimum resolution of the measuring device of each time series) for a *b=2*-splitting, only 1/0- and 0/1-splittings are possible (beginning from this time step to all finer time steps resulting from it). If the rainfall amount of a time step is higher than this threshold, x/(1-x)-splittings are possible, but under the restriction~~ $\cancel{V \cdot x \geq min}$ ~~and~~ $\cancel{V \cdot (1-x) \geq min}$~~. For disaggregation steps with *b=3*, the cascade generator is affected in the same way with a threshold of~~ $\cancel{V \geq 3 \cdot min}$ ~~to enable all splittings and~~ $\cancel{V \geq 2 \cdot min}$ ~~to enable~~ $\cancel{(0/\frac{1}{2}/\frac{1}{2})}$~~-splittings, respectively.~~

~~By MMD, the cascade generator itself is not modified. After the disaggregation, the behaviour of a measurement device is imitated. Rainfall amounts smaller than the minimum resolution of the measurement device are summated in the chronological order of the time steps until the sum~~ $\cancel{S_{thr}}$ ~~exceeds this threshold. The former wet time steps with smaller intensities are set to 0 mm, while~~ $\cancel{S_{thr}}$ ~~is moved to the last time step of the summation. Afterwards,~~ $\cancel{S_{thr}}$ ~~is set back to 0 mm. This process is carried out over the whole disaggregated time series.~~

### 3.3 Resampling algorithm

A different way to increase the autocorrelation is a resampling of the disaggregated time series. In a resampling process, two elements (here: relative diurnal cycles of the disaggregated time series) are swapped to improve an objective function (here: minimizing the deviation of the autocorrelation function of the disaggregated time series from the observed time series). With the simulated annealing algorithm as a resampling method it is possible to find the global optimum of an objective function. Simulated annealing has been used before for the optimization of the autocorrelation of rainfall time series by Bárdossy (1998). However, these time series were simulated by a different rainfall generator. The resampling algorithm in this study is applied with the aim to improve the autocorrelation function of the disaggregated time series under the following restrictions:

a) The structure of position and volume classes in the disaggregated time series generated by the cascade model should be conserved.

b) The daily rainfall amount should be conserved exactly.

For a) the restriction is fulfilled by allowing only swaps of time series elements among subsets of the same position and volume class. Restriction b) is fulfilled by swapping only relative diurnal cycles as time series elements, which does not affect the daily rainfall amount.

The objective function of the simulated annealing algorithm applied in this study is:

$$O_{auto} = \sum_{i=1}^{NoLags} (r(i) - r(i)^*)^2 \tag{5}$$

The parameters indicated by * are the prescribed values for each lag from observed time series for each station, the other parameters are the current values. *NoLags* represents the number of lags analysed for the representation of the autocorrelation function. The number and selection of lags was carried out in a sensitivity analysis before by Föt (2015), resulting in 72 lags, whereby every second lag from lag 1 (5 min) to lag 144 (12 h) is taken into account (lag 1, lag 3,…, lag 143). After 12 hours, the values of the autocorrelation of the observed time series show an asymptotic behaviour indicating a very low process memory. As proven by Müller and Haberlandt (2018), the resampling does not affect the scaling behaviour of the disaggregated time series, because the total rainfall amount as well as the number of wet time steps are kept.

In a prior study (Legler, 2017) the effect of the resampling algorithm on the extreme rainfall values was analysed. Without taking the extreme values into account explicitly in the objective function, the resampling leads to a decrease of the extreme rainfall values. Since the extreme rainfall values are represented well after the disaggregation (Fig. 9~~7~~), they would be underestimated after the resampling. Since the occurrence date and the magnitude of the extreme rainfall values differs among the investigated durations, for their identification an event-independent, general scheme has to be applied in order to take them into account in the objective function. The applied scheme in this investigation is illustrated in Fig. 5~~3~~. A threshold intensity $I_{tr}$ is chosen for the whole time series, whereby the first and the last time step of each day exceeding $I_{tr}$

determine the event and its duration $D_{event}$. During the resampling, swaps are only allowed if the following restrictions R are fulfilled:

RI) The total rainfall amount of the extreme event must not decrease.

RII) The number of dry time steps inside the extreme event must not decrease.

If $I_{tr}$ is chosen too high, extreme rainfall events of higher durations and most often lower intensities are not considered. If $I_{tr}$ is chosen too low, too many rainfall events are considered as extreme events, which leads to a rejection of too many swaps during the resampling and hence only minor improvements of the autocorrelation function.

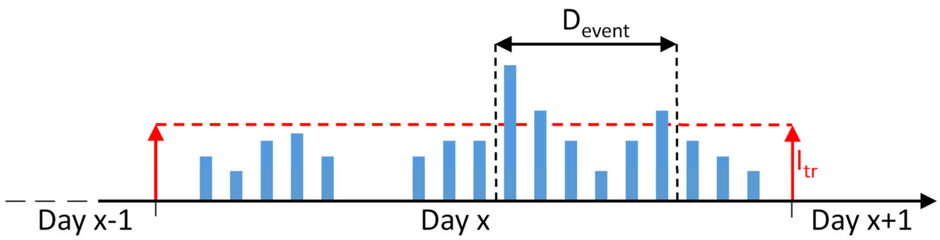



**Fig. 5~~3~~: Scheme for extreme rainfall event definition**

Hence, the choice of an appropriate $I_{tr}$ is essential for a successful resampling. Since it shall be possible to estimate $I_{tr}$ a priori without a calibration of this parameter, a transferable method was required. Müller and Haberlandt (2018) identified the existence of dry time steps inside extreme events of the disaggregated rainfall time series, while the observed extreme events consisted only of wet time steps. Since an event-based simulation of extreme rainfall events with $D$=30 min in an urban hydrological model led to satisfying results regarding flooding volume and combined sewer overflow volume in Müller and Haberlandt (2018), for the identification rainfall extreme events with $D$=60 min are considered in this study. For the observed time series of all stations, the average intensity of the extreme rainfall event with $D$=60 min and the empirical return period closest to $T_n$=1.5 years was calculated. It is assumed that the results regarding under- or overestimation for ~~T~~the return period $T_n$=1.5 years ~~is assumed to be~~ are representative for typical return periods for dimensioning purposes in urban hydrology ($T_n$={1, 2, 5, 10 years}, DWA-A 531 (2012)). The resulting average intensity $I_{tr}$=1.0 mm of the aforementioned extreme rainfall events is applied throughout the resampling and represents the 0.99 quantile for 25 % of all stations (ranging from ~0.987 to ~0.994 between all stations). Similar thresholds have been applied before in literature for tracking of convective cells in radar images ($I_{tr}$>0.7 mm, Handwerker (2002)).

Since the ~~amount~~ number of diurnal cycles is limited in the disaggregated time series, the degree of improvement is limited as well, which can be a serious problem, if only short daily rainfall time series are available. A possible solution is to enable the swapping of relative diurnal cycles between different realisations of disaggregated time series to increase the number of possible swap elements by additional realisations. Here the time series length was found to be sufficient and the resampling was carried out for each realisation separately.

The simulated annealing is carried out singular for each station as follows:

1. For each wet day the relative diurnal cycles are constructed. Subsets $y$ for each applied position-volume-class combination are created with $y=1,\dots, S$.
2. A subset $y$ is identified randomly. The probability for being identified is based on the number of elements $m$ in the subset: $$P_{y,i}=\frac{m_i}{\sum_{i=1}^{S} m_i} \quad (6)$$
3. Two days are drawn randomly from the identified subset, their diurnal cycles are swapped. If R I and R II are not fulfilled, the swap is retracted and the algorithm proceeds with step 2.
4. $O_{auto}$ (Eq. (5)) is updated.
5. The updated value $O_{auto,new}$ is compared with the former value $O_{auto,old}$. If $O_{auto,new} < O_{auto,old}$ the objective function value has improved and the swap is accepted.
6. If $O_{auto,\,new} \geq O_{auto,\,old}$ the swap is accepted with the probability $\pi$: $$\pi = \exp\left(\frac{O_{auto,old} - O_{auto,new}}{T_a}\right), \quad (7)$$ where $T_a$ is the annealing temperature that controls the acceptance of bad swaps. Local optima can be left by the acceptance of bad swaps and the global optimum can be found. The decrease of the annealing temperature during the resampling (see step 8) leads to a lower probability for accepting non-improving swaps, enabling the identification of the global optimum.
7. Steps 2-6 are repeated $K$ times.
8. Reduction of the annealing temperature: $$T_a = T_a \times dt \qquad \text{with } 0 < dt < 1 \quad (8)$$ After reducing the temperature, the algorithm proceeds to step 2.
9. Steps 7 and 8 are repeated until the algorithm converges, expressed by $M$ swaps which do not lead to an improvement of $O_{auto}$ higher than a certain threshold $thr_{O,auto}$.

~~For the sake of completeness t~~The following setup was chosen for the resampling: $T_{a,start}=1*10^{-4}$, $dt$=0.99, $K$=500, $M$=200 and $thr_{O,auto}=1*10^{-9}$.

The different variants of the cascade model can be combined with the resampling approach for the improvement of the autocorrelation. A summary of the combinations and their abbreviations used throughout the manuscript are presented in ~~Tab~~Fig. 2~~. 2~~.

### 3.4 Validation of the results

For the evaluation of the disaggregation process, the disaggregated rainfall time series are analysed regarding different event-based and continuous rainfall characteristics and their extreme values.

For an event-based evaluation, first the rainfall events are identified and then the characteristics of these events are determined. Event-based rainfall characteristics include wet and dry spell duration as well as wet spell amount. An event is hereby defined as a wet period enclosed by at least one 5 min time step without rainfall before and after the wet period.

For a continuous-based evaluation, the whole time series is considered, without differentiation into single events. As continuous time series characteristics, the average intensity, the fraction of dry intervals and the autocorrelation are analysed.

For the extreme rainfall event analyses, the event definition differs to ensure the independence of the extreme events. The definition depends on the extreme event duration under investigation. For extreme event durations shorter than 4 hours, a minimum of four dry hours before and after the event ensure the independence of the event (Schilling, 1984). For longer extreme event durations, the same duration as under investigation has to be dry before and after the event. To increase the population of extreme events, partial duration series are extracted from the time series instead of annual maxima. Partial duration series are similar to the peak-over-threshold approach, whereby the threshold is defined in order to select 3 extreme rainfall events on average per year. Since the lengths of the time series of the analysed stations differ, theoretical distribution functions are fitted to enable comparisons for the same return periods among the stations. Following the DWA-A 531 (2012), which is a technical standard in Germany, an exponential distribution is fitted to the median of the realisations for each station.

To enable comparisons of rainfall characteristics at the same location, observed 5 minute time series (*Obs*) are aggregated to daily values and then disaggregated back to 5 minute time series (*Dis*). A split-sampling into calibration and validation period was not carried out to keep the time series as long as possible for the parameter estimation (see also the discussion in Section 3.1.4).

The disaggregation is a random process. Depending on the choice of the random number generator initialization different realisations are generated. This uncertainty is taken into account by performing 30 realisations for each station. By 30 realisations the random behaviour of the disaggregation process is fairly well covered, based on analysing the impact on the mean and on the range of the event-based and continuous rainfall characteristics, which showed an asymptotic behaviour with increasing numbers of realisations.

For the validation the relative error *rE* and relative absolute error *rAE* are calculated to quantify the direction and the amount of the deviation of the rainfall characteristic *RC* with *i* as control variable ~~representing the single out of~~ of all ~~*n*~~ realisations *n*:

$$rE = \frac{1}{n} \times \sum_{i=1}^{n} \frac{RC_{Dis,i} - RC_{Obs,i}}{RC_{Obs,i}}, \tag{9}$$

$$rAE = \frac{1}{n} \times \sum_{i=1}^{n} \frac{|RC_{Dis,i} - RC_{Obs,i}|}{RC_{Obs,i}}, \tag{10}$$

## 4. Results

### 4.1 Modifications to cascade model

For an improved representation of the autocorrelation function two modifications of the multiplicative cascade models after Müller and Haberlandt (2018) have been analysed, namely method B and method C. For method B, the order of wet and dry 8 hour intervals is not assigned randomly as in Müller and Haberlandt (2015). Probabilities for each combination of wet and dry 8 hour-intervals are estimated, with a differentiation according to the position of the daily time step in the time series (starting, enclosed, ending or isolated).

The resulting probabilities are shown in Table 3, 4 and 5 (columns with position-dependent entries) in comparison to the position-independent probabilities estimated for method A (first column in each table). For starting positions, splittings with wet 8 hour-intervals at the end of a day have the highest probabilities (for both one and two wet intervals). For ending positions, a vice versa relationship can be identified with highest probabilities for wet 8 hour-intervals at the beginning of a day. For enclosed positions, probabilities for a wet 8 hour-interval at the beginning or ending of the day, so with a temporal connection to another wet day, are higher, if one interval is wet. All of these findings are similar to the findings from Olsson (1998) and Güntner et al (2001) for a splitting with *b=2*. Also, independent from the position, it can be identified that probabilities for two connected wet intervals (1-1-0 and 0-1-1) are higher than the combination with an enclosed dry time step (1-0-1). The probability, that three intervals are wet, is the highest for enclosed position and the lowest for isolated position.

The rainfall characteristics of the disaggregated time series are shown in Fig. 6~~4~~ in comparison to observations. A quantitative analysis of the deviations is provided in Table~~.~~ 6 with relative *rE* and absolute errors *rAE* (see Eq. 9 and 10) for the mean values of rainfall characteristics. ~~However, a complete overview including standard deviation and skewness values is provided in Appendix A.~~

Method A ~~without any modification to avoid too small rainfall intensities (Standard, neither MRA nor MMD)~~ represents the original model proposed by Müller and Haberlandt (2018) and will be referred to as reference for the evaluation~~ of the here tested modifications~~. Since the MMD approach is investigated for the first time, its influence on the rainfall characteristics of the disaggregated time series is analysed. If the too small rainfall intensities lower than the instrumental resolution of the measurement device are kept after the disaggregation (hence, MMD is not applied), the results are referred to as ‘A/B/C-standard’. If the too small rainfall intensities are eliminated by the MMD approach, the results are referred to as ‘A/B/C-MMD’.

While for method A-standard a slight overestimation for the average intensity is identified (*rE*=11 %), for wet spell duration (-3 %) and amount (8 %), dry spell duration (8 %), fraction of dry intervals (1 %) and lag-1 autocorrelation (-4 %) a good representation is achieved.

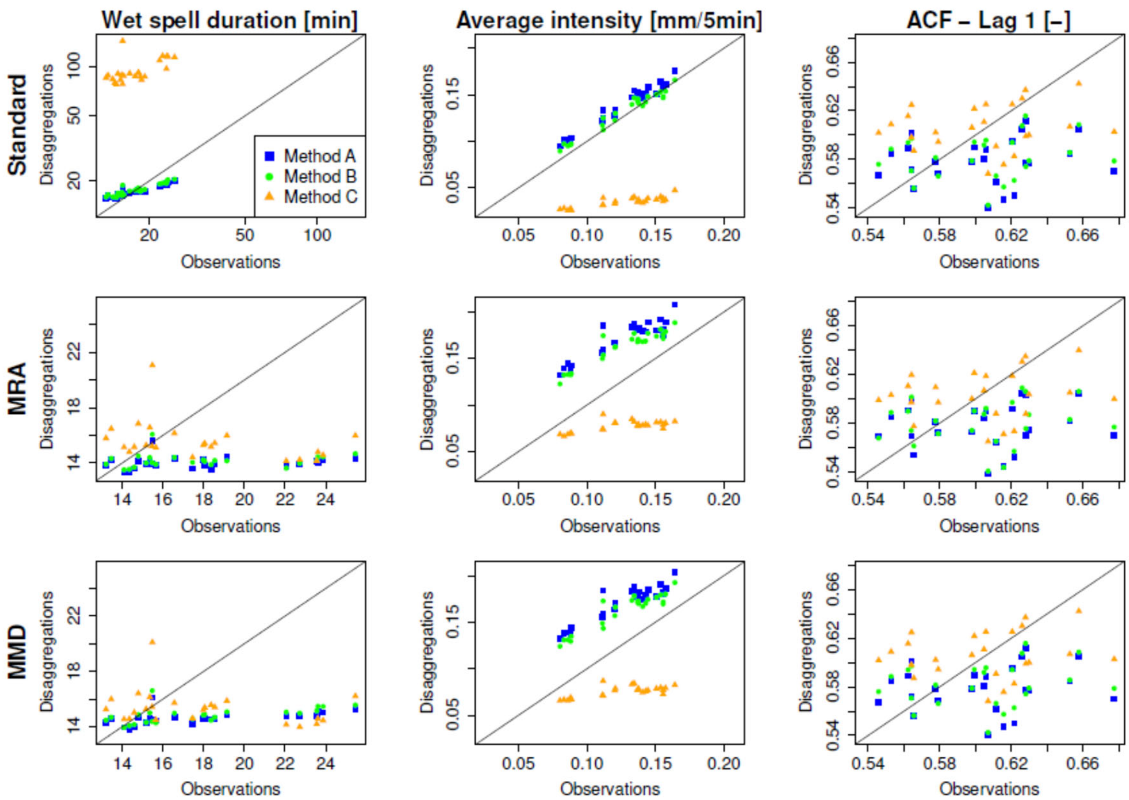
With the introduction of a position-dependence in the disaggregation step from daily values to 8 h-values in method B-standard an improvement of all rainfall characteristics can be achieved. The improvements of the wet and dry spell durations are direct consequences of the better representation of the wetness state of 8 h-intervals as is indicated by the parameter values in Table~~.~~ 3, 4 and 5. For the average intensity with $rE$=3 % a major improvement from an overestimation of $rE$=11 % (A-standard) is identified.

For method C-standard, a worsening of the majority of rainfall characteristics is identified. Wet spell duration is overestimated with $rE$=399 %. This is caused by the high probability for a *x/(1-x)*-splitting for enclosed boxes in the preceding cascade model, which decreases the probability for splitting one event into two events by the generation of dry time steps. This leads to a high number of wet time steps (underestimation of fraction of dry time steps $rE$=-15 %) and consequently to an underestimation of average rainfall intensities ($rE$=-71 %) due to the exact mass conservation of the cascade model.

Too small intensities can be avoided by the ~~modifications of the cascade model called MRA and~~ MMD approach. For method A and B, the exclusion of small rainfall intensities leads to a worsening of rainfall characteristics (Fig. 7~~5~~, Table~~.~~ 6). This indicates, that the before mentioned good representation of rainfall characteristics by method A-standard and B-standard is biased by wet time steps with rainfall amounts lower than the observed minimums (depending on the instrumental resolution of the measurement device). Since time steps with these rainfall intensities are negligible from a hydrological point of view, the line~~s~~ of ~~MRA and~~ MMD in Fig. 6~~4~~ provide a more useful insight into the disaggregated data. ~~Although differences between MRA and MMD exist (see e.g. wet spell duration for method A and B), these differences are small and will be discussed together if not mentioned explicitly otherwise.~~

The overestimation of the average rainfall intensity by methods A and B increased to $rE_{MMD\cancel{RA}}$=40 % and ~~30~~32 %, respectively, while the underestimation by method C is reduced to $rE_{MMD\cancel{RA}}$=-35~~3~~ %. An improvement for wet spell duration is also identified ($rE_{MMD\cancel{RA}}$=-16~~5~~ %). Although the fraction of dry intervals improved with ~~MRA and~~ MMD ($rE$=-3 %), a worsening of the dry spell duration is identified ($rE_{MMD\cancel{RA}}$=-47~~5~~ %), indicating higher fraction of short dry spells inside former events on a coarser time scale.

Method A and B result in similar values for wet spell duration as for method C for MMD~~, while for MRA the underestimation is slightly higher~~. For wet spell amount and duration, average intensity, dry spell duration and fraction of dry intervals a decrease in performance is identified by ~~MRA and~~ MMD in comparison to the standard approach.



Wet spell duration [min]
Average intensity [mm/5min]
ACF – Lag 1 [–]
Standard
MRA
MMD
Disaggregations
Observations
Method A
Method B
Method C

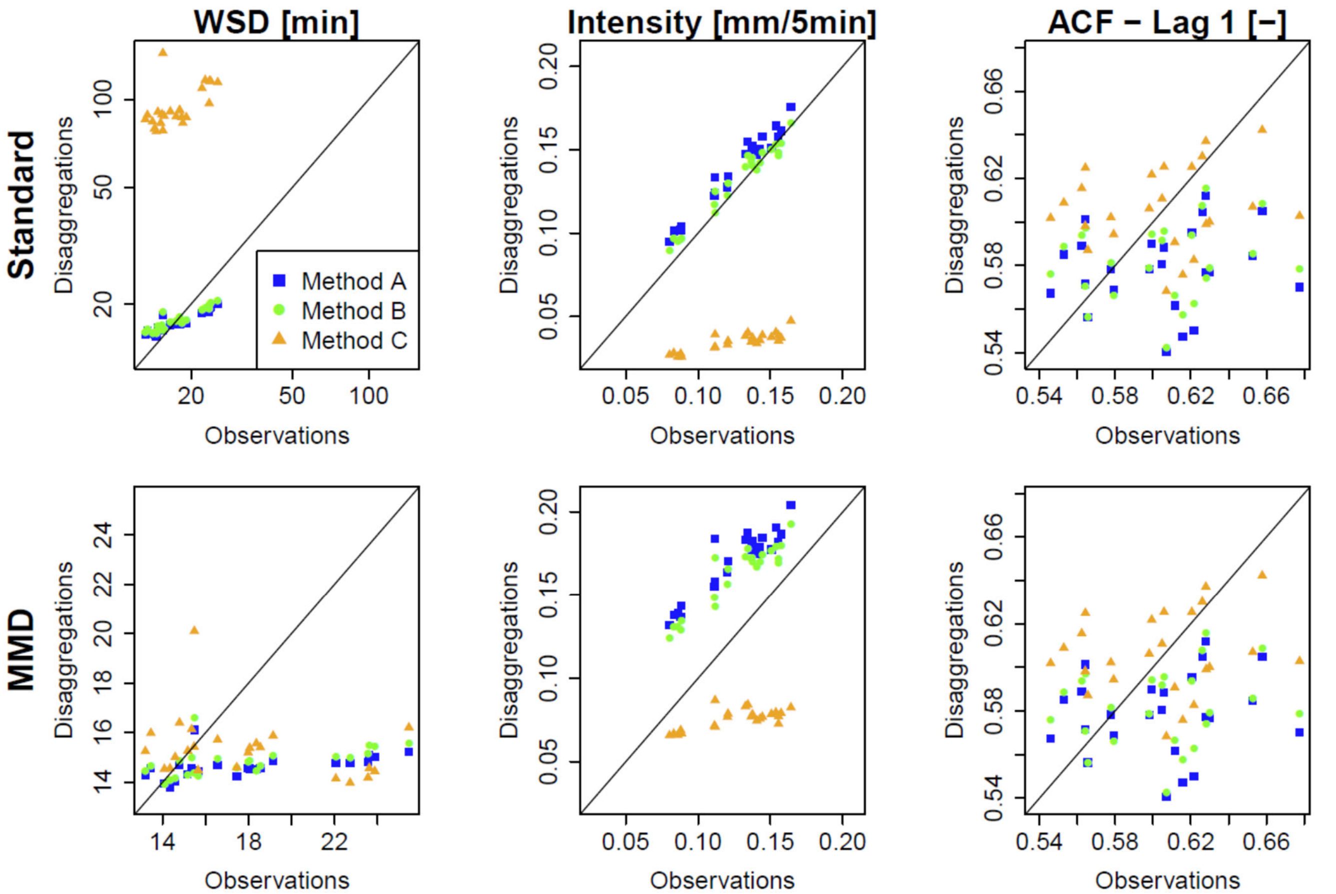



**Fig. 6~~4~~: Rainfall characteristics of observed and disaggregated time series as x-y plots for all 24 stations (WSD=wet spell duration, ACF=autocorrelation function; note the different scales for wet spell duration)**

Since the focus of this study is the improvement of the autocorrelation, the impact~~s~~ of ~~MRA and~~ MMD on method A, B and C ~~are~~ is investigated as well. From a visual inspection of the lag-1 autocorrelation in Fig. 6~~4~~, a systematic underestimation as mentioned in the Sect. 1 is not visible, since for some stations even overestimations occur. However, a comparison between observations and disaggregated time series resulting from different methods until lag 144 (representing a time shift of 720 min=12 h) shows differences and a clear underestimation by the disaggregation for station Braunlage (Fig. 7~~5~~). For other stations the relationship is similar, although for some the differences between method A and B are smaller. In Fig. 8~~6~~ the relative error between the median of the autocorrelation function of all 30 realisations for each method and the observed time series is shown for all stations regarding lag 1 (5 min), 6 (30 min) and 36 (180 min). Independently of the applied methods, the deviation is increasing from lag 1 to lag 6, while for lag 36 the deviation has decreased. Also, the range of deviations is decreasing for an increasing number of lags. This is visually confirmed by the results for station Braunlage (Fig. 7~~5~~), where the autocorrelation of the disaggregated time series decreases strongly with the first lags, while it decreases much smoother for the observed time series. The choice of the disaggregation method (method A, B or C) has a higher impact on the resulting autocorrelation than the choice of treatment of the too small rainfall intensities (Standard~~, MRA and~~ or MMD). In fact, the MMD approach has only a slight effect on the autocorrelation function values. The smallest deviations of the autocorrelation function are achieved with method C, independent from the treatment of the too small rainfall intensities.

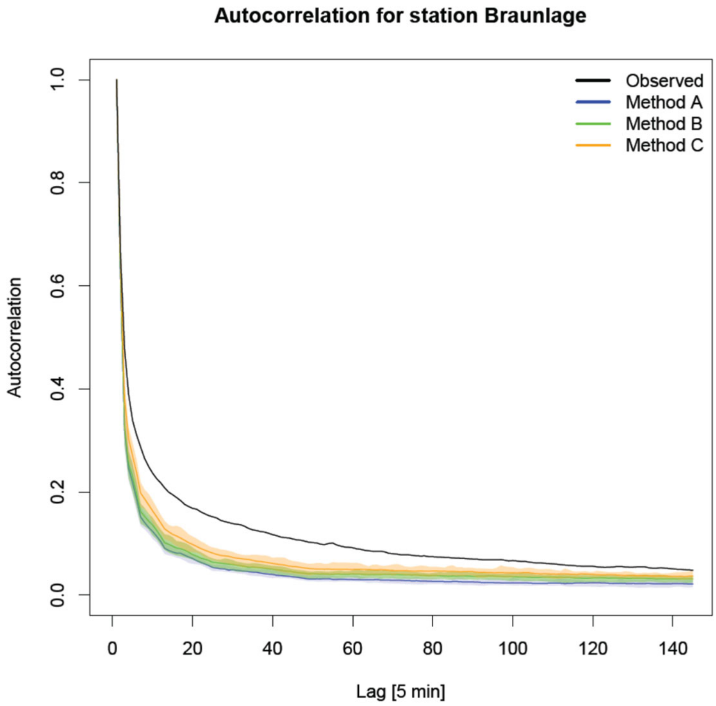
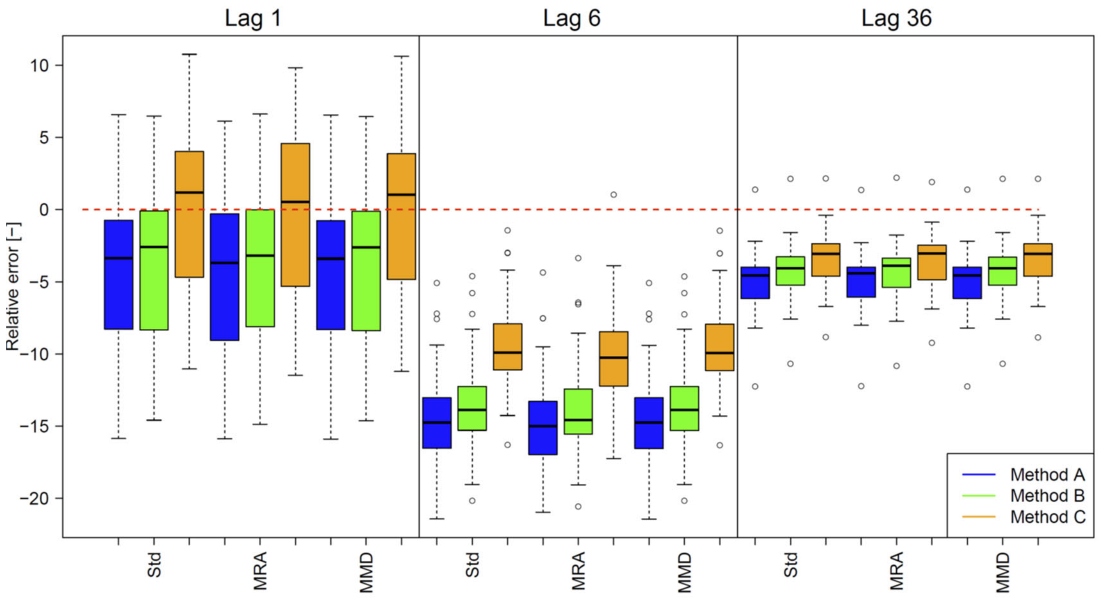



**Fig. 7~~5~~: Autocorrelation of observed and disaggregated time series using the standard approach for each method with no modification regarding too small rainfall intensities. The range for each method results from 30 realisations, the solid line represents the median.**



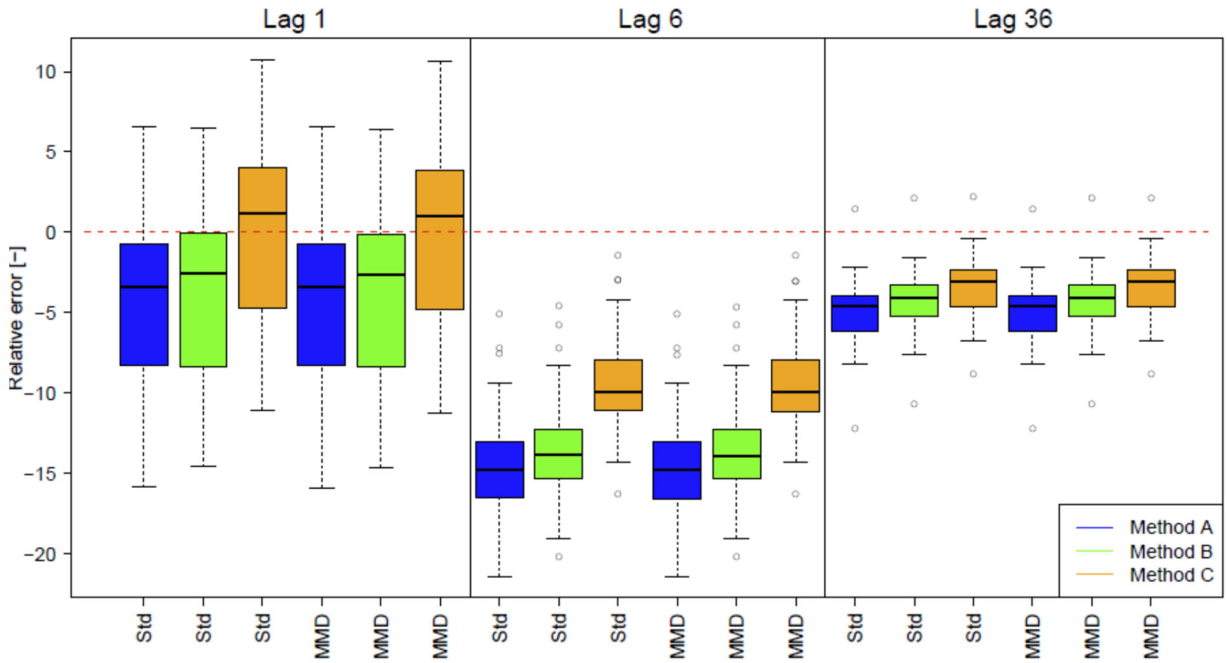




**Fig. 8~~6~~: Deviations of autocorrelation from disaggregated to observed time series as relative error for lags 1, 6 and 36. The red dashed line indicates a *rE*=0 (Std is used as abbreviation for Standard)**

The results of the extreme rainfall value analysis are illustrated in Fig. 9~~7~~ for two durations *D* (5 minutes and 1 hour) and two return periods *T* (1 year and 5 years). For the extreme events, only between methods A, B and C is differentiated. The modifications regarding the minimum rainfall intensity are not taken into account since they do not affect the rainfall extreme events.

For a return period of $T$=1 year extreme rainfall values are slightly overestimated by less than $rE$=10 % for the half of all stations and less than approximately $rE$=20 % for 75 % of all stations for both analysed durations, independent of the applied modification of the cascade model. For $T$=5 years, the range of results is increasing, leading to a worse representation in comparison to $T$=1 year. While for $D$=5 min a slight overestimation of approx. $rE$=10 % for half of all stations can still be identified, for $D$=1 hour an underestimation of $rE$=50 % is identified for half of all stations. However, increasing deviations with increasing return periods can be expected, since for a few of the time series with lengths of only 9 years the return period is limited to $T$=3 years (1/3 of time series length) to ensure plausibility from a hydrological point of view.

Nevertheless, it should be noted that over all return periods and durations, method C lead to the smallest range of relative errors over all stations in combination with the best fit to the distribution of the observed extreme rainfall values.

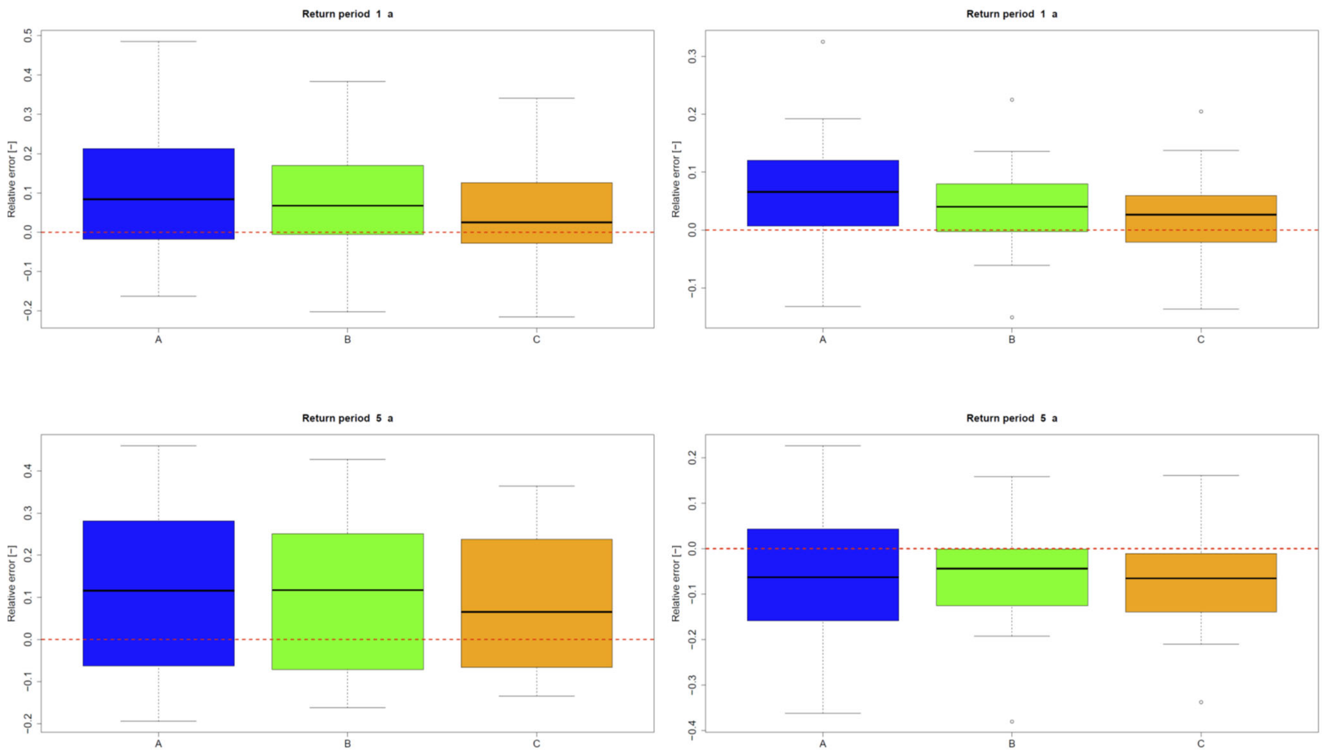



**Fig. 9~~7~~: Mean relative errors of extreme values of the disaggregated time series for all stations (the dashed line represents an error of 0). Results are shown for durations of 5 min (left) and 1 h (right) and for return periods of 1 year (top) and 5 years (bottom).**

### 4.2 Resampling results

For the resampling, only disaggregated time series ~~disaggregated~~ modified by the MMD approach ~~-modification~~ are used due to their slight better representation of the autocorrelation. The autocorrelation of the disaggregated time series before and after the resampling are shown in Fig. 10~~8~~ for lag 1, lag 6 and lag 36. A general increase of the autocorrelation along with smaller deviations for the median of all stations compared to before the resampling can be identified for all three methods A, B and C. Only for the lag 1-autocorrelation of the rainfall time series disaggregated with method C does the resampling lead to a worsening regarding the median value. However, the range of the lag 1-autocorrelation results is reduced, indicating that the under- and overestimations were reduced by the resampling approach.

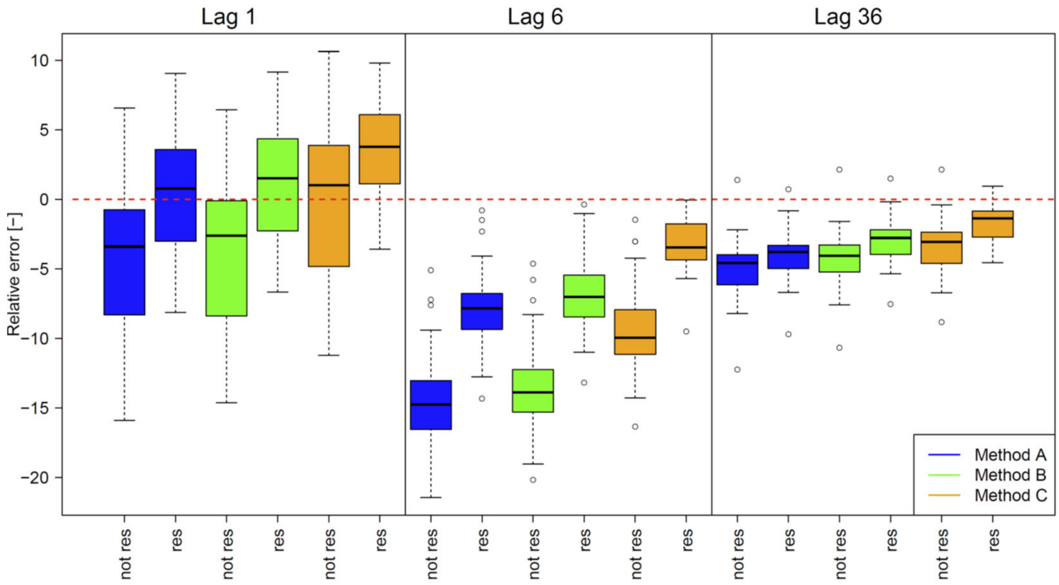



**Fig. 10~~8~~: Deviations of autocorrelation from disaggregated to observed time series before and after the resampling as relative error for lags 1, 6 and 36. All results are based on the MMD approach. The red dashed line indicates a *rE*=0, results for the resampled time series are labelled with 'res'.**

As mentioned before, the improvement of the autocorrelation depends on the chosen threshold for extreme rainfall value definition, $I_{tr}$. An increase of $I_{tr}$ leads to a decrease of the number of rejected swaps during the resampling, since less time steps are involved in the extreme value analysis. An unrealistically high value of $I_{tr}$ (identical with leaving out both restrictions RI and RII regarding extreme rainfall values) leads to almost perfect fits for lag 1 and lag 36 ($|rE|<1$ % for the majority of the stations), and for lag 6 deviations up to $|rE|<3.5$ % occur (not shown here). However, the extreme rainfall values are underestimated strongly if $I_{tr}$ is chosen too high.

Hence, both restrictions RI and RII are applied during the resampling by the choice of $I_{tr}$=1 mm. In Fig. 11~~9~~, the extreme rainfall event series for station Osnabrück is shown for *D*=5min. Although the extreme event series changed slightly after the resampling, the overall extreme series characteristics regarding range, under- and overestimation in comparison to the observations remain the same for all return periods.

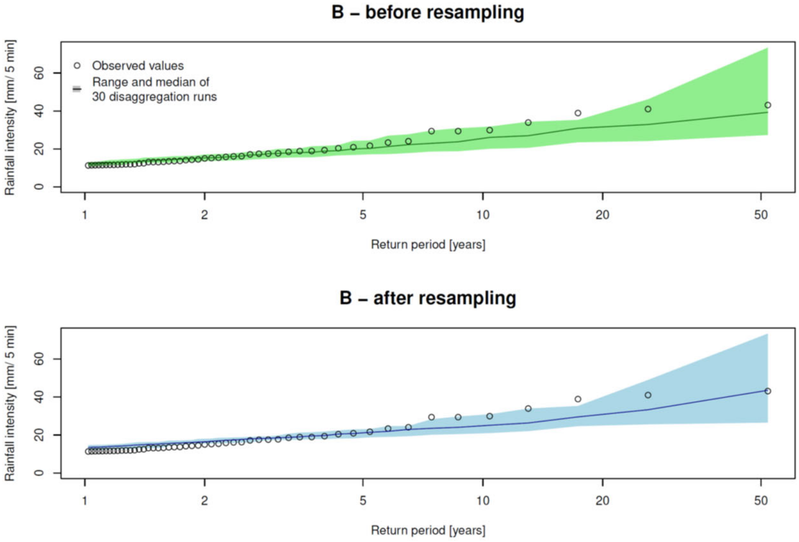



**Fig. 11~~9~~ Extreme rainfall values for $D$=5 min for station Osnabrück based on 30 disaggregation realisations with method B before (upper part) and after (lower part) the resampling**

For extreme rainfall events with longer durations ($D$={15 min, 1 h, 2 h}) the impact of the resampling is quantified in Table 7. The impact of the resampling depends on the analysed duration of the extreme rainfall events. While for $D$=15 min the median of $rE$ has decreased after the resampling with smaller $|rE|$ for the smaller return periods ($T_n$={1, 2 years}) and higher $|rE|$ for $T_n$=10 years, for $D$=2 h the median of $rE$ has increased after the resampling with higher $|rE|$ for the smaller return periods and smaller $|rE|$ for $T_n$=10 years.

Since these findings are independent from the disaggregation method, the differences are caused only by the resampling. Extreme rainfall events with $D$=15 min represent convective events with only a few wet time steps preceding and succeeded by dry time steps. Due to the short event duration, the possibility of dry time steps in between is small and RI is the active restriction, which requires the total rainfall amount not to decrease, resulting in an increase. Extreme rainfall events with $D$=2 h originate from long-lasting, stratiform events with a high fraction of wet time steps in the current day. Since this fraction of wet time steps can also be found in the disaggregated time series, the rainfall amount will be distributed on more wet time steps to fulfil the active restriction RII to increase the autocorrelation.

However, the majority of $rE$ values presented in Table 7 is smaller than 10 %, which indicates a good representation of the extreme rainfall events in the disaggregated time series in general, independent of the application of the resampling algorithm.

## 5. Discussion

### 5.1 Impact of the cascade model modifications

The two new introduced micro-canonical cascade model variants methods B and C differ regarding their way of how the disaggregation process depends on a position of a wet time step in a rainfall time series and how it is defined. Both, the position dependency in the first disaggregation step with $b$=3 for method B and the position definition after Lombardo et al. (2012, 2017) for method C, are based on additional parameters of the disaggregation model (see Table 2). However, all new parameters are process-based and physically interpretable, since they describe the rainfall dependency of past time steps. Hence, an improvement of the autocorrelation, which describes the process memory, was expected. While method B differs from method A only in the first disaggregation step, a smaller improvement of the autocorrelation can be identified, while method C differs in every disaggregation step and thus a higher improvement is identified.

As all parameters of the micro-canonical cascade model, including the newly introduced parameters can be estimated from the aggregation of observed high-resolution rainfall time series (Carsteanu and Foufoula-Georgiou, 1996), no additional calibration has to be carried out. To reduce the increase of number of parameters by method B and C, several possibilities exist. Olsson (1998), Güntner et al. (2001) and Müller and Haberlandt (2018) identified similarities between cascade model parameters of different position classes which can be used for simplification. These similarities are e.g. P(0/1) for starting and P(1/0) for ending positions (and vice versa) as well as P(0/1) and P(1/0) for both, enclosed positions and isolated positions. Another possibility is to apply a semi-bounded cascade model instead of a bounded cascade model. While in a bounded cascade model for each step of the disaggregation process the corresponding parameter set is used (as it is done in this study), in a semi-bounded cascade model the same parameter set could be applied over a range of disaggregation levels as long as a mono-fractal scaling behaviour can be assumed. Based on Veneziano et al. (2006) typical ranges for mono-fractal behaviour are from daily to hourly resolution and from hourly to 5 minute resolution.

It should also be noted that the analyses of only the lag-1 autocorrelation is not sufficient, since it provides a limited insight into the process memory. Here, for some stations an overestimation of the lag-1 autocorrelation was identified, but underestimations for lag-6 and lag-36. Hence, a multi-lag analyses is recommended for further studies.

Especially for method C, the general problem of the micro-canonical cascade model of generating time steps with too small rainfall intensities (Molnar & Burlando, 2005, Müller and Haberlandt, 2018) worsened. The MMD approach is introduced to solve this issue. ~~Two modifications, MRA and MMD, are introduced and analysed to solve this issue. While MRA affects the disaggregation process itself by changing the branching generator for rainfall amounts lower than a chosen threshold (for $b$=2 and $b$=3 it is two and three times the minimum rainfall amount, respectively),~~ MMD simulates the behaviour of a measurement device (minimum rainfall amount required to cause a registration) after the disaggregation process, eliminating too small rainfall intensities by summing them up to future time steps until the minimum rainfall amount is achieved or exceeded. The ~~choice between MRA and~~ MMD approach has a smaller impact on the resulting rainfall characteristics than the choice of method A, B or C. However, the application of MMD leads to disaggregated time series with a slight better representation of the autocorrelation function. Hence, for the subsequent analyses only the MMD approach is considered. This selection has ~~two~~ also the additional advantages~~. The cascade generator has not to be changed from its original version in Olsson (1998) and all rainfall splittings are possible throughout the whole disaggregation process, independently from the rainfall amount of the current time step to disaggregate. With the MMD modification,~~ that the process of the rainfall measurement itself is simulated.

### 5.2 Impact of the resampling

After the disaggregation process a subsequent resampling approach as post-processing strategy was investigated to improve the autocorrelation ~~was investigated~~ as well. While the generated time series structure (defined by the position-volume-class combinations) is conserved during the resampling process, a special focus has to be given to the conservation of the extreme rainfall values. Without this focus, the resampling algorithm aims to swap diurnal cycles in a way to distribute high daily rainfall amounts on many wet time steps to generate less intense events, which leads to an underestimation of the extreme rainfall values. The universal definition of extreme rainfall values introduced here is required to conserve these extreme values a priori without any information about their date of occurrence or their magnitude. This definition and the connected restrictions for the resampling can be modified in multiple ways to improve the conservation. For example, the applied threshold intensity $I_{tr}$ can be based on a different or an additional (required) return period or duration. Also, a definition of $I_{tr}$ as a quantile of all wet time steps of a disaggregated time series instead of an absolute value would be helpful if there is a high variation of mean rainfall intensities among the investigated stations for an extreme event with a certain duration and return period (which was not the case in this study). However, the extreme values were conserved during the resampling process. The autocorrelation was improved for almost all lags, independent of whether method A, B or C was applied before for the disaggregation. Also, a higher improvement was achieved by the resampling than by the modifications of method B or C.

### 5.3 Study limitations

This study is focused on the methodological development of the micro-canonical cascade model and on the ~~subsequent~~ improvement of the disaggregated time series by a resampling approach as post-processing strategy. Hence, the study is limited in several aspects, which will be stated below.

First of all, the rainfall data set, with 24 rain gauges, is rather small. Although the study area covers different topographical region and climate classes, the resulting time series characteristics are similar and do not cover a wide range. A generalization of the results has to be proven for regions which are very different from this study area. To draw general conclusions from the point of comparative hydrology future research should include rain gauges from different climate regions and topologies.

Second, based on the similar rainfall characteristics and extreme values, the introduction of the universal extreme value definition was possible and representative for all stations in the study. If stations from different climate regions and topologies are studied as recommended before, it has to be proven i) if the introduced universal extreme value definition still has the potential to conserve the extreme rainfall values throughout the resampling process and ii) if $I_{tr}$ has to be redefined (see also the discussion in Sect. 5.2).

Third, as mentioned in the Sect. 1, Pearson's autocorrelation is a measure of linear dependency. It only captures the complete dependence structure between random variables if they are jointly Gaussian. An alternative criterion would be Spearman's rank correlation (capturing monotonic but not necessarily linear relationships). In both cases, autocorrelation as a function of lags is only meaningful in the context of second-order stationary stochastic processes (or weakly stationary processes). Rainfall intensities are most likely not normally distributed. Also rainfall time series present a mixture of processes due to the high intermittency of rainfall amplified by the disaggregation process, changing between the two states of rainfall occurrence and non-occurrence. Still, every kind of autocorrelation measurement can provide a measure for the similarity of e.g. two time series. The Pearson's autocorrelation coefficient is widely used for autocorrelation analyses in hydrology. It is applied in this study to achieve a comparable similarity in the disaggregated time series as it is estimated from the observations. Besides the mixture of processes and the limitations of Pearson's autocorrelation as a measure of dependence, the Hurst-phenomenon might also offer an additional perspective for the analysis at hand (see Koutsoyiannis (2009) for an introduction).

Fourth, although method C is based on a finding in Lombardo et al. (2012, 2017), the disaggregation method differs from the additive cascade model in Lombardo et al. (2012, 2017). Hence, the by Lombardo et al. identified problem of non-stationarity of the disaggregation is not solved by the introduced cascade model variants and remains an open challenge for further studies.

Finally, a comment on the applied resampling algorithm. Simulated annealing was implemented in a computationally efficiently way suggested by Bárdossy (1998). After each swap the objective function is not completely newly calculated, rather updated only for the modified elements of the time series affected by the swap. Nevertheless, the resampling process remains very time-demanding, depending on the chosen parameter setup. More recently published optimization algorithms are very promising regarding less computational times e.g. the quantum annealing approach (Heim et al., 2015, Crosson and Harrow, 2016), enabling the optimization of longer disaggregated rainfall time series or more realisations in the same time.

## 6. Conclusions

Three variants of the micro-canonical cascade model (method A-reference from Müller and Haberlandt (2018), B and C) were assessed regarding their ability to represent the autocorrelation in the disaggregated, 5 minute rainfall time series, starting from daily totals. The methods differ regarding the position dependency in the first disaggregation step and the definition of a wet time step during the disaggregation process. The study was carried out for 24 stations in Lower Saxony, Germany, and results were analysed additionally for continuous and event-based characteristics as well as extreme rainfall values. The following conclusions are drawn based on the results:

1. The introduction of a position dependency in the first disaggregation step (method B) and especially the introduction of the position-dependency (method C) after Lombardo et al. (2012, 2017) lead to an improvement of the autocorrelation.
2. While method A and B lead to quite similar event-based and continuous rainfall characteristics, the results from method C differ significantly.
3. Method C leads to a high fraction of time steps with too small rainfall intensities in the disaggregated time series.

To avoid time steps with too small rainfall intensities, ~~two approaches were analysed, i) the conservation of a minimum rainfall amount (MRA) during the disaggregation approach and ii)~~ a process to ~~the~~ mimicry the behaviour of a measurement device (MMD) was applied after the disaggregation process~~. Both, MRA and MMD, were applied~~ in combination with method A, B and C. ~~The resulting rainfall characteristics differ only slightly.~~ For the following investigations, only method combinations with MMD were analysed, since the results indicated a slight better representation of the autocorrelation function.

After the disaggregation process the resampling algorithm Simulated Annealing was applied to improve the autocorrelation. The following conclusions are drawn:

4. The resampling leads to an improvement of the autocorrelation, independent of the applied disaggregation method or the investigated lag.
5. The improvement of the autocorrelation by the resampling was higher than by the choice of the cascade model modification.
6. The extreme rainfall values have to be considered during the resampling, otherwise they will be underestimated after the resampling process.
7. With the newly introduced universal definition the extreme rainfall events can be considered without the a priori knowledge of their occurrence and magnitude. Hence, the extreme rainfall values are represented after the resampling process as well as before.

The overall best representation of the autocorrelation was achieved by method C in combination with ~~a subsequent~~ a resampling approach as post-processing strategy. Urban hydrological simulations would provide additional information about the impact of the different disaggregation methods and the resampling process on simulated hydrographs and flood events, but this is beyond the scope of this study.

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**Competing interests**

The author declares that he has no conflict of interest.

**Code/Data availability**

The rainfall data is accessible from the Climate Data Center web portal of the German Weather Service (https://cdc.dwd.de/portal/). The rainfall disaggregation program as well as the resampling program are both written in Fortran, so only executable files can be shared. However, the author is happy to share them on request.

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Tab. 1: Attributes of all 24 rainfall stations, based on a temporal resolution of 5 minutes.

| ID | Name | Altitude [m.a.s.l.] | Mean annual precipitation [mm] | Fraction of wet 5 minute-intervals [%] | Average wet spell duration [min] | Average wet spell amount [mm] | Average dry spell duration [min] | Autocorrelation lag-1 [-] |
|---|---|---|---|---|---|---|---|---|
| 1 | Braunlage | 607 | 1397 | 8.1 | 15.5 | 0.51 | 175.3 | 0.66 |
| 2 | Braunschweig-Voel. | 81 | 638 | 4.4 | 15.7 | 0.43 | 336.8 | 0.62 |
| 3 | Cuxhaven | 5 | 869 | 6.2 | 19.1 | 0.51 | 291.9 | 0.61 |
| 4 | Diepholz | 39 | 690 | 4.6 | 15.2 | 0.43 | 314.8 | 0.56 |
| 5 | Emden | 0 | 825 | 5.2 | 15.5 | 0.47 | 281.2 | 0.55 |
| 6 | Freiburg/Elbe | 2 | 888 | 6.4 | 18.5 | 0.49 | 272.9 | 0.57 |
| 7 | Gardelegen | 47 | 581 | 6.2 | 22.7 | 0.40 | 340.2 | 0.63 |
| 8 | Göttingen | 167 | 631 | 4.3 | 14.1 | 0.40 | 315.3 | 0.62 |
| 9 | Hannover | 55 | 641 | 3.9 | 13.2 | 0.41 | 323.0 | 0.63 |
| 10 | Harzgerode | 404 | 612 | 7.3 | 23.9 | 0.38 | 304.3 | 0.65 |
| 11 | Jork-Moorende | 1 | 727 | 5.7 | 18.4 | 0.44 | 302.0 | 0.58 |
| 12 | Leinefelde | 356 | 942 | 8.0 | 25.5 | 0.57 | 291.1 | 0.60 |
| 13 | Lingen | 22 | 789 | 5.5 | 16.6 | 0.46 | 286.6 | 0.60 |
| 14 | Lüchow | 17 | 569 | 3.9 | 14.3 | 0.39 | 349.3 | 0.61 |
| 15 | Magdeburg | 76 | 496 | 5.5 | 22.1 | 0.38 | 373.3 | 0.62 |
| 16 | Norderney | 11 | 744 | 4.5 | 14.6 | 0.46 | 309.5 | 0.56 |
| 17 | Oldenburg | 11 | 809 | 6.4 | 18.1 | 0.43 | 263.1 | 0.63 |
| 18 | Osnabrück | 95 | 874 | 5.4 | 14.8 | 0.45 | 258.3 | 0.56 |
| 19 | Bad Salzuflen | 135 | 825 | 5.0 | 13.5 | 0.42 | 253.0 | 0.63 |
| 20 | Soltau | 76 | 804 | 5.3 | 15.4 | 0.44 | 274.1 | 0.61 |
| 21 | Uelzen | 50 | 643 | 5.5 | 17.5 | 0.39 | 300.1 | 0.58 |
| 22 | Ummendorf | 162 | 549 | 5.9 | 23.6 | 0.41 | 367.2 | 0.60 |
| 23 | Wendisch Evern | 62 | 686 | 5.8 | 18.0 | 0.40 | 290.2 | 0.55 |
| 24 | Wernigerode | 234 | 625 | 7.1 | 23.6 | 0.39 | 305.1 | 0.68 |

| ~~ID~~ | ~~Name~~ | ~~Altitude [m.a.s.l.]~~ | ~~Mean annual precipitation [mm]~~ | ~~Fraction of wet 5 minute-intervals [%]~~ | ~~Average wet spell duration [min]~~ | ~~Average wet spell amount [mm]~~ | ~~Average dry spell duration [min]~~ |
|---|---|---|---|---|---|---|---|
| ~~1~~ | ~~Braunlage~~ | ~~607~~ | ~~1397~~ | ~~8.1~~ | ~~15.5~~ | ~~0.51~~ | ~~175.3~~ |
| ~~2~~ | ~~Braunschweig-Voel.~~ | ~~81~~ | ~~638~~ | ~~4.4~~ | ~~15.7~~ | ~~0.43~~ | ~~336.8~~ |
| ~~3~~ | ~~Cuxhaven~~ | ~~5~~ | ~~869~~ | ~~6.2~~ | ~~19.1~~ | ~~0.51~~ | ~~291.9~~ |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ~~4~~ | ~~Diepholz~~ | ~~39~~ | ~~690~~ | ~~4.6~~ | ~~15.2~~ | ~~0.43~~ | ~~314.8~~ |
| ~~5~~ | ~~Emden~~ | ~~0~~ | ~~825~~ | ~~5.2~~ | ~~15.5~~ | ~~0.47~~ | ~~281.2~~ |
| ~~6~~ | ~~Freiburg/Elbe~~ | ~~2~~ | ~~888~~ | ~~6.4~~ | ~~18.5~~ | ~~0.49~~ | ~~272.9~~ |
| ~~7~~ | ~~Gardelegen~~ | ~~47~~ | ~~581~~ | ~~6.2~~ | ~~22.7~~ | ~~0.40~~ | ~~340.2~~ |
| ~~8~~ | ~~Göttingen~~ | ~~167~~ | ~~631~~ | ~~4.3~~ | ~~14.1~~ | ~~0.40~~ | ~~315.3~~ |
| ~~9~~ | ~~Hannover~~ | ~~55~~ | ~~641~~ | ~~3.9~~ | ~~13.2~~ | ~~0.41~~ | ~~323.0~~ |
| ~~10~~ | ~~Harzgerode~~ | ~~404~~ | ~~612~~ | ~~7.3~~ | ~~23.9~~ | ~~0.38~~ | ~~304.3~~ |
| ~~11~~ | ~~Jork-Moorende~~ | ~~1~~ | ~~727~~ | ~~5.7~~ | ~~18.4~~ | ~~0.44~~ | ~~302.0~~ |
| ~~12~~ | ~~Leinefelde~~ | ~~356~~ | ~~942~~ | ~~8.0~~ | ~~25.5~~ | ~~0.57~~ | ~~291.1~~ |
| ~~13~~ | ~~Lingen~~ | ~~22~~ | ~~789~~ | ~~5.5~~ | ~~16.6~~ | ~~0.46~~ | ~~286.6~~ |
| ~~14~~ | ~~Lüchow~~ | ~~17~~ | ~~569~~ | ~~3.9~~ | ~~14.3~~ | ~~0.39~~ | ~~349.3~~ |
| ~~15~~ | ~~Magdeburg~~ | ~~76~~ | ~~496~~ | ~~5.5~~ | ~~22.1~~ | ~~0.38~~ | ~~373.3~~ |
| ~~16~~ | ~~Norderney~~ | ~~11~~ | ~~744~~ | ~~4.5~~ | ~~14.6~~ | ~~0.46~~ | ~~309.5~~ |
| ~~17~~ | ~~Oldenburg~~ | ~~11~~ | ~~809~~ | ~~6.4~~ | ~~18.1~~ | ~~0.43~~ | ~~263.1~~ |
| ~~18~~ | ~~Osnabrück~~ | ~~95~~ | ~~874~~ | ~~5.4~~ | ~~14.8~~ | ~~0.45~~ | ~~258.3~~ |
| ~~19~~ | ~~Bad Salzuflen~~ | ~~135~~ | ~~825~~ | ~~5.0~~ | ~~13.5~~ | ~~0.42~~ | ~~253.0~~ |
| ~~20~~ | ~~Soltau~~ | ~~76~~ | ~~804~~ | ~~5.3~~ | ~~15.4~~ | ~~0.44~~ | ~~274.1~~ |
| ~~21~~ | ~~Uelzen~~ | ~~50~~ | ~~643~~ | ~~5.5~~ | ~~17.5~~ | ~~0.39~~ | ~~300.1~~ |
| ~~22~~ | ~~Ummendorf~~ | ~~162~~ | ~~549~~ | ~~5.9~~ | ~~23.6~~ | ~~0.41~~ | ~~367.2~~ |
| ~~23~~ | ~~Wendisch Evern~~ | ~~62~~ | ~~686~~ | ~~5.8~~ | ~~18.0~~ | ~~0.40~~ | ~~290.2~~ |
| ~~24~~ | ~~Wernigerode~~ | ~~234~~ | ~~625~~ | ~~7.1~~ | ~~23.6~~ | ~~0.39~~ | ~~305.1~~ |

Tab. 2: Comparison of model parameters for methods A. B, and C in dependence of the applied branching number.

| | | Method | | |
|---|---|---|---|---|
| | | A | B | C |
| *b*=3 | Basic parameters | 3 | 7 | 7 |
| | Position classes | - | 4 | 16 |
| | Volume classes | 2 | 2 | 2 |
| | **Parameters per disaggregation step** | **6** | **56** | **224** |
| *b*=2 | Basic parameters | 4 | 4 | 4 |
| | Position classes | 4 | 4 | 8 |
| | Volume classes | 2 | 2 | 2 |
| | **Parameters per disaggregation step** | **32** | **32** | **64** |

~~Tab. 2: Datasets as result of combinations of cascade model variants, modifications for minimum rainfall intensities and application of the resampling algorithm~~

| ~~Cascade model variant~~ | ~~A~~ | | | | ~~B~~ | | | | ~~C~~ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ~~Minimum rainfall intensity modifications~~ | ~~Standard~~ | ~~MRA~~ | ~~MMD~~ | | ~~Standard~~ | ~~MRA~~ | ~~MMD~~ | | ~~Standard~~ | ~~MRA~~ | ~~MMD~~ | |
| ~~Application of the resampling algorithm~~ | ~~no~~ | ~~no~~ | ~~no~~ | ~~yes~~ | ~~no~~ | ~~no~~ | ~~no~~ | ~~yes~~ | ~~no~~ | ~~no~~ | ~~no~~ | ~~yes~~ |
| ~~Nomenclature~~ | ~~A-Std~~ | ~~A-MRA~~ | ~~A-MMD~~ | ~~A-MMD-res~~ | ~~B-Std~~ | ~~B-MRA~~ | ~~B-MMD~~ | ~~B-MMD-res~~ | ~~C-Std~~ | ~~C-MRA~~ | ~~C-MMD~~ | ~~C-MMD-res~~ |

Tab. 3: Position-dependent and –independent probabilities for one wet 8 hour interval in the uniform splitting (mean of all 24 stations for lower volume class, all values in percent [%]). The combination of ‘1’ (wet) and ‘0’ (dry) illustrates the order of wet and dry 8 hour intervals in a day.

| **One wet interval, position-independent** | **One wet interval, position dependent** | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **starting position** | | | | **enclosed position** | | | | **ending position** | | | | **isolated** | | | |
| | **001** | **010** | **100** | **Σ** | **001** | **010** | **100** | **Σ** | **001** | **010** | **100** | **Σ** | **001** | **010** | **100** | **Σ** |
| **40** | 33 | 9 | 8 | **50** | 13 | 6 | 12 | **31** | 9 | 10 | 28 | **47** | 21 | 19 | 20 | **60** |

Tab. 4: Position-dependent and –independent probabilities for two wet 8 hour interval in the uniform splitting (mean of all 24 stations for lower volume class, all values in percent [%]). The combination of '1' (wet) and '0' (dry) illustrates the order of wet and dry 8 hour intervals in a day.

| **Two wet intervals, position-independent** | **Two wet intervals, position dependent** | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **starting position** | | | | **enclosed position** | | | | **ending position** | | | | **isolated** | | | |
| | **011** | **101** | **110** | **Σ** | **011** | **101** | **110** | **Σ** | **011** | **101** | **110** | **Σ** | **011** | **101** | **110** | **Σ** |
| **35** | 20 | 5 | 9 | **34** | 14 | 9 | 13 | **36** | 9 | 5 | 22 | **36** | 14 | 3 | 14 | **31** |

Tab. 5: Position-dependent and –independent probabilities for ~~two~~ three wet 8 hour interval in the uniform splitting (mean of all 24 stations for lower volume class, all values in percent [%]). The combination of ‘1’ (wet) and ‘0’ (dry) illustrates the order of wet and dry 8 hour intervals in a day.

| **Three wet intervals, position-independent** | **Three wet intervals, position dependent** | | | |
|---|---|---|---|---|
| | **starting position** | **enclosed position** | **ending position** | **isolated** |
| | **111** | **111** | **111** | **111** |
| **25** | 17 | 33 | 16 | 9 |

Tab. 6: Relative and absolute error of rainfall characteristics between disaggregated and observed time series (mean for 24 stations)

| | | ~~*rE* [%]~~ | | | | | | ~~*rAE* [%]~~ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ~~Wet spell duration~~ | ~~Average intensity~~ | ~~Wet spell amount~~ | ~~Dry spell duration~~ | ~~Fraction of dry intervals~~ | ~~Autocorrelation lag 1~~ | ~~Wet spell duration~~ | ~~Average intensity~~ | ~~Wet spell amount~~ | ~~Dry spell duration~~ | ~~Fraction of dry intervals~~ | ~~Autocorrelation lag 1~~ |
| ~~Standard~~ | ~~Method A~~ | ~~-3~~ | ~~11~~ | ~~8~~ | ~~8~~ | ~~1~~ | ~~-4~~ | ~~3~~ | ~~11~~ | ~~8~~ | ~~8~~ | ~~1~~ | ~~6~~ |
| | ~~Method B~~ | ~~0~~ | ~~3~~ | ~~4~~ | ~~4~~ | ~~0~~ | ~~-3~~ | ~~1~~ | ~~3~~ | ~~4~~ | ~~4~~ | ~~0~~ | ~~6~~ |
| | ~~Method C~~ | ~~399~~ | ~~-71~~ | ~~44~~ | ~~22~~ | ~~-15~~ | ~~1~~ | ~~399~~ | ~~71~~ | ~~44~~ | ~~22~~ | ~~15~~ | ~~5~~ |
| ~~MRA~~ | ~~Method A~~ | ~~-22~~ | ~~40~~ | ~~10~~ | ~~12~~ | ~~2~~ | ~~-4~~ | ~~22~~ | ~~40~~ | ~~10~~ | ~~12~~ | ~~2~~ | ~~6~~ |
| | ~~Method B~~ | ~~-21~~ | ~~33~~ | ~~5~~ | ~~7~~ | ~~2~~ | ~~-4~~ | ~~21~~ | ~~33~~ | ~~5~~ | ~~7~~ | ~~2~~ | ~~6~~ |
| | ~~Method C~~ | ~~-15~~ | ~~-33~~ | ~~-43~~ | ~~-45~~ | ~~-3~~ | ~~0~~ | ~~15~~ | ~~33~~ | ~~43~~ | ~~45~~ | ~~3~~ | ~~5~~ |
| ~~MMD~~ | ~~Method A~~ | ~~-18~~ | ~~40~~ | ~~14~~ | ~~16~~ | ~~2~~ | ~~-4~~ | ~~18~~ | ~~40~~ | ~~14~~ | ~~16~~ | ~~2~~ | ~~6~~ |
| | ~~Method B~~ | ~~-17~~ | ~~32~~ | ~~9~~ | ~~11~~ | ~~1~~ | ~~-3~~ | ~~17~~ | ~~32~~ | ~~9~~ | ~~11~~ | ~~1~~ | ~~6~~ |
| | ~~Method C~~ | ~~-16~~ | ~~-35~~ | ~~-45~~ | ~~-47~~ | ~~-3~~ | ~~1~~ | ~~16~~ | ~~35~~ | ~~45~~ | ~~47~~ | ~~3~~ | ~~5~~ |

| | *rE* [%] | | | | | | *rAE* [%] | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Wet spell duration | Average intensity | Wet spell amount | Dry spell duration | Fraction of dry intervals | Autocorrelation lag 1 | Wet spell duration | Average intensity | Wet spell amount | Dry spell duration | Fraction of dry intervals | Autocorrelation lag 1 |
| **Method A** | -18 | 40 | 14 | 16 | 2 | -4 | 18 | 40 | 14 | 16 | 2 | 6 |
| **Method B** | -17 | 32 | 9 | 11 | 1 | -3 | 17 | 32 | 9 | 11 | 1 | 6 |
| **Method C** | -16 | -35 | -45 | -47 | -3 | 1 | 16 | 35 | 45 | 47 | 3 | 5 |

Tab. 7 The median of *rE* of extreme rainfall events (over all stations and realisations ) for different return periods and disaggregation methods before and after the resampling

| | | ***rE* [%]** | | | | | |
|---|---|---|---|---|---|---|---|
| | **D** | **15 min** | | **1 h** | | **2 h** | |
| | **Tn** | **Before res.** | **After res.** | **Before res.** | **After res.** | **Before res.** | **After res.** |
| **Method A** | 1 | 17 | 4 | 6 | 19 | 2 | 24 |
| | 2 | 13 | -3 | -2 | 7 | -4 | 12 |
| | 5 | 10 | -9 | -8 | -2 | -7 | 4 |
| | 10 | 8 | -11 | -10 | -6 | -9 | 0 |
| **Method B** | 1 | 13 | 2 | 4 | 17 | 0 | 21 |
| | 2 | 10 | -4 | -3 | 6 | -5 | 11 |
| | 5 | 8 | -9 | -7 | -2 | -8 | 3 |
| | 10 | 7 | -11 | -9 | -6 | -9 | 0 |
| **Method C** | 1 | 10 | 3 | 3 | 15 | -1 | 18 |
| | 2 | 8 | -2 | -4 | 4 | -5 | 8 |
| | 5 | 6 | -5 | -8 | -3 | -9 | 2 |
| | 10 | 5 | -7 | -10 | -7 | -10 | -1 |

## ~~Appendix A~~

~~Tab. A1: Absolute error of rainfall characteristics between disaggregated and observed time series (mean for 24 stations)~~

| | | | ~~Wet spell duration~~ | | | ~~Average intensity~~ | ~~Wet spell amount~~ | | | ~~Dry spell duration~~ | | | ~~Fraction of dry intervals~~ | ~~Autocorrelation lag 1~~ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ~~Average~~ | ~~Standard deviation~~ | ~~Skewness~~ | ~~-~~ | ~~Average~~ | ~~Standard deviation~~ | ~~Skewness~~ | ~~Average~~ | ~~Standard deviation~~ | ~~Skewness~~ | ~~-~~ | ~~-~~ |
| ~~Absolute error [%]~~ | ~~Standard~~ | ~~Method A~~ | ~~3~~ | ~~52~~ | ~~47~~ | ~~11~~ | ~~8~~ | ~~19~~ | ~~13~~ | ~~8~~ | ~~3~~ | ~~5~~ | ~~1~~ | ~~6~~ |
| | | ~~Method B~~ | ~~1~~ | ~~50~~ | ~~46~~ | ~~3~~ | ~~4~~ | ~~20~~ | ~~7~~ | ~~4~~ | ~~2~~ | ~~3~~ | ~~0~~ | ~~6~~ |
| | | ~~Method C~~ | ~~399~~ | ~~413~~ | ~~50~~ | ~~71~~ | ~~44~~ | ~~20~~ | ~~25~~ | ~~22~~ | ~~13~~ | ~~10~~ | ~~15~~ | ~~5~~ |
| | ~~MRA~~ | ~~Method A~~ | ~~22~~ | ~~62~~ | ~~43~~ | ~~40~~ | ~~10~~ | ~~20~~ | ~~11~~ | ~~12~~ | ~~2~~ | ~~4~~ | ~~2~~ | ~~6~~ |
| | | ~~Method B~~ | ~~21~~ | ~~61~~ | ~~42~~ | ~~33~~ | ~~5~~ | ~~22~~ | ~~9~~ | ~~7~~ | ~~1~~ | ~~2~~ | ~~2~~ | ~~6~~ |
| | | ~~Method C~~ | ~~15~~ | ~~29~~ | ~~28~~ | ~~33~~ | ~~43~~ | ~~34~~ | ~~21~~ | ~~45~~ | ~~27~~ | ~~40~~ | ~~3~~ | ~~5~~ |
| | ~~MMD~~ | ~~Method A~~ | ~~18~~ | ~~58~~ | ~~43~~ | ~~40~~ | ~~14~~ | ~~17~~ | ~~15~~ | ~~16~~ | ~~1~~ | ~~1~~ | ~~2~~ | ~~6~~ |
| | | ~~Method B~~ | ~~17~~ | ~~56~~ | ~~43~~ | ~~32~~ | ~~9~~ | ~~19~~ | ~~8~~ | ~~11~~ | ~~2~~ | ~~1~~ | ~~1~~ | ~~6~~ |
| | | ~~Method C~~ | ~~16~~ | ~~28~~ | ~~30~~ | ~~35~~ | ~~45~~ | ~~35~~ | ~~23~~ | ~~47~~ | ~~28~~ | ~~42~~ | ~~3~~ | ~~5~~ |