Review by anonymous referee, Referee #3

1 Summary

The paper deals with the improvement of a given disaggregation model using microcanonical cascade. In general the topic is relevant for the community. The paper is interesting, but cannot be published in its current state, and requires major modifications.
Reviewer #3 is gratefully acknowledged for her/his efforts and the time spend on the manuscript. I think the comments of the reviewer refer to the original submitted version, not to the „current“ version. The current version was uploaded later and includes already the reviews of Elena Volpi (reviewer #2) and reviewer #1 and covers some of the issues and concerns raised by the reviewer. Nevertheless, please find below a detailed point-by-point reply under the assumption, that the review was based on the original submitted manuscript. All page and line numbers refer to the original submission.

2. General comments:

- The paper is quite hard to read with many models being compared. Explanations for the slight variations between the various models are sometimes hard to follow. There is a lack of mathematical details in the presentations of the various models.
The reviewer points out several issues about the method description and the complexity of the manuscript. These issues are repeated as detailed comments, hence please find there a detailed reply.

- Only comparison between variations of a given model are provided. Comparisons with other type of cascade models should at least be discussed.
I agree with the reviewers suggestion that the manuscript would benefit from comparisons of the disaggregation results with results from other cascade models. However, most references compare only the value for the lag-1 autocorrelation, which is not representative (e.g. as shown for method C in Fig. 5 with an overestimation of the lag-1 autocorrelation, while in Fig. 6 a clear underestimation for the majority of lags can be identified). Hence, a comparison is hardly possible, especially because a comparison based only on lag-1 values could be misleading.
Also, the autocorrelation depends on the rainfall processes and genesis in the study area, which enables comparison of absolute values from other studies. A fair comparison could only be carried out with the direct application of another cascade model, but this would be contrary to the reviewers comment on the already very large complexity of the manuscript with too much model version comparisons.

- There are numerous parameters to be estimated per model (not even very clear which number according to the model choice). It is not clear whether a calibration period and a validation period were used.
To provide a better overview an additional figure and a table were added to the manuscript. Fig. 3 provides an explanation of position definitions for method A, B and C in dependence of the branching number. Table 2 lists the cascade model parameters for methods A, B and C in dependence of the applied branching number. From Table 2 it is visible, that for method C a high number of parameters is required. Since the maximum length of the observed time series is 20 years, no split-sampling has been applied, to ensure a best-possible parameter estimation. Also, for later “real-life” applications, the whole high-resolution time series is used for parameter estimation for the disaggregation of the daily time series. Hence, a shortening of the time series could lead to a worsening of the disaggregation results, which would not occur otherwise. The following explanation was added in the validation section 3.1.4:
„A split-sampling into calibration and validation period was not carried out to keep the time series as long as possible for the parameter estimation (see also the discussion in Section 3.1.4).“

3. Detailed comments:

1) Introduction

- p.2 l.21 : “since time series with 1280 minutes do not exist as observation do not exist”. I do not understand this statement and this does not seem a real issue. Anyway, if needed, you can disaggregate at a higher resolution and up-scale to the desired one.

The aim of real-life applications is to disaggregate daily values to achieve high-resolution rainfall time series. The implication of putting a1440 min rainfall amount (1 d) into 1280 min and start the disaggregation is that you will end up with a day which is 160 min (~2.5 h) too short. So either you assume that the missing time steps have a rainfall amount of 0 mm (the question is then: Where to put them?) or you apply any transformation to the daily time series before the disaggregation. Both approaches will affect the resulting rainfall time series characteristics. The following sentence was added for clarification:

„Of course, by the application of an additional transformation process a desired temporal resolution can be achieved, whereby the transformation process affects the characteristics of the disaggregated time series.“

2) Rainfall data

- p. 3 l.28-29 : “from a practical. . . have an impact on the autocorrelation function”. Why not trying the compute the autocorrelation using higher moments to limit the influence of smaller values ?

The reviewer suggests another way of computing the autocorrelation. Indeed, by doing so the influence of the too small intensities would be limited, but it solves not the problem itself. Also a threshold could be introduced to “ignore” the lower values. However, especially for method C very long wet spells with too small rainfall intensities are generated. This underestimation of the average intensity (Fig. 5) poses a serious problem for later applications. Hence, the introduction of MRA and MMD to avoid too small rainfall intensities is also done for practical applications, not only for a more representative computation of the autocorrelation.

- p.4 l.10 : “how can a minimum rainfall intensity be ensured during the disaggregation process?”. It is not very clear to me the need for this, since as pointed out by the author and references cited, it might very well be simply due to the rain gauge measurement limitations. It might be worth testing a time series obtained with a disdrometer which enables better representation of small values of rainfall.

Indeed, a disdrometer time series would gain more insights in this relation of observed and disaggregated rainfall intensities, but is not available to me for this study area. However, the long wet spells with very low rainfall intensities generated by method C have to be avoided (please see also my reply to your previous comment). This is the motivation for the introduction of MRA and MMD (see P14L25-32).

3) Methods

- p.5 l.8 : “actual” - - > “Actually”

This has been corrected.

- Eq. 1 : it should be added how lambda is related to t1 and t2 (I guess lambda = t2-t1)

This has been added in the paragraph before Eq. 1.

- p. 6 l.24 : “on” - - > “no” ?

This has been corrected.

- Eq. 2 : shouldn’t it be P(0/0/1)/3 in the first line since there are three possibilities (100,010,001) for the same P(0/0/1) ? This remark is also valid for all the other probabilities except P(1/3,1/3,1/3)

The reviewer is right, the way Eq. 2 was written was misleading. Eq. 2 has been changed to:

$$W_1, W_2, W_3 = \begin{cases} \{1,0,0;\ 0,1,0\ or\ 0,0,1\} & with\ P(0/0/1) \\ \left\{\frac{1}{2},\frac{1}{2},0;\ \frac{1}{2},0,\frac{1}{2}\ or\ 0,\frac{1}{2},\frac{1}{2}\right\} & with\ P(0/\frac{1}{2}/\frac{1}{2}) \\ \frac{1}{3},\frac{1}{3},\frac{1}{3} & with\ P(\frac{1}{3}/\frac{1}{3}/\frac{1}{3}) \end{cases}$$

- p.7 l.11 : "an empirical function", please be more specific (see also general comment on the lack of mathematical details).

The description of the empirical distribution function was extended to:
„An empirical distribution function is used to represent *f(x)*, with a maximum of 14 equidistant bins (based on the number of available splittings, see Storm (1988, p. 86))."

- p.7 l.15-18 : a summary table or scheme would be helpful.

Fig. 3 has been added to illustrate all position definitions.

- It remains weird to have different branching number and probabilities weights for the first cascade steps which seems to be in contradiction with the underlying scaling properties.

The different branching numbers are a compromise between scaling theory and practical applications. Indeed, it can be questioned if scaling properties are met by b=3 and especially by the uniform distribution in this first disaggregation step. Although the number of wet 8 h-time steps is realistic, the resulting rainfall amounts are maybe less realistic. There are two reasons for this assumption.
First, the number of wet 8 h-time steps tells us something about the genesis of the rainfall event. If it is only one wet time step, it is likely a convective event and hence the whole daily rainfall amount is put into this single 8 h-time step. If the rainfall event lasts longer than 8 h, it is likely a stratiform event, so a long-lasting and less intense event (with often more or less uniform rainfall occurrence). The uniform distribution can be found only on the 8 h-level, on finer temporal resolutions intensities vary due to the b=2-splitting so the resulting final time series does not show any uniform rainfall intensity distribution anymore.
Second, the assumption with the uniform distribution demands only a few parameters. For method A, two parameters are required (P(0/0/1) and P(0/0.5/0.5)). The only other tested approach with a b=3-splitting in the first disaggregation step was introduced by Lisniak et al. (2013), who use for example 8 distribution functions for a splitting with 3 wet 8 h intervals. To keep the cascade model parameter parsimonious, the uniform distribution has been chosen. Also, in previous publications (Müller and Haberlandt, 2015, 2018, Müller-Thomy et al., 2018, Müller-Thomy and Sikorska-Senoner, 2019) with this assumption good representation of rainfall characteristics have been achieved.

- Section 3.1.3 : I found the paragraph quite hard to read. may be a more precise scheme could be helpful. It should be mentioned that it adds a lot of parameters. In general, a summary table with the number of parameters according to the model would be helpful.

The existing scheme in Fig. 2 was improved and Table 3 including all parameters in dependence of the applied branching number was added as well as section 3.1.4.

- Section 3.2 : why presenting two different models (especially given that they provide rather similar results) ? It adds complexity to a paper with already a lot of comparison. I would keep only the MMD which is the more realist I believe.

This suggestion was also made by reviewer #1. He suggested as well to leave out the MRA approach to reduce the complexity of the manuscript. However, both approaches represent possible solutions and none of them has been tested before to the authors knowledge. To reduce the complexity, only the MMD approach has been applied for the resampling investigations. I prefer to keep both approaches in the current study, especially since reviewer 2 identifies this issue as the more fundamental one. The comparison of the two approaches is also useful for other researchers, because they know that the outcome is very similar and don't have to carry out a study on their own.

- Section 3.3 : The process with Ir and more generally the swapping seems rather ad hoc. It seems that the underlying physical meaning of cascade process is lost. I think that this issue should at least be discussed.

Maybe the swapping seems ad hoc because it is not related to cascade models in general. However, in a previous publication Müller and Haberlandt (2018, Fig. 12) have shown, that the scaling behaviour of the disaggregated time series is not changed by the resampling process. The following sentence has been added:

„As proven by Müller and Haberlandt (2018), the resampling does not affect the scaling behaviour of the disaggregated time series, because the total rainfall amount as well as the number of wet time steps are kept.“

- p. 13 l. 18 : "30 realisations". Why such a small number, it seems that much more could have been performed.

The disaggregation is a random process. In a prior study it was analysed, which number of realisations is required to cover the stochastic behaviour of the disaggregation based on the rainfall characteristics of the disaggregated time series analysed in this study. It was found that 30 realisations are sufficient. Also, as pointed out as last point in section "5.3 Study limitations", the simulated annealing is an optimization process that demands a high computational effort. Hence, 30 realisations are kept for the study.

4) Results

- Table 3 and 4 are really hard to follow. I think a scheme representing the various cases could be really helpful.

A scheme of all possible positions is provided in Fig. 3 for a better interpretation of both tables.

- Why the average rainfall intensity changes is such a micro-canonical cascade ?

I'm not sure if I understand the question correctly. The average rainfall intensity depends only on the number of wet time steps generated by the cascade model, since the total rainfall amount remains the same for the whole daily time series. Hence an overestimation of wet time steps leads to an underestimation of the average intensity and vice versa.

- p. 19 l. 5-10 : may be a graph showing the sensitivity of the results to Ir would be be needed.

The reviewer is right, this would be useful. However, a 2D-graph could only include the results for one lag and one extreme event (of a certain duration and return period). Since the problem is more complex, a simplification as a 2D-graph was not added to the manuscript.

5) Discussion - p. 21 l. 14-15 : "identified similarities. . . used for simplification", please clarify and be more explicit.

The reviewer is right, due to the high number of parameters every possibility to reduce their number should be listed in detail. The following sentence has been added:

„These similarities are e.g. P(0/1) for starting and P(1/0) for ending positions (and vice versa) as well as P(0/1) and P(1/0) for both, enclosed positions and isolated positions.

<u>References:</u>

Müller-Thomy, H., Sikorska-Senoner, A. (2019): Does the complexity in temporal precipitation disaggregation matter for a lumped hydrological model?, Hydrological Sciences Journal, accepted

Storm, R.: Wahrscheinlichkeitsrechnung, mathematische Statistik und statistische Qualitätskontrolle, VEB Fachbuchverlag, Leipzig, 9th edition, 360 pages, 1988.