Author Response to Reviewer 2

——General Comments————————

While I agree with the authors that there is “work yet to be done” in the field of streamflow recession analysis, and really appreciate some of the authors’ discussion points on the practical implications of using point cloud analysis vs. individual event analysis (Page 8), there are a number of parts in this manuscript that I find difficult to understand, or that I believe impose “baked in” sources of variability that may not reflect the forms of variability imposed by actual physical processes.

1) In some respects, it seems that Case 1 really encapsulates the main point of this paper (which numerous authors have already argued; though I think the point is worthy of reiteration); that individual recession events typically have steeper slopes than the best fit slope of a line through a point cloud generated by a collection of individual events. Case 1 demonstrates how this might happen; if the recession scale parameter (a) scales with the initial flow condition of the recession (Q0), the intercept of the recession curve in log-log space will shift up or down with Q0, and so a collection of steep recession curves will “stack” in a such a way as to create a point cloud that is less steep than the individual curves of which it is composed (see specific comment #5 for additional comment on imposing this form of variability in ‘a’). Cases 2 and 3 are primarily used in the Discussion and Conclusion to demonstrate this same point. For this reason, I do not see how these cases are useful. These cases might be advantageous if the authors were able to systematically explore the effects of the magnitude-frequency distribution of recharge events on individual recession curves (for example, convincingly attributing the spread in “b” to the magnitude-frequency statistics of recharge). However, I would argue, there is no clear way to perform such a systematic exploration given our present understanding of the physical origins of power law recession dynamics.

Authors Reply: Thank you for raising this important point. We agree that Case 1 illustrates a key point that individual recessions can have a steeper slope than the best fit line through the point cloud that many authors have already presented. However, individual recessions in Case 1 do not have variability in b values. The example given in the introduction of Lookout Creek (Figure 1) shows a wide distribution of b values, consistent with other studies showing a range of individual b values. Case 2 and 3 are presented with the hypothesis that superposition of flow events changes the value of w which can produce a distribution of b values. We agree that a systematic exploration of the physical origins of recession analysis can be valuable and should be explored in the future. While we have not provided a relationship between the variability of b and recharge magnitude and frequency, we believe that the cases presented by controlling for event hydrology are valuable for determining the effects on individual events. We suggest that the event magnitudes distribution produces an offset of individual events while event spacing changes the event length and antecedent flow contribution while antecedent events change the effective decay constant and thus the b value for individual recessions.

We have developed a paragraph in the conclusions to explain why each of the three Cases contribute of the findings of this paper and give more equal weight to the contribution of each of the three cases: “For Case 1, the recession analysis parameter a is equal to $w/(\tau Q_o^{1/w})$ and thus the intercept of the individual recession curve will scale with Q_o. The result is a collection of individual recession curves that are horizontally offset based on the initial discharge producing a smaller b value for the point cloud compared to the individual recessions. Case 1 illustrates that the slope of individual recession events can greater than the best fit line through the point cloud, consistent with previous studies (Biswal and Marani, 2010; Mutzner et al., 2013; Shaw and Riha, 2012). However, the point cloud in Case 1 is generated by a collection of multiple individual recessions all with the same slope and does not have the variability in *b* values presented in these same studies and shown for Lookout Creek in Figure 1. Case 2 and 3 are presented using superposition of antecedent flow events that consequentially changes the individual recession *b* values, providing a possible explanation for the variability in *b* values for individual recessions. For Case 2, the superposition of events takes into account antecedent conditions which results in a distribution of individual recession *b* values where *b* values are associated with the baseflow contribution. For Case 3, the horizontal offset of individual recession from Case 1 and the effects of antecedent conditions from Case 2 result in the variability of individual recession *b* values that are horizontally offset."

2) The methods need to (a) more thoroughly explain exactly how to generate the various forms of synthetic recession, and (b) how these different forms might reflect the impacts of real, physical processes in a watershed. On the first point:

(a)

i. How do the authors translate a recharge magnitude (presumably with units of [L]?) into a flow increment (with units of [L/T])? In the case of nonlinear recessions, the flow increment is a nonlinear, flow dependent function of the recharge depth.

Authors Reply: Thank you for this interesting comment. We now have included the relationship between recharge and flow in the synthetic hydrograph methods section. A pulse recharge can be calculated by integrating the equation for the falling limb of the hydrograph such that the recharge volume is: $V = DA = \tau Q_o/(w-1)$ for w > 1, where *D* is depth of recharge over area *A*. Interestingly for w<=1, the recharge is an infinite volume. In the previous version of the manuscript w was 0.7 which would result in infinite recharge. As a result, we have modified the value of w in all examples to be 1.2 representing non-linear reservoir behavior and updated the associated figures.

ii. What are the parameterizations used for the various distributions from which flow increments and inter-arrival times are sampled?

Authors Reply: Cases 1&3 have log-normally distributed inter-arrival times (μ = 2.5, σ = 1) and event magnitudes (μ = 1, σ = 1), compared to Case 2 with uniform event inter-arrival time (μ = 450/τ), and magnitudes (μ = 1). The time series is based on concatenating successive individual events based on the guidelines for each case (see Table 1). We have developed a paragraph in the methods section where we discuss the distributions of how the synthetic curves were generated.

iii. How am I to interpret the .mat file uploaded to Hydroshare? I loaded this file, and I see there are columns "mag", "start_locs", and "value". How do I use this information to reconstruct the recession curves the authors analyzed? It's undocumented, and not described in the text.

Authors Reply: Thank you for bringing this to our attention. We now have provided a description to the Hydroshare file that includes the file structure information and a subset of Matlab code that identified how the columns were constructed including the parameters used for the distributions. To increase accessibility, we have also included the information in a CSV in addition to the Matlab files on the Hydroshare site.

iv. For Cases 2 and 3, can the authors more clearly define their superposition procedure? Going off of Figure 2, how is the “underlying second event” (QC) constructed? Is the recharge increment added to the value of flow at the end of the previous recession? Or is this how QA is generated? One possibility for QC (once the authors clarify how it is constructed) is that we have effectively created a second “reservoir” with an initial storage equal to the magnitude of the recharge event. Then, QD would equal the sum of the discharge from the continued first event and the discharge from the second reservoir.

Authors Reply: Thank you for this comment, we agree that the superposition procedure needed to be better defined. We have now better described the superposition procedure used to create Case 2 & 3 following the nomenclature in Figure 2. “For Cases 2 and 3, individual recessions were linearly superimposed on antecedent flows. The baseflow from the first event, Q_B, is an extrapolation from the first event using a constant power law decay constant. The underlying second event, Q_C, is defined by the event magnitude given by the random number generator and a defined power law decay constant. The peak flow for Q_A is based on adding $Q_B(t_0)$ to $Q_C(t_0)$ while the decay constant is based on the underlying second even, Q_C. Case 1 is based on the hydrograph represented in Q_A. The resulting flow from superposition, Q_D, defines the peak flow the same as Q_A but the decay constant changes based on the linear superposition of Q_B and Q_C. For Cases 2 and 3, Q_D represents the hydrograph structure.”

(b)

Referring to comment (iv.) above, it’s not clear how this appearance of a second reservoir represents any physical process, or why it’s a meaningful way to generate variability. The idea that the previous event recession somehow continues unabated and superposed with the current event effectively splits the watershed into two parallel components that, owing simply to the occurrence of a recharge event, now operate independently of one another. The procedure amounts to taking the sum of two nonlinear reservoirs with identical values of ‘b’ (page 5, Line 23), and varying value of ‘a’ imposed by Page 4, Line 17. I don’t disagree that this will generate a new recession curve with entirely different power law parameters which depend on previous flow conditions, but the authors do not provide a rationale for imposing this form of memory. A more defensible approach (in my opinion) taken by previous authors is to explicitly acknowledge physical mechanisms that might give rise to parallel reservoirs throughout a landscape (for example, conceptualizing a watershed as a collection of contributing hillslopes with varying hydraulic response times). In such cases, parallel reservoirs may generate increased nonlinearity, as demonstrated by Harman et al (2009) and Gao et al (2017). While it is true that these previous authors use superposition of linear reservoirs, the actual dynamics that give rise to increased nonlinearity are similar to those operating in the present work. On a related note, I think the authors should be citing these previous manuscripts, which I believe are very closely related to the present work.

Authors Reply: The reviewer brings up an important question about the physical representation of the independent reservoirs which gives us the opportunity to clarify the motivation for how we combined individual events to create the hydrograph. Appealing to the simplest model presented by the instantaneous unit hydrograph method by Dooge (1973), we utilize the simplest model of linear superposition. We acknowledge that there are other ways to create watershed memory that would also generate variability, and the effects of parameter estimation from different reservoir models for combing events into a hydrograph would be worthwhile research to peruse. However, for the purposes of this paper providing a linear superposition for the hydrograph shows generated variability between recessions as a simple possible representation. We have included a discussion in the synthetic hydrograph methods section that describes the choice for a simple model based on the instantaneous unit hydrograph and the kinematic wave model. In the discussion and conclusions section, we have included text that introduces the concerns about the linear superposition the reviewer raised and expressed the need for future work on parameter estimation sensitivity on the different reservoir models for synthetic hydrographs.

We agree that citations for reservoir theory relating to recession analysis should be included as it is intimately related to how the synthetic hydrograph is created and interpreted. We have included the citations on reservoir theory to values for w and b with regards to the relationship between recharge and flow (see author reply for general comment 2ai for the relationship between recharge and flow). For w>1 based on the recharge equation, a value for b>2 can be achieved by combining discharge from multiple linear reservoirs in parallel, (e.g., Clark et al., 2009; Gao et al., 2017; Harman et al., 2009; Rupp et al., 2009), multiple linear reservoirs in series (e.g., Rupp et al., 2009; Wang, 2011), or multiple nonlinear reservoirs (e.g., McMillan et al., 2011). We have taken this comment into consideration and included reference to b values by using reservoir theory.

3) I do not understand the purpose of the "early" vs. "late" fitting method in the context of this work. The early/late time methodology derives from the analysis of Brutsaert and Nieber [1977], who show that a shift from a recession slope of 3 to 1.5 is a direct consequence of the dynamics of a Boussinesq-style hillslope groundwater table. The physical implications of the authors choices in construction of synthetic hydrographs (e.g. existence of parallel reservoirs in the previous comment) are not necessarily consistent with the dynamics of a single hillslope groundwater table, so why use a form of analysis that is specific to the Boussinesq framework?

Authors Reply: Thank you for this comment. The introduction of early vs time-time was intended to be used for comparison of parameter fitting methods and not to be applied towards the synthetic hydrographs. For clarification, we have elaborated on the choice of using early and late-time for the parameter estimation fitting methods section and not for the synthetic hydrographs. In the parameter estimation fitting methods section, a description has been added to clarify that because a change of hydraulic regime was suggestive in Figure 1 between high flow ranges and low flow ranges, recession analysis parameters were estimated for two flow ranges, early-time and late-time. The differentiation of recession analysis parameters into early and late-time was chosen to capture the change in hydraulic regime in Figure 1 and to provide b values for the lower envelope fitting methods. In the text when the LE fit is discussed, we have noted that a limitation of using a pre-defined value for b assumes that the watershed responds like a homogenous Boussinesq hillslope with behavior similar to a single hillslope, which isn't known priori for the LE of a watershed composed of multiple heterogeneous hillslopes of unknown b. For the synthetic hydrographs, they do not exhibit a change in hydraulic regime and thus a single fit is used for each fitting method because w is a constant value.

———Specific comments————————

1) Page 2, Lines 16 – 18: Do the authors intend to say that sources of variability in a,b between events may derive from these sources? Also, it is not clear what the authors mean by "flow superposition from previous events".

Authors Reply: We have changed the wording to avoid confusion.

2) Page 2, Line 29: This statement is vague; of course the hydrology of a recession event affects the recession event.

Authors Reply: We refocused this statement and made it more aligned with our hypothesis.

3) Page 3, Line 9: Use of superposition without defining the term.

Authors Reply: We have replaced the term in this instance to avoid confusing before the term is defined.

4) Page 3, Line 22: “theoretical”

Corrected as suggested.

5) Page 4, Line 15 – 17: While I agree that this is certainly one way to introduce variability in the recession scale parameter, it is nevertheless arbitrary to impose this particular relationship pinned to a 45 day timescale. Subsequent interpretation should be qualfied with “where a = -w/(t0*Q0ˆ(1/w)) holds. . .”. While it is a convenient expression for imposing variability in ‘a’, I am unaware of any process-oriented result that shows the recession scale parameter should be determined in this way. Related to this, on Page 6, Lines 3 – 12, this discussion is difficult to follow. I think the authors are making the point that within their imposed timescale framework, the recession scale parameter ‘a’ must collapse to a single value that no longer depends on the flow initial condition in the limit as b=1. I agree that, mathematically, this is what happens, but the authors don’t provide a compelling case that this is physically what should happen with real recessions; so the conclusion, “yet this result suggests that a condition where b=1. . .” should be qualified with the requirement that this would be true in circumstances where the authors’ imposed form of variability for ‘a’ holds.

Authors Reply: Corrected as suggested.

6) I assume the authors meant to put “3 Results” not “3 Methods” on page 5.

Authors Reply: Corrected as suggested.

7) Page 7 Line 23: “We hypothesize. . .” is an almost tautological statement.

Authors Reply: We have elaborated on this sentence to include our hypothesis about the variability of individual recession analysis parameters.

——-Figures——————————-

- Figure 2: Why is there a "t0" at the top of the plot? Isn’t t0 the 45 day timescale imposed to generate the recession scale parameter?

Authors Reply: Thank you for this comment. We have corrected this unclear notation. Now he characteristic timescale has been changed to the symbol τ to avoid confusion. The t0 in Figure 2 signifies the start of the recession, and the equation evaluates the discharge as a time since the recession start.