

Dear Authors,

This is an interesting paper on the simulation of daily streamflow through power-spectrum-based Gaussian implicit scheme and the Kappa distribution. The results indicate that the proposed scheme can very well preserve the marginal distribution and certain aspects of the frequency-domain second-order dependence structure.

The simulation algorithm is applied through the phase spectrum randomization (instead of the white noises used in autoregressive moving-average stochastic models), while it preserves the expected frequency spectrum (and thus, the expected second-order dependence structure, e.g. autocovariance function) of each process as well as the cross-correlation between two processes, whereas the Kappa marginal distribution is preserved through the Gaussian-implicit method (i.e. by transforming the target distribution to the Gaussian one and at the end, use the inverse transformation to go back to the target one).

Please see below some short/long comments to initiate a discussion which I hope they can contribute to the proposed methodology and further highlight it.

1) In the main text, the Authors mention several existing streamflow generation approaches that have focused on the frequency domain (instead on the more traditional time-domain), with the earliest cited work of Theiler et al. (1992, see ref. in the text).

As mentioned by the Authors the Gaussian-implicit (also mentioned as copula) can be implemented not only through the autocorrelation function (lag-domain) but also to the power-spectrum (frequency-domain), which are theoretically linked through the Fourier transform. Another application could be through the variance (scale-domain) of the process, else known as climacogram (i.e. the variance of the averaged process vs. time scale of averaging, see also the 6th comment). These three stochastic tools are linked theoretically and if one is known the rest can be easily obtained (Koutsoyiannis, 2016):

$$c(h) = \int_0^{\infty} s(w) \cos(2\pi wh) dw \leftrightarrow s(w) = 4 \int_0^{\infty} c(h) \cos(2\pi wh) dh \leftrightarrow$$
$$s(w) = -2 \int_0^{\infty} (2\pi wk)^2 \gamma(k) \cos(2\pi wk) dk \leftrightarrow \gamma(k) = \int_0^{\infty} s(w) \frac{\sin^2(\pi wk)}{(\pi wk)^2} dw$$

where c , s and γ are the autocovariance function, the power-spectrum and the climacogram, and h , w and k are the continuous time-lag, time-frequency and time-scale (i.e. the three major time domains for statistical analysis), respectively.

However, although the above three stochastic tools can be analytically and equivalently-easy obtained from one another, *they do not exhibit the same statistical robustness in parameter estimation from data*. Like for example the central moments are considered less accurate than the L-moments (i.e. with a lower statistical bias), the identification of the second-order dependence structure is more accurately estimated by the climacogram (for comparison of several short and long-term processes and the reasons why the performance of climacogram in stochastic modelling should be preferred see Dimitriadis and Koutsoyiannis, 2015a).

In other words, one could estimate the model of the second-order dependence structure from the empirical climacogram and then, find the same model expressed through the power-spectrum from the above equations (or even directly from data through the climacospectrum; Koutsoyiannis, 2016, 2019a). In this way, the selected model is expected to be closer to the theoretical model.

2) Another issue maybe worth discussing is the *statistical bias* of the above estimators of the autocovariance, power-spectrum and climacogram. In case of a long-term process the autocovariance is highly biased (e.g., Dimitriadis and Koutsoyiannis, 2015a) and therefore, the expectation of the Fourier estimator (linked to the power-spectrum) will be also highly biased leading to underestimation (as the Authors observed in the cross-correlations among stations). I would suggest either to add this to the Discussion for future research or to fit a model to the observed power-spectrum (or better to the climacogram as discussed above), then estimate the bias of the power-spectrum, and then perform the simulation based on the latter (as suggested by a general framework for Stochastics in Koutsoyiannis, 2000).

3) Also, by fitting a model to the power-spectrum the Authors will be able to estimate the *Hurst parameter*, which characterizes the long-term behaviour (e.g. Koutsoyiannis, 2008). Such a comment about this Hurst parameter could be added to the text, since the Authors specifically focus on processes with long-term behaviour, which is also called Hurst-Kolmogorov behaviour (Koutsoyiannis, 2016). Interestingly this behaviour has been identified through long-term stochastic similarities in 13 processes in global and local scales in the PhD thesis of Dimitriadis (2017, see also Koutsoyiannis et al., 2018), where it is also proposed an integrated stochastic view among various geophysical and hydrometeorological processes which may differ in their physical properties but not in their stochastic behaviour). Also, for the long-term or HK processes see an additional recent literature review of O'Connell et al. (2016).

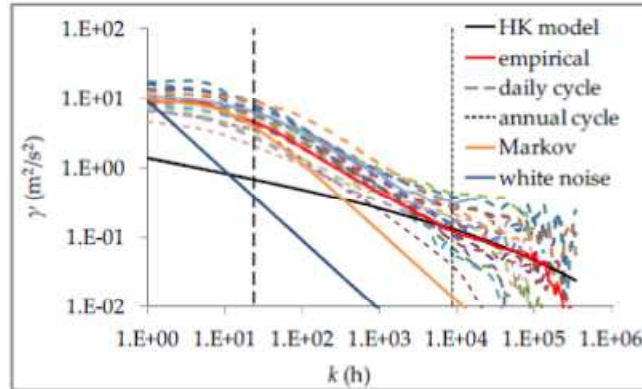
4) Concerning the concept of de-seasonalization the Authors may find worth mentioning the term of *homogenization* (e.g. Dimitriadis and Koutsoyiannis, 2018, sect. 4.2), which is actually the same concept as de-seasonalization but also includes cases of double periodicities (i.e. diurnal and seasonal) as well any other implicit transformation, i.e. transform the process to a desired

distribution for simulation $G(x_{ij})$, where i and j are indices for the seasonal and diurnal processes, and then go back to the original distribution $F(x_{ij})$ using the inverse transformation $F(G^{-1}(x_{ij}))$. These transformations are usually based on a target marginal distribution (e.g. maximized-entropy distributions in case the distribution function is unknown or other non-Gaussian distributions), and because this method uses the inverse transformation of $G(x_{ij})$ it is called implicit algorithm to be separated from the explicit algorithms (Koutsoyiannis and Dimitriadis, 2016) which preserve simultaneously (not in an indirect manner) an arbitrary second-order dependence structure and a desired marginal distribution through its marginal moments (for such algorithms see the recent work of Dimitriadis and Koutsoyiannis, 2018).

5) Maybe could be helpful for the Readers to add some *Tables* with the fitted parameters of the Kappa distribution for each season and each river. It has to be noted though that the Kappa distribution with $\xi > 0$ seem to fit the streamflow for $x > \xi$ and thus, does not include the zero values. Maybe a bivariate two-state approach (streamflow – no-streamflow, as for example in Li et al., 2012 for precipitation, where they also use the Kappa distribution). In case the Authors choose a one-state approach then they may find useful the discussion and justification for the threshold parameter to daily precipitation in Dimitriadis and Koutsoyiannis (2018, sect. 4.3), where a one-state approach is applied for the Pareto-Burr-Feller (PBF) distribution with a threshold parameter h .

6) The Authors may find useful for extending their literature review the work of Cugar and Kavanagh (1968), who seem to be the first ones in literature to apply the Gaussian-implicit scheme at the frequency-domain. Also, there seems to be an earlier work of Hoeffding (1940) that has a more general expression for implicit algorithms (please see further relevant literature in Dimitriadis and Koutsoyiannis, Appendix D).

Also, maybe is worth noticing that the *Gaussian-implicit through the scale-domain* (climacogram) for the simulation of a single timeseries has been discussed in Dimitriadis (2017, sect. 3.3.3, and references therein) and Dimitriadis and Koutsoyiannis (2018, Appendix D), and earlier illustrated for the maximized entropy distribution function (in case the marginal distribution is unknown) in Dimitriadis and Koutsoyiannis (2015b, see also Koutsoyianni et al., 2008), as well as for the extended Pareto marginal distribution (through a simple application) of a double periodic hourly wind process in Deligiannis et al. (2016, see Fig. 8a with the transformation of the empirical climacogram following an extended Pareto distribution -red line- to the climacogram following a Gaussian distribution -black line-):



Finally, the Authors may find interesting recent work of Iliopoulou et al. (2018) on seasonal precipitation extremes and Iliopoulou et al. (2018) on seasonal streamflow correlations.

7) A final comment worth discussing is that the Gaussian-implicit methodology may sometimes severely *overestimate the variability/uncertainty* of highly skewed distributions (such as in daily river discharge as discussed by the Authors, see also Koutsoyiannis, 2019b). When transforming the generated Gaussian process to the target distribution through the inverse function, the expectation of the 2nd order dependence structure (e.g. mean value of the power spectrum estimator) maybe well preserved but its variability is likely that it is not adequately preserved. This is a basic limitation of the Gaussian-implicit schemes and has been highlighted in Dimitriadis and Koutsoyiannis (2018). The reason is that the dependence structure depends on the high-order moments by definition and thus, a transformation could not simultaneously alter both the marginal and the dependence structure. This may be tackled by explicit methodologies of stochastic simulation (Dimitriadis and Koutsoyiannis, 2018; also see an illustrative example in Appendix D) but the latter cannot preserve in an theoretically exact way the marginal distribution since it is based on the preservation of moments rather than the marginal distribution function itself (like in implicit schemes). A fancy solution could be the use of higher-order copula (as also mentioned in Dimitriadis and Koutsoyiannis, 2018, Appendix D).

As a final comment I would like to congratulate their Authors for their work, for their ideas discussed, and for not being carried away using vague expressions in their text like *to anything, unified theory, ill transformation* etc. As always in Science every algorithm has merits and demerits and only through open discussion, we may invent new algorithms for engineering practices that keep the merits and overcome the demerits.

Sincerely,

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