Interactive comment on "Technical note: Stochastic simulation of streamflow time series using phase randomization" *by* Manuela I. Brunner et al.

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This is a useful and interesting technical note on simulating stream flow time series preserving observed characteristics. Several techniques exist to approximate time series such as preserving moments, using marginal-back transformations, bootstrap, amplitude adjusted Fourier transformations methods, etc. All of them have advantages and disadvantages. This technical note is well-structured, well-written, and the real-world case is nicely demonstrated. The authors' intention to provide an easy-to-apply solution is clear, and although this is a technical note, the added value against previews works on the amplitude adjusted Fourier transformation method should be better high-

lighted in order to strengthen the publication.

Before providing a detailed review, I'm expressing my gratitude to the commenters referring to my 2018 work. Clarifying, I worked on this framework much earlier in 2009 for a multivariate and cyclostationary simulation of daily rainfall (13 stations in Greece) aiming to preserve marginals (the Burr type XII was used), correlations and intermittency. The method was described in detail in a document (Papalexiou, 2010) that includes also mixed-type distributions inflated at arbitrary points and not just at zero. The extended work was published in arXiv (Papalexiou, 2017), followed by the journal publication some months later (Papalexiou, 2018). The scheme also applied in a stationary/nonstationary disaggregation framework preserving marginals and correlations (DiPMaC) (Papalexiou et al., 2018). It was an attempt to provide a simple framework for univariate and multivariate modeling preserving continuous, discrete, binary or mixed-type marginals having positive definite autocorrelation structure (including long memory). The focus was specifically on hydroclimatic variables such as precipitation, streamflow, wind, humidity, etc.

Yet as the saying goes, most things have been already found before; indeed, the first attempts in other scientific fields to simulate time series preserving marginals and correlations using inflated correlations date back in the works of Conner and Hammond (1972), and probably much earlier. A clear presentation for continuous marginals using AR models was given by Li & Hammond in (1975). The authors may wish to check also an old paper entitled "Generation of random signals with specified probability density function and power density spectra" by Gugar and Kavanagh published in 1968. Following Li & Hammond (1975) several papers got published in different fields (e.g., Cario & Nelson, 1997, 1998; Kugiumtzis, 2002; Macke et al., 2009; Demirtas, 2014, 2017; Emrich & Piedmonte, 1991; Macke et al., 2009 to mentioned a few) dealing with several cases. These and many other interesting works yet were not suitable for simulating intermittent processes, like precipitation or stream flows, and most of them suggested demanding iterative procedures to estimate the parent Gaussian correlations

or the cross-correlation upper limits (Papalexiou, 2018). In few papers the same link between the gaussian correlation and the bivariate copula with prescribed marginals is also established (even earlier) by using the same technique, i.e., the fundamental 2-dimensional theorem of the expected value of a transformed random variable (Nataf, 1962; Liu & Der Kiureghian, 1986; H. Li et al., 2008; Lebrun & Dutfoy, 2009; Xiao, 2014). This link can also be established by using the Jacobian of the transformation (Papalexiou, 2018, Eq. 7).

The authors here focus on another method, i.e., the amplitude adjusted Fourier transformation, with the same intent and aiming to generate consistent stream flow time series. They can also see a simulation of daily stream flow preserving a heavy tailed marginal (Burr type III) and a slowly decaying autocorrelation structure in Section 4.3 in Papalexiou (2018). Specific comments that authors may find useful to improve the manuscript are:

1. The authors write: "We hereafter refer to such methods, which are also known as amplitude-adjusted Fourier transformations (Lancaster et al., 2018), as phase randomization simulations. In hydrology, phase randomization simulation has rarely been applied for purposes other than hypothesis testing (Fleming et al., 2002), even though it has desirable properties which make it suitable for a wider range of applications. (...) However, its application is limited to Gaussian data. We here propose the use of phase randomization simulation for the stochastic generation of streamflow time series at individual and multiple sites. To allow for non-Gaussian distributions, as commonly observed for daily streamflow values, we combine the data simulated by phase randomization with the Kappa distribution," Probably missing something here, but amplitude-adjusted Fourier transformations can account for non-Gaussian amplitude distributions. The method proposed by Prichard and Theiler (1994) (authors cite indeed this paper) accounts for non-Gaussian marginal distributions as stated by the authors at page 953 "We account for non-Gaussian amplitude distribution by using the amplitude adjusting algorithm described in in Ref. [8] for each component". Prichard

and Theiler (1994) use the algorithm described in Sec 2.4.3 of Theiler et al (1992), which reads as follows "The idea is to first rescale the values in the original time series so they are gaussian. Then the FF or WFT algorithm can be used to make surrogate time series which have the same Fourier spectrum as the rescaled data. Finally, the gaussian surrogate is then rescaled back to have the amplitude distribution as the original time series.". These techniques (Amplitude adjusted Fourier transformations) are known to preserve the linear correlations of the parent Gaussian process rather than those of the target. Indeed, they were further refined as Iterative Amplitude adjusted Fourier transformations in order to match ACF and marginal distributions of the target variables as closely as possible (Kugiumtzis, 1999; Schreiber & Schmitz, 1996; e.g., Serinaldi & Lombardo, 2017; Venema et al., 2006). So, if not missing something here I see that the methodology proposed in this technical note is related to the procedure applied by Prichard and Theiler (1994) in their second example with a difference spotted in the rescaling of the marginal distribution, where empirical CDF is replaced by a parametric Kappa distribution. Thus, maybe the statement that AAFT cannot deal with non-Gaussian marginals should be revised as my understanding is that AAFT it was devised to deal exactly with this. Of course, replacing empirical CDF with Kappa is more appropriate for stream flow simulations.

2. An important point regards the underestimation of cross-correlation reported by the authors (it could be also observed also in the autocorrelation). It was proven mathematically long time ago (Kendall & Stuart, 1979, p. 600) (Embrechts et al., 2002) that any nonlinear transformation of a gaussian time series reduces the strength of the linear correlations as expressed by the Pearson correlation coefficient. Obviously, this does not affect the rank correlations. Since the authors are not calculating the inflated autocorrelations (or inflated spectrum) it is expected the generated time series to have lower correlations. However, in practice this depends on the transformation used. If the target marginal is bell-shaped then typically it has a small effect in reducing the autocorrelation, yet the effect can be very large for j-shaped target marginals with heavy tails and zero inflated (see Fig. 1 and Fig. 2 and the simulation examples in Fig4-Fig7

in Papalexiou (2018) and the discussion therein). Therefore, the fact that the authors don't observe large differences in the autocorrelation of the simulated time series is case specific and not generally true. This point is important and should be mentioned as there are streamflow series highly inflated at zero and highly skewed; without using inflated correlations for the parent gaussian process it is certain that the transformed series will not match the target process.

3. As previously mentioned early approaches in simulating nongaussian timeseries preserving marginals and correlations date back long time ago (e.g. S. T. Li & Hammond, 1975), yet this framework as it was formulated didn't include the modeling of mixed-type marginals (see Papalexiou 2018) which allows easy simulation of intermittent processes such as precipitation or stemflows of ephemeral streams. To increase the novelty of the paper the authors can easily include mixed-type quantiles (see Eq. 17 in Papalexiou 2018) with the kappa distribution. As far as I know this approach has not been implemented in amplitude adjusted Fourier transformations.

Minor points (p = page, L = Line)

p2L33-p3L4: We can easily use large order AR models to simulate time series having long memory or any other strong autocorrelation structure. Especially AR models have an analytical solution (Yule-Walker system) and the fit and the application e.g., of an AR of order 10000 is a matter of less than a second. This means that we can reproduce exactly the autocorrelation structure up to order p. It should be clear that fitting an AR of any large order to a long memory autocorrelation structure is parsimonious and efficient as all the AR parameters are analytically and without uncertainty estimated by the autocorrelation structure, e.g., if an AR of order 10000 is fitted to an fGn correlation then it is an one-parameter model and not a 10000 parameter model. More details can be found in Papalexiou (2018), where the authors can also see examples of long memory process simulations using AR models and preserving marginals. Also long memory can be approximated by the sum of independent AR(1) processes as suggested by Mandelbrot (1971). So, it is a matter of how these models are applied and definitely

they can reproduce long memory or any other autocorrelation structure.

p4L6-p4L9: The Kappa distribution is a well-established distribution in hydrology and since Hosking (1994) introduced the four-parameter version it has been applied countless times (e.g., Hanson & Vogel, ; Kjeldsen et al., 2017; Park et al., 2009)). Its importance and flexibility stems for the fact that generalizes important distribution such as the GP, GEV, GLO etc, but it can also be seen as a special case and generalization at the same time of the Burr type XII. Maybe the great disadvantage of the four-parameter version is the location parameter which can end up in supporting a range of values which is inconsistent with the variable under study. For stream flows expected to range in the positive axis this can be problematic. If distributions like the Generalized Gamma or the Burr type XII didn't work, maybe the authors should try with the Burr type III (1 scale, 2 shape pars) (Burr, 1942) or the Generalized beta of the second kind (Mielke & Johnson, 1974) (1 scale, 3 shapes) which has great flexibility and is defined in (0, Infinity). This would solve the issue of negative values or of a lower positive limit but of course the authors may neglect this suggestion.

p5L19 and P6L9-11: To clarify regarding the normal transformation. The authors have fitted the Kappa in each day and then use the Kappa cdf to transform to uniform and then apply the gaussian quantile? Even if they did so the final time series might have normal marginal but the autocorrelation may be different in each week/month or season.

p6L3: The KS-test is not a very robust test. It will not change anything to the analysis, but maybe more robust tests should be used and promoted, e.g., the Anderson-Darling. If it's not much of a trouble the authors could test the fit based on the AD test.

p6L20: I might be missing something here but why is 0 the lower bound of the four-parameter Kappa?

Summarizing, this is well-written and useful technical note that deserves publication after some amendments and literature updates.

Regards,

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