Dear Dr. Peleg,

We thank the reviewers and the commentators for acknowledging the value of our work, their feedback, and their constructive comments. We appreciate the wide range of inputs, which allowed us to enrich our introduction and discussion section. In addition to many useful references, the reviewers and commentators point out where and how the stochastic streamflow generator could be reformulated or extended. While each of the points risen is valid, most of them would make the model more complex, less flexible, and less generalizable. With the stochastic simulator presented in the manuscript, we aim at proposing a simple and flexible tool, which can be adapted to different contexts. This is facilitated by the provision of the simulation procedure as an R-package. In order to guarantee flexibility and generalizability, we combine phase randomization, which is a nonparametric approach for the generation of a time dependence structure, with the flexible four-parameter kappa distribution. Making the model more complex or more parametric would imply a loss in flexibility. We therefore would like to retain the main features of the model proposed. However, we agree that a more profound discussion of its limitations and potential extensions is necessary and valuable. We also agree that the issue with the replacement of negative values by zero values needs to be addressed and that the use of an empirical marginal distribution can in some cases be sufficient. The R-package PRSim (versions > 1.0) was made even more flexible by adding the possibility to use any type of distribution (empiricial or theoretical distribution) in the back transformation process.

Below, we address the points risen by the three reviewers Ashish Sharma, Demetris Koutsoyiannis, and Simon Papalexiou and state how we addressed them in a revised version of the manuscript. Our replies to the reviewers' comments are written in blue and italic to distinct them from the reviewers' comments.

On the behalf of all co-authors,

Yours sincerely,

Manuela Brunner

Reviewer 1 (Ashish Sharma)

My congratulations to the authors on this excellent paper. Very glad to see a clever adopted to frequency domain alternatives in formulating a stochastic streamflow generator. My comments below are aimed to enhance the presentation and I am in support of publication once these have been addressed. Comments are:
Reply: *Thank you for acknowledging the value of our work and for the constructive comments, which help to enrich the introduction and discussion section.*

line 2/9 - The authors are missing the works by Keylock (10.1029/2012WR011923). This work performed resampling to an existing time series using phase randomization in the frequency domain. If I remember correctly, it had some nice inclusion of ICA to tackle the multivariate issue, and wavelets to get around nonstationarity in the data that cannot be handled using a fourier transformation alone. I think they need to read those papers (I am familiar with the above one but there may be more since) and acknowledge them here, and also try and show how their work distinguishes itself from the above paper.
Reply: *The work by Keylock (2007) indeed shows many parallels to the approach presented in this paper. His approach is not directly based on the Fourier transformation but rather based on the wavelet decomposition of a signal. Instead of the phases of the Fourier transform, the*

wavelet coefficients are (partly) randomized. The randomized series are then backtransformed to the time domain by using a rank-ordering procedure as presented in the approach used in our manuscript. Keylock (2012) later extended the procedure to the joint simulation at multiple sites. The work by Keylock was acknowledged in the introduction and discussion section.

Modification: p:4, l:7-8, p:15, l:26-28

line 3/21: I think the work by Mehrotra (10.1029/2005JD006637) should be acknowledged here as it represents essentially something analogous to a ARMAX type of a model even though it is cast as a stochastic downscaling approach. A mention should be made on the ability to preserve low frequency variability, which I believe the proposed approach will be able to address as well.

Reply: *The work by Mehrotra and Sharma (2006) was acknowledged as an approach allowing for the extension of Markov chains to multiple sites by using spatially correlated random numbers.*

Modification: p:3, l:22-23

Line 3/35: Even though it relates to the problem of correcting systematic biases, given the use of phase transformation (not randomisation), the approaches of Nguyen should perhaps be acknowledged for completeness. The rationale behind these approaches and the one here has a lot in common. (10.1007/s00382-018-4191-6, 10.1016/j.jhydrol.2016.04.018).

Reply: *Thank you for pointing out these references. We acknowledged the work of Nguyen et al. (2019) in the discussion section where we talk about options of how to improve the representation of the cross-correlation in simulated series.*

Modification: p:15, l:30-32

line 5/21: The authors may want to look through the details of (10.1007/s00382-018-4191-6, 10.1016/j.jhydrol.2016.04.018) as they performed another level of preprocessing - they fit a Thomas Feiring type model to the monthly data and after that structure was removed, the Fourier transformation was performed. This was done after trying with the steps referred to above, as it was found to exhibit clear advantages.

Reply: *We experimented with different types of deseasonalization techniques and found that the normalization at daily scale served the purpose of removing seasonality in the data well. Compared to using a Thomas-Fiering model, the approach used here is non-parametric and does not assume any temporal seasonality structure. Deseasonalizing by a Thomas-Fiering model and re-adding this seasonality at the end, might be valuable if the reproduction of the lag-1 autocorrelation was an issue, which was not the case here. However, it requires the fitting of a parametric model which is data dependent. Our routine works independent of the time resolution of the data and is easily adjustable to different contexts. We show that the ACF of the observed data is nicely preserved by the approach employed in our study.*

line 6/21: Setting negatives to zero is not a clean option. Please refer to the Keylock paper above again on how they restricted their approach to resampling to avoid having to set negatives to zero.

Reply: *We agree that setting negative values to zero is indeed not very elegant. We changed the algorithm in order to avoid this. Instead of replacing negative values by zero, we replaced these values by a value sampled from a uniform distribution in the interval [0, min(Q_obs_day)], where min(Q_obs_day) represents the minimum of the observed values corresponding to the day under consideration.*

Modification: p:6, l:26-28

line 11/10: Underestimation of cross-correlations is I think addressed well in (10.1007/s00382-018-4191-6). The trick that is used is to not randomly generate phases for all variables, but for a "key" variable (say biggest streamflow mean location). And then maintain the phase difference between alternate sites. The phase difference in space helps capture the cross-dependence attributes.

Reply: *The approach proposed by* Nguyen et al. (2019) *for a good representation of the cross-correlation between two or multiple time series in the context of bias correction could also be adopted in the stochastic simulation framework presented in our manuscript. The discussion section was extended by the phase-difference correction functions introduced by* Nguyen et al. (2019).

Modification: p:15, l:30-32

Lastly, I feel not addressing the issue of non-stationarity in a stochastic generation paper under our present climate should be discouraged. The issue of nonstationarity can be addressed in the sense of a discussion by thinking of adding an exogenous predictor variable set in the formulation, which can impart the changes needed. Some discussion to that effect would be good to include in the paper before it is published.

Reply: *We agree that addressing non-stationarity, if present, is important. The manuscript therefore contains a note stating that the stochastic generator could be applied using discharge time series simulated with a hydrological model driven by meteorological data simulated with a GCM (and RCM) (p 13. L20-22 in the original manuscript). We slightly extended the discussion by discussing more options of how to adjust the phase randomization approach to non-stationary conditions.*

Modification: p:16, l:7-9

Reviewer 2 (Demetris Koutsoyiannis)

1. The Technical Note by Brunner et al. (2019) implements a useful idea for easy stochastic simulation of daily streamflow, based on spectral representation and phase randomization. The method has several limitations (see below) but it is practical and useful, and it certainly deserves publication. I believe several issues can be improved before final publication and therefore I am providing some suggestions. I also appreciate the commentaries by Francesco Serinaldi, Ioannis Tsoukalas and Panayiotis Dimitriadis, who provided a lot of information to the authors. I think this information is useful to optimize their Note and also to put it in the context of modern and older literature, some of which is missing in the literature review. I believe that not everything suggested in the commentaries needs to be addressed, as this would change the orientation of the Note. However, with several changes in the formulations and a few expansions, rather than additional analyses, the Note could be improved. My own suggestions, which I list in the following points, fall in two categories: (a) recognition of the limitations of the method and (b) improvements in formulations, phraseology and terminology.

Reply: *Thank you for acknowledging the usefulness and practicality of our approach. We retained the main characteristics of the stochastic simulation procedure proposed here, which make it a flexible and generalizable tool. However, we extended the discussion section in order to discuss its limitations more in depth.*

2. A first limitation, which in the current version is not stated clearly, is the severe dependence of the method on the sample size of observations. The synthetic series has the same length as the observed series. The authors properly recognize the importance of respecting long-range dependence (LRD) in simulation. However, to study its effect in hydrosystems we need synthetic series much longer than

the observed. The use of ensembles of small-length time series may not be equivalent with using a long time series as each member of the ensemble is independent from the others.

Reply: *Thank you for pointing out the need to address this limitation. We agree that producing an ensemble of time series of the same length as the observed one might not be equivalent to the generation of one very long time series if long-range dependence features are present which exceed the length of the observed series. However, such features cannot be generated anyway since the model is fitted based on a limited number of years of observations. We added this issue to the discussion.*

Modification: p:14, l:5 – p:15, l:1

3. A second limitation is the absence of a model for time dependence. While the authors correctly adopt a model for the marginal distribution (e.g. they state "Using the empirical distribution instead of the Kappa distribution would prevent us from obtaining values that go beyond the range of observed data…") their method misses to do so for the dependence structure. The empirical autocorrelogram and periodogram are affected by significant bias and huge noise (see references provided by Panayiotis Dimitriadis) and if we do not use a model, then we reproduce a particular random realization, in terms of autocorrelogram and periodogram, in all our simulated series. I believe authors' statement "The periodogram, the empirical counterpart of the power spectrum, shows high values at those frequencies which correspond to strong periodic components" is only partly true and perhaps misleading. The periodogram could be regarded a realization of a stochastic process per se (on the frequency domain) and its peaks do not necessarily reflect a real peak in the "true" power spectrum. The same thing happens with the autocorrelogram. For example, the ups and downs in the empirical autocorrelograms in Fig. 5 may well be sampling artefacts, which we do not need to reproduce—but the method does reproduce them. If the authors have difficulty to accept my comment, I would suggest doing an experiment with a particular (smooth) autocorrelation function and see the ups and downs in the produced autocorrelogram and periodogram of a single realization.

Reply: *As correctly stated above, the approach brought forward here employs a nonparametric approach for the stochastic generation of the time dependence structure. In theory, one could try to find a suitable parametric model to represent this time dependence. We are convinced, however, that doing so would be far from straightforward since it would be very difficult to represent the complexity in the time dependence structure at different time scales ranging from short to long range. Furthermore, the choice of a model would be dependent on the catchment area. Besides its flexibility in reproducing dependence structures at different ranges, the approach presented here has the advantage of being applicable to any dataset of interest without having to fit a parametric time dependence model. The nonparametric time dependence model is flexible, applicable in any catchment, and easy to apply.*

4. A third limitation is the lack of parsimony of the entire methodology. From the statement "We fit a separate distribution for each day to take into account seasonal differences in the distribution of daily streamflow values" one can imagine that the overall method encompasses lots of parameters. Apparently, it is nowadays easy to do calculations with lots of parameters but, in my view, stochastics goes beyond calculations and algorithms. Parsimony in stochastic modelling is always important (see Koutsoyiannis 2016).

Reply: *We agree that the stochastic model presented in our study involves a fair amount of parameters. However, this is necessary if we would like to achieve a proper representation of the distributions of the daily discharge values in addition to being able to generate values outside the range of the observed values. If the user is satisfied with values within the observed values, he/she might forgo the use of a theoretical distribution and use the* *empirical distribution of the observed values for backtransformation instead. This option was implemented in the new version of the R-package PRsim and the issue was addressed in the revised version of the manuscript.*

Modification: p:15, l:10-11

5. A final limitation for the particular time scale of modelling, i.e. daily, is the lack of explicit modelling of time irreversibility (an issue also mentioned in the comment by Francesco Serinaldi). This would not be an issue if the time scale was monthly or longer, but I suspect that it is for the daily scale (see Koutsoyiannis 2019 and also Müller et al. 2017). I clarify here that I do not suggest changing the method to overcome the limitations (e.g. to become more parsimonious or to take irreversibility into account). Rather, I just recommend stating them in a clear and explicit manner.

Reply: *Thank you for pointing out the issue of time irreversibility. We agree that this aspect is not explicitly considered in the modeling strategy. Neither is it considered in most existing modeling approaches. The lack of consideration of the time irreversibility issue was mentioned in the discussion section.*

Modification: p:16, l:2-3

6. Now coming to the second category of my suggestions, I would recommend avoiding the name kappa distribution for the chosen distribution. It is true that in hydrological literature this name is in common use, but if we wish to facilitate communication with other disciplines, we should be aware that the name kappa distribution has another meaning in statistical thermodynamics—namely it is used to describe Cauchy-type (or Student-like) distributions in motion of particles (e.g. Olbert, 1968; Livadiotis and McComas 2013). The specific distribution used in the Note (which I do not think is a generalization of GEV as suggested by Ioannis Tsoukalas), is commonly (in most disciplines) referred to as the Dagum distribution—see https://en.wikipedia.org/wiki/Dagum_distribution. In addition, in terms of sign conventions in eqn. (4), I would suggest changing the signs of k and h and replacing the two minus signs in front of them with plus signs. This will make the expression more convenient and intuitive, and also complying to the standard notation used in other disciplines (e.g. as seen in the above web site).

Reply: *Thank you for highlighting that there was some confusion about the use of the term kappa distribution. The development of the name of the kappa distribution introduced by Hosking in 1994 has indeed an interesting history and there is a huge potential for confusion.* Mielke (1973) *introduced a three-parameter kappa distribution.* Hosking (1994) *generalized this distribution to a four-parameter distribution and called it "the four-parameter kappa distribution", which is a generalization of the generalized logistic, GEV, and generalized Pareto distributions. The Dagum distribution is also related to the three-parameter distribution by Mielke (1973) in the sense that it has the same properties but uses a different parameterization* (Kleiber, 2008)*. We here used the four-parameter kappa distribution by Hosking, which offers more flexibility compared to the three-parameter distribution. We retained the notation introduced by Hosking (1994) to stress the link to this original publication. We added a remark to the text highlighting the link between the article by Hosking and the article by Mielke.*

Modification: p:4, l:14-16

7. The phrase "Stochastically generated time series mimic the characteristics of observed data and represent sets of plausible but as yet unobserved streamflow sequences" (my emphasis) may distort the meaning of what stochastic simulation is. It is not a matter of something that is "yet unobserved" but expected to be observed in the future. It is a matter of producing artificial "realizations" from the stochastic model. A model, stochastic or otherwise, is not identical to the real world.

Reply: *The corresponding sentence was rephrased.*
Modification: p:2, l:5

8. The term "deseasonalization" needs to be used with care and clarification; otherwise it may mislead people to think that, by techniques like that used in the Note, we can get rid of seasonality. This, however, is quite difficult—if ever possible. With transformations of the time series, either linear (as in standardizing by mean and variance of each period) or nonlinear (as in fitting a separate distribution for each period, like what is done in this Note), we can only remove the seasonal effect on the marginal distribution, not that of the joint distribution of a cyclostationary stochastic process. (For example, differences in autocorrelation coefficients in different seasons are not removed by techniques such as the above mentioned). Therefore I suggest replacing "deseasonalization" with "deseasonalization of the marginal distribution."
Reply: *Thank you for suggesting this clarification. Deseasonalization will be replaced by deseasonalization of the marginal distribution.*
Modification: p:6, l:14,17

9. The notion of "nonparametric" techniques referred to in the literature review is, in my opinion, problematic when we deal with stochastic processes with time dependence. As opposite to iid statistics, in which the first "i" (independent) is taken for granted, in stochastics there cannot be "nonparametric" methods; something of parametric type is always present, albeit sometimes hidden. Furthermore, the "bootstrap approaches" also mentioned in the Note are unsuitable for stochastic processes as they distort the stochastic structure—particularly in the presence of LRD. Therefore, I suggest making these clarifications and limiting the references to such types of models (as well as to ARMA-type models whose value is only historical, I believe). Instead, I suggest extending the review to other models, more appropriate for hydrological applications, such as those suggested by other commenters.
Reply: *The term nonparametric has been used in the literature for certain types of models used for the generation of stochastic time series (Salas and Lee, 2010). We highlight that these approaches are not suitable for the reproduction of long-range dependence (see p:3, l:16-17). The disadvantages of the ARMA models are also clearly stated (see p:3, l:1-4). We extended the literature review by more advanced models, which allow for more flexible time dependence structures (e.g.* Tsoukalas et al. (2018)*).*
Modification: p:3, l:27-28

10. Could the authors double check their equations? Is an imaginary unit missing somewhere in equation (2)? Could they correct the notation in eqn (3)? (Is 'rand' meant to be a subscript?).
Reply: *We checked the equations and there was indeed an imaginary unit missing (we misspelled j instead of i). The equations were corrected in the revised version of the manuscript.*
Modification: Equations 2-4

11. Finally, I uphold the other commenters in congratulating the authors and I particularly second Panayiotis Dimitriadis in congratulating them for using modest phraseology. I would add in the reasons for congratulation the fact that they do not follow the cliches and fashionable paths: for example they limit their mentions to climate impacts and nonstationarity, a notion that has become a must in hydrological papers—often by authors who do not know what it actually is (see Koutsoyiannis and Montanari 2015; Serinaldi and Kilsby, 2015).
Reply: *Thank you.*

Selected comments from the comments by Francesco Serinaldi, Ioannis Tsoukalas, and Panayiotis Dimitriadis

The comments provided by the three commentators are highly appreciated and were used to enrich the introduction, methods, and discussion sections. More specifically, we addressed several approaches which could be employed to allow for an improved representation of the cross-correlation in simulated time series; we addressed the problem of temporal asymmetry; we added a comment on the SPARTA model by Tsoukalas et al. (2018) to the introduction; we specified the estimation method used for the estimation of the parameters of the kappa distribution; we tested the Burr type XII (validation results see Figure 1) and generalized Gamma distributions, which were, however, not flexible enough to model the marginal distributions of the daily discharge values and led to unrealistically extreme high flows in the simulations and were therefore not directly considered as alternatives to the kappa distribution. However, we added an additional flexibility to the R-package PRSim, which now allows for using any type of marginal distribution which is found suitable to represent the distribution of the input time series. The implementation of any type of theoretical distribution is illustrated in PRSim for the GEV and the Generalized Beta distribution of the second kind.

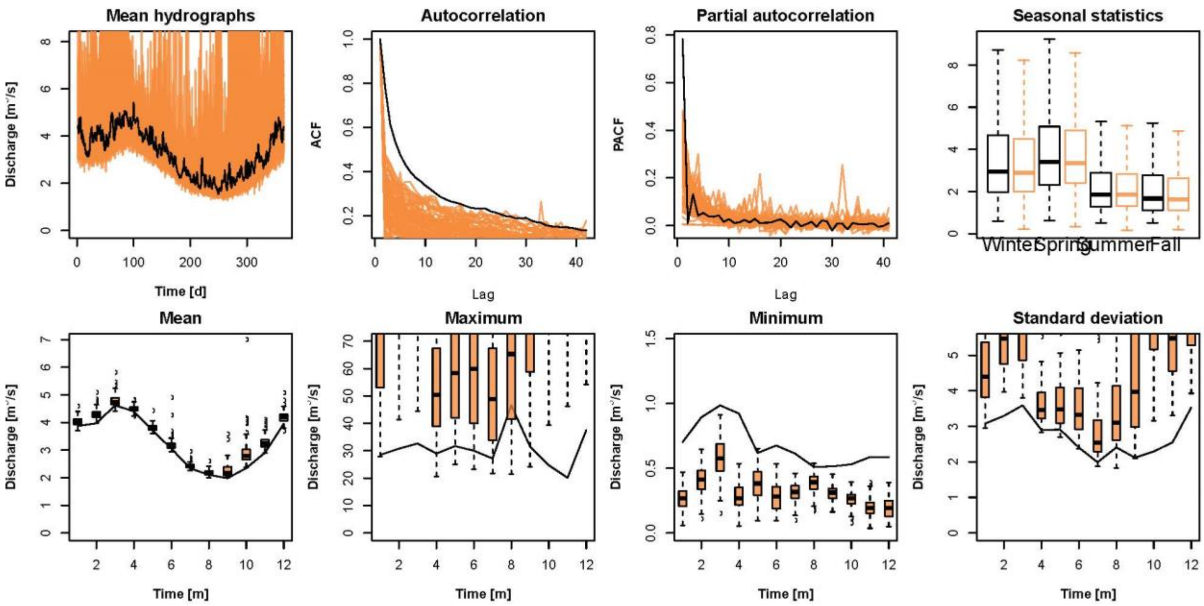

Figure 1: Validation plots for the Birse catchment on discharge time series generated using the Burr type XII distribution.

Reviewer 3 (Simon Papalexiou)

This is a useful and interesting technical note on simulating stream flow time series preserving observed characteristics. Several techniques exist to approximate time series such as preserving moments, using marginal-back transformations, bootstrap, amplitude adjusted Fourier transformations methods, etc. All of them have advantages and disadvantages. This technical note is well-structured, well-written, and the real-world case is nicely demonstrated. The authors' intention to provide an easy-to-apply solution is clear, and although this is a technical note, the added value against previews works on the amplitude adjusted Fourier transformation method should be better highlighted in order to strengthen the publication.

Before providing a detailed review, I'm expressing my gratitude to the commenters referring to my 2018 work. Clarifying, I worked on this framework much earlier in 2009 for a multivariate and cyclostationary simulation of daily rainfall (13 stations in Greece) aiming to preserve marginals (the Burr type XII was used), correlations and intermittency.

The method was described in detail in a document (Papalexiou, 2010) that includes also mixed-type distributions inflated at arbitrary points and not just at zero.

The extended work was published in arXiv (Papalexiou, 2017), followed by the journal publication some months later (Papalexiou, 2018). The scheme also applied in a stationary/nonstationary disaggregation framework preserving marginals and correlations (DiPMaC) (Papalexiou et al., 2018). It was an attempt to provide a simple framework for univariate and multivariate modeling preserving continuous, discrete, binary or mixed-type marginals having positive definite autocorrelation structure (including long memory). The focus was specifically on hydroclimatic variables such as precipitation, streamflow, wind, humidity, etc.

Yet as the saying goes, most things have been already found before; indeed, the first attempts in other scientific fields to simulate time series preserving marginals and correlations using inflated correlations date back in the works of Conner and Hammond (1972), and probably much earlier. A clear presentation for continuous marginals using AR models was given by Li & Hammond in (1975). The authors may wish to check also an old paper entitled "Generation of random signals with specified probability density function and power density spectra" by Gugar and Kavanagh published in 1968.

Following Li & Hammond (1975) several papers got published in different fields (e.g., Cario & Nelson, 1997, 1998; Kugiumtzis, 2002; Macke et al., 2009; Demirtas, 2014, 2017; Emrich & Piedmonte, 1991; Macke et al., 2009 to mentioned a few) dealing with several cases. These and many other interesting works yet were not suitable for simulating intermittent processes, like precipitation or stream flows, and most of them suggested demanding iterative procedures to estimate the parent Gaussian correlations or the cross-correlation upper limits (Papalexiou, 2018). In few papers the same link between the gaussian correlation and the bivariate copula with prescribed marginals is also established (even earlier) by using the same technique, i.e., the fundamental 2-dimensional theorem of the expected value of a transformed random variable (Nataf, 1962; Liu & Der Kiureghian, 1986; H. Li et al., 2008; Lebrun & Dutfoy, 2009; Xiao, 2014). This link can also be established by using the Jacobian of the transformation (Papalexiou, 2018, Eq. 7).

The authors here focus on another method, i.e., the amplitude adjusted Fourier transformation, with the same intent and aiming to generate consistent stream flow time series. They can also see a simulation of daily stream flow preserving a heavy tailed marginal (Burr type III) and a slowly decaying autocorrelation structure in Section 4.3 in Papalexiou (2018). Specific comments that authors may find useful to improve the manuscript are:

1. The authors write: "We hereafter refer to such methods, which are also known as amplitude-adjusted Fourier transformations (Lancaster et al., 2018), as phase randomization simulations. In hydrology, phase randomization simulation has rarely been applied for purposes other than hypothesis testing (Fleming et al., 2002), even though it has desirable properties which make it suitable for a wider range of applications. (. . .) However, its application is limited to Gaussian data. We here propose the use of phase randomization simulation for the stochastic generation of streamflow time series at individual and multiple sites. To allow for non-Gaussian distributions, as commonly observed for daily streamflow values, we combine the data simulated by phase randomization with the Kappa distribution," Probably missing something here, but amplitude-adjusted Fourier transformations can account for non-Gaussian amplitude distributions. The method proposed by Prichard and Theiler (1994) (authors cite indeed this paper) accounts for non-Gaussian marginal distributions as stated by the authors at page 953 "We account for non-Gaussian amplitude distribution by using the amplitude adjusting algorithm described in in Ref. [8] for each component". Prichard and Theiler (1994) use the algorithm described in Sec 2.4.3 of Theiler et al (1992), which reads as follows "The idea is to first rescale the values in the original time series so they are gaussian. Then the FF or WFT algorithm can be used to make surrogate time series which have the same Fourier spectrum as the rescaled data. Finally, the gaussian surrogate is then rescaled back to have the amplitude distribution as the original time series.". These techniques (Amplitude adjusted Fourier transformations) are known to preserve the linear correlations of the parent Gaussian process rather than those of the target. Indeed, they were further refined as Iterative Amplitude adjusted Fourier transformations in order to match ACF and marginal distributions of the target variables as closely as possible (Kugiumtzis, 1999; Schreiber & Schmitz, 1996; e.g., Serinaldi & Lombardo, 2017; Venema et al., 2006). So, if not missing something here I see that the methodology proposed in this technical note is related to the procedure applied by Prichard and Theiler (1994) in their second example with a difference spotted in the rescaling of the marginal distribution, where empirical CDF is replaced by a parametric Kappa distribution. Thus, maybe the statement that AAFT cannot deal with non-Gaussian marginals should be revised as my understanding is that AAFT it was devised to deal exactly with this. Of course, replacing empirical CDF with Kappa is more appropriate for stream flow simulations.

Reply: *We agree that the amplitude-adjusted Fourier transformation procedure proposed by Prichard and Theiler (1994) can deal with non-Gaussian data. However, it does usually not allow for the use of parametric distributions in the back-transformation process. We adjusted the text accordingly by acknowledging that amplitude-adjusted Fourier transform allows for the generation of non-Gaussian data. We specified that our approach differs from amplitude-adjusted Fourier transformation by that it allows for the generation of values beyond the values in the empirical distribution.*

Modification: p:4, l:11-12

2. An important point regards the underestimation of cross-correlation reported by the authors (it could be also observed also in the autocorrelation). It was proven mathematically long time ago (Kendall & Stuart, 1979, p. 600) (Embrechts et al., 2002) that any nonlinear transformation of a gaussian time series reduces the strength of the linear correlations as expressed by the Pearson correlation coefficient. Obviously, this does not affect the rank correlations. Since the authors are not calculating the inflated autocorrelations (or inflated spectrum) it is expected the generated time series to have lower correlations. However, in practice this depends on the transformation used. If the target marginal is bell-shaped then typically it has a small effect in reducing the autocorrelation, yet the effect can be very large for j-shaped target marginals with heavy tails and zero inflated (see Fig. 1 and Fig. 2 and the simulation examples in Fig4-Fig7 in Papalexiou (2018) and the discussion therein). Therefore, the fact that the authors don't observe large differences in the autocorrelation of the simulated time series is case specific and not generally true. This point is important and should be mentioned as there are streamflow series highly inflated at zero and highly skewed; without using inflated correlations for the parent gaussian process it is certain that the transformed series will not match the target process.

Reply: *We agree that the autocorrelation of a stochastically generated time series might not in all cases well reproduced the autocorrelation of observed time series. We added a section to the discussion section, that discusses this issue: "While the reproduction of the temporal dependence was well reproduced here, this is not necessarily the case under all conditions.* Embrechts et al., 2010 *have shown that any nonlinear transformation of a Gaussian time series, which is done during backtransformation, reduces the strength of the linear correlations in the time series as expressed by Pearson's correlation coefficient. If one is working with heavy-tailed and zero inflated marginals, it can happen that autocorrelations are reduced during backtransformation* (Papalexiou, 2018)*."*

Modification: p:15, l:1-5

3. As previously mentioned early approaches in simulating nongaussian timeseries preserving marginals and correlations date back long time ago (e.g. S. T. Li & Hammond, 1975), yet this framework as it was formulated didn't include the modeling of mixed-type marginals (see Papalexiou 2018) which allows easy simulation of intermittent processes such as precipitation or stemflows of ephemeral streams. To increase the novelty of the paper the authors can easily include mixed-type quantiles (see Eq. 17 in Papalexiou 2018) with the kappa distribution. As far as I know this approach has not been implemented in amplitude adjusted Fourier transformations.

Reply: *We agree that the use of mixed-type marginals can be beneficial in certain cases, where the process to simulate from is intermittent. We therefore made PRSim even more flexible by introducing user-defined distributions to be used in the backtransformation process. The software illustrates the functionality with GEV and GB2 distributions. This user-defined distribution can potentially be a mixture distribution. We did not include an example of a mixture distribution in our technical note because the time series chosen for the analysis were not characterized by intermittency. Furthermore, the use of mixtures of a discrete and a continuous part is delicate as we cannot resort on classical/standard definitions of* *likelihood tests or confidence intervals.*

Modification: p:16, l:12-14

Minor points (p = page, L = Line)

p2L33-p3L4: We can easily use large order AR models to simulate time series having long memory or any other strong autocorrelation structure. Especially AR models have an analytical solution (Yule-Walker system) and the fit and the application e.g., of an AR of order 10000 is a matter of less than a second. This means that we can reproduce exactly the autocorrelation structure up to order p. It should be clear that fitting an AR of any large order to a long memory autocorrelation structure is parsimonious and efficient as all the AR parameters are analytically and without uncertainty estimated by the autocorrelation structure, e.g., if an AR of order 10000 is fitted to an fGn correlation then it is an one-parameter model and not a 10000 parameter model. More details can be found in Papalexiou (2018), where the authors can also see examples of long memory process simulations using AR models and preserving marginals. Also long memory can be approximated by the sum of independent AR(1) processes as suggested by Mandelbrot (1971). So, it is a matter of how these models are applied and definitely they can reproduce long memory or any other autocorrelation structure.

Reply: *We agree that it is possible to approximate an arbitrary spectrum with either a large order AR or many AR(1) processes. However, this approach remains an approximation. A long-memory process can be characterized with a polynomial decay of the spectrum. AR processes have an exponential decay. Hence, it will not be possible to generate a «truly» long range process. It is possible to approximate arbitrarily precisely an empirical spectrum. If one observes a spectrum of length n, n AR(1) processes will allow for the approximation of an empirical spectrum. We specified in the introduction that AR models can be used to generate seemingly long-memory processes if a parametric autocorrelation structure is used to fit the data.*

Modification: p:3, l:4-5

p4L6-p4L9: The Kappa distribution is a well-established distribution in hydrology and since Hosking (1994) introduced the four-parameter version it has been applied countless times (e.g., Hanson & Vogel, ; Kjeldsen et al., 2017; Park et al., 2009)). Its importance and flexibility stems for the fact that generalizes important distribution such as the GP, GEV, GLO etc, but it can also be seen as a special case and generalization at the same time of the Burr type XII. Maybe the great disadvantage of the four-parameter version is the location parameter which can end up in supporting a range of values which is inconsistent with the variable under study. For stream flows expected to range in the positive axis this can be problematic. If distributions like the Generalized Gamma or the Burr type XII didn't work, maybe the authors should try with the Burr type III (1 scale, 2 shape pars) (Burr, 1942) or the Generalized beta of the second kind (Mielke & Johnson, 1974) (1 scale, 3 shapes) which has great flexibility and is defined in (0, Infinity). This would solve the issue of negative values or of a lower positive limit but of course the authors may neglect this suggestion.

Reply: *Thank you for this suggestion. We tested the Generalized beta distribution of the second kind. It seems to be rather flexible and works fine for certain catchments. In other catchments, however, it produces very extreme, and rather implausible high-flow values. As the distribution is defined in the interval [0,Infinity], it solves the problem with the zero* *values, but introduces a problem with infinity values. While this and other distributions tested (GEV, Burr type XII, generalized Gamma, Wakeby) were not found to be suitable in our application example, they might be appropriate in other cases. We therefore adjusted the PRSim R-package (version > 1.0) to allow for any type of distribution specified by the user in the back-transformation process.*

Modification: p:16, l:21

p5L19 and P6L9-11: To clarify regarding the normal transformation. The authors have fitted the Kappa in each day and then use the Kappa cdf to transform to uniform and then apply the gaussian quantile? Even if they did so the final time series might have normal marginal but the autocorrelation may be different in each week/month or season.

Reply: *It is correct that the kappa distribution was fitted to each day individually and that the autocorrelation shows monthly variations.*

p6L3: The KS-test is not a very robust test. It will not change anything to the analysis, but maybe more robust tests should be used and promoted, e.g., the Anderson-Darling. If it's not much of a trouble the authors could test the fit based on the AD test.

Reply: *We agree that the KS-test is not very robust and added the AD test as a goodness-of-fit test. The PRSim package now allows for choosing one of the two tests.*

Modification: p:6, l:8

p6L20: I might be missing something here but why is 0 the lower bound of the four parameter Kappa?

Reply: *It is correct that the kappa distribution does not have a lower bound. The sentence was rephrased to "Negative simulated values are replaced by values sampled from a uniform distribution in the interval [0, min(x)], where min(x) represents the minimum of the observed values corresponding to the day under consideration".*

Modification: p:6, l:26-28

Summarizing, this is well-written and useful technical note that deserves publication after some amendments and literature updates.

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Technical note: Stochastic simulation of streamflow time series using phase randomization

Manuela I. Brunner[1], András Bárdossy[2], and Reinhard Furrer[3]

[1]Mountain Hydrology and Mass Movements, Swiss Federal Institute for Forest, Snow and Landscape Research WSL, Birmensdorf ZH, Switzerland
[2]Institute for Modelling Hydraulic and Environmental Systems, University of Stuttgart, Stuttgart, Germany
[3]Institute of Mathematics, University of Zurich, Zurich, Switzerland

Correspondence: Manuela Brunner (manuela.brunner@wsl.ch)

Abstract. Stochastically generated streamflow time series are widely used in water resource planning and management. Such series represent sets of plausible yet unobserved streamflow realizations which should reproduce the main characteristics of observed data. These characteristics include the distribution of daily streamflow values and their temporal correlation as expressed by short- and long-range dependence. Existing streamflow generation approaches have mainly focused on the time domain, even though simulation in the frequency domain provides good properties. These properties comprise the simulation of both short- and long-range dependence, as well as extension to multiple sites. Simulation in the frequency domain is based on the randomization of the phases of the Fourier transformation. We here combine phase randomization simulation with a flexible, four-parameter kappa distribution, which allows for the extrapolation to yet unobserved low and high flows. The simulation approach consists of seven steps: 1) fitting the theoretical kappa distribution, 2) normalization and deseasonalization of the marginal distribution, 3) Fourier transformation, 4) random phase generation, 5) inverse Fourier transformation, 6) back transformation, and 7) simulation. The simulation approach is applicable both to individual and multiple sites. It was applied to and validated on a set of four catchments in Switzerland. Our results show that the stochastic streamflow generator based on phase randomization produces realistic streamflow time series with respect to distributional properties and temporal correlation. However, cross-correlation among sites was in some cases found to be underestimated. The approach can be recommended as a flexible tool for various applications such as the dimensioning of reservoirs or the assessment of drought persistence.

Keywords: Fourier transformation, spectral analysis, generator, frequency domain, catchment, temporal dependence, correlation

Keypoints:

1. Stochastic simulation of streamflow time series for individual and multiple sites by combining phase randomization and the kappa distribution

2. Simulated time series reproduce temporal correlation, seasonal distributions, and extremes of observed time series

3. Simulation procedure suitable for use in water resource planning and management

1 Introduction

Stochastically generated streamflow time series are used in various applications of water resource planning and management. These applications include water and reservoir management, the determination of the dimensions of hydraulic structures such as reservoirs, and the estimation of hydrological extremes such as droughts and floods. Stochastically generated time series mimic the characteristics of observed data and represent sets of plausible realizations of streamflow sequences (Ilich, 2014; Borgomeo et al., 2015; Tsoukalas et al., 2018b). They are essential for many uncertainty studies in hydrology because they can serve as input for deterministic water system models in which they allow for the propagation of natural variability and uncertainty (Tsoukalas et al., 2018b).

Stochastic models for the generation of synthetic streamflow time series need to fulfill certain requirements. They should reproduce both the marginal distribution of observed streamflow time series as well as their temporal dependence structure (Sharma et al., 1997; Salas and Lee, 2010). Temporal dependence encompasses both short- and long-range dependence. While short-range dependence typically refers to the dependence of daily streamflow values measured within a few days, long-range dependence refers to dependencies across months or years. This temporal dependence has been found to depend on magnitude, in that low values have stronger dependence than high values (Lee and Salas, 2011). A proper representation of this long-range dependence is of particular importance in studies where storage in reservoirs is of interest (Tsoukalas et al., 2018b). If one is interested in extreme events, the model should allow for the generation of values that go beyond the magnitude of those observed (Herman et al., 2016). This requires the choice of a suitable theoretical marginal distribution. Streamflow typically exhibits a skewed distribution, which requires the use of a three or more-than-three-parameter distribution (Koutsoyiannis, 2000; Blum et al., 2017). Studies looking at individual hydrological events such as floods or droughts require a daily resolution. Therefore, the stochastic model should allow for outputs at such a fine temporal resolution. Often, study regions encompass several sites whose streamflows are correlated. Consequently, the model ideally not only allows for the simulation of streamflow at individual sites, but also for the joint simulation of streamflow at multiple sites, taking into account their spatio-temporal dependence. From a practitioner's point of view, the model should not only reproduce the characteristics of the observed data, but it should also be simple (Sharma et al., 1997).

Many different approaches have been proposed for the stochastic simulation of streamflow time series, each able to fulfill some but usually not all of the desired properties listed above. One commonly used approach is the use of a synthetic weather generator in combination with a rainfall-runoff model (Pender et al., 2015). This approach is affected by uncertainties due to hydrological model selection and calibration, which can be avoided by using direct synthetic streamflow generation approaches (Herman et al., 2016). According to Stedinger and Taylor (1982), the development of such direct approaches consists of the following steps: 1) selection of a model for seasonal marginal distributions, 2) selection of a model for spatial and temporal dependence, and 3) validation of the model. Different groups of direct approaches exist which are distinct in terms of their flexibility regarding marginal distributions and temporal dependence structures.

A first group of models consists of parametric models such as autoregressive moving average (ARMA) models and their modifications. While these models are commonly used in stochastic hydrology, they only allow for modeling short-range dependence because their autocorrelation decreases strongly with increasing lag time (Sharma et al., 1997). This means they guarantee neither the reproduction of observed persistence of annual flows nor the correlation structure among flows in different months (Stedinger and Taylor, 1982). This makes them unsuitable for applications where long-range dependence is important (Koutsoyiannis, 2000). However, AR models can be used to generate seemingly long-memory processes if a parametric autocorrelation structure is used to fit the data (Papalexiou, 2018). A second group of parametric models is based on the temporal disaggregation of annual series and enables the representation of long-range dependence (Stedinger and Taylor, 1982; Salas and Lee, 2010). These models include fractional Gaussian noise models (Mandelbrot, 1965), fast fractional Gaussian noise models (Mandelbrot, 1971), broken line models (Mejia et al., 1972), and fractional autoregressive integrated moving average models (Hosking, 1984). Disaggregation models can be extended to multi-site applications (Grygier and Stedinger, 1988). However, this group of models has been shown to exhibit parameter estimation problems, and only allows for the representation of a narrow range of autocorrelation functions (Koutsoyiannis, 2000). A third group of models is nonparametric in its approach and includes kernel density estimation (Lall and Sharma, 1996; Sharma et al., 1997) and various bootstrap approaches. The latter include simple bootstrap, which is only useful if data are uncorrelated, moving block-bootstrap, nearest-neighbor bootstrap (Salas and Lee, 2010; Herman et al., 2016), matched-block bootstrap (Srinivas and Srinivasan, 2006), and maximum-entropy bootstrap (Srivastav and Simonovic, 2014), which also take lagged correlations into account. These nonparametric techniques resample from the data with perturbations and directly reproduce the characteristics of the original data (Sharma et al., 1997). However, the reproduction of long-range dependence is difficult and variance can be under- or overestimated (Salas and Lee, 2010). To allow for values that go beyond the observed distribution, Salas and Lee (2010) proposed a model employing k-nearest neighbor resampling with a gamma kernel perturbation. A further group of models consists of models that employ Markov chains and their variations. These models account for transition probabilities between different hydrological states (Stagge and Moglen, 2013; Bracken et al., 2014; Pender et al., 2015) and can be combined with nonparametric approaches such as k-nearest neighbors (Prairie et al., 2008). They can be extended to multiple sites by scaling the simulated values at individual sites with spatially correlated random numbers (Mehrotra and Sharma, 2006).

Several alternatives to these well-established simulation procedures have been proposed, which allow for a flexible choice of marginal distributions. These include models where the temporal dependence structure is modelled with copula functions, which are, however, difficult to apply for higher orders of autocorrelation (Lee and Salas, 2011). Examples of new simulation procedures based on the Autoregressive to Anything (ARTA) model proposed by Cario and Nelson (1996) are the SMARTA model by Tsoukalas et al. (2018b) or the SPARTA model by Tsoukalas et al. (2018a), which employ Nataf's joint distribution model for the simulation of stochastic time series, representing both short- and long-range dependence. In addition, simulation schemes based on wavelet decomposition, which avoid assumptions about the temporal dependence structure, have been proposed by Kwon et al. (2007); Wang et al. (2010) and Erkyihun et al. (2016). Borgomeo et al. (2015) have shown how simulated annealing can be used to generate synthetic streamflow time sequences that represent possible climate-induced changes in user-specified streamflow properties.

All these previously mentioned models are based on the time domain. An alternative to time-domain models is frequency-domain models (Shumway and Stoffer, 2017), which allow for the simulation of surrogate data with the same Fourier spectra as the raw data (Theiler et al., 1992). Such methods are based on the randomization of the phases of the Fourier transformation and have been commonly applied in hypothesis testing, when identifying nonlinearity in time series (Schmitz and Schreiber, 1996; Kugiumtzis, 1999; Venema et al., 2006; Maiwald et al., 2008), and in trend detection (Radziejewski et al., 2000). We hereafter refer to such methods, which are also known as amplitude-adjusted Fourier transformations (AAFT) (Lancaster et al., 2018), as *phase randomization* simulations. Serinaldi and Lombardo (2017) used an iterative AAFT method to generate binary series of rainfall occurrence and non-occurrence. An extension of the amplitude-adjusted Fourier transformation has been presented by Keylock (2007) who employed randomization procedures to wavelet decomposed signals to generate surrogate data. In hydrology, phase randomization simulation has rarely been applied for purposes other than hypothesis testing (Fleming et al., 2002) even though it has desirable properties which make it suitable for a wider range of applications. Indeed, its implementation is relatively simple, it can simulate time series with both short- and long-range dependence, and it can be extended to multiple sites. However, its application is often limited to the reproduction of the empirical distribution of the data. We here propose the use of phase randomization simulation for the stochastic generation of streamflow time series at individual and multiple sites. To allow for non-empirical distributions, we combine the data simulated by phase randomization with the flexible, four-parameter kappa distribution introduced by Hosking (1994) as a generalization of the three-parameter kappa distribution suggested by Mielke (1973). The stochastic streamflow generation approach shall represent a flexible tool, which is easy to apply, and generalizable to different contexts. This is enabled by combining a nonparametric time dependence model with a flexible four-parameter distribution. The simulation approach can be tailored to the specific problem at hand and be used for various water resource management applications.

We now turn to some theoretical background on Fourier transformation and phase randomization. For a more detailed introduction to the Fourier transformation, the reader is referred to textbooks by Morrison (1994) or Shumway and Stoffer (2017). We then discuss the use of phase randomization for the stochastic generation of streamflow time series. For illustration purposes, we apply and validate the approach on a set of four catchments in Switzerland. Finally, we discuss potential applications of the simulation approach.

2 Theoretical background

The basic idea behind all surrogate methods is to randomize the Fourier phases of the underlying (hydrological) process. The Fourier transformation converts a time-domain signal into a frequency-domain signal, which is complex-valued. This transformation may be depicted as a decomposition of the time series into sine and cosine waves of different amplitude, phase, and period (Fleming et al., 2002; Shumway and Stoffer, 2017). In the frequency domain, the power spectral density (power spectrum) expresses the same information in cycles as the autocovariance function expresses in lags in the time domain. The periodogram, the empirical counterpart of the power spectrum, shows high values at those frequencies which correspond to strong periodic components (Shumway and Stoffer, 2017).

The surrogate approach utilizes the property that realizations of linear Gaussian processes differ only in their Fourier phases and not their power spectrum. It preserves the autocorrelation structure of the raw series by conserving its power spectrum through phase randomization. The procedure consists of three main steps (Radziejewski et al., 2000; Maiwald et al., 2008; Kim et al., 2010). In the first step, the streamflow series is converted from the time domain to the frequency domain by Fourier transformation (Morrison, 1994). In this frequency domain, the data are represented by the phase angle and the power spectrum, as represented by the periodogram. The phase angle θ of the power spectrum is uniformly distributed over the range of $-\pi$ to π. In the second step, the phases in the phase spectrum are randomized while the power spectrum is preserved. In the third step, the inverse Fourier transformation is applied to transform the data from the frequency domain back to the temporal domain (Maiwald et al., 2008).

The Fourier transformation of a given time series $x = (x_1, \ldots, x_t, \ldots, x_n)$ of length n is

$$f(\omega) = \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} e^{-i\omega t} x_t, -\pi \leq \omega \leq \pi, \tag{1}$$

where $i = \sqrt{-1}$ is the imaginary unit. The original time series can be recovered by the back transformation

$$x_t = \sqrt{\frac{2\pi}{n}} \sum_{j=1}^{n} e^{i\omega_j t} f(\omega_j), t = 1, 2, ..., n, \tag{2}$$

if the transformation is calculated for discrete frequencies $\omega_j = 2\pi/n, j = 1, 2, ..., n$. The Fourier transformation surrogate method constructs a new time series y_t with the same periodogram as the observations. Apart from this, the new series are statistically independent of x_t. This can be achieved by fixing the Fourier amplitudes $|f(\omega_j)|$ and replacing the Fourier phases $\phi(\omega_j) = \arg(f(\omega_j))$ by uniformly distributed random numbers $\phi_{\text{rand}}(\omega_j) \in [-\pi, \pi]$. A new realization is given by

$$y_t = \sqrt{\frac{2\pi}{n}} \sum_{j=1}^{n} e^{i\omega_j t} |f(\omega_j)| e^{i\phi_{\text{rand}}(\omega_j)}. \tag{3}$$

The surrogate data consist of the same values as the original data in another temporal order but with the same time dependence structure as the original data (Schreiber and Schmitz, 2000). The approach can be extended to multiple sites by multiplying the phases of each site by the same set of random phases. This is possible because the cross-spectrum, which describes the cross-correlation of the data in the frequency domain, reflects relative phases only (Prichard and Theiler, 1994; Schreiber and Schmitz, 2000).

3 Methods

3.1 Stochastic streamflow simulation

Here, we use phase randomization to simulate stochastic streamflow time series to be used in various water resource management studies. The stochastic series generated using phase randomization are combined with a theoretical distribution to allow extrapolation to unobserved values, which still realistically represent daily streamflow values. The observed streamflow time series require pre-treatment before phase randomization can be applied. First, they need to be normalized because phase randomization assumes Gaussianity (Maiwald et al., 2008). Second, they need to be deseasonalized in order to remove monthly/daily fluctuations (Pender et al., 2015). The stochastic simulation procedure consists of the following seven steps.

1. **Fitting of theoretical kappa distribution:** The four-parameter kappa distribution (Hosking, 1994) is fit to the daily values of the observed input time series using L-moments. This distribution will be used for the back transformation in Step 7, and permits extreme values going beyond the empirical distribution to be obtained. It has four parameters and its cumulative distribution function is expressed as

$$F(x) = \left\{1 - h[1 - k(x-\xi)/\alpha]^{1/k}\right\}^{1/h}, \tag{4}$$

where ξ is the location parameter, α is the scale parameter which must be positive, and k and h are the shape parameters. The kappa distribution was found suitable for fitting observed streamflow data in U.S. catchments (Blum et al., 2017). A suitable fit was also found for our data as confirmed by the Kolmogorov–Smirnov and Anderson–Darling tests which did not reject the null hypothesis at $\alpha = 0.05$ for most catchments. We fit a separate distribution for each day to take into account seasonal differences in the distribution of daily streamflow values. To do so, we used the daily values in a 30-day window around the day of interest. This procedure guarantees a large enough sample for the parameter fitting procedure, and allows for smoothly changing distributions along the year. For leap years, flows from February 29 were removed to maintain constant sample sizes across years as in Blum et al. (2017).

2. **Normalization and deseasonalization of the marginal distribution:** The input time series are normalized using the normal transform, i.e., values corresponding to a certain rank are replaced with respective values from a standard normal distribution. The normal transform is applied to each day of the year separately, which results in the deseasonalization of the marginal distribution of the data.

3. **Fourier transformation:** The normalized and deseasonalized data are transferred to the frequency domain using the Fourier transformation (Equation 1). The Fourier phases (i.e., the arguments of the Fourier transformation) are computed.

4. **Random phase generation:** Random phase series are generated by sampling from the uniform phase distribution. The observed spectrum (i.e., the modulus of the Fourier transformation) is preserved.

5. **Inverse Fourier transformation:** The random phases are combined with the observed spectrum and inverse Fourier transformation is applied (Equation 2) to transform the data back to the time domain.

6. **Back transformation:** The data are back transformed from the normal to the kappa domain using the fitted daily kappa distributions (Equation 4), which achieves reseasonalization. This is done by generating a sample of length n (length of observed time series) and reassigning values according to the ranks in the simulated series. Negative simulated values are replaced by values sampled from a uniform distribution in the interval [0, min(x)], where min(x) represents the minimum of the observed values corresponding to the day under consideration. Using the empirical distribution instead of the kappa distribution would prevent us from obtaining values that go beyond the range of observed data (Srinivas and Srinivasan, 2006). Depending on the input time series, other suitable theoretical distributions than the kappa distributions could be used for back transformation.

7. **Simulation:** Steps 4-7 are repeated m times to generate m time series of the same length as the observed time series.

The method is extended to the simulation of stochastic streamflow time series at multiple sites. To model the cross-correlation between sites, the phase randomization performed in Step 4 of the procedure is performed in the same way for all the stations in the dataset (Prichard and Theiler, 1994). In contrast, the parameters of the monthly kappa distributions and the power spectrum are calculated for each individual site separately.

3.2 Model validation

The simulation was validated on the observed streamflow time series of a set of four catchments in Switzerland (Figure 1), namely, Plessur-Chur, Birse-Moutier, Thur-Jonschwil, and Cassarate-Pregassona. The catchments are characterized by diverse catchment characteristics and flow regimes (Table 1). Their catchment areas range between 74 and 493 km^2 and their mean elevations between 930 and 1,850 m a.s.l. Plessur represents a catchment with a melt-dominated flow regime with high flows in summer but low flows in winter. In contrast, the flow regimes of Birse and Thur are dominated by precipitation with high flows in winter and low flows in summer. The regime of Cassarate shows two peaks, one in spring due to melt processes, and one in fall due to precipitation.

The model outlined in the previous section was fit to the observed time series over 50 years (1960-2009) for each individual catchment. The application of this approach is only recommended for records longer than 30 years to reduce uncertainty in the estimation of the parameters of the kappa distribution. The model was then run, on the one hand, for each individual catchment and, on the other hand, for the four sites jointly. In both cases, 100 sets of stochastic streamflow time series of the same length as the observed series were generated as in Salas and Lee (2010) and Pender et al. (2015).

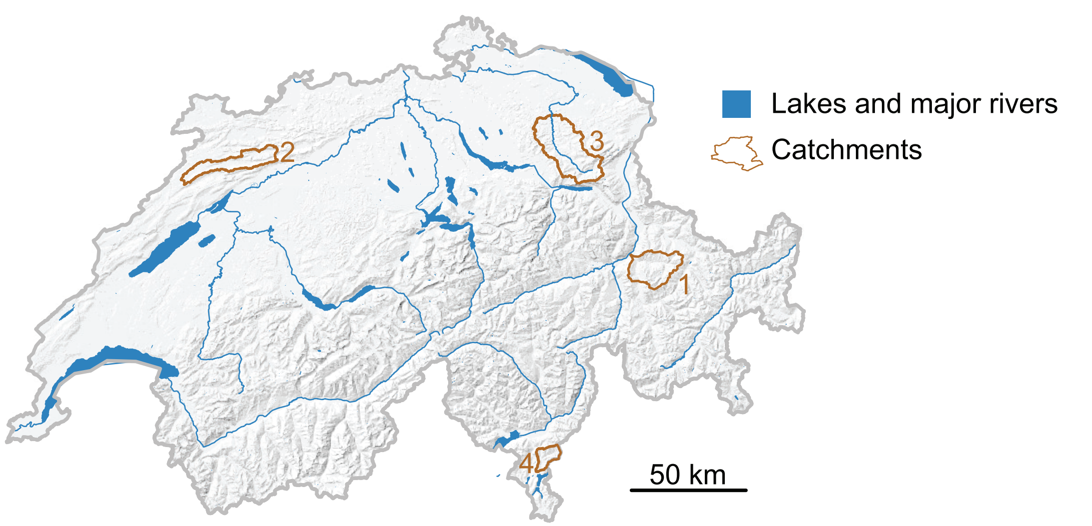

Figure 1. Map showing the four Swiss catchments: 1) Plessur, 2) Birse, 3) Thur, and 4) Cassarate.

Table 1. List of catchments and catchment summary including ID, river name, gauging station, catchment area, station elevation, mean elevation, and flow regime.

ID	River	Gauging station	Area (km^2)	Station elevation (m a.s.l.)	Mean elevation (m a.s.l.)	Flow regime
1	Plessur	Chur	263	573	1,850	Melt-dominated
2	Birse	Moutier	183	519	930	Rainfall-dominated
3	Thur	Jonschwil	493	534	1,030	Rainfall-dominated
4	Cassarate	Pregassona	74	291	990	Mixed

Both the temporal correlation structure and seasonal streamflow statistics were used to compare observed and simulated streamflow time series in order to assess the validity of the stochastic streamflow generation model. As in Kim et al. (2010), we used the autocorrelation function on daily values to represent the short-range temporal correlation. Further, we also used the partial autocorrelation function (Stedinger and Taylor, 1982). In addition to short-range dependence, long-range dependence was assessed by looking at the autocorrelation function of annual discharge sums. The seasonal statistics were validated with respect to the seasonal distributions (winter: Dec–Feb, spring: Mar–May, summer: Jun–Aug, and fall: Sep–Nov) and the monthly means, maxima, minima, and standard deviations. In addition to general distribution characteristics, the approach was validated for low and high flows because these characteristics are often of interest in hydrological simulation studies (Borgomeo et al., 2015). High and low flows were defined as above or below threshold values, respectively. For high flows, the 95th percentile was used as a threshold, while the 5th percentile was used for low flows.

4 Results

4.1 Simulation at individual sites

The stochastic streamflow generator was found to produce realistic annual hydrograph realizations as illustrated in Figure 2 for the Plessur catchment (Figure 1). This is confirmed visually by observing the temporal correlation structure, as well as the seasonal statistics (see Figure 3 for Thur and Plessur).

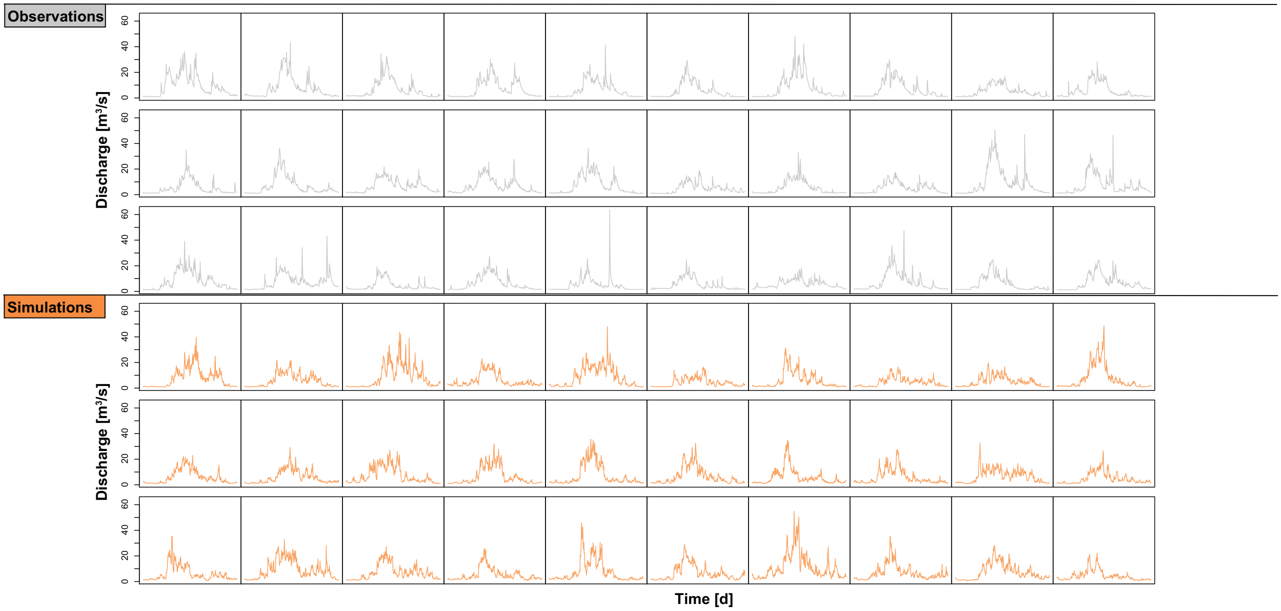

Figure 2. Observed (grey) and stochastically generated (orange) annual hydrographs at daily resolution over 30 years for the Plessur catchment.

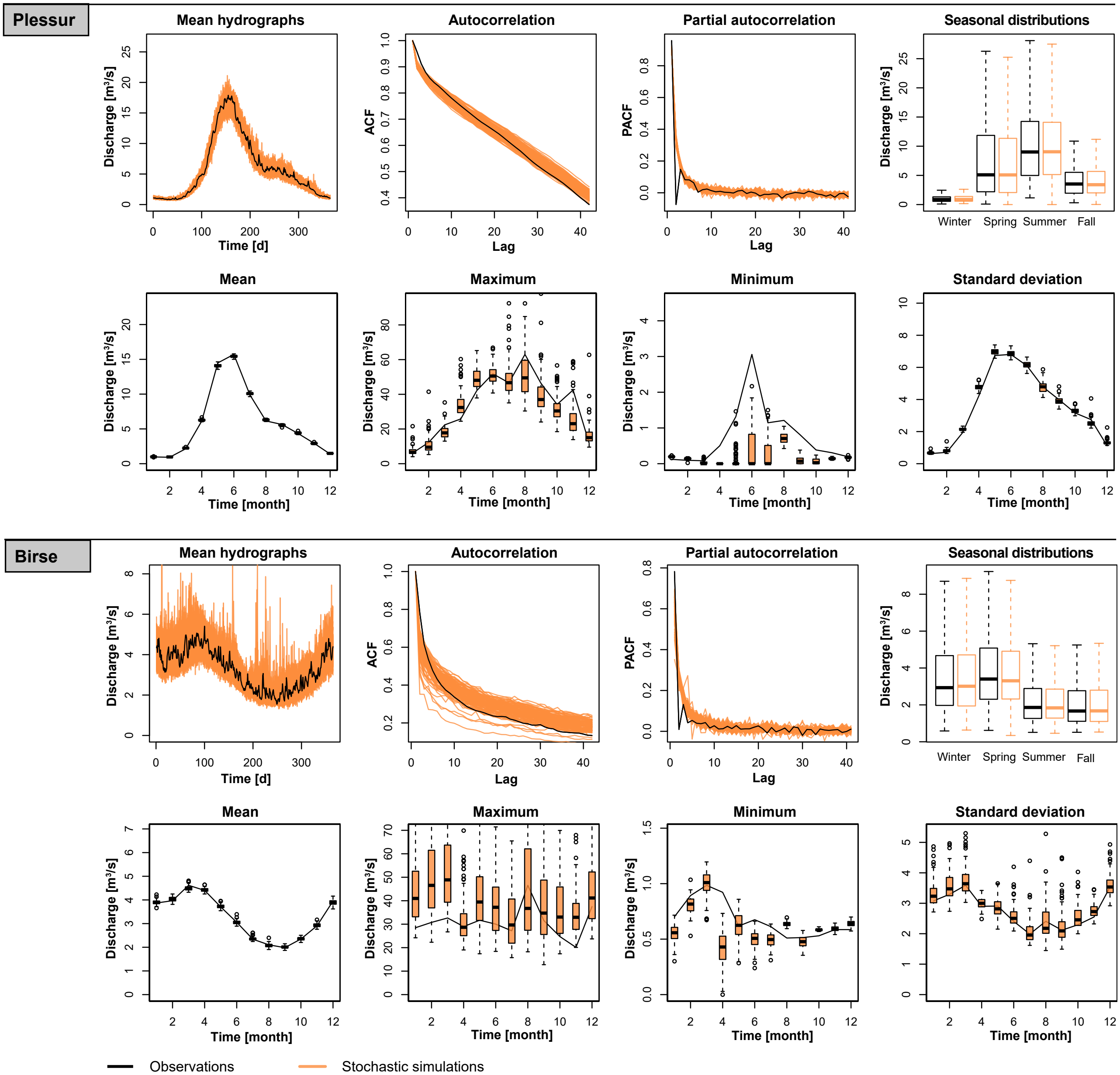

Figure 3. Comparison of observed and stochastically generated time series for the melt-dominated Plessur catchment (upper panels) and the rainfall-dominated Birse catchment (lower panels) for the following characteristics: Mean hydrograph over 50 years, autocorrelation function, partial autocorrelation function, seasonal distributions, monthly means, monthly maxima, monthly minima, and monthly standard deviations. Black lines represent observations while orange lines represent simulations.

The stochastic generator produces time series with mean regimes similar to the observed mean regime, and reproduces both the autocorrelation (ACF) and partial autocorrelation functions (PACF). Seasonal distributions match well thanks to the good fit of the kappa distribution to the data. Monthly means and standard deviations match particularly well, while monthly maxima and minima show some deviations from the observed maxima and minima, as was intended by using a theoretical instead of an empirical distribution. The suitability of the kappa distribution to produce realistic high and low flows is confirmed in Figure 4. The distribution produces low flows similar to observed low flows but with different outliers. In two catchments (Thur and Cassarate) however, observed low flows were rather overestimated. High flow distributions match well in all catchments, and values exceeding observed values are generated. The four-parameter kappa distribution (Houghton, 1978; Griffiths, 1989) was found to be more suitable for representing daily streamflow values compared to distributions with even more parameters, which are rather prone to over-fitting. Similarly, tests on distributions with only three parameters (e.g. Burr type XII (Burr, 1942) and generalized Gamma distributions (Stacy and Mihram, 1965)) were here not satisfactory because the distributions were not flexible enough. In cases, where distributions with less parameters provide a satisfactory fit, they could, however, be used instead of the kappa distribution to ensure model parsimony.

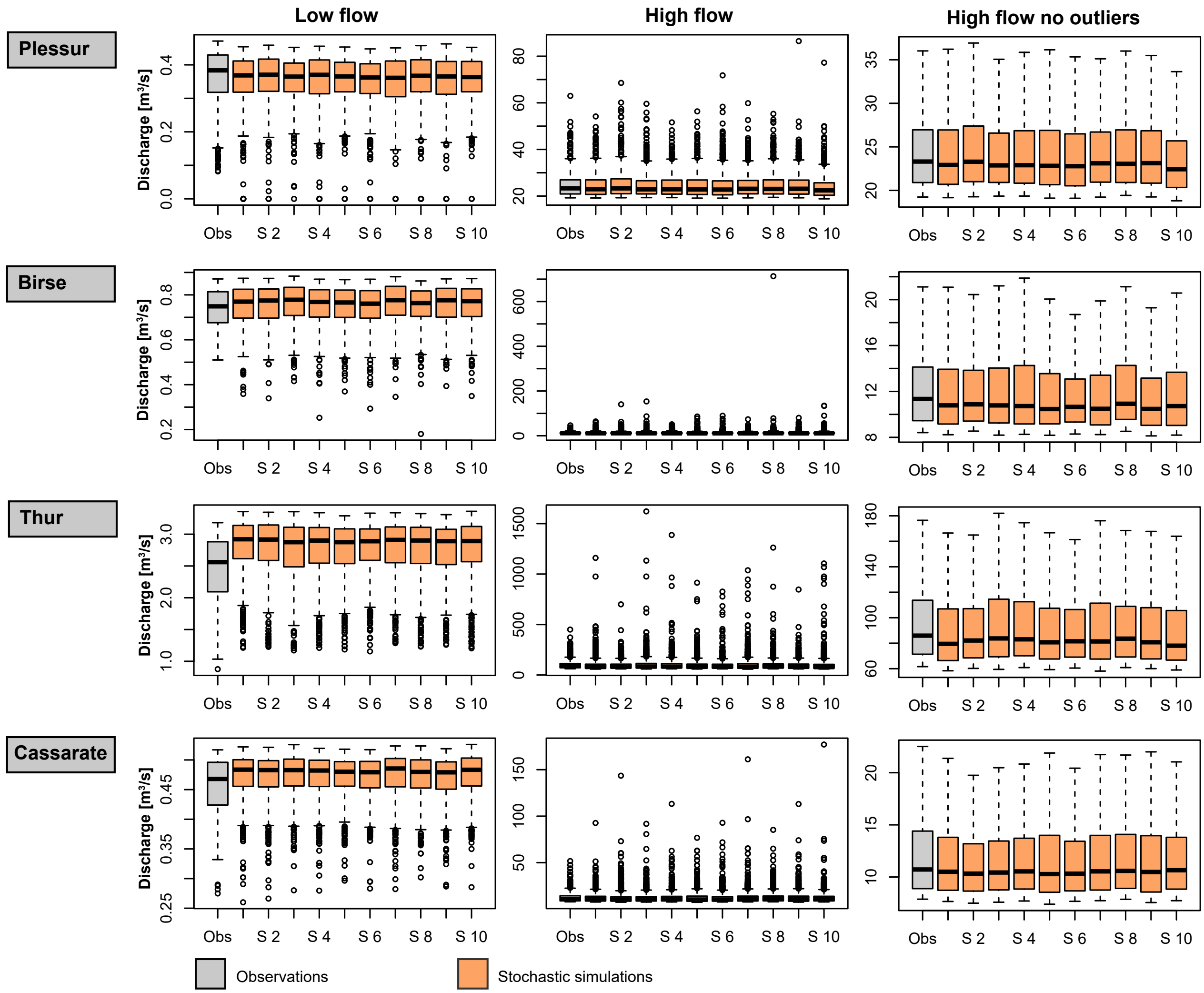

Figure 4. Low and high flows for observed (grey) and simulated (orange) time series for the four catchments Plessur, Birse, Thur, and Cassarate. The results are given for ten simulation runs (S1-S10), and high flows are plotted with (middle panel) and without outliers (right panel). Whiskers extend to the lowest/highest data point which is still within 1.5 times the interquartile range.

The stochastic streamflow generator is not only able to reproduce the streamflow distribution and the short-range dependence in the data, but also the long-range dependence over several years (Figure 5). Both the rapid decrease in the ACF at short lags (up to five years) and the cyclical behavior at lags longer than five years are reproduced as well.

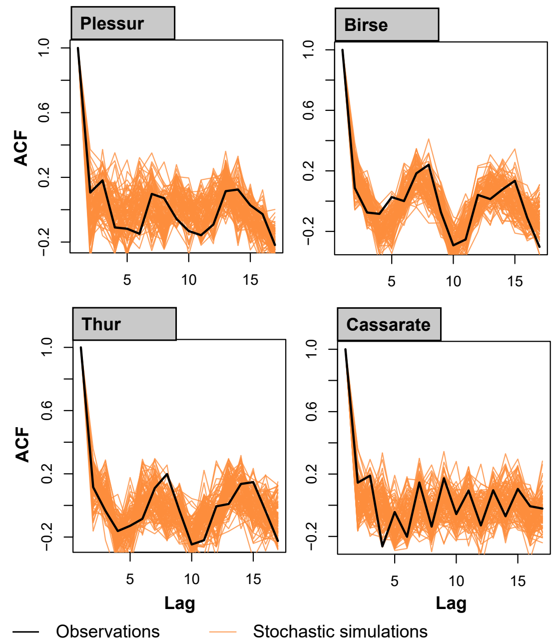

Figure 5. Autocorrelation (ACF) of annual streamflow sums of the observed and simulated streamflow time series for the catchments Plessur, Birse, Thur, and Cassarate.

The good performance of the stochastic streamflow generator with respect to streamflow distribution and temporal correlation - both short and long range - is not limited to these four example catchments but generalizes to other data sets used as input.

4.2 Simulation at multiple sites

The stochastic streamflow generator can be extended from the simulation at individual sites to the joint simulation at multiple sites. In addition to reproducing distribution and temporal correlation at individual sites, it should then be able to reproduce the cross-correlation among sites, which describes the similarity of time series at two sites. Figure 6 shows the cross-correlation function (CCF) for pairs of stations among the example catchments for the observed time series and the 100 simulation runs. Cross-correlation is already generally low for observations because the selected sample catchments are characterized by diverse discharge regimes and seasonality. The shape of the cross-correlation is reproduced for all pairs of stations. However, the magnitude of cross-correlation is underestimated for certain pairs of stations in the simulated time series compared to the observed series independently of the simulation run considered. For the catchment pair Birse-Thur, whose discharge behavior is rainfall-dominated, the simulated cross-correlation is much lower than the observed one. In the observations, spatially consistent rainfall events lead to a joint rise in discharge at both stations. This behavior is not captured by the stochastic discharge generator. The underestimation of cross-correlation is also visible when looking at the cross-correlation of below- or above-threshold events (not shown).

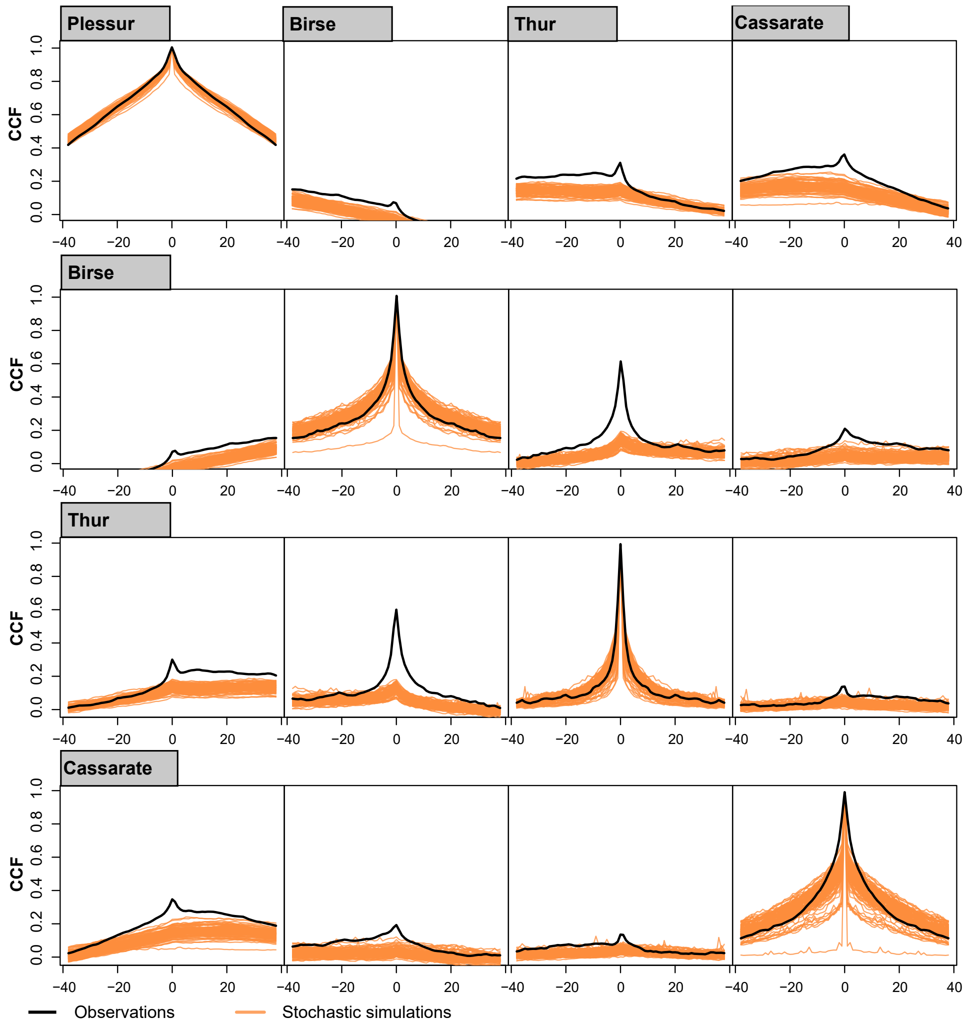

Figure 6. Cross-correlation function (CCF) of observed (black line) and simulated (orange lines) daily streamflow for pairs of stations at Plessur, Birse, Thur, and Cassarate.

5 Discussion and Conclusions

The stochastic streamflow generator based on phase randomization has been shown to produce realistic streamflow time series with respect to both distributional properties and temporal correlation. Compared to models commonly used for the stochastic generation of streamflow time series, such as autoregressive moving average models, the simulation approach presented here not only reproduces short-range but also long-range dependence. However, the representation of this dependence is limited to ranges within the length of the observed time series. Instead of producing one long time series, the simulation procedure allows for the simulation of multiple series of the same length as the original series. The use of ensembles of the same length of the observed time series might not be equivalent to using a long time series. Still, long-range dependence features may not be generated in either case since the model is fitted based on a limited number of years of observations. While the reproduction of the temporal dependence was well reproduced here, this is not necessarily the case under all conditions. Embrechts et al. (2010) have shown that any nonlinear transformation of a Gaussian time series, which is done during backtransformation, reduces the strength of the linear correlations in the time series as expressed by Pearson's correlation coefficient and preserves only rank correlations. If one is working with heavy-tailed and zero inflated marginals (as present when looking at intermittent processes), it can happen that autocorrelations are reduced during backtransformation (Papalexiou, 2018).

Phase randomization was here combined with the flexible four-parameter kappa distribution, which was found to effectively represent daily streamflow values. The distribution of daily flows was found to be modelled well in all seasons. However, the use of one distribution per day has the disadvantage of introducing a lot of parameters, which makes the model non-parsimonious (Koutsoyiannis, 2016). If the user is not reliant on the generation of unobserved values, he/she might use the empirical instead of the theoretical kappa distribution for backtransformation instead. The use of the kappa distribution allows us to generate values that go beyond the range of observed values, which would not be the case if the empirical distribution was used. This ability of the generator to extrapolate extremes makes it suitable for applications where extreme events such as floods and droughts are of interest.

The generator can, on the one hand, be used to simulate streamflow at individual sites, and, on the other, to simulate jointly at multiple sites, which is not necessarily the case for other existing models. Its application to the example catchments, however, resulted in somewhat underestimated cross-correlations between stations. This underestimation can be explained by the fact that phase randomization preserves the cross-correlation in the normal domain but not necessarily in the domain of the original distribution. This cannot be overcome even if the simulation run which best reproduces these cross-correlations is extracted from a large set of simulations. However, Stedinger and Taylor (1982) showed that estimators of the autocorrelation and cross-correlation of flows which do not match the historical sample estimates often provide more accurate estimates of the true but unknown correlations. Still, there are several potential avenues for improving the representation of cross-correlation. A first possibility would be the use of phase annealing (Hörning and Bárdossy, 2018). Phase annealing modifies the Fourier phases in an iterative way in order to optimize certain statistics, such as the cross-correlation function, and makes it possible to take covariates into consideration for the generation of time series. However, using phase annealing increases the computational effort. A second possibility was presented by Keylock (2012) who only randomized the phases corresponding the wavelet coefficients lying above a certain threshold. He suggested to fix the large wavelet scales if one wanted to ensure that the low-frequency behavior between the observations and simulated series remains the same. This can indeed be a solution for retaining the cross-correlation between two series. However, it comes with the disadvantage that the temporal structure of the simulated series is not very variable from the one of the observed series anymore. A third possibility is the introduction of functions correcting for the phase differences between two series as done by Nguyen et al. (2019) who applied this approach to correct for biases across multiple atmospheric variables derived from global circulation models. Another possibility for addressing the underestimation of cross-correlation would be the inflation of the cross-spectrum in the original domain in order to allow for a certain target cross-correlation after backtransformation. To do so, transformation approaches have been introduced, which inflate the original process, which should after the backtransformation to the original domain result in a process with a target distribution and correlation structure (Papalexiou, 2018; Tsoukalas et al., 2018b). An additional disadvantage of the method presented here (and of most other approaches presented in the literature) is that time irreversibility, which has been shown to be significant at a daily scale (Koutsoyiannis, 2019), is not explicitly modelled.

The streamflow generator was here used on observed streamflow time series. The input time series, however, do not necessarily need to consist of observed values. One could also use the generator on streamflow simulated with a hydrological model. This extends its application to climate impact studies where a hydrological model is driven by meteorological time series generated with global and/or regional climate models. Alternatively, the representation of non-stationary conditions in the properties of the marginal distribution or the temporal dependence structure could also be achieved by adjusting the parameters of the marginal distribution or the frequency spectrum, respectively. Phase randomization simulation can potentially not only accommodate changing climate conditions but also changes in land use or water extractions. The approach is not limited to the simulation of streamflow time series but extends to other hydro-meteorological variables such as precipitation, evapotranspiration, or snowmelt. This would require the test and identification of a suitable marginal distribution. In the case of intermittent processes, mixed-type marginal distributions would need to be used (Papalexiou, 2018). Distributions other than the kappa distribution can be used in PRSim by specifing a suitable (mixture) distribution. Spatio-temporal modelling of precipitation fields, for example, may be performed using a technique based on phase randomization. However, it must be noted that due to the large number of zero observations (specifically with fine temporal resolution) the normal score transformation can become non-unique. In this case, additional efforts are needed to preserve the spatial structure of precipitation.

The stochastic streamflow generator presented here represents a flexible tool for streamflow simulation at individual or multiple sites. It can be used for various applications such as the design of hydropower reservoirs, the assessment of flood risk, or the assessment of drought persistence and the estimation of the risk of multi-year droughts.

Code availability. The stochastic simulation procedure for a single site using the empirical, kappa, or any other distribution and some of the functions used to generate the validation plots are provided in the R-package PRSim. The stable version can be found in the CRAN repository https://cran.r-project.org/web/packages/PRSim/index.html, and the current development version is available at https://git.math.uzh.ch/reinhard.furrer/PRSim-devel.

Data availability. The observational discharge data was provided by the Federal Office for the Environment (FOEN), and can be ordered from http://www.bafu.admin.ch/wasser/13462/13494/15076/index.

Author contributions. AB and MB jointly developed the concept and methodology of the study. MB and RF set up the simulation approach. MB did the data analysis, produced the figures, and wrote the first draft of the manuscript. The manuscript was revised by RF and AB and edited by MB.

Competing interests. The authors do not have any competing interests

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