Responses to Anonymous Referee #1

This technical notes compares two solutions to a Nash cascade model. The note is generally well-written but content is highly mathematical and probably only accessible to a small number of people. This is of course not necessarily a problem, but in this case I am struggling to understand what is the exact nature of the problem, and how the proposed work constitutes an improvement. It also appear that the model has previously been published in a recent J. Hydrology paper.

Reply: We thank Referee #1 for his/her positive evaluations. In the paper published in J. Hydrology, the detailed theoretical derivation of the Generalized Nash Model (GNM) was made. In our recent research, we found that the discrete linear cascade model (DLCM) proposed by Szollosi-Nagy and then developed by Szilagyi is similar to GNM. These two models are both derived from a same problem, i.e., the Nash cascade model with a non-zero initial condition. Theoretically, these two models should have a same expression and a same result. However, whether the expressions or the simulation results of these two models are differently exhibited. This may confuse the model users. As the proposer of the GNM, we should have the responsibility to clarify these confusions. In this manuscript, the relationship between these two models was first investigated by the reconstruction of the DLCM. Then the difference was found in the process of the reconstruction. This comparison can help the model users make a clearer understanding of these two models. Furthermore, along with the reconstruction of the DLCM, the interpretation of the DLCM was also made, which makes it more conceptual in hydrology and not only a mathematical formulation any more.

The improvements seems only to be only very minor as reported in Section 4, and the comparison of the hydrographs in Figure 1 are almost indistinguishable. Finally, the conclusion is a general summary, and not really related to the results presented in the note - it could have been written before anything else in the note.

Reply: The main purpose of this manuscript is to clarify the relationship and difference between these two models. The essential difference between these two models lies in the identification of the initial state. In the DLCM, the initial state is estimated, while that in the GNM is observed. As a result, the DLCM will have multiple solutions to the Nash cascade model, and the GNM can provide the unique result. For a long-time simulation in a linear system, the influence of the initial state can be ignored, as shown in Fig.1. However, in the real-time forecasting, the updated precision of initial state will have a great impact on the following predicted one, just as the first few predictions of the hydrographs in Fig.1. The conclusion has been rewritten as follows:

Both the DLCM and GNM are derived from the Nash cascade model with a non-zero initial condition. The DLCM formulated the Nash cascade model in a matrix form based on the principles of state space analysis, while the GNM was written in a simpler algebraic expression after the complicated theoretical derivation. To clarify these two *models, the relationship and difference have been investigated mathematically and experimentally. The main conclusions are summarized as follows:*

(1) The DLCM can be transformed to the GNM when the initial storage state is directly calculated by the linear storage-outflow relationship suggested in the Nash cascade model.

(2) The essential difference between these two models lies in the identification of the initial state. In the DLCM, the initial state is estimated, while that in the GNM is observed.

(3) The DLCM is an approximate solution of the Nash cascade model but not the exact solution due to its nonuniqueness of the initial estimated state. The GNM is the unique analytical solution of the Nash cascade model, whose initial state is implicitly written in a form of derivative and does not need to be estimated separately.