Supplement for

Using the Maximum Entropy Production approach to integrate energy budget modeling in a hydrological model

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Introduction

Figures S1 to S3 supplement the manuscript by providing time series of observed and modelled soil moisture at multiple depths, rather than a single one, as presented in the manuscript. There is one figure per study site (S1: US-Wkg, S2: US-Ton and S3: US-WBW).

Table S1 provides the equation of performance metrics (RMSE, NSE, BE, R², PBIAS) using to compare simulated and observed values.



Figure S1. Time series of observed and modelled soil moisture at a depth of a) 5 cm, b) 15 cm and c) 30 cm at US-Wkg (climate: semiarid, vegetation: grassland).



Figure S2. Time series of observed and modelled soil moisture at a depth of a) 0 cm, b) 20 cm and c) 50 cm at US-Ton (climate: Mediterranean, vegetation: woody savanna).



Figure S3. Time series of observed and modelled soil moisture at a depth of a) 5 cm, b) 20 cm and c) 60 cm at US-WBW (climate: temperate, vegetation: deciduous broadleaf forest).

Table S1. Equation of performance metrics to compare observed and simulated values.

metric	equation
root mean square error	$RMSE = \left[\frac{1}{N} \sum_{t=1}^{N} \left[x_{sim}(t) - x_{obs}(t)\right]^2\right]$
Nash-Sutcliffe efficiency (NSE)	NSE = 1 - $\begin{bmatrix} \frac{N}{t=1} [x_{sim}(t) - x_{obs}(t)]^2 \\ \frac{N}{t=1} [x_{obs}(t) - x_{obs}]^2 \end{bmatrix}$
normalized benchmark efficiency (BE)	$BE = 1 - \left[\frac{\sum_{t=1}^{N} [x_{sim}(t) - x_{obs}(t)]^{2}}{\sum_{t=1}^{N} [x_{obs}(t) - x_{bench}(t)]^{2}}\right]$
coefficient of determination (R ²)	$R^{2} = \frac{\frac{1}{N} \sum_{t=1}^{N} \left[\left(x_{obs}(t) - x_{obs} \right) \left(x_{sim}(t) - x_{obs} \right) \right]}{\left[\frac{N \sum_{t=1}^{N} x_{obs}^{2} - \left[\sum_{t=1}^{N} x_{obs}(t) \right]^{2}}{N(N-1)} \sqrt{\frac{N \sum_{t=1}^{N} x_{sim}^{2} - \left[\sum_{t=1}^{N} x_{sim}(t) \right]^{2}}{N(N-1)}} \right]}$
percent bias (PBIAS)	PBIAS = $\frac{\frac{N}{t=1} \left[x_{sim}(t) - x_{obs}(t) \right]}{\frac{N}{t=1} \left[x_{obs}(t) \right]} * 100$

where $x_{obs}(t)$ is the observed value at time step t, $x_{obs}(t)$ is the simulated value, \bar{x}_{obs} is the mean observed value over the simulation period of length N, x_{bench} is the benchmark model, in this case the interannual mean of observed values for each calendar day.