

## ***Interactive comment on “Derivation of the mean annual water-energy balance equation based on an Ohms-type approach” by X. Shan et al.***

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This manuscript claims to give a new, “more physical” derivation of the MCY model of the Budyko equation describing long-term catchment evapotranspiration. The authors attempt to make an analogy between water flow from one catchment to another via transport in the atmosphere, followed by precipitation in the receiving catchment, and Ohm’s law as applied to two resistors arranged in series in an electrical circuit. No justification based on catchment hydrology is given for this analogy or for the three assumptions and two corollaries that it is purportedly derived from. The authors’ resulting “Ohm’s law expression” (Equation 18) is then converted into the MCY model by making another assumption, that the functional form of each term is a power-law in the independent variable. The manuscript also contains many erroneous statements

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(for example, that homogeneity is the basis for Equation 19). Experts on modeling the Budyko equation will find nothing “new” or “more physical” in this manuscript.

Response:

We really appreciate the reviewer taking time to review our manuscript and giving important comments.

Regarding the “more physical” derivation, I think that the derivation for Equation (18) has more physical meaning, namely proposing a catchment network using Lagrangian particle tracking method, establishing the equations based on the Ohm-type law, and giving the boundary conditions of catchment hydrology. According to the Ohm-type law, we assumed water vapor being forced by some potential difference, similar to the movement of soil moisture.

Regarding “new” understanding, one is that the previous derivations for the MCY equation had an underlying assumption, i.e. the homogeneous assumption, the generalized flux having the same form for both water vapor transportation and chase transformation, and in other words precipitation and potential evaporation having an equalized effect on evaporation. It indicates that the assumption needs further test or another form is more suitable for the real catchment.

As the review pointed out, Equation (19) and the text aren’t rigorous. Instead, the homogeneity assumption is included in Equation (18). We will revise the manuscript.

In addition, we have revised the conclusion as: “Previous studies have analytically derived the mean annual water-energy balance equation for the Budyko hypothesis mainly by mathematical reasoning, such as Fu (1981), Yang et al. (2008), and Zhou et al. (2015). To give a new derivation with more physical meaning, this study focused on subsequent transportation and transformation of the precipitation fallen down to a certain catchment using the Lagrangian particle tracking method, proposed a catchment network in which water vapor was transformed and transported through evaporation-

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precipitation processes, defined the generalized flux of water vapor, and expressed the generalized flux as the ratio of potential difference with resistance by using an Ohms-type approach. On the base of these, the relationship among potential evaporation ( $E_0$ ), precipitation ( $P$ ) and evaporation ( $E$ ),  $1/(f(E))=1/(f(E_0))+1/(f(P))$ , was achieved, in which  $f()$  represents the generalized flux (i.e. a function of flux). Furthermore, the MCY equation  $E=(PE_0)/((P^n+E_0^n)^{1/n})$  was derived based on mathematic reasoning. Remarkably, an implicit homogeneity assumption for the MCY equation was exposed, i.e., the generalize function has the same form for both vapor transportation and phase transition, and in other words, precipitation and potential evaporation have an equalized effect on evaporation. In addition, without the homogeneity assumption, this study suggested a general form  $E=P(b+kE_0)/(P^n+(b+kE_0)^n)^{1/n}$ , where  $b$  and  $k$  are constants. The equation can be simplified to  $E=((kE_0 * P)/(P^n+(kE_0)^n)^{1/n})$  proposed by Zhou et al. (2015) if setting  $b = 0$ ; and the MCY equation if setting  $b = 0$  and  $k = 1$ ."

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