Anonymous Reviewer #1

Reply explanation: The reviewers' comments are shown in black, while the author's replies and revises are shown in blue.

Comments: The authors present a reasonable supplement to the conductivity two-component hydrograph separation method.

Reply: We appreciate the positive comment for this study.

Comments: I have found only one smaller calculation error. Page 3, equation 14, and page 8, equations A8 and C1: In the second term in the numerator of the partial derivative dBFI/dyk a factor n is missing. It comes from the partial derivative of the sum of yk with respect to yk: (sum yk)’ = d(sum yk)/dyk = sum dyk/dyk = n times 1 = n.

Reply and Revise: Well taken, thank you very much for finding out our errors. We have rechecked all the equations in the manuscript and revised the errors (Page 4, equations 12, 14 and16; Page 9, equations A8; Page 10 equation C1, and line 10).

Comments: Some formulations are linguistically or technically incorrect. Please consider the following suggestions: page 2, lines 10 - 11: Parameter sensitivity, as I understand this term, is the sensitivity of model output to the varying values of model input and not "the sensitivity of the parameters". Also, better than "fluctuation parameters" would perhaps be "varying parameter values". Replace "Eckhradt" by "Eckhardt".

Reply and Revise: Well taken, we also understand that parameter sensitivity is the sensitivity of model output to the varying values of model input. There may be some errors in the expression of the manuscript, and we have revised it (Page 2, lines 24--25). And we have replaced “fluctuation parameters” by “varying parameter values” (Page 2, lines 24--25), also “Eckhradt” by “Eckhardt” (Page 2, line 26).

Comments: page 2, line 12: Replace "An empirical sensitivity analysis is only an analytical calculation of the error propagation through the model, is not feasible." by "An empirical sensitivity analysis is only a makeshift if an analytical sensitivity analysis, that is an analytical calculation of the error propagation through the model, is not feasible".

Reply and Revise: Well taken, we have replaced the sentence as suggested (Page 2, lines 26--28).

Comments: page 2, line 14: Replace "However, the" by "Until now, the".

Reply and Revise: Well taken, we have replaced the words as suggested (Page 2, line 30).

Comments: page 3, lines 5-6: Replace "the BFI' errors caused by tiny errors of BF_C and RO_C can be expressed as" by "the errors of BFI caused by small errors of BF_C and RO_C can be approximated by".

Reply and Revise: Well taken, we have replaced the sentence as suggested (Page 3, lines 24--25).

Comments: Throughout the paper, the sensitivity indices should be noted with vertical bars, and not with slashes (e. g. S(BFI|BFc) instead of S(BFI/BFc))

Reply and Revise: Well taken, we have revised the sensitivity indices throughout the paper as suggested.

Comments: page 3, lines 16 - 17: Replace "e.g. S(BFI/BFc) = 1.5, the relative error of BFc is 5%, then the relative error of BFI should be 1.5 times 5% (7.5%)" by "e.g. if S(BFI|BFc) = 1.5, and the relative error of BFc is 5%, then the relative error of BFI is 1.5 times 5% = 7.5%"

Reply and Revise: Well taken, we have replaced the sentence as suggested (Page 4, lines 4--5).

Comments: page 3, line 26: If the unit of Qck is µs/cm, then the unit of the partial derivative of BFI with respect to Qck is cm/µs.

Reply and Revise: Well taken, we have added the unit of the partial derivative of BFI with respect to Q_{ck} (Page 4, line 17).

Comments: page 3, line 27, page 8, lines 16 and 22: If the unit of yk is m3/d, then the unit of the partial derivative of BFI with respect to yk is d/m3.

Reply and Revise: Well taken, we have added the unit of the partial derivative of BFI with respect to y_k (Page 4, line 17; Page 10, lines 8 and 14).

Comments: page 4, line 3: Omit "usually".

Reply and Revise: Well taken, we have deleted the "usually".

Comments: page 4, lines 2-5, and lines 6-10: These two paragraphs express one and the same ("the error of BFI caused by the errors of Qck and yk can be neglected"). Then, this is empirically shown again in rest of this section, including figures 1 and 2. Is this necessary? If the sum of delta Qck and the sum of delta yk were not zero for n to infinity, then delta Qck and delta yk did not stand for random errors, but for systematic errors.

Reply and Revise: Well taken, we have reduced the description of this section and have added Fig. 1 and Fig. 2 and related descriptions to the Supplement S1 (Page 5, lines 3--31).

Comments: page 5, line 3: Replace "a parameter g is calculated" by "a variable g is calculated".

Reply and Revise: Well taken, we have replaced the words as suggested (Page 5, line 34).

Comments: page 5, line 5: Equation 17 is the Gaussian error propagation. The citation "(Taylor, 1982; Kline, 1985; Genereux, 1998)" is not appropriate in this context.

Reply and Revise: Well taken, we have adjusted the description and citation of equation 17 (Page 5, lines 34--37).

Comments: page 6, line 24: Replace "The sensitivity index" by "The absolute value of the sensitivity index".

Reply and Revise: Well taken, we have replaced the words as suggested (Page 7, line 31).

Comments: page 6, line 25: Replace "-1.39 times of uncertainty in BFI (-6.95%), while RO_C leads to -0.98 times (4.9%)" by "-1.39 times 5 % of uncertainty in BFI (-6.95%), while RO_C leads to -0.98 times 5 % (4.9%)".

Reply and Revise: Well taken, we have replaced the sentence as suggested (Page 7, lines 33--34).

Comments: Fig. 3: Replace "normal axes" by "linear axes".

Reply and Revise: Well taken, we have replaced the words as suggested (Fig. 1)

Language Improve: We have asked an English native language agency to check and correct the grammar and structure of the manuscript.

Reviewer #2

Reply explanation: The reviewers' comments are shown in black, while the author's replies and revises are shown in blue.

Comments: The authors present an interesting analysis of sensitivity of the two-component chemical mass balance method based on specific conductivity (SC). The paper is generally clear, although the final version should be read carefully for grammar and understanding.

Reply and Revise: We appreciate the positive comment for this study. And we have asked an English native language agency to check and correct the grammar and structure of the manuscript.

Comments: My main concern is how the errors in the baseflow SC are dealt with. As noted by the authors, this has a major impact on the results of the chemical mass balance. Aside from the question as to whether to use the 99th percentile or the maximum SC, there are several common ways of estimating the SC of baseflow in chemical mass balance studies, these include:

1) Measurement in near-river groundwater bores

2) Using a single value based on the highest SC of the river throughout the study period

3) For multi-year studies, assigning a constant value for each water year (generally based on the highest SC in low summer flows)

4) Assuming that the baseflow SC varies linearly between the SC of successive low flow periods (the paper of Miller et al., 2014 uses that strategy).

These strategies can produce very different estimates of baseflow from the same river SC data. This is especially true for catchments where the contrast between the SC of surface runoff and baseflow are large and where the maximum SC in the river varies between successive low flows.

In practice, it is very difficult to estimate the SC of baseflow due to

* Groundwater having spatially variable SC and the fluxes of groundwater from different areas of the catchment varying over time as water tables rise and fall

* Baseflow being comprised of different components (groundwater, interflow, bank return waters), all of which have different SC, that contribute to river flow in different proportions at different times.

An uncertainty of 5% (section 4.2) is probably over optimistic. In section 4.1, it would be better to calculate an uncertainty based on the last three strategies noted above (perhaps with or without the 99th percentile constraint as well). While there is no foolproof methodology for estimating the SC of baseflow, this would yield a better estimate of what the realistic uncertainties are.

Reply and Revise: We are very grateful for your explanation of the common strategies of estimating the conductivity of baseflow and the complexity of the conductivity of baseflow. We fully agree with your description.

We have recalculated the sensitivity indices and the uncertainty based on the fourth strategy (the baseflow conductivity varies linearly between the conductivity of successive low flow periods) you mentioned with the 99th percentile constraint (Sect. 4.1; Sect. 4.2; Table 1; Figures 1&2).

Based on the results of the recalculation, we found that the mean uncertainty of BF_C is about 10%, and the original use of 5% is indeed too optimistic. We have changed the uncertainty (Page 7, lines 32--33).

Other minor comments:

Comments: Equation (1). Suggest changing the nomenclature – Q is commonly used for streamflow in papers.

Reply and Revise: Well taken, we have changed the nomenclature and used SC as the variable name of specific conductance throughout the paper.

Comments: Somewhere in the introduction, you should outline the necessary conditions for chemical mass balance

a) Contributions from end-members other than baseflow and surface runoff are negligible

b) The SC of runoff and baseflow are constant (or vary in a known way) over the period of record

c) Instream processes (such as evaporation) do not change SC makedly

d) Baseflow and surface runoff have significantly different SC

Reply and Revise: Well taken, we have added the assumptions of conductivity two-component hydrograph separation method (chemical mass balance method) as suggested (Page 2, lines 3--7).

Comments: Check consistency with spelling of Eckhardt throughout

Reply and Revise: Well taken, we have checked the spelling and revised the mistake (Page 2, line 26).

Comments: Page 2 lines 12-15 is not very clearly written – try to rephrase it

Reply and Revise: Well taken, we have rephrased the sentences (Page 2, lines 26--28).

Comments: Section 2.2. The errors in streamflow y are only briefly discussed. The value of 3% may be fine but this value looks to come from a thesis and it is not certain whether the gauges studied are relevant to this study. Presumably someone has addressed this for the USGS gauges? More justification of this value is needed.

Reply and Revise: Well taken, the value of 3% came from the uncertainty analysis in streamflow of Yellow River, China. This value may be unfair for the USGS gauges, so we have read some articles to find a more reasonable value.

Olson et al. (2007) and Sauer et al. (2010) indicated that the continuous records of water level in USGS gauges are translated to streamflow by applying the rating curve. And the water level measurements are accurate to the nearest 0.01 foot or 0.2 percent of water level. However, we did not find a description of the uncertainty in streamflow on the website of USGS. Hamilton et al. (2012) indicated that streamflow data from USGS are often assumed by analysts to be accurate and precise to within ±5% at the 95% confidence interval.

Based on the above, we have revised the value of 3% to 5%, and have revised the references and related content (Page 4, lines 32--33).

Comments: Page 4, Lines 10-30. Do you need this amount of detail for these minor errors? Perhaps keep the text as is, but I do not think that the figures are strictly necessary.

Reply and Revise: Well taken, we have reduced the description of this section and have added Fig. 1 and Fig. 2 and related descriptions to the Supplement S1 (Page 5, lines 3--31).

Comments: Page 5, line 6. State the assumptions that the uncertainties are uncorrelated and have a Gaussian distribution.

Reply and Revise: Well taken, we have added the assumptions as suggested (Page 5, line 35).

Comments: Section 4. This application is appropriate but as noted above, the uncertainties in the SC of the baseflow (and possibly yk) are understated.

Reply and Revise: Well taken, we have recalculated the sensitivity indices and the uncertainty based on the fourth strategy (the baseflow conductivity varies linearly between the conductivity of successive low flow periods) you mentioned above with the 99^{th} percentile constraint (Sect. 4.1; Sect. 4.2; Table 1; Figures 1&2).

Based on the results of the recalculation, we found that the mean uncertainty of BF_C is about 10%, and the original use of 5% is indeed too optimistic. We have changed the uncertainty (Page 7, lines 32-33).

Comments: Conclusions. You should add a sentence or two stating what the main sources of error are and how practitioners can go about reducing those. For example, better rating curves are probably more important than better loggers and more work on understanding the SC of baseflow (although it is not clear how you might do that) is more important than understanding the SC of surface runoff.

Reply and Revise: Well taken, we have added the relevant content as suggested (Page 9, lines 6--9).

References:

Olson, S.A., and Norris, J.M., 2007, U.S. Geological Survey streamgaging…from the National Streamflow Information Program: U.S. Geological Survey Fact Sheet 2005–3131, 4 p. (Also available at http://pubs.usgs.gov/fs/2005/3131/.)

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Hamilton, A.S., and Moore R.D., 2012, Quantifying Uncertainty in Streamflow Records , Canadian Water Resources Journal / Revue canadienne des ressources hydriques, 37:1, 3-21, DOI: 10.4296/cwrj3701865

Anonymous Reviewer #3

Reply explanation: The reviewers' comments are shown in black, while the author's replies and revises are shown in blue.

General comments:

Comments: The paper by Yang et al. presents a methodology to compute the uncertainty in the estimation of the long-term baseflow index (BFI) from streamflow and conductance timeseries in rivers. The paper develops equations on the sensitivity of the BFI that, to my knowledge, are new. However, I find that the overall significance of the paper is rather limited. In particular:

1) The authors mention in the title the "two-component hydrograph separation". This is a rather broad and active field of research but the authors narrow their focus on one single index (the BFI, which expresses the long-term ratio between baseflow and streamflow) and they compute it with a very specific methodology.

2) The methodology for the hydrograph separation (equation 1) is based on several assumptions (not mentioned in the manuscript) that are typically not met in the field. One of these is the fact that the parameters of equation (1) are supposed to be fixed during an event (or for an entire time series, as done by the authors). Finding a methodology to relax those assumptions is, in my view, more useful than evaluating the sensitivity of the present methodology to small measurement errors.

In other words, I feel that the authors improve the uncertainty evaluation of an index that, as currently defined, has major constraints and limited reliability.

Reply: We are very grateful to the anonymous reviewer for reviewing the paper and affirming the sensitivity equation. There are two main purposes in this paper. One is to analyze the sensitivity of the long-term series of baseflow separation results (BFI) to the parameters and variables in the conductivity two-component hydrograph separation equation (Sect. 2), and the other is to derive the uncertainty of BFI (Sect.3).(Page 3, lines 1--3).

Sensitivity analysis can quantitatively describe the impact of parameters and variables on the results of base flow separation (so that future users can clearly know which parameter has a greater impact, and should be more cautious when choosing the value). Uncertainty analysis can quantitatively describe the trusted range of BFI calculated by the conductivity two-component hydrograph separation method. The sensitivity analysis equation and uncertainty analysis equation of this paper can be easily applied to the two-component baseflow separation methods of other tracers because they usually have a unified equation form.

Reply to 1): BFI is a form of presentation of long-term series of baseflow separation results. And BFI is a hydrogeological parameter useful in modeling un-gaged basins (Lott and Stewart, 2016) and is believed to represent the effect of geology on basin low flows (Gustard et al., 1992). The total amount of baseflow in a long-term series can be easily obtained by multiplying the BFI by the total streamflow, where the long-term series can be months or years. Researchers in water resources management and assessment usually want to analyze the transformation between groundwater and streamflow by determining the baseflow under a long-term series, so this paper is necessary for them.

Stewart et al. (2007) applied the conductivity two-component hydrograph separation method (also known as CMB) to 10 real-time USGS gauging stations in Florida, Georgia, Texas, and Kentucky. Cartwright et al. (2014) applied this method to the Barwon River catchment, southeast Australia. Miller et al. (2014) applied this method to estimate the baseflow in 14 snowmelt-dominated streams and rivers in the Upper Colorado River Basin. Mei and Anagnostou (2015) indicated that the tracer based hydrograph separation method yields the most realistic results among various methods, because the tracer based method with the highest physical basis. Lott and Stewart (2016) applied this method to 35 basins in USA to calibrate five other baseflow separation methods. The conductivity two-component hydrograph separation method has its own limitations, but there seems to be no better method at present (Miller et al., 2014; Lott and Stewart, 2016).

Reply and Revise to 2): The conductivity two-component hydrograph separation method is indeed based on some assumptions. We have added these assumptions to the manuscript (Page 2, lines 3--7).

The field test (which was located within a 12km^2 drainage basin in southeast Hillsborough County, Florida.) of Stewart et al. (2007) showed that the maximum conductivity of streamflow can be used to replace BF_C, and the minimum conductivity can be used to replace RO_C. The field test may be limited, but the conclusions have been applied to many basins in USA (as mentioned above). Miller et al. (2014) pointed out that the maximum conductivity of streamflow may exceed the real BF_C, so they suggested that the 99th percentile of the conductivity of each year should be used as the BF_C and assumed that the baseflow conductivity varies linearly between years.

Considering that the parameters of the conductivity two-component hydrograph separation method may be varied during the time series, we have changed the application of sensitivity analysis and uncertainty estimation based on the strategy of Miller et al. (2014) (Sect. 4.1 & 4.2).

Specific comments:

Comments: Variable names are rather confusing to a hydrologic community, as Q is conventionally used for streamflow. I invite the authors to adopt a notation based on the papers they refer to (e.g. Miller et al. 2014, Genereux 1998).

Reply and Revise: Well taken, we have changed the nomenclature and used SC as the variable name of specific conductance throughout the paper.

Comments: Besides English grammar errors, the language needs to be improved as the text is often difficult to understand. I invite the authors to revise the use of the term "specific": it seems that they use "specific" to say computed/available. (e.g. specific discharge appears to be just an available timeseries of discharge). Similarly, the use of "specific values" at page 3 Line 17 and "specific" conductivity values (the correct form is specific conductance or electrical conductivity).

Reply and Revise: Well taken, we have asked an English native language agency to check and correct the grammar and structure of the manuscript. We have revised the mistake use of the term "specific" throughout the paper.

Comments: Section 2.2. What is, ultimately, the purpose of this section? Is it to show that the sensitivity of BFI on streamflow and conductance measurements is low (and so it can be removed from subsequent equations like eq 20)?. If so, please make it clearer. What sounds interesting to me is that BFI sensitivity only depends on the integral of the (little) errors on Q and y. But once this is clear from the formula (eq. 15 and 16), then there is no need to show Figures 1 and 2 as the result is implicit from the definition of random errors on Q and y. Instead of the current Figures 1 and 2, why not showing an example of the methodology applied to a case study time series? It would make it easier to understand the usefulness of the approach.

Reply and Revise: Well taken, the ultimately purpose of Section 2.2 is indeed to show that the sensitivity of BFI on streamflow and specific conductance measurements is low (which can be ignored in estimating the uncertainty of BFI). We have made it clearer (Page 5, lines 3--6).

This section really does not need so much description. We have reduced the description and have added Fig. 1 and Fig. 2 and related descriptions to the Supplement S1 (Page 5, lines 3--31).

Comments: Section 3.1: Please make explicit assumptions on the requirement to apply the error propagation formula (eq 17). For example, "tiny" errors means that errors on Q and y should be small random errors related to the analytic uncertainty of the instrument, i.e. no systematic error.

Reply and Revise: Well taken, we have added the assumptions of the error propagation formula (Gaussian error propagation) (Page 5, lines 34--37).

Comments: Page 1 Line 22: rather than "can effectively identify" use "aims to identify"

Reply and Revise: Well taken, we have replaced the words as suggested (Page 1, line 26).

Comments: Page 1 Line 25: "is considered the most effective separation method". By which standards?

Reply and Revise: Kendall et al. (1998) and Miller et al. (2014) indicated that stable isotopes are generally considered to be the most accurate chemical tracers for hydrograph separation. Klaus et al. (2013) and Lott and Stewart (2016) indicated that stable isotope tracers are considered to be the best geochemical method for hydrograph separation. Mei and Anagnostou (2015) indicated that the tracer based hydrograph separation method yields the most realistic results among various methods, because the tracer based method with the highest physical basis.

This paper follows the statement in the above articles. Our statement may not be objective enough, so we have made some changes (Page 1, lines 29--32).

Comments: Page 2 Line 1: I guess this is limited to the particular conditions at which Stewart et al (2007) applied the method. But this is not enough to generalize.

Reply and Revise: The field test site of Stewart et al. (2007) was located within a $12km^2$ drainage basin in southeast Hillsborough County, Florida. The conclusions (the maximum conductivity of streamflow can be used to replace BF_C, and the minimum conductivity can be used to replace RO_C) of the test were applied to 10 real-time USGS gauging stations in Florida, Georgia, Texas, and Kentucky. Then, the conclusions were applied to 35 basins in USA to calibrate five other baseflow separation methods (Lott and Stewart, 2016).

The field test may be limited, but the conclusions have been applied to many basins in USA. Considering these, we have added the particular conditions for the field test to the manuscript (Page 2, line 13).

Comments: Page 2 Line 30: here and after I guess it should be equation (A1) rather than Appendix A1

Reply and Revise: Well taken, we have replaced the presentation of the citations as suggested (Page 3, line 10; Page 3, lines 20 and 30; Page 4, line 10).

Comments: Page 4 Line 2: unclear what is meant by "random analysis errors". Please define what you mean by "tiny errors in Qck and yk".

Reply and Revise: Well taken, we have revised the description of this paragraph (Page 4, lines 27--35).

Comments: Page 4 Line 2-5: This statement is unjustified. Please either formulate it as a hypothesis (e.g., if the errors follow a normal distribution: : :) or remove it.

Reply and Revise: Well taken, we have removed it.

Comments: Page 4 Lines 6-7: "The uncertainty of [: : :] is: : :": please avoid these unjustified general statements. Instrument precision depends on the particular instrument at hand and streamflow precision depends on a very large number of factors. You can simply reformulate the sentence stating that you assumed errors of Lines 11-18 is particularly unclear

Reply and Revise: Well taken, we have changed the statements.

Comments: Page 4 Line 17: which "average error"

Reply and Revise: We have moved this paragraph to the Supplement S1. It should be "relative error (%) of $\sum_{k=1}^{n} SC_k$ and $\sum_{k=1}^{n} y_k$" not "average error", and we have revised it.

Comments: Page 5 Line 13-17: what is the rationale behind the choice of these particular types of uncertainty (W terms)?

Reply and Revise: The uncertainty terms in Gaussian error propagation should be of the same type. One has some choice in the type of uncertainty to propagate, but all the uncertainty values must be the same kind of quantity: either all average errors, all standard deviations, etc. (Genereux, 1998; Ernest, 2005).

"While any set of consistent uncertainty (W) values may be propagated using Gaussian error propagation, using standard deviations multiplied by *t* values from the Student's *t* distribution (each *t* for the same confidence level, such as 95%) has the advantage of providing a clear meaning (tied to a confidence interval) for the computed uncertainty would correspond to, for example, 95% confidence limits on BFI" (Genereux, 1998).

We have added this rationale to the manuscript (Page 6, lines 5--8).

Comments: Page 7 Line 4-5: "During the rainstorm [: : :] the streamflow is almost entirely from the rainfall runoff": this is a serious misinterpretation of hydrological processes. It is well known since at least 15 years that in most catchments the event-water is not a major component of streamflow (and very often it only accounts for a few percent of total flow). See e.g. the commentary by Kirchner (2003) on Hydrological Processes (https://doi.org/10.1002/hyp.5108).

Reply and Revise: We are very grateful to the reviewer for pointing out this problem and giving the reference. Previously, we were mainly concerned about the effect of water-rock interaction on the chemical composition of groundwater and baseflow, and missed the brilliant debate on the “old water paradox”.

After reading Kirchner’s (2003) article, we thought the “old water paradox” is that in the same basin, most isotope tracers show that the flood streamflow contains a large amount of pre-event water, while dissolved salt tracers show that the flood streamflow contains less pre-event water (Isotopic fluctuations are very small in the process of flood streamflow, while the fluctuation of dissolved salts is more obvious).

Van Verseveld (2009) tried to explain the “old water paradox” through a hillside sprinkling experiment, the results show that “mass transfer to the immobile domain, dispersive mixing and rapid transport via lateral subsurface flow explained rapid mobilization of old water and thus the first part of the double paradox in a plausible mechanistic way”, “the supply limitation of DOC, in combination with the vertical and lateral flow paths, controlled the variable DOC chemistry in lateral subsurface flow”. Kienzler (2010) also tried to explain the first part of the “old water paradox” through some hillside sprinkling experiments, the results show that “shallow soil may already contain significant amounts of pre-event water, which can be rapidly released from small, saturated patches of the soil matrix”, “an intensive exchange between overland flow and shallow subsurface flow might be quite common, … overland flow and fast subsurface flow, may, at the same time, produce rapid discharge responses and deliver substantial amounts of pre-event water to the stream”. We have not found a strong theory to solve this paradox, and the existing theories are mostly plausible.

Just our opinion, the isotope composition of rainfall runoff has changed significantly in the surface or shallow soil, while the change in dissolved salt is not obvious. Therefore, the isotope tracer may classify the soil flow and the return flow into the subsurface runoff, while the dissolved salt tracer may classify the soil flow and the return flow into the surface runoff. And we do not think that soil flow and return flow are strictly subsurface runoff (which should be the water flow through the aquifer with uniform hydraulic connection). The above is just our experience. We believe that it is difficult to say that event-water is not a major component of streamflow until this paradox is thoroughly explained or proven to be correct in one part and biased in the other.

Considering the existence of the “old water paradox”, we have removed the description that may be wrong in the paper (Page 8, lines 14--16).

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Technical note: Analytical sensitivity analysis and uncertainty estimation of a two-component hydrograph separation method ~~which uses~~ with conductivity as a tracer

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Abstract. The ~~conductivity~~ two-component hydrograph separation method with conductivity as a tracer is favored by hydrologists owing to its low cost and easy application~~is cheap and easy to operate and is favored by hydrologists~~. This ~~paper~~ study analyzes the sensitivity of the baseflow index (BFI, ~~the~~ long-term ratio of baseflow to streamflow) calculated ~~by~~using this method to errors or uncertainties of ~~the~~ two parameters (BF_C, the conductivity of baseflow, and~~;~~ RO_C, the conductivity of surface runoff) and ~~of the~~ two variables (y_k, ~~the specific~~ streamflow~~;~~, and $SC\cancel{Q}_{\cancel{c}k}$, ~~the~~ specific conductance~~ivity~~ of streamflow, where *k* is the time step), and then estimates the uncertainty of BFI. The analysis shows that ~~when the~~for time series ~~is~~ longer than 365 days, ~~the~~ random measurement errors of y_k or $SC\cancel{Q}_{\cancel{c}k}$ will cancel each other, and their influence on BFI can be neglected. ~~Dimensionless sensitivity indices (the ratio of the relative error of BFI to the relative error of BF_C or RO_C) can well express the propagation of errors or uncertainties of BF_C or RO_C into BFI. Based on the sensitivity analysis, the~~ An uncertainty estimation method of BFI is derived on the basis of the sensitivity analysis. Representative sensitivity indices (the ratio of the relative error of BFI to that of BF_C or RO_C) and BFI' uncertainties are determined~~yielded~~ by applying~~ication of~~ the resulting equations to 24 watersheds in the United States. These dimensionless sensitivity indices can well express the propagation of errors or uncertainties of BF_C or RO_C into BFI. The results indicate that BFI is more sensitive to BF_C, and the conductivity two-component hydrograph separation method may be more suitable for the long time series in a small watershed. ~~After considering~~When the mutual offset of the measurement errors of conductivity and streamflow is considered, the uncertainty of BFI is reduced by half.

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1 Introduction

Hydrograph separation (also called baseflow separation), aims to identify ~~can effectively identify~~ the proportion of water in different runoff pathways in ~~a basin's~~the export flow of a basin, which helps ~~to~~ in identifying the conversion relationship between groundwater and surface water;~~,~~ ~~and~~ in addition, it is a necessary condition for optimal allocation of water resources (Cartwright et al., 2014; Miller et al., 2014; Costelloe et al., 2015). Some researchers indicated that tracer-based hydrograph separation methods yield the most realistic results~~aIisotope (tracer) hydrograph separation method is considered to be the most effective separation method,~~ because they are the most physically based methods ~~which can reflect the actual characteristics of a basin~~ (Miller et al., 2014; Mei and Anagnostou, 2015; Zhang et al., 2017). Many hydrologists have suggested~~indicated~~ that electrical conductivity can be used as a tracer ~~to perform~~in hydrograph separation (Stewart et al., 2007; Munyaneza et al., 2012; Cartwright et al., 2014; Lott and Stewart, 2016; Okello et al., 2018). ~~The measurement of c~~Conductivity is a suitable tracer because its measurement is simple and inexpensive, and it has ~~a~~ distinct applicability in ~~a long~~ long-series ~~of~~ hydrograph separation (Okello et al., 2018).

The ~~conductivity~~ two-component hydrograph separation method with conductivity as a tracer (also called conductivity mass balance method (CMB) (Stewart et al. 2007)) ~~uses conductivity as a tracer to~~ calculates baseflow through a two-component mass balance equation. The general equation is shown in Eq. (1).~~,~~ which is based on the following assumptions:

a) Contributions from end-members other than baseflow and surface runoff are negligible.

b) The specific conductance of runoff and baseflow are constant (or vary in a known manner) over the period of record.

c) Instream processes (such as evaporation) do not change specific conductance makedly.

d) Baseflow and surface runoff have significantly different specific conductance. ~~The general equation is shown in Eq. (1).~~

$$b_k = \frac{y_k(\cancel{Q}SC_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} \quad (1)$$

where b is ~~the~~ baseflow (L^3/t), y is ~~the~~ streamflow (L^3/t), $SC\cancel{Q}_{\cancel{c}}$ is the electrical conductivity of streamflow, and k is ~~the~~ time step number. The two parameters BF_C and RO_C ~~respectively~~ represent the electrical conductivity of baseflow and surface runoff, respectively.

Stewart et al. (2007) conducted a~~The~~ field test in a drainage basin of 12km^2 area in southeast Hillsborough County, Florida ~~of Stewart et al. (2007)~~ and showed that the maximum conductivity of streamflow can be used to replace BF_C, and the minimum conductivity can be used to replace RO_C. However, Miller et al. (2014) pointed out that the maximum conductivity of streamflow may exceed the real BF_C~~,~~; Therefore,~~so~~ they suggested that the 99th percentile of the conductivity of ~~a long series of streamflow~~each year should be used as ~~the~~ BF_C to avoid the impact of high BF_C estimates on the separation results and assumed that baseflow conductivity varies linearly between years. ~~There is uncertainty in~~The determin~~ing~~ation of the parameters (BF_C, RO_C) of the conductivity two-component hydrograph separation method involves some uncertainties (Miller et al., 2014; Okello et al., 2018). Therefore, sensitivity analysis of parameters and ~~the uncertainty~~ quantitative analysis of the uncertainties~~separation results are helpful to~~ will contribute towards further optimization~~e~~ of the conductivity two-component hydrograph separation method and improving~~e~~ the accuracy of hydrograph separation.

Most ~~of the~~ existing parameter sensitivity analysis methods ~~use~~ are experimental methods ~~sensitivity analysis method, which~~ that usually substitute ~~s the fluctuation~~ varying values of a certain parameter into the separation model~~,~~ and then ~~analyzes the sensitivity of the parameters by~~ compar~~e~~ings the range of the separation results produced by these ~~fluctuation~~ varying parameter values~~s~~ (Eckhar~~ra~~dt, 2005; Miller et al., 2014; Okello et al., 2018). Eckhardt (2012) indicated that ~~"~~"An empirical sensitivity analysis is only a makeshift if an analytical sensitivity analysis, that is an analytical calculation of the error propagation through the model, is not feasible"~~An empirical sensitivity analysis is only an analytical calculation of the error propagation through the model, is not feasible.~~" Eckhardt (2012) derived ~~the~~ sensitivity indices of ~~the~~ equation parameters by the partial derivative of a two-parameter recursive digital baseflow separation filter equation. Until now~~However~~, the parameters' sensitivity indices of the conductivity two-component hydrograph separation equation have not been derived.

At present, the uncertainty of the separation results of the conductivity two-component hydrograph separation method is mainly estimated ~~by~~using an uncertainty transfer equation based on the uncertainty of BF_C, RO_C, and $SC\cancel{Q}_{\cancel{c}k}$ (Genereux, 1998; Miller et al., 2014). See Sect. 3.1 for details. ~~This~~ In this uncertainty estimation method, ~~can only estimate~~ the uncertainty of the baseflow ratio (f_{bf}, the ratio of baseflow to streamflow in a single calculation process) is estimated, and ~~then use~~ the average uncertainty of multiple calculation processes is then used to estimate the uncertainty of the baseflow index (BFI, ~~the~~ long-term ratio of baseflow to total streamflow). This ~~uncertainty estimation~~ method can neither directly estimate the uncertainty of BFI nor consider the randomness and mutual offset of conductivity measurement errors, and thus, it does not provide accurate estimates of BFI uncertainty ~~the uncertainty estimation of BFI is not appropriate enough~~.

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The main objectives of this study are as follows: (i) analyze the sensitivity of long-term series of baseflow separation results (BFI) to parameters and variables of the conductivity two-component hydrograph separation equation (Sect. 2); (ii) derive the uncertainty of BFI (Sect.3).~~The purpose of this paper is to derive the parameters' sensitivity indices of the conductivity two-component hydrograph separation equation by calculating the partial derivative of Eq. (1) (Sect. 2), and further derive the direct estimation method of BFI' uncertainty (Sect. 3).~~ The derived solutions~~methods~~ were applied to 24 basins in the United States, and the parameter~~s'~~ sensitivity indices and BFI~~'~~ uncertainty characteristics were analyzed (Sect. 4).

2 ~~Analytical~~ Ssensitivity analysis

2.1 Parameters BF_C and RO_C

In order to calculate the sensitivity indices of the parameters, the partial derivatives of b_k in Eq. (1) with respect to BF_C and RO_C ~~the partial derivatives of b_k in Eq. (1) to BF_C and RO_C~~ are required ~~respectively~~ (~~for~~ the derivation process is expressed as~~, see Appendix~~ Eq. (A1) and~~,~~ (A2)):

$$\frac{\partial b_k}{\partial \mathrm{BF_c}} = -y_k \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{2}$$

$$\frac{\partial b_k}{\partial \mathrm{RO_c}} = y_k \frac{\cancel{Q}SC_{\cancel{c}k} - \mathrm{BF_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{3}$$

For the convenience of comparison, the baseflow index (BFI) is selected as the baseflow separation result for long time series to analyze the influence of parameter~~s'~~ uncertainty on BFI,

$$\mathrm{BFI} = \frac{\sum_{k=1}^{n} b_k}{\sum_{k=1}^{n} y_k} = \frac{b}{y} \tag{4}$$

where b ~~denotes the total baseflow~~ and y denote the total baseflow and~~the~~ total streamflow, respectively, over the whole available streamflow sequences, and n is the number of available streamflow data.

Then, the partial derivatives of BFI to BF_C and RO_C should be calculated~~,~~ (~~for~~ the derivation process~~, see Appendix~~ is presented in Eq. (A3) and~~,~~ (A4)):

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} = \frac{y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{5}$$

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k} - y\mathrm{BF_c}}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{6}$$

~~It can be seen from~~ T~~t~~he definition of the partial derivative suggests that the influence of the errors of the parameters (ΔBF_C and ΔRO_C) in Eq. (1) on the BFI can be expressed by the product of the errors and its partial derivatives. Then the errors of BFI caused by small errors of BF_C and RO_C can be approximated by ~~the BFI' errors caused by tiny errors of BF_C and RO_C can be expressed as~~:~~:~~

$$\Delta_{\mathrm{BF_c}}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} \Delta \mathrm{BF_c} = \frac{y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \Delta \mathrm{BF_c} \tag{7}$$

$$\Delta_{\mathrm{RO_c}}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} \Delta \mathrm{RO_c} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k} - y\mathrm{BF_c}}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \Delta \mathrm{RO_c} \tag{8}$$

The dimensionless sensitivity indices (S) can be obtained by comparing the relative error of BFI caused by the small~~tiny~~ errors of BF_C and RO_C with that of BF_C and RO_C, (see ~~Appendix~~ Eq. (B1), (B2)):

$$\mathrm{S}(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}}) = \frac{\Delta_{\mathrm{BF_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta \mathrm{BF_c}}{\mathrm{BF_c}} = \frac{\mathrm{BF_c}(y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k})}{y\mathrm{BFI}(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{9}$$

$$\mathrm{S}(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}}) = \frac{\Delta_{\mathrm{RO_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta \mathrm{RO_c}}{\mathrm{RO_c}} = \frac{\mathrm{RO_c}(\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k} - y\mathrm{BF_c})}{y\mathrm{BFI}(\mathrm{BF_c} - \mathrm{RO_c})^2} \tag{10}$$

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where S(BFI|BF$_c$ ~~BFI/BF$_c$~~) represent the dimensionless sensitivity index of BFI (output) with BF$_c$ (uncertain input), and S(BFI|RO$_c$ ~~BFI/RO$_c$~~) with RO$_c$.

The dimensionless sensitivity index is also called the "elasticity index", and it reflects the proportional relationship between the relative error of BFI and the relative error of parameters (e.g. if S(BFI|BF$_c$ ~~BFI/BF$_c$~~) = 1.5, and the relative error of BF$_c$ is 5%, then the relative error of BFI ~~should be~~is 1.5 times 5% =~~(~~ 7.5%~~)~~). After determining the ~~specific~~ values of BF$_C$, RO$_C$, BFI, y, y_k and $SC\cancel{Q}_{\cancel{c}k}$, the sensitivity indices S(BFI|BF$_c$ ~~BFI/BF$_c$~~) and S(BFI|RO$_c$ ~~BFI/RO$_c$~~) can be calculated and compared.

2.2 Variables y_k and $SC\cancel{Q}_{\cancel{c}k}$

In addition to the two parameters, there are two variables ($SC\cancel{Q}_{\cancel{c}k}$ and y_k) in Eq. (1). This section ~~will analyze~~describes the sensitivity analysis of BFI to these two variables. Similar to Sect. 2.1, the partial derivatives of b_k in Eq. (1) to $SC\cancel{Q}_{\cancel{c}k}$ and y_k are obtained (see ~~Appendix~~Eq. ~~(~~-A5), (A6)), and the partial derivatives of BFI to $SC\cancel{Q}_{\cancel{c}k}$ and y_k are further obtained (see ~~Appendix~~Eq. (A7), (A8)),

$$\frac{\partial \mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}} \tag{11}$$

$$\frac{\partial \mathrm{BFI}}{\partial y_k} = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})}$$

(12)

According to previous studies (Munyaneza et al., 2012; Cartwright et al., 2014; Miller et al., 2014; Okello et al., 2018) and this study (Table 1), the difference between BF$_C$ and RO$_C$ is often greater than 100 μs/cm. Therefore, ~~so~~ $\partial\mathrm{BFI}/\partial SC\cancel{Q}_{\cancel{c}k}$ is usually less than 0.01 cm/ μs. Appendix C shows that the value of $\partial\mathrm{BFI}/\partial y_k$ is usually far less than 1 d/m^3.

~~Tiny~~ Small errors in $SC\cancel{Q}_{\cancel{c}k}$ and y_k cause errors in BFI ~~of~~

$$\Delta_{SC\cancel{Q}_{\cancel{c}k}}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{c}k}}\Delta SC\cancel{Q}_{\cancel{c}k} = \frac{\Delta SC\cancel{Q}_{\cancel{c}k}}{\mathrm{BF_c}-\mathrm{RO_c}}$$

(13)

$$\Delta_{y_k}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial y_k}\Delta y_k = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})}\Delta y_k \tag{14}$$

The errors of BFI caused by $SC\cancel{Q}_{\cancel{c}k}$ and y_k are summed up to obtain~~get~~ the error of BFI caused by $\sum_{k=1}^{n} SC\cancel{Q}_{\cancel{c}k}$ and $\sum_{k=1}^{n} y_k$ in the whole time series:

$$\Delta_{\sum_{k=1}^{n} SC\cancel{Q}_{\cancel{c}k}}\mathrm{BFI} = \sum_{k=1}^{n}\Delta_{SC\cancel{Q}_{\cancel{c}k}}\mathrm{BFI} = \sum_{k=1}^{n}\frac{\Delta SC\cancel{Q}_{\cancel{c}k}}{\mathrm{BF_c}-\mathrm{RO_c}} = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}}\sum_{k=1}^{n}\Delta SC\cancel{Q}_{\cancel{c}k}$$

(15)

$$\Delta_{\sum_{k=1}^{n} y_k}\mathrm{BFI} = \sum_{k=1}^{n}\Delta_{y_k}\mathrm{BFI} = \sum_{k=1}^{n}\left(\frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})}\Delta y_k\right) = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})}\sum_{k=1}^{n}\Delta y_k \tag{16}$$

Wagner et al. (2006) reported that ~~The tiny errors in Q_{ck} and y_k are mainly composed of random analysis errors. Random errors mostly follow a normal distribution or a uniform distribution. The magnitude and direction of the random errors are usually not fixed. As the number of measurements increases, the positive and negative errors can compensate each other, and the average value of the errors will gradually trend to zero (Huang and Chen, 2011).~~

~~T~~the uncertainty of ~~the~~ instruments is usually less than~~<~~ 5% for $SC\cancel{Q}_{\cancel{c}k}$ ~~less than~~(<~~-~~100 μs/cm) and less than~~<~~3% for $SC\cancel{Q}_{\cancel{c}k}$ ~~greater than~~ (>100 μs/cm) ~~(Wagner et al., 2006; Miller et al., 2014)~~. According to Hamilton et al. (2012) streamflow data from USGS are often assumed by analysts to be accurate and precise within ±5% at the 95% confidence interval. ~~The measurement uncertainty of streamflow is usually <3% (Zhang, 2005)~~. In this ~~paper~~study, the error ranges of $SC\cancel{Q}_{\cancel{c}k}$ and y_k are all considered to be ±5% ~~and ±3%, respectively~~. The errors in SC$_k$ and y$_k$ mainly comprise random analysis errors which mostly follow a normal

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distribution or a uniform distribution (Huang and Chen, 2011). Considering the mutual offset of random errors, when the time series (n) is sufficiently long ~~enough~~, $\sum_{k=1}^{n} \Delta SC\cancel{Q}_{ck}$ in Eq. (15) and $\sum_{k=1}^{n} \Delta y_k$ in Eq. (16) will approach zero.

The analysis of $\sum_{k=1}^{n} \Delta SC_k$ and $\sum_{k=1}^{n} \Delta y_k$ under different time series (n) and different error distributions (normal distribution or uniform distribution) of a surface water station (USGS site number 0297100) showed that the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series is greater than 365days (See Supplement S1 for detail).~~Therefore, when n is large enough, the error of BFI caused by the errors of Q_{ck} and y_k can be neglected.~~

~~To verify this phenomenon, the study collected the daily average conductivity and daily average streamflow of the surface water station with the USGS site number 0297100 (Table 1) from 2001 to 2010 (2979 days in total). Then, office Excel was used to generate 10 sets (2979 per set) of random numbers between -0.05 and 0.05 that obey normal distribution and uniform distribution respectively to simulate the errors (%) of the daily average conductivity. And 10 sets (2979 per set) of random numbers obeying normal distribution and uniform distribution between -0.03 and 0.03, respectively, were used to simulate the errors (%) of the daily average streamflow. Finally, according to different time series (n) (e.g. 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 365, 730, 1095, … , 2979, days) sum the errors value ($\sum_{k=1}^{n} \Delta Q_k$ and $\sum_{k=1}^{n} \Delta y_k$) and analyze the trend of the average error (%) with n.~~

~~The trend of the average error (%) of conductivity with n is shown in Fig. 1. The average errors of the uniform distribution (Fig. 1(a)) and the normal distribution (Fig. 1(b)) are all gradually approach zero with the increase of the time series (n), and the uniform distribution converges faster than the normal distribution. The average errors of the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.49% when n is greater than 365.~~

~~Similar to the conductivity, the trend of the average error (%) of the streamflow with n is shown in Fig. 2. The average errors of the uniform distribution (Fig. 2(a)) and the normal distribution (Fig. 2(b)) all gradually approach to zero as the time series (n) increases, and the uniform distribution converges faster than the normal distribution. The average errors of different n under the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.67% when n is greater than 365.~~

~~From the above analysis, when the time series (n) is greater than 365 days (1 year), $\Delta_{\sum_{k=1}^{n} Q_{ck}}$BFI will be less than 0.0049% (0.01 times 0.49%), and $\Delta_{\sum_{k=1}^{n} y_k}$BFI will be much less than 0.76% (1 times 0.76%). Therefore, the random errors of daily average conductivity and streamflow have a negligible effect on BFI.~~

~~**Figure 1. Average conductivity error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.**~~

~~**Figure 2. Average streamflow error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.**~~

3 Uncertainty estimation

3.1 Previous attempts

According to previous studies, in the case where a ~~parameter~~ variable g is calculated as a function of several factors x_1, x_2, x_3, …, x_n (e.g. g= G(x_1, x_2, x_3, …, x_n)) ~~.~~ and based on the assumptions that the factors are uncorrelated and have a Gaussian distribution, t~~T~~he transfer equation (also known as Gaussian error propagation) between the uncertainty of the independent factors and the uncertainty of g is ~~(Taylor, 1982; Kline, 1985; Genereux, 1998)~~:

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$$W_g = \sqrt{(\frac{\partial g}{\partial x_1} W_{x_1})^2 + (\frac{\partial g}{\partial x_2} W_{x_2})^2 + \cdots + (\frac{\partial g}{\partial x_n} W_{x_n})^2} \quad (17)$$

where W_g, W_{x1}, W_{x2}, and W_{xn} are the same type of uncertainty values (e.g. all average errors or all standard deviations) for g, x_1, x_2, and x_n, respectively. A more detailed description of this equation can be found in Taylor (1982), Kline (1985), and Ernest (2005).

According to Genereux (1998), "While any set of consistent uncertainty (W) values may be propagated using Gaussian error propagation, using standard deviations multiplied by *t* values from the Student's *t* distribution (each *t* for the same confidence level, such as 95%) has the advantage of providing a clear meaning (tied to a confidence interval) for the computed uncertainty would correspond to, for example, 95% confidence limits on BFI".

Based on the above principle, Genereux (1998) substituted Eq. (18) into Eq. (17) to derive the uncertainty estimation equation (Eq. (19)) of the two-component mass balance baseflow separation method:

$$f_{bf} = \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{\mathrm{BF_c} - \mathrm{RO_c}} \quad (18)$$

$$W_{f_{bf}} = \sqrt{(\frac{f_{bf}}{\mathrm{BF_c}-\mathrm{RO_c}} W_{\mathrm{BF_c}})^2 + (\frac{1-f_{bf}}{\mathrm{BF_c}-\mathrm{RO_c}} W_{\mathrm{RO_c}})^2 + (\frac{1}{\mathrm{BF_c}-\mathrm{RO_c}} W_{SC\cancel{Q}_{\cancel{c}}})^2} \quad (19)$$

where f_{bf} is the ratio of baseflow to streamflow in a single calculation process, W_{fbf} is the uncertainty in f_{bf} at the 95% confidence interval, W_{BFC} is the standard deviation of the BF$_C$ ~~highest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, W_{ROC} is the standard deviation of the RO$_C$ ~~lowest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and $W_{S\cancel{Q}C}$ is the analytical error in ~~the~~ conductivity multiplied by the t-value (α=0.05; two-tail) (Miller et al., 2014). ~~(Miller et al., 2014).~~

~~Equation (19) can b~~Better estimates of the uncertainty of f_{bf} within a single calculation step can be obtained using Eq. (19). Hydrologists usually approximate~~estimate~~ the uncertainty of BFI ~~approximately~~ by averaging the uncertainty of all steps (Genereux, 1998; Miller et al., 2014). However, this method does not consider the mutual offset of the conductivity measurement errors~~,~~ and cannot accurately reflect the uncertainty of BFI. In this ~~paper~~study, ~~based on the parameter sensitivity analysis, the~~an uncertainty estimation equation of BFI is derived on the basis of the parameter sensitivity analysis. ~~See the next section for details.~~

3.2 Uncertainty estimation of~~in~~ BFI

BFI is a function of BF$_c$, RO$_c$, $SC\cancel{Q}_{\cancel{c}k}$ and y_k. In addition,~~And~~ the uncertaint~~y~~ies ~~of~~ of BF$_c$, RO$_c$, $SC\cancel{Q}_{\cancel{c}k}$ and y_k ~~is~~ are independent of each other. As explained earlier (Sect. 2.2),~~Sect. 2.2 has explained that~~ the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series (n) is greater than 365 days (1 year), ~~so~~Therefore, the uncertainty of BFI can be expressed as:

$$W_{BFI} = \sqrt{(\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} W_{\mathrm{BF_c}})^2 + (\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} W_{\mathrm{RO_c}})^2} \quad (20)$$

~~where (see Eq. 5 and Eq. 9; Eq. 6 and Eq. 10)~~

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} = \mathrm{S}(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}})\frac{\mathrm{BFI}}{\mathrm{BF_c}}$$

(21)

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} = \mathrm{S}(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}})\frac{\mathrm{BFI}}{\mathrm{RO_c}}$$

(22)

Then, ~~the~~ Eq. (20) can be rewritten as:

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$$W_{BFI} = \sqrt{\left(S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}})\frac{\mathrm{BFI}}{\mathrm{BF_c}}W_{\mathrm{BF_C}}\right)^2 + \left(S(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}})\frac{\mathrm{BFI}}{\mathrm{RO_c}}W_{\mathrm{RO_C}}\right)^2} \qquad (23)$$

where W_{BFI}, W_{BFC}, and W_{ROC} are the same type of uncertainty values for BFI, $\mathrm{BF_C}$, and $\mathrm{RO_C}$, respectively. For instance, W_{BFI} is the uncertainty in BFI at the 95% confidence interval, W_{BFC} is the standard deviation of the ~~highest 1% of measured conductivity~~ $\mathrm{BF_C}$ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and W_{ROC} is the standard deviation of the $\mathrm{RO_C}$ ~~lowest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution.

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4 Application

4.1 Data and processing

The above sensitivity analysis and uncertainty estimation methods were applied to 24 catchments in the United States (Table 1). All basins used in this study are perennial streams, with drainage areas ranging from 10 km^2 to 1258481 km^2. Each gage has about at least 1 year of continuous streamflow and conductivity ~~at~~ for the same period of records. All streamflow and conductivity data are daily average values retrieved from the United States Geological Survey's (USGS) National Water Information System (NWIS) website, http://waterdata.usgs.gov/nwis.

The daily baseflow of each basin was calculated using Eq. (1). The 99th percentile of the conductivity of each year was used as $\mathrm{BF_C}$, and linear variation of baseflow conductivity between years was assumed. The ~~-~~ 1st percentile ~~99th percentile~~ of the conductivity of the whole series of streamflow in each basin was used as the ~~$\mathrm{BF_C}$ and the 1st percentile as the~~ $\mathrm{RO_C}$. The total baseflow b, ~~the~~ total streamflow y and ~~the~~ baseflow index BFI of each watershed were then calculated. According to the results of the hydrograph separation, the parameter sensitivity indices of BFI for mean $\mathrm{BF_C}$ ($S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}})$) and $\mathrm{RO_C}$ ($S(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}})$) were calculated using ~~by~~ Eq. (9) and Eq. (10), respectively.

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Finally, the uncertainty of f_{bf} in each step was calculated using ~~by~~ Eq. (19) and averaged to obtain the ~~M~~mean W_{fbf} ~~in~~ for each basin. The uncertainty (W_{BFI}) of BFI was directly calculated using ~~by~~ Eq. (23), and then the values of ~~M~~mean W_{fbf} and W_{BFI} were compared. For each basin, W_{BFC} is the standard deviation of the $\mathrm{BF_C}$ of the whole series ~~highest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, W_{ROC} is the standard deviation of the lowest 1% of measured conductivity multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and W_{SC} ~~W_{QC}~~ is the analytical error in the conductivity (5%) multiplied by the t-value (α=0.05; two-tail).

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4.2 Results and discussion

The calculation results are shown in Table 1. The average baseflow index of the 24 watersheds is ~~0.29~~0.34, the average sensitivity index of BFI for mean $\mathrm{BF_C}$ ($S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}})$) is -1.40~~39~~, and the average sensitivity index of BFI for $\mathrm{RO_C}$ ($S(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}})$) is -0.~~98~~89. The negative sensitivity indices indicate a negative correlation between BFI and $\mathrm{BF_C}$, $\mathrm{RO_C}$. The absolute value of the sensitivity index ~~The sensitivity index~~ for $\mathrm{BF_C}$ is generally greater than that for $\mathrm{RO_C}$, indicating that BFI is more affected by $\mathrm{BF_C}$ (for example, if there are 10~~5~~% uncertaint~~y~~ies in both $\mathrm{BF_C}$ and $\mathrm{RO_C}$, then $\mathrm{BF_C}$ leads to -1.~~.39~~ 40 times 10% of uncertainty in BFI (~~-6.95~~14.0%), while $\mathrm{RO_C}$ leads to -0.~~.98~~ 89 times 10% (~~4.9~~ 8.9%)). Therefore, the determination of $\mathrm{BF_C}$ requires more caution, and any small error may lead to greater uncertainty in BFI. Miller et al. (2014) reported ~~have indicated~~ that anthropogenic activities over long periods of time, or year to year changes in the elevation of the water table may result in temporal changes~~ly changing~~ in ~~the~~ BF_C. They~~He~~ recommended taking different BF_C values per year based on the conductivity values during~~at~~ low flow periods to avoid the effects of temporal fluctuations in BF_C~~' temporally fluctuations~~.

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote "a" in the "Area" column indicates that the values are estimated based on data from adjacent sites.

The sensitivity index of BFI for BF_C shows~~has~~ a decreasing trend with the increase of time series (n) (Fig. 1~~3~~(a)) and ~~has~~ an increasing trend with~~ the~~ increasing~~e of~~ watershed area (Fig. 1~~3~~(b)), ~~the~~ with correlation coefficients ~~are~~ of 0.~~1698~~ 1492 and 0.~~4468~~3577, respectively. Although the correlations are not obvious, ~~it still has~~they have important guiding significance. ~~In the l~~Large basins, comprise~~there are~~ many different subsurface flow paths contributing to streams (Okello et al., 2018), each of which has a unique conductivity value (Miller et al., 2014). It is difficult to represent the conductivity characteristics of subsurface flow with a special value. Therefore, the conductivity two-component hydrograph separation method has~~ a~~ higher applicability to long time series of~~ in a~~ small watershed~~ of long time series~~.

The sensitivity index of BFI for RO_C did not change significantly with the increase of time series and watershed area (Fig. 1~~3~~(c), Fig. 1~~3~~(d)). During~~ the~~ rainstorms, ~~the water level of the stream rises sharply, the subsurface flow is suppressed, and the streamflow is almost entirely from the rainfall runoff. At this time,~~ the conductivity of~~ the~~ streams ~~is~~ became similar to ~~the~~ that ~~conductivity~~ of the ~~local~~ rainfall (Stewart et al., 2007). The electrical conductivity of regional rainfalls vary~~ies~~ slightly, usually at a fixed value, and it has no significant relationship with the basin area and year (Munyaneza et al., 2012). Therefore, the temporal and spatial variation characteristics of BFI for RO_C are not obvious.

Figure 1~~3~~. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear~~normal~~ axes.

Genereux's method (Eq.19) estimates the average uncertainty of BFI in the 24 basins (a~~A~~verage of m~~M~~ean W_{fbf}) to be 0.~~13~~20, whereas the average uncertainty of BFI (a~~A~~verage of W_{BFI}) calculated directly using the proposed ~~by this paper'~~ method (Eq. 23) is 0.~~06~~ 11 (Table 1). Mean W_{fbf} in each basin is generally larger than W_{BFI} (W_{BFI} is about 0.51 times of m~~M~~ean W_{fbf}), and there is a significant linear correlation (Fig. 2~~4~~). This shows that the two methods have the same volatility characteristics for BFI uncertainty estimation~~ results~~, but Genereux's method (Eq. 19) often overestimates the uncertainty of BFI. This also means that when the time series is longer than 365 days~~ (1 year)~~, the measurement errors of conductivity and streamflow will cancel each other and thus reduce the uncertainty of BFI (about half of the original).

Figure 2~~4~~. Scatter plot of uncertainty in BFI (W_{BFI}) and mean uncertainty in f_{bf} (m~~M~~ean W_{fbf}).

5 Conclusions

This study analyzed the sensitivity of BFI calculated using the conductivity two-component hydrograph separation method to errors or uncertainties of parameters BF_C and RO_C and variables y_k and SC_k. In addition, the uncertainty of BFI was calculated. The equations derived in this study (~~Equation~~ Eq. (9) and Eq. (10)) ~~can well~~could calculate the sensitivity indices of BFI for~~-~~ BF_C and RO_C. For time series longer than 365 days, the measurement errors of conductivity and streamflow exhibited an obvious mutual offset effect, and their influence on BFI could be neglected. Considering the mutual offset, the uncertainty of BFI would be reduced to half. From this viewpoint, Eq. (23) ~~can~~could estimate the uncertainty of BFI ~~when the~~for time series ~~is larger~~longer than 365 days~~, taking into account the mutual cancellation of conductivity measurement errors~~. The application of the method to

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~~Applications in~~ 24 basins in the United States showed that BFI is more sensitive to BF_C, and future studies should dedicate~~devote~~ more effort to determining the value of BF_C. In addition, the conductivity two-component hydrograph separation method may be more suitable for ~~the~~ long time series ~~in a~~of small watersheds.

~~When the time series is greater than 365 days, the measurement errors of conductivity and streamflow have obvious mutual offset, and its influence on BFI can be neglected. After considering the mutual offset of random errors, the uncertainty of BFI will be reduced to half.~~Systematic errors in specific conductance and streamflow as well as temporal and spatial variations in baseflow conductivity may be the main sources of BFI uncertainty. Better rating curves are probably more important than better loggers, and more work on understanding the specific conductance of baseflow is more important than understanding that of surface runoff.

The above conclusions are only from the average of the 24 basins in the United States, and further researches ~~is needed~~ in other countries or in more watersheds are thus required. This~~e~~ study~~research in this paper~~ only focused~~s~~ on the two-component hydrograph separation method with conductivity as a tracer, but ~~the~~ parameter sensitivity analysis and uncertainty analysis methods involving ~~of~~ other tracers are very similar ~~to this paper,~~. ~~and~~ Therefore, ~~it is easy to derive~~ similar equations can be easily derived by referring to the findings of this study.

Appendix A

Calculation of the partial derivatives

$$\frac{\partial b_k}{\partial \mathrm{BF_c}} = \frac{\partial}{\partial \mathrm{BF_c}} \frac{y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} = y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}) \frac{\partial}{\partial \mathrm{BF_c}} \frac{1}{\mathrm{BF_c} - \mathrm{RO_c}} = -y_k \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2} \quad \text{(A1)}$$

$$\frac{\partial b_k}{\partial \mathrm{RO_c}} = \frac{\partial}{\partial \mathrm{RO_c}} \frac{y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} = y_k \frac{\partial}{\partial \mathrm{RO_c}} \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{\mathrm{BF_c} - \mathrm{RO_c}} = y_k \frac{-(\mathrm{BF_c} - \mathrm{RO_c}) + (SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{(\mathrm{BF_c} - \mathrm{RO_c})^2} = y_k \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{BF_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2}$$

(A2)

$$\frac{\partial BFI}{\partial BF_c} = \frac{\partial}{\partial BF_c} \frac{b}{y} = \frac{1}{y} \sum_{k=1}^{n} \frac{\partial b_k}{\partial BF_c} = \frac{1}{y} \sum_{k=1}^{n} \left(-y_k \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2}\right) (\text{see Eq. A1}) = \frac{1}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \sum_{k=1}^{n} (y_k \mathrm{RO_c} - y_k SC\cancel{Q}_{\cancel{c}k}) = \frac{y RO_c - \sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}}{y(BF_c - RO_c)^2}$$

(A3)

$$\frac{\partial BFI}{\partial RO_c} = \frac{\partial}{\partial RO_c} \frac{b}{y} = \frac{1}{y} \sum_{k=1}^{n} \frac{\partial b_k}{\partial RO_c} = \frac{1}{y} \sum_{k=1}^{n} \left(y_k \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{BF_c}}{(\mathrm{BF_c} - \mathrm{RO_c})^2}\right) (\text{see Eq. A2}) = \frac{1}{y(\mathrm{BF_c} - \mathrm{RO_c})^2} \sum_{k=1}^{n} (y_k SC\cancel{Q}_{\cancel{c}k} - y_k \mathrm{BF_c}) = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k} - yBF_c}{y(BF_c - RO_c)^2}$$

(A4)

$$\frac{\partial b_k}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{\partial}{\partial \mathrm{Q_{ck}}} \frac{y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} = \frac{1}{\mathrm{BF_c} - \mathrm{RO_c}} \frac{\partial}{\partial SC\cancel{Q}_{\cancel{c}k}} y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}) = \frac{y_k}{\mathrm{BF_c} - \mathrm{RO_c}}$$

(A5)

$$\frac{\partial b_k}{\partial y_k} = \frac{\partial}{\partial y_k} \frac{y_k(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} = \frac{(SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} \frac{\partial}{\partial y_k} y_k = \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{\mathrm{BF_c} - \mathrm{RO_c}}$$

(A6)

$$\frac{\partial \mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{\partial}{\partial SC\cancel{Q}_{\cancel{c}k}} \frac{b}{y} = \frac{1}{y} \sum_{k=1}^{n} \frac{\partial b_k}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{1}{y} \sum_{k=1}^{n} \frac{y_k}{\mathrm{BF_c} - \mathrm{RO_c}} (\text{see Eq. A5}) = \frac{1}{y(\mathrm{BF_c} - \mathrm{RO_c})} \sum_{k=1}^{n} \mathrm{y}_k = \frac{1}{\mathrm{BF_c} - \mathrm{RO_c}}$$

(A7)

$$\frac{\partial \mathrm{BFI}}{\partial y_k} = \frac{\partial}{\partial y_k} \frac{b}{y} = \frac{\partial}{\partial y_k} \frac{\sum_{k=1}^{n} b_k}{\sum_{k=1}^{n} y_k} = \frac{(\sum_{k=1}^{n} b_k)'(\sum_{k=1}^{n} y_k) - (\sum_{k=1}^{n} b_k)(\sum_{k=1}^{n} y_k)'}{(\sum_{k=1}^{n} y_k)^2} = \frac{y(\sum_{k=1}^{n} b_k)' - b(\sum_{k=1}^{n} y_k)'}{\mathrm{y}^2} = \frac{y \sum_{k=1}^{n} \left(\frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{\mathrm{BF_c} - \mathrm{RO_c}}\right) - \mathrm{n}b}{\mathrm{y}^2} (\text{see Eq. A6}) = \frac{y \sum_{k=1}^{n} (SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}) - \mathrm{n}b(\mathrm{BF_c} - \mathrm{RO_c})}{\mathrm{y}^2(\mathrm{BF_c} - \mathrm{RO_c})} = \frac{\sum_{k=1}^{n} (SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}) - \mathrm{n}\mathrm{BFI}(\mathrm{BF_c} - \mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c} - \mathrm{RO_c})} \quad \text{(A8)}$$

Appendix B

Calculation of the sensitivity indices

$$S(\mathrm{BFI}|\mathrm{BF_c}\,\cancel{\mathrm{BFI}/\mathrm{BF_c}}) = \frac{\Delta_{\mathrm{BF_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta\mathrm{BF_c}}{\mathrm{BF_c}} = \frac{y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q_c}_k}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{BF_c}\frac{\mathrm{BF_c}}{\mathrm{BFI}\Delta\mathrm{BF_c}}(\text{see Eq. 7}) = \frac{\mathrm{BF_c}(y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q_c}_k)}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2} \quad (B1)$$

$$S(\mathrm{BFI}|\mathrm{RO_c}\,\cancel{\mathrm{BFI}/\mathrm{RO_c}}) = \frac{\Delta_{\mathrm{RO_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta\mathrm{RO_c}}{\mathrm{RO_c}} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q_c}_k - y\mathrm{BF_c}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{RO_c}\frac{\mathrm{RO_c}}{\mathrm{BFI}\Delta\mathrm{RO_c}}(\text{see Eq. 8}) = \frac{\mathrm{RO_c}(\sum_{k=1}^{n} y_k SC\cancel{Q_c}_k - y\mathrm{BF_c})}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2} \quad (B2)$$

Appendix C

Prove that $\partial\mathrm{BFI}/\partial y_k$ is far less than 1 d/m^3.

$$\frac{\partial\mathrm{BFI}}{\partial y_k} = \frac{\sum_{k=1}^{n}(SC\cancel{Q_c}_k - \mathrm{RO_c}) - n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})}(\text{see Eq. A8}) \quad (C1)$$

Because of n>0, BFI>0, (BF_C-RO_C)>0, the above formula can be simplified:

$$\frac{\partial\mathrm{BFI}}{\partial y_k} < \frac{\sum_{k=1}^{n}(SC\cancel{Q_c}_k - \mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})} \quad (C2)$$

Since BF_C is usually much larger than $SC\cancel{Q_c}_k$, the above formula can be rewritten as:

$$\frac{\partial\mathrm{BFI}}{\partial y_k} < \frac{\sum_{k=1}^{n}(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})} = \frac{n(\mathrm{BF_c}-\mathrm{RO_c})}{y(\mathrm{BF_c}-\mathrm{RO_c})} = \frac{n}{y} = \frac{1}{\bar{y}} \quad (C3)$$

The daily average streamflow ($\bar{y}$) is usually much larger than 1 m^3/d, so $\partial BFI/\partial y_k$ is far less than 1 d/m^3.

Data availability

All streamflow and conductivity data can be retrieved from the United States Geological Survey's (USGS) National Water Information System (NWIS) website use the special gage number, http://waterdata.usgs.gov/nwis.

Author contributions

Weifei Yang, Changlai Xiao and Xiujuan Liang designed the research train of thought. Weifei Yang and Changlai Xiao completed the parameters' sensitivity analysis. Xiujuan Liang completed the uncertainty estimate of BFI. Weifei Yang carried out most of the data analysis and prepared the manuscript with contributions from all co-authors.

Competing interests

The authors declare that they have no conflict of interest.

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Tables

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote “a” in the “Area” column indicates that the values are estimated based on data from adjacent sites.

~~State~~	~~Gage Number~~	~~N~~	~~Area~~	~~BF_C~~	~~RO_C~~	~~Mean Baseflow~~	~~BFI~~	~~$S(BFI/BF_C)$~~	~~$S(BFI/RO_C)$~~	~~W_{BFI}~~	~~Mean W_{bf}~~
~~-~~	~~-~~	~~days~~	~~km^2~~	~~µs/cm~~	~~µs/cm~~	~~m^3/s~~	~~-~~	~~-~~	~~-~~	~~-~~	~~-~~
~~FL~~	~~2298202~~	~~1808~~	~~966~~	~~1190.0~~	~~292.5~~	~~3.05~~	~~0.29~~	~~-1.31~~	~~-0.78~~	~~0.04~~	~~0.11~~
~~FL~~	~~2310545~~	~~1218~~	~~119[a]~~	~~7150.5~~	~~531.5~~	~~0.10~~	~~0.15~~	~~-1.09~~	~~-0.44~~	~~0.05~~	~~0.06~~
~~FL~~	~~2310650~~	~~779~~	~~77[a]~~	~~7195.0~~	~~3210.0~~	~~0.08~~	~~0.45~~	~~-1.79~~	~~-0.98~~	~~0.06~~	~~0.14~~
~~FL~~	~~2303000~~	~~728~~	~~570~~	~~462.0~~	~~120.5~~	~~3.28~~	~~0.30~~	~~-1.30~~	~~-0.82~~	~~0.08~~	~~0.17~~
~~FL~~	~~2298488~~	~~1303~~	~~76~~	~~810.0~~	~~194.0~~	~~0.20~~	~~0.33~~	~~-1.30~~	~~-0.63~~	~~0.05~~	~~0.09~~
~~FL~~	~~2298554~~	~~899~~	~~207[a]~~	~~1155.0~~	~~320.5~~	~~0.25~~	~~0.20~~	~~-1.36~~	~~-1.55~~	~~0.03~~	~~0.08~~
~~FL~~	~~2298492~~	~~1478~~	~~16~~	~~1425.0~~	~~304.0~~	~~0.04~~	~~0.21~~	~~-1.26~~	~~-1.01~~	~~0.03~~	~~0.07~~
~~FL~~	~~2298495~~	~~330~~	~~10~~	~~1905.0~~	~~662.0~~	~~0.05~~	~~0.24~~	~~-1.51~~	~~-1.66~~	~~0.03~~	~~0.08~~
~~FL~~	~~2298527~~	~~807~~	~~23~~	~~1640.0~~	~~201.5~~	~~0.04~~	~~0.14~~	~~-1.10~~	~~-0.83~~	~~0.06~~	~~0.16~~
~~FL~~	~~2298530~~	~~1510~~	~~17~~	~~1520.0~~	~~348.0~~	~~0.13~~	~~0.27~~	~~-1.27~~	~~-0.80~~	~~0.07~~	~~0.12~~
~~FL~~	~~2297100~~	~~2979~~	~~342~~	~~1460.0~~	~~221.5~~	~~1.54~~	~~0.21~~	~~-1.17~~	~~-0.69~~	~~0.04~~	~~0.09~~
~~FL~~	~~2313000~~	~~787~~	~~4727~~	~~449.0~~	~~173.0~~	~~8.62~~	~~0.43~~	~~-1.62~~	~~-0.84~~	~~0.06~~	~~0.13~~
~~FL~~	~~2300500~~	~~821~~	~~386~~	~~470.0~~	~~83.0~~	~~0.49~~	~~0.19~~	~~-1.19~~	~~-0.90~~	~~0.11~~	~~0.20~~
~~ND~~	~~5057000~~	~~1401~~	~~16757~~	~~1520.0~~	~~610.0~~	~~1.73~~	~~0.46~~	~~-1.64~~	~~-0.81~~	~~0.09~~	~~0.15~~
~~ND~~	~~5056000~~	~~1277~~	~~5361~~	~~1770.0~~	~~546.0~~	~~2.50~~	~~0.42~~	~~-1.41~~	~~-0.61~~	~~0.04~~	~~0.11~~
~~TX~~	~~8068275~~	~~2801~~	~~482~~	~~368.0~~	~~65.0~~	~~4.20~~	~~0.15~~	~~-1.18~~	~~-1.23~~	~~0.06~~	~~0.13~~
~~GA~~	~~2336300~~	~~1235~~	~~225~~	~~230.0~~	~~63.0~~	~~4.00~~	~~0.29~~	~~-1.36~~	~~-0.93~~	~~0.24~~	~~0.42~~
~~GA~~	~~2207120~~	~~1383~~	~~417~~	~~381.0~~	~~59.0~~	~~3.97~~	~~0.18~~	~~-1.17~~	~~-0.86~~	~~0.03~~	~~0.06~~
~~SC~~	~~2160105~~	~~1363~~	~~1966~~	~~150.0~~	~~51.0~~	~~40.27~~	~~0.25~~	~~-1.49~~	~~-1.56~~	~~0.03~~	~~0.10~~
~~SC~~	~~2160700~~	~~1392~~	~~1150~~	~~181.0~~	~~51.0~~	~~24.02~~	~~0.26~~	~~-1.37~~	~~-1.13~~	~~0.05~~	~~0.11~~
~~MO~~	~~6894000~~	~~1375~~	~~477~~	~~1110.0~~	~~334.0~~	~~0.86~~	~~0.21~~	~~-1.40~~	~~-1.59~~	~~0.09~~	~~0.13~~
~~MO~~	~~6895500~~	~~802~~	~~1258481~~	~~800.0~~	~~428.0~~	~~904.39~~	~~0.55~~	~~-2.14~~	~~-0.95~~	~~0.05~~	~~0.18~~
~~ND~~	~~5082500~~	~~1274~~	~~77959~~	~~1670.0~~	~~427.0~~	~~41.48~~	~~0.27~~	~~-1.33~~	~~-0.95~~	~~0.06~~	~~0.09~~
~~KS~~	~~7144780~~	~~575~~	~~1847~~	~~1550.0~~	~~678.0~~	~~0.52~~	~~0.44~~	~~-1.60~~	~~-1.08~~	~~0.09~~	~~0.17~~
~~Mean~~							~~0.29~~	~~-1.39~~	~~-0.98~~	~~0.06~~	~~0.13~~
~~Standard deviation (STDEV)~~							~~0.11~~	~~0.24~~	~~0.32~~	~~0.04~~	~~0.07~~

State	Gage Number	N	Area	Mean BF_C	RO_C	Mean Baseflow	BFI	$S(BFI\|BF_C)$	$S(BFI\|RO_C)$	W_{BFI}	Mean W_{fbf}
		days	km^2	µs/cm	µs/cm	m^3/s					
FL	2298202	1808	966	1149.1	292.5	2.12	0.31	-1.32	-0.76	0.05	0.12
FL	2310545	1218	119[a]	6404.7	531.5	0.65	0.17	-1.11	-0.44	0.05	0.06
FL	2310650	779	77[a]	6558.7	3210.0	0.90	0.57	-1.84	-0.79	0.18	0.27
FL	2303000	728	570	432.7	120.5	2.32	0.34	-1.30	-0.77	0.06	0.14
FL	2298488	1303	76	737.3	194.0	0.14	0.38	-1.32	-0.58	0.14	0.18
FL	2298554	899	207[a]	969.2	320.5	0.50	0.30	-1.25	-1.22	0.13	0.27
FL	2298492	1478	16	1238.2	304.0	0.05	0.30	-1.11	-0.82	0.13	0.31
FL	2298495	330	10	1870.0	662.0	0.29	0.25	-1.52	-1.65	0.03	0.08
FL	2298527	807	23	1410.7	201.5	0.10	0.19	-1.03	-0.74	0.06	0.18
FL	2298530	1510	17	1460.8	348.0	0.14	0.29	-1.27	-0.77	0.08	0.13
FL	2297100	2979	342	1260.6	221.5	0.92	0.25	-1.18	-0.64	0.08	0.20
FL	2313000	787	4727	407.2	173.0	5.89	0.51	-1.71	-0.71	0.19	0.28
FL	2300500	821	386	447.9	83.0	0.30	0.20	-1.21	-0.89	0.05	0.11
ND	5057000	1401	16757	1420.6	610.0	2.08	0.51	-1.75	-0.74	0.14	0.21
ND	5056000	1277	5361	1681.4	546.0	3.61	0.44	-1.50	-0.60	0.07	0.14
TX	8068275	2801	482	361.7	65.0	0.57	0.15	-1.18	-1.23	0.06	0.11
GA	2336300	1235	225	230.4	63.0	0.79	0.31	-1.28	-0.88	0.16	0.33
GA	2207120	1383	417	312.5	59.0	1.48	0.24	-1.14	-0.76	0.09	0.20
SC	2160105	1363	1966	124.7	51.0	6.36	0.36	-1.56	-1.30	0.14	0.27

SC	2160700	1392	1150	148.7	51.0	4.45	0.37	-1.40	-0.94	0.15	0.28
MO	6894000	1375	477	1031.9	334.0	0.79	0.25	-1.40	-1.50	0.13	0.22
MO	6895500	802	1258481	786.7	428.0	939.98	0.57	-2.17	-0.90	0.06	0.20
ND	5082500	1274	77959	1390.6	427.0	77.19	0.38	-1.30	-0.77	0.15	0.26
KS	7144780	575	1847	1389.1	678.0	1.73	0.54	-1.73	-0.91	0.14	0.26
Mean							0.34	-1.40	-0.89	0.11	0.20
Standard deviation (STDEV)							0.13	0.28	0.29	0.05	0.08

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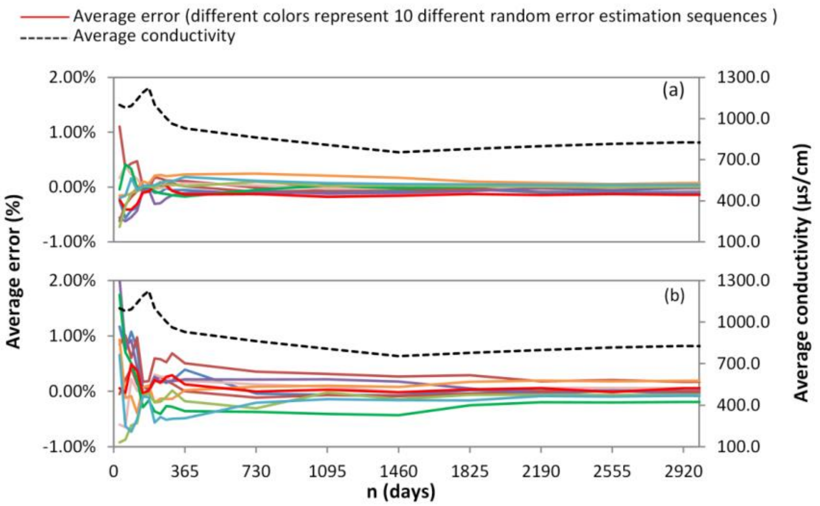
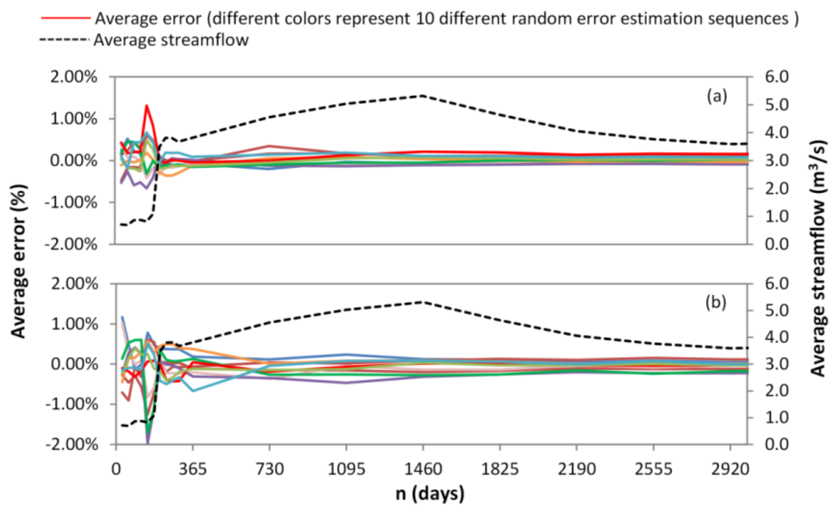
Figures



~~**Figure 1. Average conductivity error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.**~~



~~Figure 2. Average streamflow error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.~~

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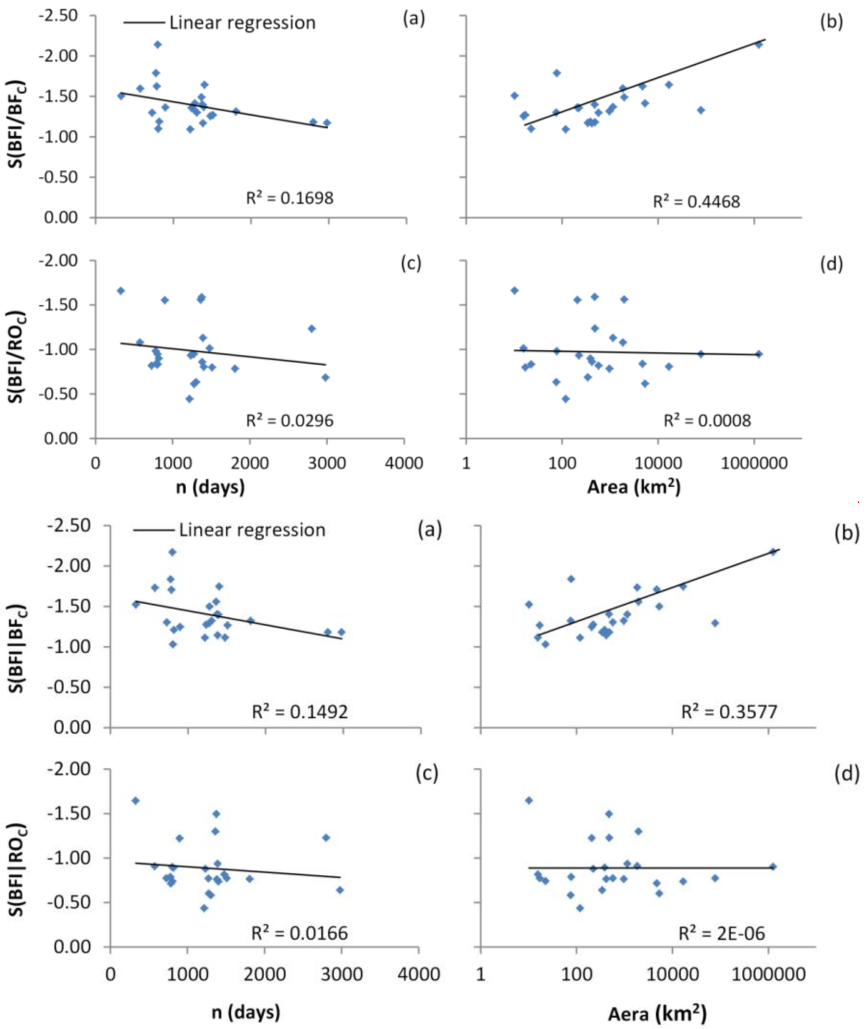

Figure 1~~3~~. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear~~normal~~ axes.

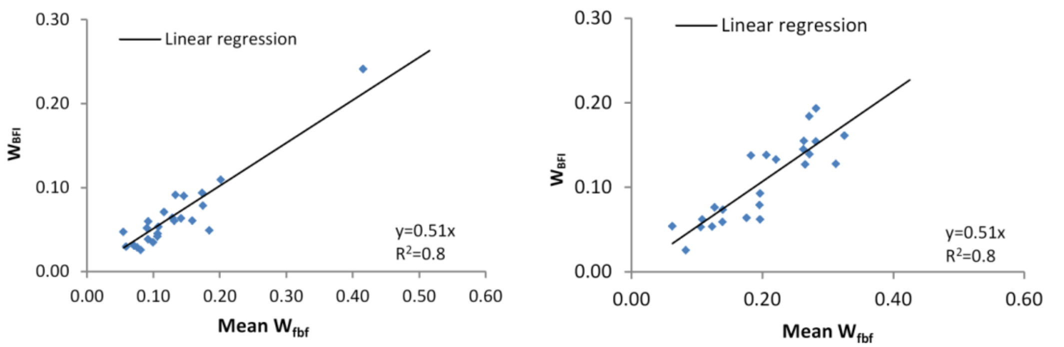

Figure 24. Scatter plot of uncertainty in BFI (W_{BFI}) and mean uncertainty in f_{bf} (Mean W_{fbf}).