Technical note: Analytical sensitivity analysis and uncertainty estimation of a two-component hydrograph separation method ~~which uses~~ with conductivity as a tracer

Weifei Yang[1], Changlai Xiao[1], Xiujuan Liang[1]

[1] Key Laboratory of Groundwater Resources and Environment, Ministry of Education, and National-Local Joint Engineering Laboratory of In-situ Conversion, Drilling and Exploitation Technology for Oil Shale, and College of New Energy and Environment, Jilin University, No 2519, Jiefang Road, Changchun 130021, PR China

Correspondence to: Changlai Xiao (xcl2822@126.com, jluywf@126.com)

Abstract. The ~~conductivity~~ two-component hydrograph separation method with conductivity as a tracer is favored by hydrologists owing to its low cost and easy application~~is cheap and easy to operate and is favored by hydrologists~~. This ~~paper~~ study analyzes the sensitivity of the baseflow index (BFI, ~~the~~ long-term ratio of baseflow to streamflow) calculated ~~by~~using this method to errors or uncertainties of ~~the~~ two parameters (BF_C, the conductivity of baseflow, and~~;~~ RO_C, the conductivity of surface runoff) and ~~of the~~ two variables (y_k, ~~the specific~~ streamflow~~;~~, and SC~~Q_c~~$_k$, ~~the~~ specific conductance~~ivity~~ of streamflow, where k is the time step), and then estimates the uncertainty of BFI. The analysis shows that ~~when the~~for time series ~~is~~ longer than 365 days, ~~the~~ random measurement errors of y_k or SC~~Q_c~~$_k$ will cancel each other, and their influence on BFI can be neglected. ~~Dimensionless sensitivity indices (the ratio of the relative error of BFI to the relative error of BF_C or RO_C) can well express the propagation of errors or uncertainties of BF_C or RO_C into BFI. Based on the sensitivity analysis, the~~ An uncertainty estimation method of BFI is derived on the basis of the sensitivity analysis. Representative sensitivity indices (the ratio of the relative error of BFI to that of BF_C or RO_C) and BFI' uncertainties are determined~~yielded~~ by applying~~ication of~~ the resulting equations to 24 watersheds in the United States. These dimensionless sensitivity indices can well express the propagation of errors or uncertainties of BF_C or RO_C into BFI. The results indicate that BFI is more sensitive to BF_C, and the conductivity two-component hydrograph separation method may be more suitable for the long time series in a small watershed. ~~After considering~~When the mutual offset of the measurement errors of conductivity and streamflow is considered, the uncertainty of BFI is reduced by half.

1 Introduction

Hydrograph separation (also called baseflow separation), aims to identify ~~can effectively identify~~ the proportion of water in different runoff pathways in ~~a basin's~~the export flow of a basin, which helps ~~to~~ in identifying the conversion relationship between groundwater and surface water;~~,~~ ~~and~~ in addition, it is a necessary condition for optimal allocation of water resources (Cartwright et al., 2014; Miller et al., 2014; Costelloe et al., 2015). Some researchers indicated that tracer-based hydrograph separation methods yield the most realistic results~~alisotope (tracer) hydrograph separation method is considered to be the most effective separation method,~~ because they are the most physically based methods ~~which can reflect the actual characteristics of a basin~~ (Miller et al., 2014; Mei and Anagnostou, 2015; Zhang et al., 2017). Many hydrologists have suggested~~indicated~~ that electrical conductivity can be used as a tracer ~~to perform~~in hydrograph separation (Stewart et al., 2007; Munyaneza et al., 2012; Cartwright et al., 2014; Lott and Stewart, 2016; Okello et al., 2018). ~~The measurement of c~~Conductivity is a suitable tracer because its measurement is simple and inexpensive, and it has ~~a~~ distinct applicability in ~~a~~ ~~long~~ long-series ~~of~~ hydrograph separation (Okello et al., 2018).

Formatted: Font: (Asian) Times New Roman, Superscript

Formatted: Font: Italic

Formatted: Font: Italic

The ~~conductivity~~ two-component hydrograph separation method with conductivity as a tracer (also called conductivity mass balance method (CMB) (Stewart et al. 2007)) ~~uses conductivity as a tracer to~~ calculates baseflow through a two-component mass balance equation. The general equation is shown in Eq. (1), ~~,~~ which is based on the following assumptions:

a) Contributions from end-members other than baseflow and surface runoff are negligible.

b) The specific conductance of runoff and baseflow are constant (or vary in a known manner) over the period of record.

c) Instream processes (such as evaporation) do not change specific conductance makedly.

d) Baseflow and surface runoff have significantly different specific conductance. ~~The general equation is shown in Eq. (1).~~

$$b_k = \frac{y_k(\cancel{Q}SC_{\cancel{c}k} - \mathrm{RO_c})}{\mathrm{BF_c} - \mathrm{RO_c}} \quad (1)$$

where b is ~~the~~ baseflow (L^3/t), y is ~~the~~ streamflow (L^3/t), SC~~Q_c~~ is the electrical conductivity of streamflow, and k is ~~the~~ time step number. The two parameters BF_C and RO_C ~~respectively~~ represent the electrical conductivity of baseflow and surface runoff, respectively.

Stewart et al. (2007) conducted a~~The~~ field test in a drainage basin of $12km^2$ area in southeast Hillsborough County, Florida ~~of Stewart et al. (2007)~~ and showed that the maximum conductivity of streamflow can be used to replace BF_C, and the minimum conductivity can be used to replace RO_C. However, Miller et al. (2014) pointed out that the maximum conductivity of streamflow may exceed the real BF_C~~,~~; Therefore, ~~so~~ they suggested that the 99th percentile of the conductivity of ~~a long series of streamflow~~each year should be used as ~~the~~ BF_C to avoid the impact of high BF_C estimates on the separation results and assumed that baseflow conductivity varies linearly between years. ~~There is uncertainty in~~The determination~~ing~~ of the parameters (BF_C, RO_C) of the conductivity two-component hydrograph separation method involves some uncertainties (Miller et al., 2014; Okello et al., 2018). Therefore, sensitivity analysis of parameters and ~~the uncertainty~~ quantitative analysis of the uncertainties~~separation results are helpful to~~ will contribute towards further optimization~~e~~ of the conductivity two-component hydrograph separation method and improving~~e~~ the accuracy of hydrograph separation.

Most ~~of the~~ existing parameter sensitivity analysis methods ~~use~~ are experimental methods ~~sensitivity analysis method, which~~ that usually substitute ~~s the fluctuation~~ varying values of a certain parameter into the separation model~~,~~ and then ~~analyzes the sensitivity of the parameters by~~ compar~~e~~ings the range of the separation results produced by these ~~fluctuation~~ varying parameter values~~s~~ (Eckhar~~ra~~dt, 2005; Miller et al., 2014; Okello et al., 2018). Eckhardt (2012) indicated that ~~“~~"An empirical sensitivity analysis is only a makeshift if an analytical sensitivity analysis, that is an analytical calculation of the error propagation through the model, is not feasible"~~An empirical sensitivity analysis is only an analytical calculation of the error propagation through the model, is not feasible.”~~ Eckhardt (2012) derived ~~the~~ sensitivity indices of ~~the~~ equation parameters by the partial derivative of a two-parameter recursive digital baseflow separation filter equation. Until now~~However~~, the parameters’ sensitivity indices of the conductivity two-component hydrograph separation equation have not been derived.

At present, the uncertainty of the separation results of the conductivity two-component hydrograph separation method is mainly estimated ~~by~~using an uncertainty transfer equation based on the uncertainty of BF_C, RO_C, and SC~~Q_c~~$_k$ (Genereux, 1998; Miller et al., 2014). See Sect. 3.1 for details. ~~This~~ In this uncertainty estimation method, ~~can only estimate~~ the uncertainty of the baseflow ratio (f_{bf}, the ratio of baseflow to streamflow in a single calculation process) is estimated, and ~~then use~~ the average uncertainty of multiple calculation processes is then used to estimate the uncertainty of the baseflow index (BFI, ~~the~~ long-term ratio of baseflow to total streamflow). This ~~uncertainty estimation~~ method can neither directly estimate the uncertainty of BFI nor consider the randomness and mutual offset of conductivity measurement errors, and thus, it does not provide accurate estimates of BFI uncertainty ~~the uncertainty estimation of BFI is not appropriate enough~~.

Formatted: Font: Italic

Formatted: Superscript

Formatted: Font: Italic

The main objectives of this study are as follows: (i) analyze the sensitivity of long-term series of baseflow separation results (BFI) to parameters and variables of the conductivity two-component hydrograph separation equation (Sect. 2); (ii) derive the uncertainty of BFI (Sect.3).~~The purpose of this paper is to derive the parameters' sensitivity indices of the conductivity two-component hydrograph separation equation by calculating the partial derivative of Eq. (1) (Sect. 2), and further derive the direct estimation method of BFI' uncertainty (Sect. 3).~~ The derived solutions~~methods~~ were applied to 24 basins in the United States, and the parameter~~s'~~ sensitivity indices and BFI~~'~~ uncertainty characteristics were analyzed (Sect. 4).

2 ~~Analytical~~ S~~s~~ensitivity analysis

2.1 Parameters $\mathrm{BF_C}$ and $\mathrm{RO_C}$

In order to calculate the sensitivity indices of the parameters, the partial derivatives of b_k in Eq. (1) with respect to $\mathrm{BF_C}$ and $\mathrm{RO_C}$ ~~the partial derivatives of b_k in Eq. (1) to $\mathrm{BF_C}$ and $\mathrm{RO_C}$~~ are required ~~respectively~~ (~~for~~ the derivation process is expressed as~~, see Appendix~~ Eq. (A1) and~~,~~ (A2)):

$$\frac{\partial b_k}{\partial \mathrm{BF_c}} = -y_k \frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{2}$$

$$\frac{\partial b_k}{\partial \mathrm{RO_c}} = y_k \frac{\cancel{Q}SC_{\cancel{c}k}-\mathrm{BF_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{3}$$

For the convenience of comparison, the baseflow index (BFI) is selected as the baseflow separation result for long time series to analyze the influence of parameter~~s'~~ uncertainty on BFI,

$$\mathrm{BFI} = \frac{\sum_{k=1}^{n} b_k}{\sum_{k=1}^{n} y_k} = \frac{b}{y} \tag{4}$$

where b ~~denotes the total baseflow~~ and y denote the total baseflow and~~the~~ total streamflow, respectively, over the whole available streamflow sequences, and n is the number of available streamflow data.

Then, the partial derivatives of BFI to $\mathrm{BF_C}$ and $\mathrm{RO_C}$ should be calculated~~,~~ (~~for~~ the derivation process~~, see~~ ~~Appendix~~ is presented in Eq. (A3) and~~,~~ (A4)):

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} = \frac{y\mathrm{RO_c}-\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{5}$$

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}-y\mathrm{BF_c}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{6}$$

~~It can be seen from~~ T~~t~~he definition of the partial derivative suggests that the influence of the errors of the parameters ($\Delta\mathrm{BF_C}$ and $\Delta\mathrm{RO_C}$) in Eq. (1) on the BFI can be expressed by the product of the errors and its partial derivatives. Then the errors of BFI caused by small errors of $\mathrm{BF_C}$ and $\mathrm{RO_C}$ can be approximated by ~~the BFI' errors caused by tiny errors of $\mathrm{BF_C}$ and $\mathrm{RO_C}$ can be expressed as~~:~~:~~

$$\Delta_{\mathrm{BF_c}}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}}\Delta\mathrm{BF_c} = \frac{y\mathrm{RO_c}-\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{BF_c} \tag{7}$$

$$\Delta_{\mathrm{RO_c}}\mathrm{BFI} = \frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}}\Delta\mathrm{RO_c} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}-y\mathrm{BF_c}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{RO_c} \tag{8}$$

The dimensionless sensitivity indices (S) can be obtained by comparing the relative error of BFI caused by the small~~tiny~~ errors of $\mathrm{BF_C}$ and $\mathrm{RO_C}$ with that of $\mathrm{BF_C}$ and $\mathrm{RO_C}$, (see~~Appendix~~ Eq. (B1), (B2)):

$$\mathrm{S}(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}}) = \frac{\Delta_{\mathrm{BF_c}}\mathrm{BFI}}{\mathrm{BFI}}\Big/\frac{\Delta\mathrm{BF_c}}{\mathrm{BF_c}} = \frac{\mathrm{BF_c}(y\mathrm{RO_c}-\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k})}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{9}$$

$$\mathrm{S}(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}}) = \frac{\Delta_{\mathrm{RO_c}}\mathrm{BFI}}{\mathrm{BFI}}\Big/\frac{\Delta\mathrm{RO_c}}{\mathrm{RO_c}} = \frac{\mathrm{RO_c}(\sum_{k=1}^{n} y_k SC\cancel{Q}_{\cancel{c}k}-y\mathrm{BF_c})}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2} \tag{10}$$

Formatted: Subscript

Formatted: Subscript

Formatted: Subscript

where $S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF}_{\epsilon}})$ represent the dimensionless sensitivity index of BFI (output) with $\mathrm{BF_c}$ (uncertain input), and $S(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO}_{\epsilon}})$ with $\mathrm{RO_c}$.

The dimensionless sensitivity index is also called the "elasticity index", and it reflects the proportional relationship between the relative error of BFI and the relative error of parameters (e.g. if $S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF}_{\epsilon}}) = 1.5$, and the relative error of $\mathrm{BF_c}$ is 5%, then the relative error of BFI ~~should be~~is 1.5 times 5% =~~(~~ 7.5%~~)~~). After determining the ~~specific~~ values of $\mathrm{BF_C}$, $\mathrm{RO_C}$, BFI, y, y_k and $SC\cancel{Q}_{\cancel{\epsilon}k}$, the sensitivity indices $S(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF}_{\epsilon}})$ and $S(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO}_{\epsilon}})$ can be calculated and compared.

2.2 Variables y_k and $SC\cancel{Q}_{\cancel{\epsilon}k}$

In addition to the two parameters, there are two variables ($SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k) in Eq. (1). This section ~~will analyze~~describes the sensitivity analysis of BFI to these two variables. Similar to Sect. 2.1, the partial derivatives of b_k in Eq. (1) to $SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k are obtained (see ~~Appendix~~Eq. (~~-~~A5), (A6)), and the partial derivatives of BFI to $SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k are further obtained (see ~~Appendix~~Eq. (A7), (A8)),

$$\frac{\partial\mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{\epsilon}k}} = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}} \tag{11}$$

$$\frac{\partial\mathrm{BFI}}{\partial y_k} = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{\epsilon}k}-\mathrm{RO_c})-\mathrm{nBFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})}$$

(12)

According to previous studies (Munyaneza et al., 2012; Cartwright et al., 2014; Miller et al., 2014; Okello et al., 2018) and this study (Table 1), the difference between $\mathrm{BF_C}$ and $\mathrm{RO_C}$ is often greater than 100 µs/cm. Therefore, ~~so~~ $\partial\mathrm{BFI}/\partial SC\cancel{Q}_{\cancel{\epsilon}k}$ is usually less than 0.01 cm/ µs. Appendix C shows that the value of $\partial\mathrm{BFI}/\partial y_k$ is usually far less than 1 d/m^3.

~~Tiny~~ Small errors in $SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k cause errors in BFI ~~of~~

$$\Delta_{SC\cancel{Q}_{\cancel{\epsilon}k}}\mathrm{BFI} = \frac{\partial\mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{\epsilon}k}}\Delta SC\cancel{Q}_{\cancel{\epsilon}k} = \frac{\Delta SC\cancel{Q}_{\cancel{\epsilon}k}}{\mathrm{BF_c}-\mathrm{RO_c}}$$

(13)

$$\Delta_{y_k}\mathrm{BFI} = \frac{\partial\mathrm{BFI}}{\partial y_k}\Delta y_k = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{\epsilon}k}-\mathrm{RO_c})-\mathrm{nBFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})}\Delta y_k \tag{14}$$

The errors of BFI caused by $SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k are summed up to obtain~~get~~ the error of BFI caused by $\sum_{k=1}^{n} SC\cancel{Q}_{\cancel{\epsilon}k}$ and $\sum_{k=1}^{n} y_k$ in the whole time series:

$$\Delta_{\sum_{k=1}^{n} SC\cancel{Q}_{\cancel{\epsilon}k}}\mathrm{BFI} = \sum_{k=1}^{n}\Delta_{SC\cancel{Q}_{\cancel{\epsilon}k}}\mathrm{BFI} = \sum_{k=1}^{n}\frac{\Delta SC\cancel{Q}_{\cancel{\epsilon}k}}{\mathrm{BF_c}-\mathrm{RO_c}} = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}}\sum_{k=1}^{n}\Delta SC\cancel{Q}_{\cancel{\epsilon}k}$$

(15)

$$\Delta_{\sum_{k=1}^{n} y_k}\mathrm{BFI} = \sum_{k=1}^{n}\Delta_{y_k}\mathrm{BFI} = \sum_{k=1}^{n}\left(\frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{\epsilon}k}-\mathrm{RO_c})-\mathrm{nBFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})}\Delta y_k\right) = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{\epsilon}k}-\mathrm{RO_c})-\mathrm{nBFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})}\sum_{k=1}^{n}\Delta y_k \tag{16}$$

Wagner et al. (2006) reported that ~~The tiny errors in $Q_{\epsilon k}$ and y_k are mainly composed of random analysis errors. Random errors mostly follow a normal distribution or a uniform distribution. The magnitude and direction of the random errors are usually not fixed. As the number of measurements increases, the positive and negative errors can compensate each other, and the average value of the errors will gradually trend to zero (Huang and Chen, 2011).~~

~~T~~the uncertainty of ~~the~~ instruments is usually less than~~<~~ 5% for $SC\cancel{Q}_{\cancel{\epsilon}k}$ ~~less than~~(≤~~-~~100 µs/cm) and less than~~<~~3% for $SC\cancel{Q}_{\cancel{\epsilon}k}$ ~~greater than~~ (>100 µs/cm) ~~(Wagner et al., 2006; Miller et al., 2014)~~. According to Hamilton et al. (2012) streamflow data from USGS are often assumed by analysts to be accurate and precise within ±5% at the 95% confidence interval. ~~The measurement uncertainty of streamflow is usually <3% (Zhang, 2005)~~. In this ~~paper~~study, the error ranges of $SC\cancel{Q}_{\cancel{\epsilon}k}$ and y_k are all considered to be ±5% ~~and ±3%, respectively~~. The errors in SC_k and y_k mainly comprise random analysis errors which mostly follow a normal

Formatted: Font: Italic

Formatted: Font: Bold, Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Superscript

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Subscript

Formatted: Subscript

distribution or a uniform distribution (Huang and Chen, 2011). Considering the mutual offset of random errors, when the time series (n) is sufficiently long ~~enough~~, $\sum_{k=1}^{n} \Delta SC$~~Q~~$_{ck}$ in Eq. (15) and $\sum_{k=1}^{n} \Delta y_k$ in Eq. (16) will approach zero.

The analysis of $\sum_{k=1}^{n} \Delta SC_k$ and $\sum_{k=1}^{n} \Delta y_k$ under different time series (n) and different error distributions (normal distribution or uniform distribution) of a surface water station (USGS site number 0297100) showed that the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series is greater than 365days (See Supplement S1 for detail).~~Therefore, when n is large enough, the error of BFI caused by the errors of Q_{ck} and y_k can be neglected.~~

~~To verify this phenomenon, the study collected the daily average conductivity and daily average streamflow of the surface water station with the USGS site number 0297100 (Table 1) from 2001 to 2010 (2979 days in total). Then, office Excel was used to generate 10 sets (2979 per set) of random numbers between -0.05 and 0.05 that obey normal distribution and uniform distribution respectively to simulate the errors (%) of the daily average conductivity. And 10 sets (2979 per set) of random numbers obeying normal distribution and uniform distribution between -0.03 and 0.03, respectively, were used to simulate the errors (%) of the daily average streamflow. Finally, according to different time series (n) (e.g. 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 365, 730, 1095, … , 2979, days) sum the errors value ($\sum_{k=1}^{n} \Delta Q_k$ and $\sum_{k=1}^{n} \Delta y_k$) and analyze the trend of the average error (%) with n.~~

~~The trend of the average error (%) of conductivity with n is shown in Fig. 1. The average errors of the uniform distribution (Fig. 1(a)) and the normal distribution (Fig. 1(b)) are all gradually approach zero with the increase of the time series (n), and the uniform distribution converges faster than the normal distribution. The average errors of the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.49% when n is greater than 365.~~

~~Similar to the conductivity, the trend of the average error (%) of the streamflow with n is shown in Fig. 2. The average errors of the uniform distribution (Fig. 2(a)) and the normal distribution (Fig. 2(b)) all gradually approach to zero as the time series (n) increases, and the uniform distribution converges faster than the normal distribution. The average errors of different n under the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.67% when n is greater than 365.~~

~~From the above analysis, when the time series (n) is greater than 365 days (1 year), $\Delta_{\sum_{k=1}^{n} Q_{ck}}$BFI will be less than 0.0049% (0.01 times 0.49%), and $\Delta_{\sum_{k=1}^{n} y_k}$BFI will be much less than 0.76% (1 times 0.76%). Therefore, the random errors of daily average conductivity and streamflow have a negligible effect on BFI.~~

~~Figure 1. Average conductivity error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.~~

~~Figure 2. Average streamflow error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.~~

3 Uncertainty estimation

3.1 Previous attempts

According to previous studies, in the case where a ~~parameter~~ variable g is calculated as a function of several factors x_1, x_2, x_3, …, x_n (e.g. g= G(x_1, x_2, x_3, …, x_n)) ~~.~~ and based on the assumptions that the factors are uncorrelated and have a Gaussian distribution, t~~T~~he transfer equation (also known as Gaussian error propagation) between the uncertainty of the independent factors and the uncertainty of g is ~~(Taylor, 1982; Kline, 1985; Genereux, 1998)~~:

Formatted: Font: Not Italic

$$W_g = \sqrt{(\frac{\partial g}{\partial x_1} W_{x_1})^2 + (\frac{\partial g}{\partial x_2} W_{x_2})^2 + \cdots + (\frac{\partial g}{\partial x_n} W_{x_n})^2} \quad (17)$$

where W_g, W_{x1}, W_{x2}, and W_{xn} are the same type of uncertainty values (e.g. all average errors or all standard deviations) for g, x_1, x_2, and x_n, respectively. A more detailed description of this equation can be found in Taylor (1982), Kline (1985), and Ernest (2005).

According to Genereux (1998), "While any set of consistent uncertainty (W) values may be propagated using Gaussian error propagation, using standard deviations multiplied by *t* values from the Student's *t* distribution (each *t* for the same confidence level, such as 95%) has the advantage of providing a clear meaning (tied to a confidence interval) for the computed uncertainty would correspond to, for example, 95% confidence limits on BFI".

Based on the above principle, Genereux (1998) substituted Eq. (18) into Eq. (17) to derive the uncertainty estimation equation (Eq. (19)) of the two-component mass balance baseflow separation method:

$$f_{bf} = \frac{SC\cancel{Q}_{\cancel{c}k} - \mathrm{RO_c}}{\mathrm{BF_c} - \mathrm{RO_c}} \quad (18)$$

$$W_{f_{bf}} = \sqrt{(\frac{f_{bf}}{\mathrm{BF_c} - \mathrm{RO_c}} W_{\mathrm{BF_C}})^2 + (\frac{1 - f_{bf}}{\mathrm{BF_c} - \mathrm{RO_c}} W_{\mathrm{RO_c}})^2 + (\frac{1}{\mathrm{BF_c} - \mathrm{RO_c}} W_{SC\cancel{Q}_{\cancel{c}}})^2} \quad (19)$$

where f_{bf} is the ratio of baseflow to streamflow in a single calculation process, W_{fbf} is the uncertainty in f_{bf} at the 95% confidence interval, W_{BFC} is the standard deviation of the $\mathrm{BF_C}$~~ highest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, W_{ROC} is the standard deviation of the $\mathrm{RO_C}$~~ lowest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and $W_{S\cancel{Q}C}$ is the analytical error in~~ the~~ conductivity multiplied by the t-value (α=0.05; two-tail)~~.~~ (Miller et al., 2014).~~(Miller et al., 2014).~~

~~Equation (19) can b~~Better estimates of the uncertainty of f_{bf} within a single calculation step can be obtained using Eq. (19). Hydrologists usually approximate~~estimate~~ the uncertainty of BFI ~~approximately ~~by averaging the uncertainty of all steps (Genereux, 1998; Miller et al., 2014). However, this method does not consider the mutual offset of the conductivity measurement errors~~,~~ and cannot accurately reflect the uncertainty of BFI. In this ~~paper~~study, ~~based on the parameter sensitivity analysis, the~~an uncertainty estimation equation of BFI is derived on the basis of the parameter sensitivity analysis.~~ See the next section for details.~~

3.2 Uncertainty estimation of~~in~~ BFI

BFI is a function of $\mathrm{BF_c}$, $\mathrm{RO_c}$, $SC\cancel{Q}_{\cancel{c}k}$ and y_k. In addition,~~And~~ the uncertaint~~y~~ies ~~-~~of $\mathrm{BF_c}$, $\mathrm{RO_c}$, $SC\cancel{Q}_{\cancel{c}k}$ and y_k ~~is ~~are independent of each other. As explained earlier (Sect. 2.2),~~Sect. 2.2 has explained that~~ the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series (n) is greater than 365 days (1 year), ~~so~~Therefore, the uncertainty of BFI can be expressed as:

$$W_{BFI} = \sqrt{(\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} W_{\mathrm{BF_C}})^2 + (\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} W_{\mathrm{RO_C}})^2} \quad (20)$$

~~where (see Eq. 5 and Eq. 9; Eq. 6 and Eq. 10)~~

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{BF_c}} = \mathrm{S}(\mathrm{BFI}|\mathrm{BF_c}\ \cancel{\mathrm{BFI}/\mathrm{BF_c}})\frac{\mathrm{BFI}}{\mathrm{BF_c}}$$

(21)

$$\frac{\partial \mathrm{BFI}}{\partial \mathrm{RO_c}} = \mathrm{S}(\mathrm{BFI}|\mathrm{RO_c}\ \cancel{\mathrm{BFI}/\mathrm{RO_c}})\frac{\mathrm{BFI}}{\mathrm{RO_c}}$$

(22)

Then,~~ the~~ Eq. (20) can be rewritten as:

Formatted: Font: Not Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Subscript

Formatted: Subscript

$$W_{BFI}=\sqrt{(S(\mathrm{BFI}|\mathrm{BF_c}\,\cancel{\mathrm{BFI}/\mathrm{BF}_\epsilon})\frac{\mathrm{BFI}}{\mathrm{BF_c}}W_{\mathrm{BF_C}})^2+(S(\mathrm{BFI}|\mathrm{RO_c}\,\cancel{\mathrm{BFI}/\mathrm{RO}_\epsilon})\frac{\mathrm{BFI}}{\mathrm{RO_c}}W_{\mathrm{RO_C}})^2} \quad (23)$$

where W_{BFI}, $W_{\mathrm{BFC},}$ and W_{ROC} are the same type of uncertainty values for BFI, $\mathrm{BF_{C,}}$ and $\mathrm{RO_C}$, respectively. For instance, W_{BFI} is the uncertainty in BFI at the 95% confidence interval, W_{BFC} is the standard deviation of the ~~highest 1% of measured conductivity~~$\mathrm{BF_C}$ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and W_{ROC} is the standard deviation of the $\mathrm{RO_C}$ ~~lowest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution.

Formatted: Subscript

Formatted: Subscript

4 Application

4.1 Data and processing

The above sensitivity analysis and uncertainty estimation methods were applied to 24 catchments in the United States (Table 1). All basins used in this study are perennial streams, with drainage areas ranging from 10 km^2 to 1258481 km^2. Each gage has about at least 1 year of continuous streamflow and conductivity ~~at~~ for the same period of records. All streamflow and conductivity data are daily average values retrieved from the United States Geological Survey's (USGS) National Water Information System (NWIS) website, http://waterdata.usgs.gov/nwis.

The daily baseflow of each basin was calculated using Eq. (1). The 99th percentile of the conductivity of each year was used as $\mathrm{BF_C}$, and linear variation of baseflow conductivity between years was assumed. The~~–~~ 1st percentile ~~99th percentile~~ of the conductivity of the whole series of streamflow in each basin was used as the ~~$\mathrm{BF_C}$ and the 1st percentile as the~~ $\mathrm{RO_C}$. The total baseflow b,~~ the~~ total streamflow y and~~ the~~ baseflow index BFI of each watershed were then calculated. According to the results of the hydrograph separation, the parameter sensitivity indices of BFI for mean $\mathrm{BF_C}$ ($S(\mathrm{BFI}|\mathrm{BF_c}$ ~~$\mathrm{BFI}/\mathrm{BF}_\epsilon$~~$)$) and $\mathrm{RO_C}$ ($S(\mathrm{BFI}|\mathrm{RO_c}$ ~~$\mathrm{BFI}/\mathrm{RO}_\epsilon$~~$)$) were calculated using~~by~~ Eq. (9) and Eq. (10), respectively.

Formatted: Subscript

Finally, the uncertainty of f_{bf} in each step was calculated using~~by~~ Eq. (19) and averaged to obtain the ~~M~~mean W_{fbf} ~~in~~ for each basin. The uncertainty (W_{BFI}) of BFI was directly calculated using~~by~~ Eq. (23), and then the values of ~~M~~mean W_{fbf} and W_{BFI} were compared. For each basin, W_{BFC} is the standard deviation of the $\mathrm{BF_C}$ of the whole series ~~highest 1% of measured conductivity~~ multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, W_{ROC} is the standard deviation of the lowest 1% of measured conductivity multiplied by the t-value (α=0.05; two-tail) from the Student's distribution, and W_{S}~~$_{\mathrm{Q}}$~~$_{\mathrm{C}}$ is the analytical error in the conductivity (5%) multiplied by the t-value (α=0.05; two-tail).

Formatted: Subscript

4.2 Results and discussion

The calculation results are shown in Table 1. The average baseflow index of the 24 watersheds is ~~0.29~~0.34, the average sensitivity index of BFI for mean $\mathrm{BF_C}$ ($S(\mathrm{BFI}|\mathrm{BF_c}$ ~~$\mathrm{BFI}/\mathrm{BF}_\epsilon$~~$)$) is -1.40~~39~~, and the average sensitivity index of BFI for $\mathrm{RO_C}$ ($S(\mathrm{BFI}|\mathrm{RO_c}$ ~~$\mathrm{BFI}/\mathrm{RO}_\epsilon$~~$)$) is -0.~~98~~89. The negative sensitivity indices indicate a negative correlation between BFI and $\mathrm{BF_C}$, $\mathrm{RO_C}$. The absolute value of the sensitivity index ~~The sensitivity index~~ for $\mathrm{BF_C}$ is generally greater than that for $\mathrm{RO_C}$, indicating that BFI is more affected by $\mathrm{BF_C}$ (for example, if there are 10~~5~~% uncertaintries~~y~~ in both $\mathrm{BF_C}$ and $\mathrm{RO_C}$, then $\mathrm{BF_C}$ leads to -1.~~.39~~ 40 times 10% of uncertainty in BFI (-~~6.95~~14.0%), while $\mathrm{RO_C}$ leads to -0.~~.98~~ 89 times 10% (~~4.9~~-8.9%)). Therefore, the determination of $\mathrm{BF_C}$ requires more caution, and any small error may lead to greater uncertainty in BFI. Miller et al. (2014) reported ~~have indicated~~ that anthropogenic activities over long periods of time, or year to year changes in the elevation of the water table may result in temporal changes~~ly changing~~ in ~~the~~ BF$_C$. They~~He~~ recommended taking different BF$_C$ values per year based on the conductivity values during~~at~~ low flow periods to avoid the effects of temporal fluctuations in BF$_C$~~' temporally fluctuations~~.

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote "a" in the "Area" column indicates that the values are estimated based on data from adjacent sites.

The sensitivity index of BFI for BF$_C$ shows~~has~~ a decreasing trend with the increase of time series (n) (Fig. 1~~3~~(a)) and ~~has~~ an increasing trend with~~ the~~ increasing~~e of~~ watershed area (Fig. 1~~3~~(b)), ~~the~~ with correlation coefficients ~~are~~ of 0.~~1698~~ 1492 and 0.~~4468~~3577, respectively. Although the correlations are not obvious, ~~it still has~~they have important guiding significance. ~~In the l~~Large basins, comprise~~there are~~ many different subsurface flow paths contributing to streams (Okello et al., 2018), each of which has a unique conductivity value (Miller et al., 2014). It is difficult to represent the conductivity characteristics of subsurface flow with a special value. Therefore, the conductivity two-component hydrograph separation method has~~ a~~ higher applicability to long time series of~~ in a~~ small watershed~~ of long time series~~.

The sensitivity index of BFI for RO$_C$ did not change significantly with the increase of time series and watershed area (Fig. 1~~3~~(c), Fig. 1~~3~~(d)). During~~ the~~ rainstorms, ~~the water level of the stream rises sharply, the subsurface flow is suppressed, and the streamflow is almost entirely from the rainfall runoff. At this time,~~ the conductivity of~~ the~~ streams ~~is~~ became similar to ~~the~~ that ~~conductivity~~ of the ~~local~~ rainfall (Stewart et al., 2007). The electrical conductivity of regional rainfalls vary~~ies~~ slightly, usually at a fixed value, and it has no significant relationship with the basin area and year (Munyaneza et al., 2012). Therefore, the temporal and spatial variation characteristics of BFI for RO$_C$ are not obvious.

Figure 1~~3~~. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear~~normal~~ axes.

Genereux's method (Eq.19) estimates the average uncertainty of BFI in the 24 basins (a~~A~~verage of m~~M~~ean W_{fbf}) to be 0.~~13~~20, whereas the average uncertainty of BFI (a~~A~~verage of W_{BFI}) calculated directly using the proposed ~~by this paper'~~ method (Eq. 23) is 0.~~06~~ 11 (Table 1). Mean W_{fbf} in each basin is generally larger than W_{BFI} (W_{BFI} is about 0.51 times of m~~M~~ean W_{fbf}), and there is a significant linear correlation (Fig. 2~~4~~). This shows that the two methods have the same volatility characteristics for BFI uncertainty estimation~~ results~~, but Genereux's method (Eq. 19) often overestimates the uncertainty of BFI. This also means that when the time series is longer than 365 days~~ (1 year)~~, the measurement errors of conductivity and streamflow will cancel each other and thus reduce the uncertainty of BFI (about half of the original).

Figure 2~~4~~. Scatter plot of uncertainty in BFI (W_{BFI}) and mean uncertainty in f_{bf} (m~~M~~ean W_{fbf}).

5 Conclusions

This study analyzed the sensitivity of BFI calculated using the conductivity two-component hydrograph separation method to errors or uncertainties of parameters BF$_C$ and RO$_C$ and variables y_k and SC_k. In addition, the uncertainty of BFI was calculated. The equations derived in this study (~~Equation~~ Eq. (9) and Eq. (10)) ~~can well~~could calculate the sensitivity indices of BFI for~~-~~ BF$_C$ and RO$_C$. For time series longer than 365 days, the measurement errors of conductivity and streamflow exhibited an obvious mutual offset effect, and their influence on BFI could be neglected. Considering the mutual offset, the uncertainty of BFI would be reduced to half. From this viewpoint, Eq. (23) ~~can~~could estimate the uncertainty of BFI ~~when the~~for time series ~~is larger~~longer than 365 days~~, taking into account the mutual cancellation of conductivity measurement errors~~. The application of the method to

Formatted: Subscript

Formatted: Subscript

Formatted: Font: Italic

Formatted: Font: Italic, Subscript

Formatted: Font: Italic

Formatted: Font: Italic, Subscript

~~Applications in~~ 24 basins in the United States showed that BFI is more sensitive to $\mathrm{BF_C}$, and future studies should dedicate~~devote~~ more effort to determining the value of $\mathrm{BF_C}$. In addition, the conductivity two-component hydrograph separation method may be more suitable for ~~the~~ long time series ~~in a~~of small watersheds.

~~When the time series is greater than 365 days, the measurement errors of conductivity and streamflow have obvious mutual offset, and its influence on BFI can be neglected. After considering the mutual offset of random errors, the uncertainty of BFI will be reduced to half.~~Systematic errors in specific conductance and streamflow as well as temporal and spatial variations in baseflow conductivity may be the main sources of BFI uncertainty. Better rating curves are probably more important than better loggers, and more work on understanding the specific conductance of baseflow is more important than understanding that of surface runoff.

The above conclusions are only from the average of the 24 basins in the United States, and further researches ~~is needed~~ in other countries or in more watersheds are thus required. This~~e~~ study~~research in this paper~~ only focused~~s~~ on the two-component hydrograph separation method with conductivity as a tracer, but ~~the~~ parameter sensitivity analysis and uncertainty analysis methods involving ~~of~~ other tracers are very similar~~ to this paper,~~. ~~and~~ Therefore, ~~it is easy to derive~~ similar equations can be easily derived by referring to the findings of this study.

Appendix A

Calculation of the partial derivatives

$$\frac{\partial b_k}{\partial \mathrm{BF_c}} = \frac{\partial}{\partial \mathrm{BF_c}} \frac{y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{\mathrm{BF_c}-\mathrm{RO_c}} = y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})\frac{\partial}{\partial \mathrm{BF_c}}\frac{1}{\mathrm{BF_c}-\mathrm{RO_c}} = -y_k\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2} \qquad \text{(A1)}$$

$$\frac{\partial b_k}{\partial \mathrm{RO_c}} = \frac{\partial}{\partial \mathrm{RO_c}} \frac{y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{\mathrm{BF_c}-\mathrm{RO_c}} = y_k\frac{\partial}{\partial \mathrm{RO_c}}\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{\mathrm{BF_c}-\mathrm{RO_c}} = y_k\frac{-(\mathrm{BF_c}-\mathrm{RO_c})+(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{(\mathrm{BF_c}-\mathrm{RO_c})^2} = y_k\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{BF_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2}$$

(A2)

$$\frac{\partial BFI}{\partial BF_c} = \frac{\partial}{\partial BF_c}\frac{b}{y} = \frac{1}{y}\sum_{k=1}^{n}\frac{\partial b_k}{\partial BF_c} = \frac{1}{y}\sum_{k=1}^{n}\left(-y_k\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2}\right)(\text{see Eq. A1}) = \frac{1}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\sum_{k=1}^{n}(y_k\mathrm{RO_c}-y_kSC\cancel{Q}_{\cancel{c}k}) = \frac{yRO_c-\sum_{k=1}^{n}y_kSC\cancel{Q}_{\cancel{c}k}}{y(BF_c-RO_c)^2}$$

(A3)

$$\frac{\partial BFI}{\partial RO_c} = \frac{\partial}{\partial RO_c}\frac{b}{y} = \frac{1}{y}\sum_{k=1}^{n}\frac{\partial b_k}{\partial RO_c} = \frac{1}{y}\sum_{k=1}^{n}\left(y_k\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{BF_c}}{(\mathrm{BF_c}-\mathrm{RO_c})^2}\right)(\text{see Eq. A2}) = \frac{1}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\sum_{k=1}^{n}(y_kSC\cancel{Q}_{\cancel{c}k}-y_k\mathrm{BF_c}) = \frac{\sum_{k=1}^{n}y_kSC\cancel{Q}_{\cancel{c}k}-yBF_c}{y(BF_c-RO_c)^2}$$

(A4)

$$\frac{\partial b_k}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{\partial}{\partial \mathrm{Q_{ck}}}\frac{y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{\mathrm{BF_c}-\mathrm{RO_c}} = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}}\frac{\partial}{\partial \mathrm{SC}\cancel{Q}_{\cancel{c}\mathrm{k}}}y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}) = \frac{y_k}{\mathrm{BF_c}-\mathrm{RO_c}}$$

(A5)

$$\frac{\partial b_k}{\partial y_k} = \frac{\partial}{\partial y_k}\frac{y_k(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{\mathrm{BF_c}-\mathrm{RO_c}} = \frac{(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})}{\mathrm{BF_c}-\mathrm{RO_c}}\frac{\partial}{\partial y_k}y_k = \frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{\mathrm{BF_c}-\mathrm{RO_c}}$$

(A6)

$$\frac{\partial \mathrm{BFI}}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{\partial}{\partial SC\cancel{Q}_{\cancel{c}k}}\frac{b}{y} = \frac{1}{y}\sum_{k=1}^{n}\frac{\partial b_k}{\partial SC\cancel{Q}_{\cancel{c}k}} = \frac{1}{y}\sum_{k=1}^{n}\frac{y_k}{\mathrm{BF_c}-\mathrm{RO_c}}(\text{see Eq. A5}) = \frac{1}{y(\mathrm{BF_c}-\mathrm{RO_c})}\sum_{k=1}^{n}\mathrm{y}_k = \frac{1}{\mathrm{BF_c}-\mathrm{RO_c}}$$

(A7)

$$\frac{\partial \mathrm{BFI}}{\partial y_k} = \frac{\partial}{\partial y_k}\frac{b}{y} = \frac{\partial}{\partial y_k}\frac{\sum_{k=1}^{n}b_k}{\sum_{k=1}^{n}y_k} = \frac{(\sum_{k=1}^{n}b_k)'(\sum_{k=1}^{n}y_k)-(\sum_{k=1}^{n}b_k)(\sum_{k=1}^{n}y_k)'}{(\sum_{k=1}^{n}y_k)^2} = \frac{y(\sum_{k=1}^{n}b_k)'-b(\sum_{k=1}^{n}y_k)'}{\mathrm{y}^2} = \frac{y\sum_{k=1}^{n}\left(\frac{SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c}}{\mathrm{BF_c}-\mathrm{RO_c}}\right)-\mathrm{n}b}{\mathrm{y}^2}(\text{see Eq. A6}) = \frac{y\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-\mathrm{n}b(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}^2(\mathrm{BF_c}-\mathrm{RO_c})} = \frac{\sum_{k=1}^{n}(SC\cancel{Q}_{\cancel{c}k}-\mathrm{RO_c})-\mathrm{nBFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})} \qquad \text{(A8)}$$

Appendix B

Calculation of the sensitivity indices

$$\mathrm{S}(\mathrm{BFI}|\mathrm{BF_c}\,\cancel{\mathrm{BFI}/\mathrm{BF_c}}) = \frac{\Delta_{\mathrm{BF_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta\mathrm{BF_c}}{\mathrm{BF_c}} = \frac{y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q_{c}}k}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{BF_c}\frac{\mathrm{BF_c}}{\mathrm{BFI}\Delta\mathrm{BF_c}}(\text{see Eq. } 7) = \frac{\mathrm{BF_c}(y\mathrm{RO_c} - \sum_{k=1}^{n} y_k SC\cancel{Q_{c}}k)}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2}$$

(B1)

$$\mathrm{S}(\mathrm{BFI}|\mathrm{RO_c}\,\cancel{\mathrm{BFI}/\mathrm{RO_c}}) = \frac{\Delta_{\mathrm{RO_c}}\mathrm{BFI}}{\mathrm{BFI}} \Big/ \frac{\Delta\mathrm{RO_c}}{\mathrm{RO_c}} = \frac{\sum_{k=1}^{n} y_k SC\cancel{Q_{c}}k - y\mathrm{BF_c}}{y(\mathrm{BF_c}-\mathrm{RO_c})^2}\Delta\mathrm{RO_c}\frac{\mathrm{RO_c}}{\mathrm{BFI}\Delta\mathrm{RO_c}}(\text{see Eq. } 8) = \frac{\mathrm{RO_c}(\sum_{k=1}^{n} y_k SC\cancel{Q_{c}}k - y\mathrm{BF_c})}{y\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})^2}$$

(B2)

Appendix C

Prove that $\partial\mathrm{BFI}/\partial y_k$ is far less than 1 d/m^3.

$$\frac{\partial\mathrm{BFI}}{\partial y_k} = \frac{\sum_{k=1}^{n}(SC\cancel{Q_{c}}k - \mathrm{RO_c}) - n\mathrm{BFI}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})}(\text{see Eq. A8}) \qquad \text{(C1)}$$

Because of n>0, BFI>0, ($\mathrm{BF_C}$-$\mathrm{RO_C}$)>0, the above formula can be simplified:

$$\frac{\partial\mathrm{BFI}}{\partial y_k} < \frac{\sum_{k=1}^{n}(SC\cancel{Q_{c}}k - \mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})} \qquad \text{(C2)}$$

Since $\mathrm{BF_C}$ is usually much larger than $SC\cancel{Q_{c}}k$, the above formula can be rewritten as:

$$\frac{\partial\mathrm{BFI}}{\partial y_k} < \frac{\sum_{k=1}^{n}(\mathrm{BF_c} - \mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})} = \frac{\mathrm{n}(\mathrm{BF_c}-\mathrm{RO_c})}{\mathrm{y}(\mathrm{BF_c}-\mathrm{RO_c})} = \frac{\mathrm{n}}{\mathrm{y}} = \frac{1}{\bar{\mathrm{y}}} \qquad \text{(C3)}$$

The daily average streamflow ($\bar{y}$) is usually much larger than 1 m^3/d, so $\partial BFI/\partial y_k$ is far less than 1 d/m^3.

Formatted: Superscript

Formatted: Font: Italic

Formatted: Superscript

Data availability

All streamflow and conductivity data can be retrieved from the United States Geological Survey's (USGS) National Water Information System (NWIS) website use the special gage number, http://waterdata.usgs.gov/nwis.

Author contributions

Weifei Yang, Changlai Xiao and Xiujuan Liang designed the research train of thought. Weifei Yang and Changlai Xiao completed the parameters' sensitivity analysis. Xiujuan Liang completed the uncertainty estimate of BFI. Weifei Yang carried out most of the data analysis and prepared the manuscript with contributions from all co-authors.

Competing interests

The authors declare that they have no conflict of interest.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (41572216), the Provincial School Co-construction Project Special -- Leading Technology Guide (SXGJQY2017-6), the China Geological Survey Shenyang Geological Survey Center "Changji Economic Circle Geological Environment Survey" project (121201007000150012), and the Jilin Province Key Geological Foundation Project (2014-13). We thank the anonymous reviewers for useful comments to improve the manuscript.

References

Cartwright, I., Gilfedder, B., and Hofmann, H.: Contrasts between estimates of baseflow help discern multiple sources of water contributing to rivers, Hydrol. Earth Syst. Sci., 18, 15-30, doi:10.5194/hess-18-15-2014, 2014.

Costelloe, J. F., Peterson, T. J., Halbert, K., Western, A. W., and McDonnell, J. J.: Groundwater surface mapping informs sources of catchment baseflow, Hydrol. Earth Syst. Sci., 19, 1599-1613, doi:10.5194/hess-19-1599-2015, 2015.

Eckhardt, K.: How to construct recursive digital filters for baseflow separation, Hydrol. Process., 19, 507-515, doi:10.1002/hyp.5675, 2005.

Eckhardt, K.: Technical Note: Analytical sensitivity analysis of a two parameter recursive digital baseflow separation filter, Hydrol. Earth Syst. Sci., 16, 451-455, doi:10.5194/hess-16-451-2012, 2012.

Ernest, L.: Gaussian error propagation applied to ecological data: Post-ice-storm-downed woody biomass, Ecol. Monogr., 75, 451-466, doi.org/10.1890/05-0030, 2005.

Genereux, D.: Quantifying uncertainty in tracer-based hydrograph separations, Water Resour. Res, 34, 915-919, doi:10.1029/98wr00010, 1998.

Hamilton, A.S., and Moore R.D.: Quantifying Uncertainty in Streamflow Records, Can. Water Resour. J., 37, 3-21, doi:10.4296/cwrj3701865, 2012

Huang, Z. P., Chen, Y. F.: Hydrological statistics, China Water&Power Press, Beijing, China, 2011.

Kline, S. J.: The purposes of uncertainty analysis, J. Fluids Eng., 107, 153-160, 1985.

Lott, D. A., Stewart, M. T.: Base flow separation: A comparison of analytical and mass balance methods, J. Hydrol., 535, 525-533, doi:10.1016/j.jhydrol.2016.01.063, 2016.

Mei, Y., Anagnostou, E. N.: A hydrograph separation method based on information from rainfall and runoff records, J. Hydrol., 523, 636-649, doi:10.1016/j.jhydrol.2015.01.083, 2015.

Miller, M. P., Susong, D. D., Shope, C. L., Heilweil, V. M., and Stolp, B. J.: Continuous estimation of baseflow in snowmelt-dominated streams and rivers in the Upper Colorado River Basin: A chemical hydrograph separation approach, Water Resour. Res., 50, 6986–6999, doi:10.1002/2013WR014939, 2014.

Munyaneza, O., Wenninger, J., and Uhlenbrook, S.: Identification of runoff generation processes using hydrometric and tracer methods in a meso-scale catchment in Rwanda, Hydrol. Earth Syst. Sci., 16, 1991-2004, doi:10.5194/hess-16-1991-2012, 2012.

Okello, A. M. L. S., Uhlenbrook, S., Jewitt, G. P. W., Masih, L., Riddell, E. S., and Zaag, P.V.: Hydrograph separation using tracers and digital filters to quantify runoff components in a semi-arid mesoscale catchment, Hydrol. Process., 32, 1334-1350, doi:10.1002/hyp.11491, 2018.

Stewart, M., Cimino, J., and Rorr, M.: Calibration of base flow separation methods with streamflow conductivity, Ground Water, 45, 17-27, doi:10.1111/j.1745-6584.2006.00263.x, 2007.

Taylor, J. R.: An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, Univ. Sci. Books, Mill Valley, Calif., 1982.

Wagner, R. J., Boulger R. W. Jr., Oblinger, C. J., and Smith, B. A.: Guidelines and standard procedures for continuous water-quality monitors-Station operation, record computation, and data reporting, U.S. Geol. Surv. Tech. Meth., 1-D3, 51 pp, 2006.

~~Zhang, L. Z.: On the error of hydrologic survey, HoHai University, Nanjing, China, 2005.~~

Zhang, J., Zhang, Y., Song, J., and Cheng, L.: Evaluating relative merits of four baseflow separation methods in Eastern Australia, J. Hydrol., 549, 252-263, doi:10.1016/j.jhydrol.2017.04.004, 2017.

Tables

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote "a" in the "Area" column indicates that the values are estimated based on data from adjacent sites.

~~State~~	~~Gage Number~~	~~N~~	~~Area~~	~~BF$_C$~~	~~RO$_C$~~	~~Mean Baseflow~~	~~BFI~~	~~S(BFI/BF$_C$)~~	~~S(BFI/RO$_C$)~~	~~W_{BFI}~~	~~Mean W_{fbf}~~
~~-~~	~~-~~	~~days~~	~~km^2~~	~~μs/cm~~	~~μs/cm~~	~~m^3/s~~	~~-~~	~~-~~	~~-~~	~~-~~	~~-~~
~~FL~~	~~2298202~~	~~1808~~	~~966~~	~~1190.0~~	~~292.5~~	~~3.05~~	~~0.29~~	~~-1.31~~	~~-0.78~~	~~0.04~~	~~0.11~~
~~FL~~	~~2310545~~	~~1218~~	~~119[a]~~	~~7150.5~~	~~531.5~~	~~0.10~~	~~0.15~~	~~-1.09~~	~~-0.44~~	~~0.05~~	~~0.06~~
~~FL~~	~~2310650~~	~~779~~	~~77[a]~~	~~7195.0~~	~~3210.0~~	~~0.08~~	~~0.45~~	~~-1.79~~	~~-0.98~~	~~0.06~~	~~0.14~~
~~FL~~	~~2303000~~	~~728~~	~~570~~	~~462.0~~	~~120.5~~	~~3.28~~	~~0.30~~	~~-1.30~~	~~-0.82~~	~~0.08~~	~~0.17~~
~~FL~~	~~2298488~~	~~1303~~	~~76~~	~~810.0~~	~~194.0~~	~~0.20~~	~~0.33~~	~~-1.30~~	~~-0.63~~	~~0.05~~	~~0.09~~
~~FL~~	~~2298554~~	~~899~~	~~207[a]~~	~~1155.0~~	~~320.5~~	~~0.25~~	~~0.20~~	~~-1.36~~	~~-1.55~~	~~0.03~~	~~0.08~~
~~FL~~	~~2298492~~	~~1478~~	~~16~~	~~1425.0~~	~~304.0~~	~~0.04~~	~~0.21~~	~~-1.26~~	~~-1.01~~	~~0.03~~	~~0.07~~
~~FL~~	~~2298495~~	~~330~~	~~10~~	~~1905.0~~	~~662.0~~	~~0.05~~	~~0.24~~	~~-1.51~~	~~-1.66~~	~~0.03~~	~~0.08~~
~~FL~~	~~2298527~~	~~807~~	~~23~~	~~1640.0~~	~~201.5~~	~~0.04~~	~~0.14~~	~~-1.10~~	~~-0.83~~	~~0.06~~	~~0.16~~
~~FL~~	~~2298530~~	~~1510~~	~~17~~	~~1520.0~~	~~348.0~~	~~0.13~~	~~0.27~~	~~-1.27~~	~~-0.80~~	~~0.07~~	~~0.12~~
~~FL~~	~~2297100~~	~~2979~~	~~342~~	~~1460.0~~	~~221.5~~	~~1.54~~	~~0.21~~	~~-1.17~~	~~-0.69~~	~~0.04~~	~~0.09~~
~~FL~~	~~2313000~~	~~787~~	~~4727~~	~~449.0~~	~~173.0~~	~~8.62~~	~~0.43~~	~~-1.62~~	~~-0.84~~	~~0.06~~	~~0.13~~
~~FL~~	~~2300500~~	~~821~~	~~386~~	~~470.0~~	~~83.0~~	~~0.49~~	~~0.19~~	~~-1.19~~	~~-0.90~~	~~0.11~~	~~0.20~~
~~ND~~	~~5057000~~	~~1401~~	~~16757~~	~~1520.0~~	~~610.0~~	~~1.73~~	~~0.46~~	~~-1.64~~	~~-0.81~~	~~0.09~~	~~0.15~~
~~ND~~	~~5056000~~	~~1277~~	~~5361~~	~~1770.0~~	~~546.0~~	~~2.50~~	~~0.42~~	~~-1.41~~	~~-0.61~~	~~0.04~~	~~0.11~~
~~TX~~	~~8068275~~	~~2801~~	~~482~~	~~368.0~~	~~65.0~~	~~4.20~~	~~0.15~~	~~-1.18~~	~~-1.23~~	~~0.06~~	~~0.13~~
~~GA~~	~~2336300~~	~~1235~~	~~225~~	~~230.0~~	~~63.0~~	~~4.00~~	~~0.29~~	~~-1.36~~	~~-0.93~~	~~0.24~~	~~0.42~~
~~GA~~	~~2207120~~	~~1383~~	~~417~~	~~381.0~~	~~59.0~~	~~3.97~~	~~0.18~~	~~-1.17~~	~~-0.86~~	~~0.03~~	~~0.06~~
~~SC~~	~~2160105~~	~~1363~~	~~1966~~	~~150.0~~	~~51.0~~	~~40.27~~	~~0.25~~	~~-1.49~~	~~-1.56~~	~~0.03~~	~~0.10~~
~~SC~~	~~2160700~~	~~1392~~	~~1150~~	~~181.0~~	~~51.0~~	~~24.02~~	~~0.26~~	~~-1.37~~	~~-1.13~~	~~0.05~~	~~0.11~~
~~MO~~	~~6894000~~	~~1375~~	~~477~~	~~1110.0~~	~~334.0~~	~~0.86~~	~~0.21~~	~~-1.40~~	~~-1.59~~	~~0.09~~	~~0.13~~
~~MO~~	~~6895500~~	~~802~~	~~1258481~~	~~800.0~~	~~428.0~~	~~904.39~~	~~0.55~~	~~-2.14~~	~~-0.95~~	~~0.05~~	~~0.18~~
~~ND~~	~~5082500~~	~~1274~~	~~77959~~	~~1670.0~~	~~427.0~~	~~41.48~~	~~0.27~~	~~-1.33~~	~~-0.95~~	~~0.06~~	~~0.09~~
~~KS~~	~~7144780~~	~~575~~	~~1847~~	~~1550.0~~	~~678.0~~	~~0.52~~	~~0.44~~	~~-1.60~~	~~-1.08~~	~~0.09~~	~~0.17~~
~~Mean~~							~~0.29~~	~~-1.39~~	~~-0.98~~	~~0.06~~	~~0.13~~
~~Standard deviation (STDEV)~~							~~0.11~~	~~0.24~~	~~0.32~~	~~0.04~~	~~0.07~~

State	Gage Number	N	Area	Mean BF$_C$	RO$_C$	Mean Baseflow	BFI	S(BFI|BF$_C$)	S(BFI|RO$_C$)	W_{BFI}	Mean W$_{fbf}$
		days	km^2	μs/cm	μs/cm	m^3/s					
FL	2298202	1808	966	1149.1	292.5	2.12	0.31	-1.32	-0.76	0.05	0.12
FL	2310545	1218	119[a]	6404.7	531.5	0.65	0.17	-1.11	-0.44	0.05	0.06
FL	2310650	779	77[a]	6558.7	3210.0	0.90	0.57	-1.84	-0.79	0.18	0.27
FL	2303000	728	570	432.7	120.5	2.32	0.34	-1.30	-0.77	0.06	0.14
FL	2298488	1303	76	737.3	194.0	0.14	0.38	-1.32	-0.58	0.14	0.18
FL	2298554	899	207[a]	969.2	320.5	0.50	0.30	-1.25	-1.22	0.13	0.27
FL	2298492	1478	16	1238.2	304.0	0.05	0.30	-1.11	-0.82	0.13	0.31
FL	2298495	330	10	1870.0	662.0	0.29	0.25	-1.52	-1.65	0.03	0.08
FL	2298527	807	23	1410.7	201.5	0.10	0.19	-1.03	-0.74	0.06	0.18
FL	2298530	1510	17	1460.8	348.0	0.14	0.29	-1.27	-0.77	0.08	0.13
FL	2297100	2979	342	1260.6	221.5	0.92	0.25	-1.18	-0.64	0.08	0.20
FL	2313000	787	4727	407.2	173.0	5.89	0.51	-1.71	-0.71	0.19	0.28
FL	2300500	821	386	447.9	83.0	0.30	0.20	-1.21	-0.89	0.05	0.11
ND	5057000	1401	16757	1420.6	610.0	2.08	0.51	-1.75	-0.74	0.14	0.21
ND	5056000	1277	5361	1681.4	546.0	3.61	0.44	-1.50	-0.60	0.07	0.14
TX	8068275	2801	482	361.7	65.0	0.57	0.15	-1.18	-1.23	0.06	0.11
GA	2336300	1235	225	230.4	63.0	0.79	0.31	-1.28	-0.88	0.16	0.33
GA	2207120	1383	417	312.5	59.0	1.48	0.24	-1.14	-0.76	0.09	0.20
SC	2160105	1363	1966	124.7	51.0	6.36	0.36	-1.56	-1.30	0.14	0.27

Formatted Table

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted Table

Formatted: Font: (Default) Times New Roman, 9 pt, Italic

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

Formatted ...

SC	2160700	1392	1150	148.7	51.0	4.45	0.37	-1.40	-0.94	0.15	0.28
MO	6894000	1375	477	1031.9	334.0	0.79	0.25	-1.40	-1.50	0.13	0.22
MO	6895500	802	1258481	786.7	428.0	939.98	0.57	-2.17	-0.90	0.06	0.20
ND	5082500	1274	77959	1390.6	427.0	77.19	0.38	-1.30	-0.77	0.15	0.26
KS	7144780	575	1847	1389.1	678.0	1.73	0.54	-1.73	-0.91	0.14	0.26
			Mean				0.34	-1.40	-0.89	0.11	0.20
			Standard deviation (STDEV)				0.13	0.28	0.29	0.05	0.08

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Formatted Table

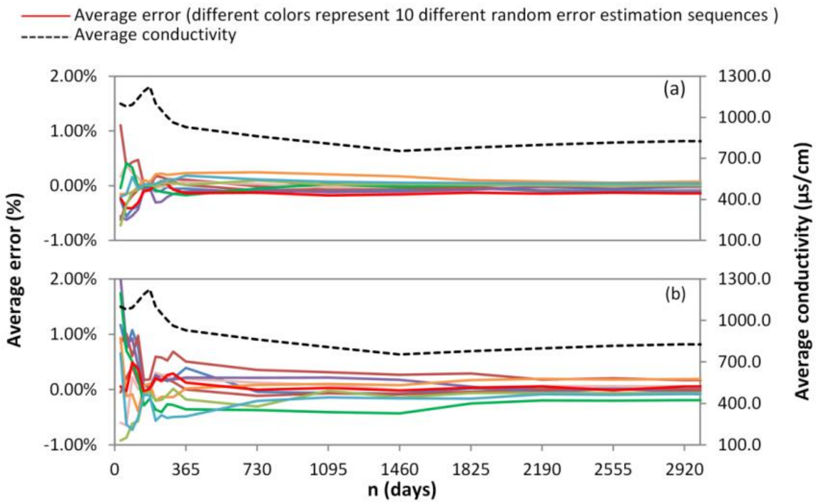
Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

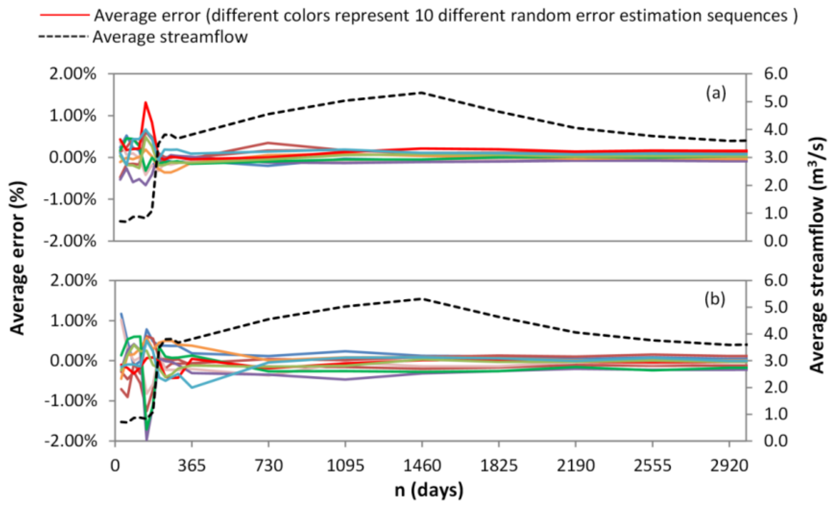
Formatted: Font: (Default) Times New Roman, 9 pt

Formatted: Line spacing: At least 6 pt

Figures



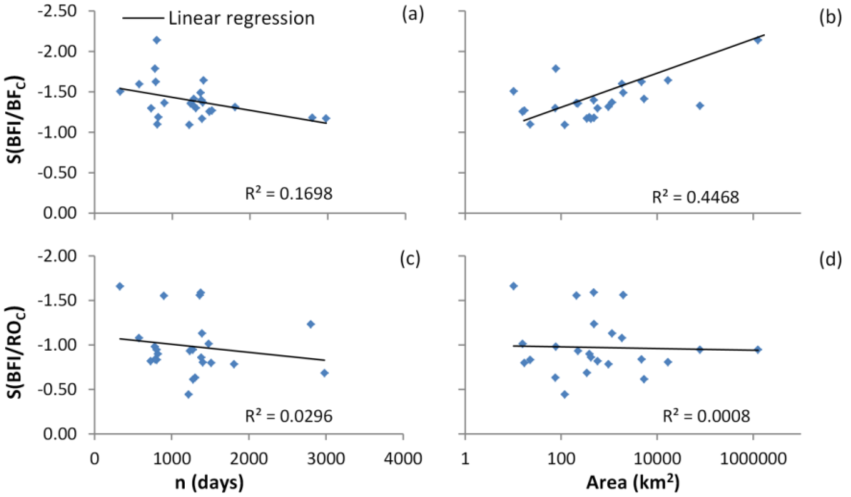
~~**Figure 1. Average conductivity error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.**~~



~~**Figure 2. Average streamflow error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.**~~

Formatted: Line spacing: single

Formatted: Left, Line spacing: single



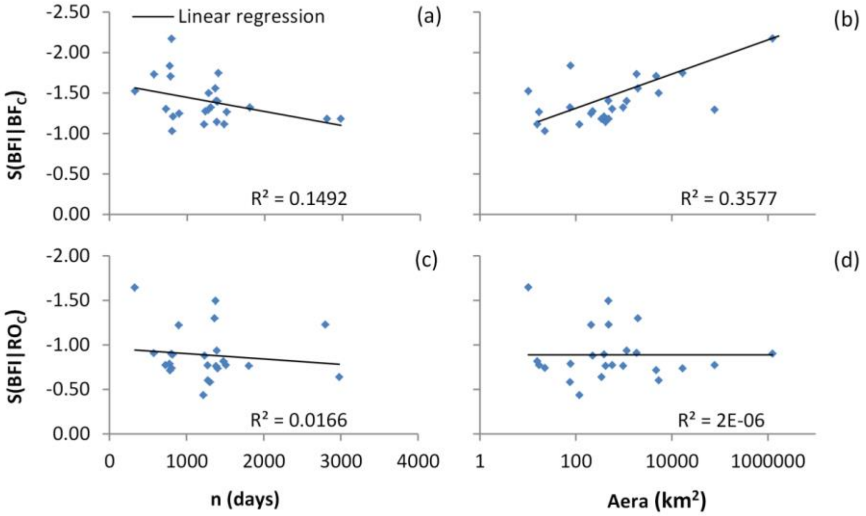


Figure 1~~3~~. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear~~normal~~ axes.

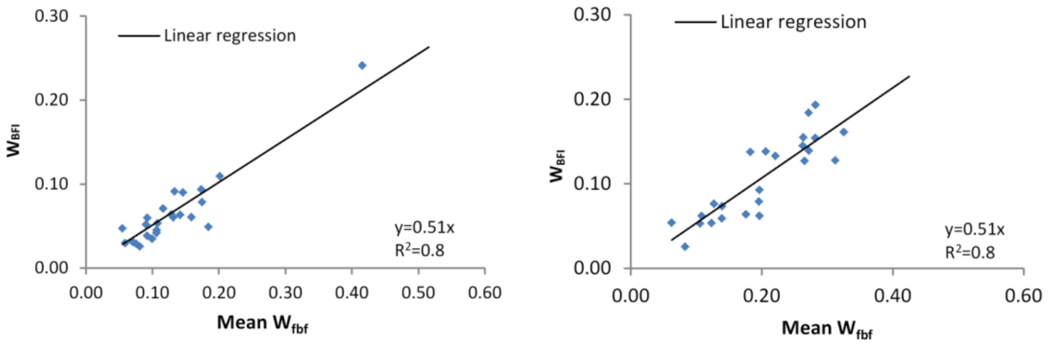


Figure 24. Scatter plot of uncertainty in BFI (W_{BFI}) and mean uncertainty in f_{bf} (Mean W_{fbf}).