Date: December 7, 2018

Review of: "A General Analytical Model for Head Response to Oscillatory Pumping in Unconfined Aquifers: Consider the Effects of Delayed Gravity Drainage and Initial Condition" by HUANG Ching-Sheng, TSAI Ya-Hsin, YEH Hund-Der, and YANG Tao (HESS-2018-482)

Review by: Todd C Rasmussen, *trasmuss@uga.edu*

General Comments

1. This manuscript examines the response of water-table aquifers to periodic (sinusoidal, oscillatory) hydraulic perturbations.

   As noted in our periodic aquifer test at the Savannah River Site (Rasmussen et al 2003), the estimated storativity of the water-table aquifer more closely represented confined (early-time) as opposed to unconfined (late-time) conditions, and we speculated that the effects of delayed yield might explain this behavior.

   This manuscript examines this effect by comparing instantaneous and delayed yield solutions against each other as well as the observed field behavior. As such, it provides valuable new insight in the physics of water-table responses to hydraulic perturbations.

   Specifically, Section 3.5 is an accurate and thoughtful analysis of our (Rasmussen et al, 2003) periodic aquifer test at the Savannah River Site. This section is a valuable contribution showing the usefulness of the proposed technique.

2. The manuscript is well-written in clear and concise English. The tables and figures are also appropriate, clear, and well notated. I provide a few suggested edits as noted in a subsequent section.

3. Agree with Reviewer 1 that detailed mathematical derivation can be placed in an appendix.

4. Your model might be better formulated using alternative parameters (e.g., Depner and Rasmussen, 2017, *Hydrodynamics of Time-Periodic Groundwater Flow: Diffusion Waves in Porous Media*):

   (a) Equation 1 can be written more parsimoniously using:

   $$D_r \left[\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \alpha \, \frac{\partial^2 h}{\partial z^2} \right] = \frac{\partial h}{\partial t}$$

   where $D_r = K_r/S_s$ and $\alpha = K_z/K_r$, which reduces the number of model parameters from three to two.

   (b) Equation 4. The vertical flux at $z = b$ is:

   $$q_z = -K_z \, \frac{\partial h}{\partial z} \tag{4a}$$

which can be defined for DGD conditions using (Boulton, 1954):

$$q_z = \frac{S_y}{\kappa} \int_o^t \frac{\partial h}{\partial \tau} \exp \frac{-(t-\tau)}{\kappa} \, d\tau \tag{4b}$$

where $\kappa = 1/\epsilon$, which has units of time rather than inverse time. Note that Eqn 4b reduces to IGD conditions as $\kappa \to \infty$:

$$q_z = S_y \frac{\partial h}{\partial t} \tag{4c}$$

Solving for the boundary gradient gives:

$$\frac{\partial h}{\partial z} = -\frac{1}{\kappa \, C_y} \int_o^t \frac{\partial h}{\partial \tau} \, e^{-\frac{r-\tau}{\kappa}} \, d\tau \tag{4d}$$

where $C_y = K_z/S_y$, with units of L/T.

(c) Note that D_r and α are domain parameters defined by Eqn 1, K_r is a boundary parameter defined by Eqn 3, and C_y and κ are boundary parameters defined by Eqn 4, where boundary parameters describe the aquifer characteristics on or near the boundary, and domain parameters describe the average characteristics within the interior of the aquifer. All other parameters (i.e., K_z, S_s, S_y) are hybrid domain-boundary parameters that are a composite of both boundary and domain characteristics.

(d) Dimensionless parameters in Eqn 7 can now be defined using:

$$\bar{t} = t\,(D_r/r_w^2) \quad \bar{P} = P\,(D_r/r_w^2) \quad \gamma = \omega\,(r_w^2/D_r) \quad \mu = \alpha\,(r_w^2/b^2) \quad a_1 = b/(\kappa\,C_y)$$

Suggested Edits

1. Title, suggest removing "Consider the"

2. Lines 22-24, suggest removing "without net water extraction" because a periodic test can be superimposed on a steady test.

   Also, "Oscillatory pumping tests (OPT) provide an alternative to constant-head and constant-rate tests for determining aquifer hydraulic parameters, with many analytical models available for parameter determination."

3. Lines 30-31, suggest revising to "The solution is derived using the Laplace, finite-integral, and Weber transforms."

4. Line 37, suggest explaining "certain time shift" here and subsequently.

5. Lines 56-58, suggest noting that periodic signals (depending on frequency) are likely to be observable at far greater distances than constant pumping because the signal-to-noise ratio for periodic testing is smaller due to the lack of noise at the testing frequency, unless there is interference from natural or artificial sources, such as solar and lunar periodicities.

6. Line 71, suggest explaining "certain period" here and subsequently.
7. Line 121, first reference to a partially penetrating pumping well; suggest highlighting in the abstract and introduction.
8. Line 165, suggest capitalizing "Section" here and subsequently (it's a proper noun).
9. Line 300, suggest capitalizing "Solution" here and subsequently (it's a proper noun).
10. Line 326, Figure 2 is the most interesting aspect of this manuscript; suggest explaining how period affects this plot. What happens when P is longer or shorter than ϵ (or $\kappa = 1/\epsilon$, with units of time)? I suspect that a $P \gg \kappa$ will provide an estimate of S_y (i.e., late-time), while $P \ll \kappa$ gives S_s (early time). Is it possible to have a dimensionless ratio of P/κ?
11. Line 445, suggest explaining "certain trough".
12. Table 2, suggest providing estimated domain (D_r, α), boundary (K_r, C_y, κ), and hybrid (K_z, S_s, S_y) parameters along with their individual standard errors. You might also provide the estimates from Rasmussen et al (2003).