Reply to Reviewer 2

Review of: "A General Analytical Model for Head Response to Oscillatory Pumping in Unconfined Aquifers: Consider the Effects of Delayed Gravity Drainage and Initial Condition" by HUANG Ching-Sheng, TSAI Ya-Hsin, YEH Hund-Der, and YANG Tao (HESS-2018-482)

Review by: Todd C Rasmussen, trasmuss@uga.edu

General Comments

1. This manuscript examines the response of water-table aquifers to periodic (sinusoidal, oscillatory) hydraulic perturbations. As noted in our periodic aquifer test at the Savannah River Site (Rasmussen et al 2003), the estimated storativity of the water-table aquifer more closely represented confined (early-time) as opposed to unconfined (late-time) conditions, and we speculated that the effects of delayed yield might explain this behavior.

   This manuscript examines this effect by comparing instantaneous and delayed yield solutions against each other as well as the observed field behavior. As such, it provides valuable new insight in the physics of water-table responses to hydraulic perturbations.

   Specifically, Section 3.5 is an accurate and thoughtful analysis of our (Rasmussen et al, 2003) periodic aquifer test at the Savannah River Site. This section is a valuable contribution showing the usefulness of the proposed technique.

2. The manuscript is well-written in clear and concise English. The tables and figures are also appropriate, clear, and well notated. I provide a few suggested edits as noted in a subsequent section.

Response: Many thanks.

3. Agree with Reviewer 1 that detailed mathematical derivation can be placed in an appendix.

Response: The derivation of the present solution has been moved to the supplementary material, and then the Methodology section is shortened. Please refer to the revised manuscript as attached.

4. Your model might be better formulated using alternative parameters (e.g., Depner and Rasmussen,

2017, Hydrodynamics of Time-Periodic Groundwater Flow: Diffusion Waves in Porous Media):

(a) Equation 1 can be written more parsimoniously using:

$$D_r\left[\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\alpha\frac{\partial^2 h}{\partial z^2}\right]=\frac{\partial h}{\partial t}$$

where $D_r = K_r/S_s$ and $\alpha = K_z/K_r$, which reduces the number of model parameters from three to two.

(b) Equation 4. The vertical flux at $z = b$ is:

$$q_z = -K_z\frac{\partial h}{\partial z} \tag{4a}$$

which can be defined for DGD conditions using (Boulton, 1954):

$$q_z = \frac{S_y}{\kappa}\int_0^t \frac{\partial h}{\partial \tau}\exp\frac{-(t-\tau)}{\kappa}\,d\tau \tag{4b}$$

where $\kappa = 1/\epsilon$, which has units of time rather than inverse time. Note that Eqn 4b reduces to IGD conditions as $\kappa \to \infty$:

$$q_z = S_y\frac{\partial h}{\partial t} \tag{4c}$$

Solving for the boundary gradient gives:

$$\frac{\partial h}{\partial z} = -\frac{1}{\kappa C_y}\int_0^t \frac{\partial h}{\partial \tau}e^{-\frac{r-\tau}{\kappa}}\,d\tau \tag{4d}$$

where $C_y = K_z/S_y$, with units of L/T.

(c) Note that D_r and α are domain parameters defined by Eqn 1, K_r is a boundary parameter defined by Eqn 3, and C_y and κ are boundary parameters defined by Eqn 4, where boundary parameters describe the aquifer characteristics on or near the boundary, and domain parameters describe the average characteristics within the interior of the aquifer. All other parameters (i.e., K_z, S_s, S_y) are hybrid domain-boundary parameters that are a composite of both boundary and domain characteristics.

(d) Dimensionless parameters in Eqn 7 can now be defined using:

$\bar{t} = t(D_r/r_w^2)$ $\quad$ $\bar{P} = P(D_r/r_w^2)$ $\quad$ $\gamma = \omega(r_w^2/D_r)$ $\quad$ $\mu = \alpha(r_w^2/b^2)$ $\quad$ $a_1 = b/(\kappa C_y)$

Response: Thank for the suggestions. The present model is reformulated as suggested. Please refer to the revised manuscript as attached.

Suggested Edits

1. Title, suggest removing "Consider the"

Response: Done as suggested.

2. Lines 22-24, suggest removing "without net water extraction" because a periodic test can be superimposed on a steady test.

   Also, "Oscillatory pumping tests (OPT) provide an alternative to constant-head and constant-rate tests for determining aquifer hydraulic parameters, with many analytical models available for parameter determination."

Response: Thanks for the comments. The sentence is rewritten as "Oscillatory pumping tests (OPTs) provide an alternative to constant-head and constant-rate pumping tests for determining aquifer hydraulic parameters when OPT data are analyzed based on an associated analytical model coupled with an optimization approach." (Page 2, lines 23 – 25 of the revised manuscript).

3. Lines 30-31, suggest revising to "The solution is derived using the Laplace, finite-integral, and Weber transforms."

Response: Done as suggested.

4. Line 37, suggest explaining "certain time shift" here and subsequently.

Response: The phrase "certain time shift" in the text is changed to "a time shift".

5. Lines 56-58, suggest noting that periodic signals (depending on frequency) are likely to be observable at far greater distances than constant pumping because the signal-to-noise ratio for periodic testing is smaller due to the lack of noise at the testing frequency, unless there is

interference from natural or artificial sources, such as solar and lunar periodicities.

Response: Thanks for the comment. The phrase “the problem of signal attenuation in remote distance from the pumping well” has been removed.

6. Line 71, suggest explaining “certain period” here and subsequently.

Response: The phrase “a certain period” is replaced by “a late period”.

7. Line 121, first reference to a partially penetrating pumping well; suggest highlighting in the abstract and introduction.

Response: The phrase “the rim of a finite-radius well” in the abstract is changed to “the rim of a partially screened well” (Page 2, line 31) and the phrase “the pumping well” in the Introduction section is replaced by “the partially screened well”. (Page 5, line 96).

8. Line 165, suggest capitalizing “Section” here and subsequently (it’s a proper noun).
9. Line 300, suggest capitalizing “Solution” here and subsequently (it’s a proper noun).

Response: Done as suggested.

10. Line 326, Figure 2 is the most interesting aspect of this manuscript; suggest explaining how period affects this plot. What happens when P is longer or shorter than ϵ (or $\kappa = 1/\epsilon$, with units of time)? I suspect that a $P \gg \kappa$ will provide an estimate of S_y (i.e., late-time), while $P \ll \kappa$ gives S_s (early time). Is it possible to have a dimensionless ratio of P/κ?

Response: Thanks for the comment. Figure 2 has been redrawn and also shown below. The associated section is rewritten as follows:

“**3.1. Delayed gravity drainage**

Previous analytical models for OPT consider either confined flow (e.g., Rasmussen et al., 2003) or unconfined flow with IGD effect (e.g., Dagan and Rabinovich, 2014). Little attention has been paid to the consideration of the DGD effect. This section addresses the diffrence among these three models. Figure 2 shows the curve of the dimensionless amplitude $\bar{A}_t$ at ($\bar{r}$, $\bar{z}$) = (1, 1) of Solution 1 versus the dimensionless parameter a_1 related to the DGD effect. The transient head fluctuations are plotted based on Solution 1 with $a_1 = 10^{-2}$, 1, 10, 500, Solution 2 for IGD and Solution 3 for confined flow. Define the relative error as

$$RE = |\bar{A}_t' - \bar{A}_t|/\bar{A}_t \quad (28)$$

where $\bar{A}_t'$ is the dimensionless amplitude predicted by Solution 2 for the case of $a_1 = 500$ or Solution 3 for the case of $a_1 = 10^{-2}$. The curves of the *RE* versus the period of oscillatory pumping rate (i.e., *P*) for these two cases are displayed. The range of $P \leq 10^5$s (1.16 d) contains most practical applications of OPT. When $10^{-2} \leq a_1 \leq 500$, the $\bar{A}_t$ gradually decreases with a_1 to the trough and then increases to the ultimate value of $\bar{A}_t = 1.79 \times 10^{-2}$. The DGD, in other words, causes an effect. When $a_1 < 10^{-2}$, Solutions 1 and 3 agree on the predicted heads; the *RE* is below 1% for $P < 10^4$s (2.78 h), indicating the unconfined aquifer with the DGD effect behaves like confined aquifer and the water table can be regarded as a no-flow boundary when $a_1 < 10^{-2}$ and $P < 10^4$s. When $a_1 > 500$, the head fluctuations predicted by both Solutions 1 and 2 are identical; the largest *RE* is about 0.45%, indicating the DGD effect is ignorable and Eq. (4b) reduces to (4a) for the IGD condition. This conclusion is applicable for any magnitude of *P* in spite of $P > 10^5$s."

(Pages 13 - 14, lines 280 - 300)

11. Line 445, suggest explaining "certain trough".

Response: The phrase "certain trough" is replaced by "trough".

12. Table 2, suggest providing estimated domain (D_r, α), boundary (K_r, C_y, κ), and hybrid (K_z, S_s, S_y) parameters along with their individual standard errors. You might also provide the estimates from Rasmussen et al (2003).

Response: Thanks for the suggestion. Table 2 is rewritten and given at the end of this reply. In order to compare the parameters reported in Rasmussen et al. (2003), we add two sentences, also given below, in the associated text.

"The estimates of T, S and D_r given in Rasmussen et al. (2003) are also presented." (Page 17, lines 381 - 382)

"In addition, the difference in T, S or D_r estimated by Solution 6 and those by the Rasmussen et al. (2003) solution may be attributed to the fact that their solution assumes isotropic hydraulic conductivity (i.e., $K_r = K_z$)." (Page 18, lines 393 - 396)

In addition, two sentences in the same paragraph are rewritten as:

"The result shows the estimated Sy is very small, and the estimated T and S by Solution 3, 6 or the Rasmussen et al. (2003) solution for confined flow are close to those by Solution 4 or 5 for unconfined flow, indicating that the unconfined flow induced by the OPT in the Surficial Aquifer is negligibly small." (Page 17, lines 382 - 385)

"On the other hand, transient Solution 3 gives smaller SEEs than PSS Solution 6 or the Rasmussen et al. (2003) solution for the Barnwell-McBean Aquifer and better fits to the observed data at the early pumping periods as shown in Fig. 7." (Page 18, lines 396 - 398)

References

Dagan, G. and Rabinovich, A.: Oscillatory pumping wells in phreatic, compressible, and homogeneous aquifers, Water Resour. Res., 50(8), 7058–7066, 2014.

Rasmussen, T. C., Haborak, K. G., and Young, M. H.: Estimating aquifer hydraulic properties using sinusoidal pumping at the Savannah River site, South Carolina, USA, Hydrogeol. J., 11(4), 466–482, 2003.

Figure

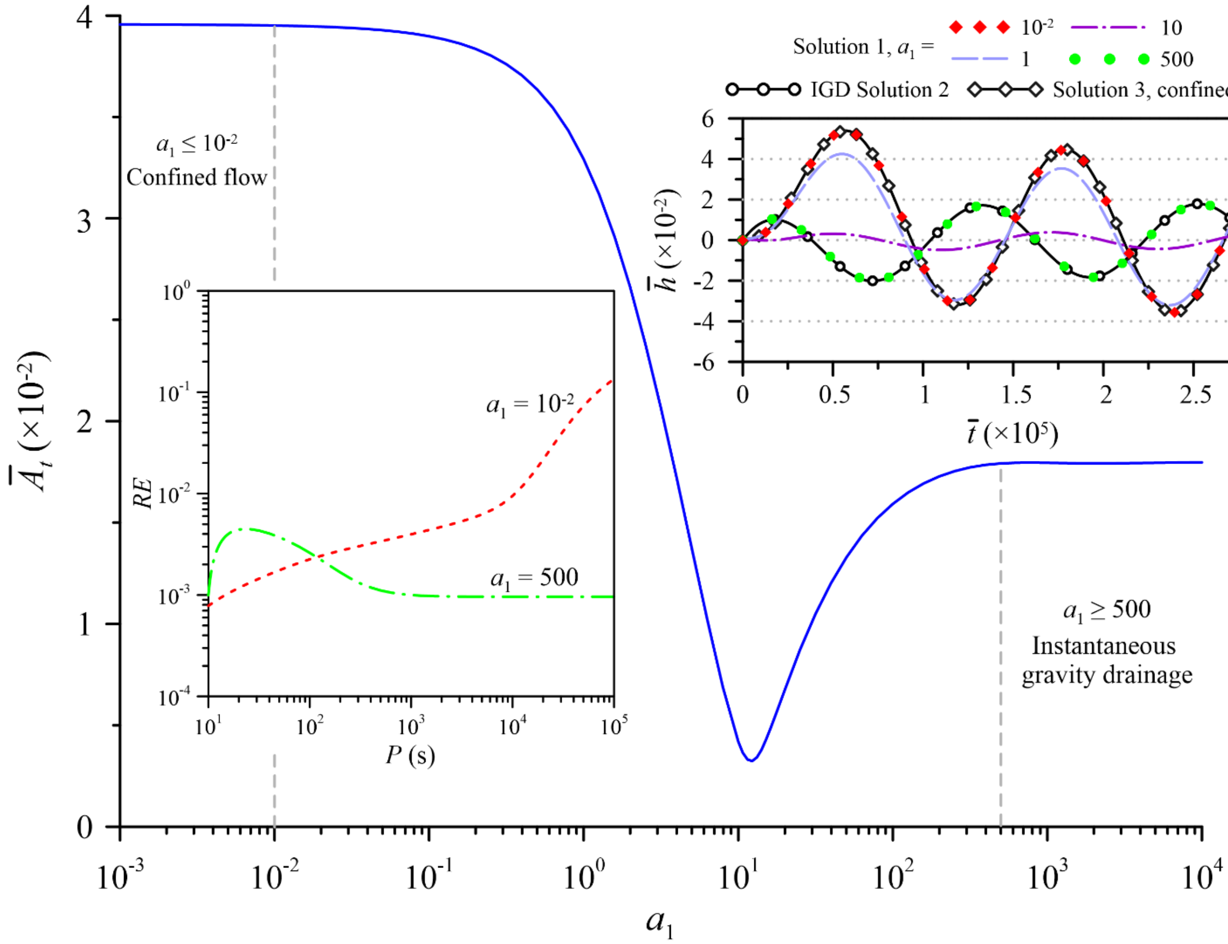

Figure 2. Influence of delayed gravity drainage on the dimensionless amplitude $\bar{A}_t$ and transient head $\bar{h}$ at $\bar{r} = 1$, $\bar{z} = 1$ predicted by Solution 1 for different magnitudes of a_1 related to the influence.

Table 2. Hydraulic parameters estimated by the present solution and the Rasmussen et al. (2003) solution for OPT data from the Savannah River site

Observation well	Solution	T (m²/s)	S	D_r (m²/s)	K_z (m/s)	S_y	C_y (m/s)	α	κ (s)	SEE	ME
					Surficial Aquifer						
101D	Solution 3[a]	9.27×10^{-4}	2.44×10^{-3}	0.380	-	-	-	-	-	0.018	-5.56×10^{-3}
	Solution 6[b]	9.18×10^{-4}	2.33×10^{-3}	0.393	-	-	-	-	-	0.018	-2.20×10^{-4}
	Solution 4[c]	4.61×10^{-4}	3.95×10^{-3}	0.117	7.38×10^{-6}	2.23×10^{-3}	3.31×10^{-3}	0.10	94.34	0.018	-2.20×10^{-4}
	Solution 5[c]	5.25×10^{-4}	1.09×10^{-3}	0.482	2.61×10^{-5}	5.49×10^{-3}	4.75×10^{-3}	0.31	-	0.019	-2.30×10^{-4}
	Rasmussen et al. (2003)[b]	2.17×10^{-3}	1.35×10^{-4}	16.074	-	-	-	-	-	0.018	-2.20×10^{-4}
102D	Solution 3[a]	9.13×10^{-4}	1.76×10^{-3}	0.519	-	-	-	-	-	0.010	-4.38×10^{-3}
	Solution 6[b]	9.17×10^{-4}	1.67×10^{-3}	0.549	-	-	-	-	-	0.011	9.57×10^{-4}
	Solution 4[c]	9.57×10^{-5}	7.85×10^{-4}	0.122	3.68×10^{-6}	4.95×10^{-3}	7.43×10^{-4}	0.24	420.17	0.011	9.57×10^{-4}
	Solution 5[c]	9.49×10^{-5}	3.25×10^{-4}	0.292	4.67×10^{-6}	4.68×10^{-3}	9.98×10^{-4}	0.31	-	0.011	9.50×10^{-4}
	Rasmussen et al. (2003)[b]	2.27×10^{-3}	2.28×10^{-4}	9.956	-	-	-	-	-	0.011	9.57×10^{-4}
					Barnwell-McBean Aquifer						
201C	Solution 3[a]	5.86×10^{-5}	7.07×10^{-4}	0.083	-	-	-	-	-	0.232	0.046
	Solution 6[b]	6.03×10^{-5}	6.54×10^{-4}	0.092	-	-	-	-	-	0.363	0.281
	Rasmussen et al. (2003)[b]	6.90×10^{-5}	4.74×10^{-4}	0.150	-	-	-	-	-	0.363	0.281

[a] transient confined flow

[b] PSS confined flow

[c] PSS unconfined flow