

Thank you, Dr. Jonas, for your valuable comments on our work. The responses to the comments are inserted below in blue color.

This technical note reports on using an existing snow density model to derive SWE from a temporally continuous record of snow depth. Given the effort required to operate and maintain a snowpillow, being able to estimate SWE from alternative snow depth measurements has its potential uses and merits, e.g. for gap filling purposes, or in case other meteorological data being unavailable to run a full snowpack model.

Obviously the authors were not the first to come up with using a snow density model in that particular context. The performance is similar (or in fact slightly worse) in comparison to two parametric models that were developed about 10 years ago with the same application in mind. However, unlike those two alternative offerings the approach presented here is capable of providing meaningful time series of SWE at high temporal resolution (at the cost of requiring complete time series of snow depth and temperature input data). This would be a good selling point of the paper if it wasn't for other publications that have already tackled this very aspect, see e.g. the excellent paper by McCreight and Small from 2014 (doi.org/10.5194/tc-8-521-2014).

We included the McCreight and Small (2014) in the review section. The existing snow density models referred in the comments (Kelly et al., 2003; Jonas et al., 2009; Sturm et al., 2010; Bormann et al., 2014; McCreight and Small, 2014) are all data-driven approach. Meanwhile, the model proposed in this article clearly belongs to a process-based approach as described in the supplement. I paste the derivation of the model for your convenience. This model is accumulation of experimental works by snow scientists, which have been largely ignored or not utilizes (to my view, N.Ohara).

Supplement of “Technical note: Snow Water Equivalence Estimation (SWEE) Algorithm from Snow Depth Time Series Using a Snow Density Model”

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Derivation of snow compaction term

Yoshida (1955) formulated the snow density change due to gravity and snow metamorphism effect.

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{w_s}{\eta} \quad (\text{S-1})$$

w_s =weight of snow above the layer in terms of SWE (cm)

η =viscosity coefficient, a constant for a given density and temperature (cm·hr)

$\rho(z, t)$ = snow density at depth z at time t (g/cm^3)

The viscosity coefficient includes two different effects.

$$\eta = \eta_c \eta_t \quad (\text{S-2})$$

η_c =viscosity coefficient for gravitational compaction
 η_t =viscosity coefficient for temperature change

Kojima (1967) obtained following expression from experiments:

$$\eta_c = \eta_{co} \exp[k_o \rho] \quad (\text{S-3})$$

Mellor (1975) formulated viscosity coefficient for temperature change as follows.

$$\eta_t = \eta_{to} \exp \left[\frac{A}{R} \frac{(T_c - T)}{T T_c} \right] \quad (\text{S-4})$$

Anderson (1976) simplified it as,

$$\eta_t = \eta_{to} \exp[0.08(T_c - T)] \quad (\text{S-5})$$

Combining (S-1) through (S-5) yields,

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{w_s}{\eta_o} \exp[-0.08(T_c - T)] \exp[-k_o \rho] \quad (\text{S-6})$$

where

$$\eta_o = \eta_{co} \eta_{to} \quad (\text{S-7})$$

Assuming the depth averaged snow density ρ_s corresponds at the 2/3 of the snow depth from the surface (von der Heydt, 1992),

$$w_s = \frac{2}{3} D \quad (\text{S-8})$$

The predictive ordinary differential equation form of the depth averaged snow density can be obtained by the depth integration (Horne and Kavvas, 1997)

$$\frac{d\rho_s}{dt} = \frac{2D\rho_s}{3\eta_o} \exp[-0.08(T_c - \bar{T})] \exp[-k_o \rho_s] \quad (\text{S-9})$$

ρ_s = depth averaged snow density

\bar{T} = depth averaged snow density

Assuming a triangular temperature profile in a snowpack, which indicates snow surface temperature $T_s = 2\bar{T}$, the compaction term in Equation (1) can be obtained.

Derivation of new snow term

When new snow sits on the snow surface, snow density must be modified as below:

$$\rho_s + \Delta\rho_s = \frac{D\rho_s + \Delta D\rho_{sn}}{\Delta D + D} \quad (\text{S-10})$$

ρ_s = depth averaged snow density (g/cm³)

$\Delta\rho_s$ = snow density change (g/cm³)

D = snow depth (cm)

ΔD = snow depth change (cm)

ρ_{sn} = new snow density (g/cm³)

$$\rho_s + \Delta\rho_s = \frac{D\rho_s + \Delta D\rho_s - \Delta D\rho_s + \Delta D\rho_{sn}}{\Delta D + D}$$

$$\rho_s + \Delta\rho_s = \frac{D\rho_s + \Delta D\rho_s}{\Delta D + D} + \frac{-\Delta D\rho_s + \Delta D\rho_{sn}}{\Delta D + D}$$

$$\rho_s + \Delta\rho_s = \rho_s + \Delta D \frac{(\rho_{sn} - \rho_s)}{\Delta D + D}$$

$$\Delta\rho_s = \Delta D \frac{(\rho_{sn} - \rho_s)}{\Delta D + D} \quad (\text{S-11})$$

Now, snow depth change can be expressed in terms of snowfall rate sn (cm/hr) as follows.

$$\Delta D = \Delta t \cdot sn \cdot \frac{\rho_w}{\rho_{sn}} \quad (\text{S-12})$$

Substituting Equation (S-12) to Equation (S-11) yields

$$\Delta\rho_s = \Delta t \cdot sn \cdot \frac{\rho_w}{\rho_{sn}} \frac{(\rho_{sn} - \rho_s)}{(\Delta D + D)} \quad (\text{S-13})$$

$$\frac{\Delta\rho_s}{\Delta t} = \frac{\rho_w}{\rho_{sn}} \frac{(\rho_{sn} - \rho_s)}{(\Delta D + D)} sn \quad (\text{S-14})$$

When Δt is sufficiently small, and $\Delta D \ll D$, one can obtain,

$$\frac{d\rho_s}{dt} = -\frac{(\rho_s - \rho_{sn})\rho_w}{\rho_{sn}D} sn \quad (\text{S-15})$$

Now the question arises, what then is the selling point of this paper? As a technical note, it might suffice to present the method as an alternative approach – if the authors manage to identify at least some differences to existing models. Better transferability to other sites without recalibration maybe? And obviously, testing the model with data from one season and one SNOTEL station only is not nearly enough.

Please refer to the previous response. This is essentially a different type of model from the “existing” models.

Line 1: the title is somewhat misleading given that your model also requires temperature data as input

We revise the title:

Technical note: Snow Water Equivalence Estimation (SWEE) Algorithm from Snow Depth and Temperature Time Series Using a Snow Density Model

Line 48-51: extend this into a convincing last paragraph of your introduction, which highlights shortcomings of existing models a/o the merits of your approach (to be demonstrated below)

Line 52: start method section here

Line 101: and the former?

We re-organized the sections accordingly. Thank you. That is a really good suggestion.

Line 117: while this assumption is what you often have to work with, it would be interesting to also deploy your model at a site where you actually do have snow temperature data to test, how big of a problem is this assumption?

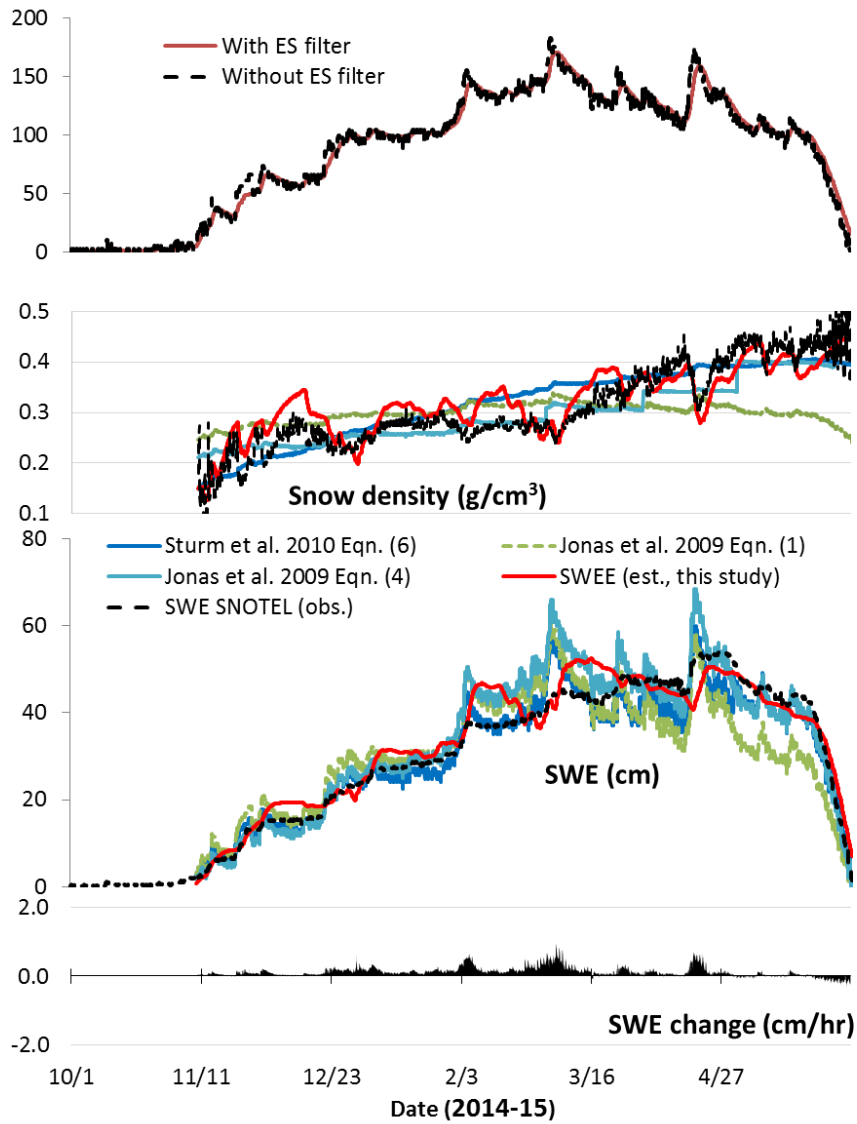
Unfortunately, we do not have concurrent snow and air temperature data. Also, this is beyond the scope of this small article.

Line 127: since you require temperature data as model input anyway, why not using a new snow density formulation that is temperature dependent?

It is a good idea. We adjusted the snow density as a linear function of the temperature data.

$$\rho_{ns} = \rho_{nso} + aT_{obs}$$

When $\rho_{nso}=0.167$ (intercept) and $a=0.0034$ (slope), we obtained better comparison as shown below. This analysis indicates that the depth averaged snow density model can describe the snow density evolution if incoming snow density is correctly estimated. In this case, the SWEE algorithm performed better than the other data-driven models, as shown below. This implies that the main source of our estimates is the density of new snow. However, we do not intend to include it in the main article in order to keep this technical note article simple.



Snow density model	R^2	NSME	Note
SWEE (this study)	0.927	0.918	Dynamic snow density model
Jonas et al. 2009, Eqn.(1)	0.716	0.707	Regression with a power function
Jonas et al. 2009, Eqn.(4)	0.931	0.927	Regression with a linear function with monthly p
Sturm et al. 2010, Eqn.(6)	0.906	0.852	Regression with an exponential with day-of-year

Line 156: "... which is independent from climate ..."? You may have a point there, but let's first see what happens if you apply your model in Japan, Siberia, Lesotho; without recalibration of course. You can refer to Matthew Sturm's snow classification scheme to back up your claim.

The process-based approach presented here is inherently such an attribute unlike the data-driven approach. However, the purpose of this article is not proving it. We added some additional model applications in Washington, California, Vermont, and Alaska in the response letter for the RC1. It shows that the SWEE

algorithm works okay and often great for these sites as well. We unfortunately could not secure sufficient time/data for the regions you suggested.