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Discussions

Quantification of Drainable Water Storage Volumes in Catchments *and* in River Networks on Global Scales using the GRACE and / or River Runoff

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Abstract.

The knowledge of storage volumes in catchments and in river networks is essential for the management of water resources and for a comprehensive description of the environment water in the context of climate change. Measurements of water storage variations by the GRACE gravity satellite or by ground based observations of river or groundwater level variation do not allow to determine the respective total storage volumes, which could be considerably larger than the mass variations themselves. In addition mass variations measured by GRACE comprise all storage compartments whether they are hydraulically coupled, contributing to river runoff, or uncoupled like soil moisture, isolated surface water or snow and ice.

The possibility to determine the hydraulic time scale from observed runoff and GRACE for the first time allows to quantify the total "Drainable Storage" i.e. the volume freely draining with gravity comprising all coupled storages in the catchment and in the river network on global scales. As investigations of the runoff-storage (R-S) relationship using GRACE have shown, the R-S relationship can be characterized as a Linear Time Invariant (LTI) System for hydraulically coupled storage compartments (Riegger and Tourian, 2014). Thus, the respective hydraulic time constant of the total system and the corresponding Drainable Storage can be determined once the observed phase shift is adapted either directly or by a model. However, even though the observed phase shift is already considered in modelling approaches its physical reason and the information it carries is not understood in detail so far.

A possible reason for the observed phase shift might be found in the river network storage, which so far has not been addressed separately in the R-S relationships. Opposite to storages draining in parallel (as for overland and groundwater flow) a sequence of storages leads to a temporal delay or a phase shift. This might explain the different phasing of the catchment, the river network storage and the total water storage. In order to investigate such a phasing effect a system of cascaded storages for the catchment and river network is set up with different hydraulic time constants and is mathematically solved by piecewise analytical solutions. Tests of the scheme with synthetic recharge time series show that the parameter estimation either versus deviations in total mass or runoff reproduces the time constants for both, the catchment τ_C and the river network τ_R in a unique way and allows to quantify the respective storage volumes individually.

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The application of the Cascaded Storage approach to the Amazon catchment leads to very good agreements of calculated and measured total mass *and* river runoff (Nash-Sutcliffe for signals > 0.96, for residuals >0.72). The signal amplitudes and the phase shift between GRACE and river runoff are reproduced very well. The calculated river network mass highly (0.96) correlates with the observed flood area from the "Global Inundation Extent from Multi-Satellites" data set (GIEMS) and corresponds to the determined flood volumes. The implementation of a river network storage in sequence to catchment storages thus describes and explains the observations w.r.t. phasing and signal amplitudes and allows a discrimination of the storage volumes in the catchment and the river network.

As the parameter optimization either versus river runoff or GRACE mass deviations leads to comparable results, river runoff and storage volumes can be determined from recharge and GRACE even for ungauged catchments. It is mainly the quality of the recharge data used that limits the quality of the results. Thus further developments in hydrometeorological data products are expected to improve the quantification of river runoff and drainable water storage volume from space.

1 Introduction

In the context of water resources management and climate change there is an ongoing discussion on how to assess available water resources i.e. the storage volumes which can be used for water supply in a dynamic way beyond the limitations of sustainable extraction rates. The maximum average extraction rate for a sustainable use of water resources is limited by the long term recharge of a catchment (Sophocleous, 1997, Bredehoeft, 1997), however, this rate based definition of groundwater stress only allows an assessment of water resources w.r.t. the long term sustainability and does not allow to consider the volume of available water resources as a basis for short term groundwater management in order to satisfy specific demands. In addition, the knowledge of the storage volumes is essential for climate studies as it might lead to limitations in the water cycle.

Thus, the attempt was made to estimate available water resources by the volume of the respective groundwater storage (under the assumption that the contribution of surface water is comparably small). Korzun, 1978 and Nace, 1969 provide estimates of total storage volumes across the global land masses (except for Greenland and Antarctica) based on very coarse assumptions for aquifers with homogeneous thickness and porosity. As a consequence the uncertainties cover orders of magnitude as Shiklomanov, 1993 Alley, 2006 and Famiglietti, 2014 are warning. The revision of these estimates by the introduction of specific yield instead of porosity for dominant soil classes together with the assumption of different saturated thicknesses in order to receive the "Extractable" storage (Richey et al., 2015a, Alley, 2006) does not solve the problem of missing information on the contributing soils or aquifers in general. Regional storage estimates for specific aquifers derived from groundwater models (Cao et al., 2013) or measured estimates of saturated thickness and porosity (Williamson et al.,1989) are considered to deliver more realistic estimates for storage volumes, yet are sparsely distributed over the globe. However, for semi / arid climate zones with very low recharge and/ or deep aquifer systems with fossil water resources they deliver the best possible estimation at present. Richey et al., 2015b try to bypass the huge uncertainties in the quantification

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of total storage by the introduction of a "Total Groundwater Stress" indicator, which is defined by the time needed for the depletion of groundwater storage to a certain extent. It is determined by the ratio of an estimated total storage of a (sufficiently large) catchment and the measured trend in the mass anomalies given by GRACE.

Very little attention is given so far to the storage volume of renewable water resources participating in the dynamic water cycle driven by precipitation P, actual evapotranspiration ET_a and river runoff R. The reason for this is seen in the problem that observations of time variant groundwater or river levels only allow to estimate volume changes yet no absolute storage volumes, which could be considerably bigger. Direct measurements of storage volumes lack of a very limited access to information on the contributing storage compartments. Natural systems consist of many different storage components like canopy, snow/ice, surface, soil, unsaturated/saturated underground, drainage system etc. Ground based measurements of storages are based on point measurements and quite rare on large spatial scales compared to the heterogeneity scale of the respective compartments. This leads to large interpolation errors. In addition, the storage coefficients for porous media describing the relationship between the measurable groundwater heads or capillary pressure on the one hand, and storage volume or absolute soil saturation on the other hand, are insufficiently known on large scales. Remote sensing data are limited to near surface storage (open water bodies, soil) up until now and thus are of limited benefit for the quantification of water storage with respect to accuracy and coverage due to methodological constraints (Schlesinger, 2007).
Opposite to dischargeless basins and/or arid areas nearly exclusively driven by precipitation and evapotranspiration, the storage dynamics of catchments draining into a river system allows to address the hydraulically coupled storage compartments via their contributions to runoff. These comprise groundwater, surface water, the river network and temporarily inundated areas. All storages draining into the river system by gravity are referred to as "Drainable" storage here. So aquifers or parts of them not draining into the river system without an energy input are not considered here.

Hydraulically coupled storage components lead to an exponential decrease in discharge for time periods with no recharge depending on the related hydraulic time constant τ:

$$Q(t) = Q(t_0) \cdot e^{-\frac{t-t_0}{\tau}} \tag{1}$$

The corresponding total Drainable Storage in terms of mass density for any given time t_0 is then given by an infinite integration over discharge Q from a catchment area A starting at time t_0 :

$$M_{storage}(t_0) = \frac{V_{tot}(t_0)}{A} = \frac{1}{A} \cdot \int_{t_0}^{\infty} Q(t)dt = \frac{Q(t_0)}{A} \cdot \int_{t_0}^{\infty} e^{-\frac{t-t_0}{\tau}} dt = \tau \cdot \frac{Q(t_0)}{A} = \tau \cdot R(t_0) \tag{2}$$

Contributions of several storage compartments (with individual time constants) superpose, if they drain in parallel and if there is no feedback from the river system. For this case, there is a wide range of time series analysis methods (Tallaksen, 1995), which allow to separate the flow components into fast, medium or slow and the corresponding surface, interflow or groundwater flow contributions according to their individual time constants. Thus, measurements of the different time constants allow to determine the Drainable Storage of the respective storage compartment and the corresponding mean Drainable Storage :

$$\overline{M}^{X} = \overline{R} \cdot \tau_{X} = \overline{N} \cdot \tau_{X} \ . \tag{3}$$

from mean runoff R or recharge N.

On global scales the absolute storage volume of the Drainable Storages can be determined from runoff time series directly, if there are distinct periods of negligible recharge like in seasonally dry regions (Niger, Tocantins, .etc.) long enough for a sufficient fit. For some catchments the sequence of different time constants taken from the discharge curve even allows a discrimination of the fast response by surface runoff and the slower response via the groundwater system. Catchments with no periods of negligible recharge, however, do not show an exponential behaviour for discharge. For these cases the hydraulic time constant cannot be taken from runoff dynamics directly, but has to be estimated either from the runoff – storage (R-S) relationship or by hydrological models. Numerical models based on climatic data in principle allow to simulate the time dependent flow components and the resulting non-exponential form of runoff via an adaption to measured runoff, or - in principle - to observations of storage.

GRACE observations of the time-variable gravity field provide monthly mass distributions for large scales >~ 200000km^2 (Tapley et al., 2004). However, as the water storage in different compartments (snow, ice, vegetation, soil, surface-, ground-water etc.) superposes with all other terrestrial (geophysical) masses, only the time variant part of the GRACE signal can be used for the quantification of the Terrestrial Water storage (TWS) not in the form of absolute storage volumes but instead by monthly deviations from the long term mean i.e. mass anomalies. This for the first time allows a direct comparison of measured TWS and river runoff R_o and an investigation of the runoff – storage relationships (R-S) on large spatial and monthly time scales which show a hysteresis of characteristic form and extent for different climatic zones (Riegger and Tourian, 2014). For Amazon the hysteresis in the R-S relationship (Fig.1) can be fully explained and described by a phase shift or time lag. For this case river runoff and storage behave like a Linear Time Invariant (LTI) system i.e. the R-S relationship is linear, if the phase shift is adapted as shown in Fig.1.

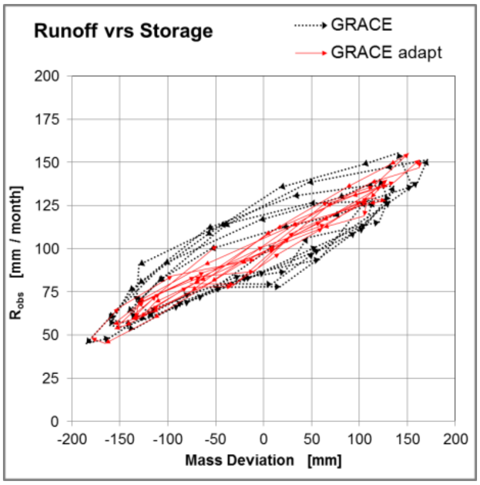

Fig.1: R-S diagram for Amazon: Observed runoff R_{obs} versus storage mass deviation from GRACE and phase adapted GRACE

This means for Amazon that all observed mass changes lead to runoff and thus are interpreted as "coupled", whereas mass changes in "uncoupled" storage compartments would not lead to runoff and thus contribute to a hysteresis which cannot be explained by a phase shift. Riegger and Tourian, 2014, also proved that this interpretation of coupled / uncoupled storages can also be applied to boreal catchments, which are temporarily dominated by snow leading to a huge hysteresis due to a superposition of masses from fully coupled (liquid) and uncoupled (solid) storage compartments. For this case remote sensing of snow coverage by MODIS allows to separate the coupled liquid storage (proportional to river runoff) and the uncoupled frozen part. The quantification of the coupled liquid part leads to a hysteresis, which corresponds to a phase shift only. Once this phase shift between runoff and storage is adapted, the slope corresponds to the hydraulic time constant via τ^{-1}. The reasonable assumption of a proportional R-S relationship (no runoff for empty storage) allows to quantify the Drainable Storage, Eq. (3), i.e. the volume related to the hydraulically coupled storage compartments, which drains with gravity.

Even though global hydrological models comprise a number of storages like soil, surface water, groundwater etc. some of them show considerable phase shifts between the calculated and measured runoff and an underestimation of the signal amplitudes (Güntner et al., 2007, Chen et al., 2007, Schmidt et al., 2008, Werth et al., 2009, Werth et al., 2010). A consideration of the phase shift between measured runoff and GRACE, however, either in the R-S relationship or in numerical models (Riegger and Tourian, 2014) can lead to a description of the system behaviour with high accuracy (Nash Sutcliffe 0.97 for Amazon), even though the reason for its occurrence is not understood in detail so far.

Opposite to storages, which are drained in parallel (as for overland and groundwater flow) leading to a superposition of contributions, a sequence of storages (cascaded storages) leads to a temporal delay i.e. a phase shift (Nash, 1957). As numerical models indicate that a simultaneous recharge over the whole catchment does not lead to a time lag in the related

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overland and groundwater flows, the observed time lag or phase shift might come from a sequence of the catchment storages and a non negligible river network storage. Observations of inundated areas in river networks provided by the GIEMS project (Prigent et al, 2007, Paiva et al., 2013) indicate a considerable contribution of river network storage for the Amazon catchment.

This paper explores the impacts of a Nash cascade comprising a catchment and a river network storage on the phasing of runoff and storage and their signal amplitudes. Even though a simple description of the system by a "Single Storage" - which could comprise several storage compartments draining in parallel, yet without a river network storage in sequence – also allows to quantify the Drainable Storage volume for the total system via the total storage time constant, the "Cascaded Storage" approach might lead to an improvement in accuracy and to a separation of the contributions from catchment and river network storages.

The paper is structured as follows: in section 2 the mathematical framework of piecewise analytical solutions of the water balance equation for a cascade of catchment and river network storages is presented. It also contains the description of observables, which allow to compare calculated and measured values. The Single Storage approach is handled as the specific case for a negligible river network time constant. In section 3, the properties of the Cascaded Storage approach and its impact on the performance of the parameter optimization are described for synthetic recharge data and compared to the "Single Storage" approach.. In section 4 the approach is applied to data from the Amazon basin and evaluated versus measurements of GRACE mass, river runoff and flood area from GIEMS. Conclusions are drawn and discussed in section 5 and an outlook on future investigations and possibilities is given in section 6.

2 Mathematical framework

In order to investigate the impact of a possible non negligible river water storage corresponding to a non negligible hydraulic time constant for the river system, the water balance of the total system comprising both the catchment and river network storage has to be considered. A conceptual model corresponding to a Nash cascade (Nash, 1957), called "Cascaded Storage" approach here, is set up with individual time constants for the different storages and the following properties:

- For simplicity the surface and the groundwater systems both fed by recharge are summarized in a first approach to one catchment storage M_C with time constant τ_C draining into the river network. (This is not necessarily appropriate for catchments in other than fully humid climate zones like seasonally dry or boreal regions)
- The river network storage M_R with time constant τ_R is assumed to be instantaneously distributed within the river network system. Internal routing effects, which might lead to an additional delay in runoff response, are not considered here.
- A possible hydraulic feedback from the river to the catchment system is assumed to be negligible.

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The following abbreviations are used in the mathematical description (Table.1):

Abbreviation	**Description**	**Units:** general / for application
N	recharge = (precipitation - actual evapotranspiration)	volume area^{-1} time^{-1} [mm month^{-1}]
M_C	Storage mass catchment	mass density in equivalent water height [mm]
τ_C	Time constant catchment	time unit [month]
R_C	Runoff catchment	volume area^{-1} time^{-1} [mm month^{-1}]
ω_{MC}	Phasing catchment mass	time unit [month]
M_R	Storage mass river network	mass density in equivalent water height [mm]
τ_R	Time constant river network	time unit [month]
R_R	Runoff river network	volume area^{-1} time^{-1} [mm month^{-1}]
ω_{RR}	Phasing river network mass	time unit [month]
M_T	Storage mass total system	mass density in equivalent water height [mm]
τ_T	Time constant total system	time unit [month]
ω_{MT}	Phasing mass total system	time unit [month]
Ro	Observed river runoff	volume area^{-1} time^{-1} [mm month^{-1}]
GRACE	GRACE mass deviation	mass density in equivalent water height [mm]
GIEMS	Flood area	area [km^2]
Prefix “d”	indicates signal deviations from long term mean	
Suffix “m”	indicates mean values on the intervals	

Table.1: Abbreviations in the mathematical descriptions:

The total system behaviour is described by two balance equations :

1. catchment storage $$\frac{\partial}{\partial t} M^C(t) = N(t) - R^C(t) = N(t) - \frac{1}{\tau} \cdot M^C(t) \quad \text{with } R^C(t) = \frac{1}{\tau_C} \cdot M^C(t) \qquad (4)\,(5)$$

2. river storage $$\frac{\partial}{\partial t} M^R(t) = R^C(t) - R^R(t) = R^C(t) - \frac{1}{\tau} \cdot M^R(t) \quad \text{with } R^R(t) = \frac{1}{\tau_R} \cdot M^R(t) \qquad (6)\,(7)$$

with a proportional R-S relationship for hydraulically coupled storages. N denotes the recharge as input, R_C the catchment runoff from the catchment storage M_C, which cannot be measured directly on large spatial scales, and R_R the river runoff from the river network storage M_R which can be measured at discharge gauging stations.

The water balance equation, Eq.(4), for the catchment is generally solved by:

$$M^C(t-t_0) = M^C(t_0)\cdot e^{-\frac{t-t_0}{\tau_C}} + \int_{t_0}^{t} N(w)\cdot e^{\frac{w-t}{\tau_C}}\cdot dw \tag{8}$$

where $M_C(t_0)$ is the initial condition and N(w) the time dependent recharge.

For recharge N(t) being given with a certain temporal resolution in time units or by periods of piecewise constant values and arbitrary length (stress periods) the recharge time series can be described as :

$$N(t) = \sum_{i=0}^{n-1} N_{i+1}\cdot\gamma_{i+1}(t) \quad \text{with} \quad \gamma_{i+1}(t) = \begin{cases}1 \\ 0\end{cases} \; for \; \begin{cases} t\in[t_i,t_{i+1}] \\ t\notin[t_i,t_{i+1}]\end{cases} \quad \text{for each interval } [t_i,t_{i+1}] \tag{9}$$

For calculation convenience Eq. (8) can be solved successively for each stress period using the values at the end of the last period as starting value, which leads to the piecewise analytical solution for catchment mass for a time $t \in [t_i,t_{i+1}]$ in stress period $_{i+1}$:

$$M_{i+1}^C(t-t_i) = M_i^C(t_i)\cdot e^{-\frac{t-t_i}{\tau_C}} + N_{i+1}\cdot\tau_C\cdot\left(1-e^{-\frac{t-t_i}{\tau_C}}\right) \quad : \tag{10}$$

The respective catchment runoff R_C from Eq. (5) and Eq. (10) is used as input for the river network water balance, Eq.(6), and leads to the general solution for the river network storage M_R :

$$M^R(t-t_i) = M^R(t_i)\cdot e^{-\frac{t-t_i}{\tau_R}} + \int_{t_i}^{t} R^C(u)\cdot e^{\frac{u-t}{\tau_R}}\cdot du \tag{11}$$

and the iterative solutions for time $t \in [t_i,t_{i+1}]$ in stress period $_{i+1}$:

$$M_{i+1}^R(t-t_i) = M_i^R(t_i)\cdot e^{-\frac{t-t_i}{\tau_R}} + N_{i+1}\cdot\tau_R\cdot\left(1-e^{-\frac{t-t_i}{\tau_R}}\right) + \left[M_i^C(t_i) - N_{i+1}\cdot\tau_C\right]\cdot\frac{\tau_R}{\tau_C-\tau_R}\cdot\left(e^{-\frac{t-t_i}{\tau_C}} - e^{-\frac{t-t_i}{\tau_R}}\right) \tag{12}$$

The total mass M_T is then given by : $M_i^T = M_i^C + M_i^R$. (13)

The mixed term in Eq. (12) and thus the total mass are commutative in (τ_C, τ_R) and show a singularity at $\tau_C = \tau_R$ with an asymptotic value. For $\tau_R > \tau_C$ solutions also exist with analogous values in total mass M_T for $M_R > M_C$.

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It has to be emphasized here, that the piecewise analytical solutions for time periods of constant recharge provide a mathematical solution for an arbitrary temporal resolution. Finite Difference solutions are limited by stability criteria $(t_{i+1}-t_i)<\tau$ and accuracy criteria $(t_{i+1}-t_i)<\tau/10$ for the smallest τ. Analytical solutions allow to calculate the response of the river network on the time interval of constant recharge (though the time constant of the river network is much shorter than the time constant of the catchment), and thus avoid the very high temporal discretization, which otherwise would be needed for a Finite Difference scheme.

The observables based on measurements by GRACE and discharge from gauging stations are the total mass deviation and the river runoff. GRACE observations with acceptable error are still limited to monthly values. Discharge as well as some of the meteorological inputs like precipitation, evapotranspiration or moisture flux divergence are often measured in daily values, some of the products in monthly values. For an optimal adaption to the monthly resolution of GRACE products, the approach presented here is based on monthly values but could also be applied to daily data without problems.

The mass values used in the calculations here are assigned to the interval boundaries while the values for monthly recharge and measured runoff are constant over the interval and temporally assigned to the centre of the interval. Thus, for a comparison of the calculated mass and runoff values versus the observed monthly values of GRACE and discharge the calculated values have to be averaged over the interval. As the dynamics follow an exponential behaviour the mean values cannot be taken from arithmetic averages at the interval boundaries but instead from an integral average over the interval.

The mean storage mass for M_X is given for each interval $\lfloor t_i, t_{i+1} \rfloor$ by :

$$\overline{M}_{i+1}^{X} = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} M_{i+1}^{X}(t-t_i)\cdot dt \qquad \text{leading to mean runoff} \qquad \overline{R}^{X}(t) = \frac{1}{\tau_X}\cdot\overline{M}^{X}(t) \tag{14}(15)$$

i.e. mean catchment mass and runoff :

$$\overline{M}_{i+1}^{C} = \left(M_i^C - N_{i+1}\cdot\tau_C\right)\cdot\frac{\tau_C}{(t_{i+1}-t_i)}\left(1-e^{-\frac{t_{i+1}-t_i}{\tau_C}}\right) + N_{i+1}\cdot\tau_C \qquad \text{and} \qquad \overline{R}_i^{C} = \frac{1}{\tau_C}\cdot\overline{M}_i^{C} \tag{16}(17)$$

and mean river mass and runoff :

$$\begin{aligned}\overline{M}_{i+1}^{R} &= \left(M_i^R - N_{i+1}\cdot\tau_R\right)\cdot\frac{\tau_R}{(t_{i+1}-t_i)}\cdot\left(1-e^{-\frac{t_{i+1}-t_i}{\tau_R}}\right) + N_{i+1}\cdot\tau_R \\ &+ \frac{\left[M_i^C - N_{i+1}\cdot\tau_C\right]}{(t_{i+1}-t_i)}\cdot\frac{\tau_R}{\tau_C-\tau_R}\cdot\left(\tau_R\cdot e^{-\frac{t_{i+1}-t_i}{\tau_R}} - \tau_C\cdot e^{-\frac{t_{i+1}-t_i}{\tau_C}} + (\tau_C-\tau_R)\right)\end{aligned} \tag{18}$$

The Observables, which allow a comparison to measured data are :

- average river runoff $\overline{R}_i^R = \frac{1}{\tau_R} \cdot \overline{M}_i^R$ corresponding to measured monthly runoff (19)
- average total mass $\overline{M}_i^T = \overline{M}_i^C + \overline{M}_i^R$ corresponding to monthly GRACE data (20)

The equations Eq. (10) - Eq. (20) are self-consistent, i.e. the corresponding balance equations are fulfilled with :

$$\frac{M_{i+1}^T (t-t_i) - M_i^T}{(t-t_i)} + \overline{R}_{i+1}^R (t-t_i) = N_{i+1} \tag{21}$$

For the Single Storage approach the above piecewise analytical solutions of the Cascaded Storage approach, Eq. (8) - Eq. (21), are used for $\tau_R << \tau_C$ (here $\tau_R = 10^{-3}$). For this case the river network mass is negligible compared to the catchment mass.

3 Properties and optimization performance

For the evaluation of the parameter optimization performance of the Cascaded Storage approach an example with synthetic recharge as input is investigated. This allows to quantify the uniqueness and accuracy of the parameter estimation undisturbed by noise. It also allows to distinguish errors in the calculation scheme itself and impacts arising from undescribed processes when compared to real world data. For an application to GRACE measurements the main question is if and why the time constants τ_C and τ_R can be determined independently by an optimization versus deviations in total mass and/or river runoff. Thus, in order to understand the optimization results with respect to uniqueness the general properties of the approach are presented and discussed first. For the synthetic case a recharge time series of sinusoidal form with a period of 12 arbitrary time units and a unit amplitude and mean value is used as the driving force and the calculation is run until equilibrium is reached. The example in Fig.2 shows the effect of a non negligible river network time constant τ_R =2.5 time units for a catchment time constant τ_C = 3 time units which leads to an increase in total mass $M_T(t) = M_C(t)+M_R(t)$ w.r.t. the average level and signal amplitude and to a phase shift between total mass M_T and river mass M_R i.e. the corresponding river runoff R_R.

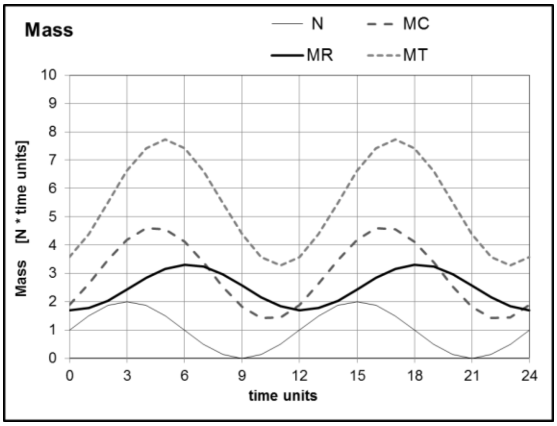


Fig.2: Time series of Recharge N, catchment mass M_C, river network mass M_R, and Total masses M_T at equilibrium

In order to describe the general behaviour of the mass and runoff time series and their dependence on τ_C and τ_R, their properties are summarized here in the form of statistical values for the synthetic case with the sinusoidal recharge in equilibrium. This helps to understand why unique values for the time constants are achieved in the parameter optimization process. The values of time constants τ_C and τ_R used for the statistical description cover a wide range from 0.1 to 100 time units and are combined independently.

3.1 Catchment and river mass

Based on the mean mass values, Eq.(14), (16), (18), of each stress period the long term averages for the storage compartments are given by :

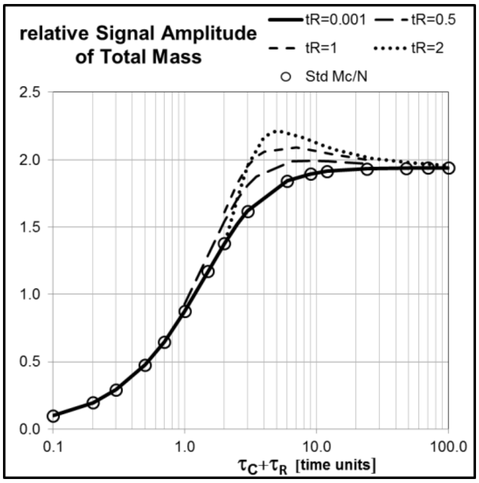
$$\overline{M}^C = \overline{N} \cdot \tau_C \qquad \overline{M}^R = \overline{N} \cdot \tau_R \qquad \overline{M}^T = \overline{N} \cdot (\tau_C + \tau_R)$$

(22a,b,c)

For $\tau_R << \tau_C$ (here $\tau_R = 10^{-3}$) the river network mass is negligible and the solution corresponds to a Single Storage approach. For a non negligible river network storage the given average values for total mass M_T mean that the effective "total" time constant is given by the sum of the catchment and river time constants $\tau_T = \tau_C + \tau_R$, which means that the total mass M_T observed by GRACE is bigger than the mass M_C calculated for the catchments alone. However, Equation (22c) cannot be used for the determination of $\tau_T = \tau_C + \tau_R$ from GRACE measurements directly as GRACE only provides mass anomalies.

The relative signal amplitudes (normalized with the respective input) of both the catchment mass M_C or river mass M_R show the same functional form $\sigma_{MC} / \sigma_N \sim \sigma_{MR} / \sigma_{RC} = stdev(M_C) / N$ for the respective time constants τ_C or τ_R (Fig.3, $\tau_R = 10^{-3}$) with a monotonous increase to an asymptotic value $\sigma_{MC} / \sigma_N \sim \sigma_{MR} / \sigma_{RC} = 2$ which is reached at about one full period of the input.

The superposition of the signal amplitudes for the observable total mass $M_T(t) = M_C(t) + M_R(t)$ leads to a complex behaviour for $\sigma_{MT} / \sigma_N(\tau_C, \tau_R)$ (Fig.3), if the river time constant τ_R is not negligible ($\tau_R = 10^{-3}$) and especially if it gets close to τ_C.



5 **Fig.3: Relative Signal Amplitudes of Total mass σ_{MT} / σ_N versus $\tau_T = \tau_C + \tau_R$ for different τ_R**

3.2 Catchment and river runoff

The calculated long term averages of the runoff contributions R_C and R_R correspond to the ones of the water balance equations, Eq.(4), (6), given by the mean recharge and thus are not dependent on the time constants.

$$\bar{R}^R(t) = \bar{R}^C(t) = \bar{N} \tag{23}$$

Thus, observed long term average of runoff does not allow to determine the time constant and thus the storage volume, Eq. (22).

The relative signal amplitudes of both, catchment and river runoff (normalized with the respective input σ_{RC} / σ_N and $\sigma_{RR} / \sigma_{RC}$ show the same functional form corresponding to a Single Storage approach (Fig.4, $\tau_R = 10^{-3}$) and decrease monotonously with the respective time constants τ_C and τ_R to an asymptotic zero. However, the signal amplitude of the observable river runoff $\sigma_{RR} / \sigma_N(\tau_C + \tau_R)$, normalized with recharge N, shows a 2D dependence for combinations in τ_C and τ_R (Fig.4).

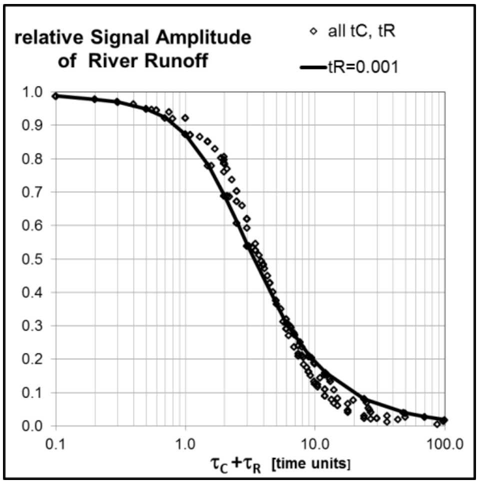


Fig.4: Relative Signal Amplitudes for river runoff σ_{RR} / σ_N versus $\tau_T = \tau_C + \tau_R$ for combinations in (τ_C, τ_R)

Both observables, total mass and river runoff, show a non unique behaviour with respect to combinations in (τ_C, τ_R) for the same $\tau_T = \tau_C + \tau_R$ and considerable deviations from the Single Storage approach ($\tau_R = 10^{-3}$). Measurements of the signal amplitudes thus only provide coarse estimates of the total time constant τ_T, yet do not allow to distinguish τ_R versus τ_C and to separate catchment and river network storage.

However, so far, only the signal amplitudes are examined, yet not the specific properties of the time series, i.e. the dynamic response to input signals in form and phase. The convolution in the solution of the balance equation, Eq.(8) and (11), leads to a different phasing w.r.t. the input N(t), which can be utilized for a separation of the respective time constants.

3.3 Phasing

For the synthetic example with a sinusoidal recharge time series N(t) as input the phasing ω of the different response signals is determined by the fit of a sinusoidal function (Fig.5). This allows to easily determine the phasing and thus the relative phase shift $\Delta\omega$ between the signals. Masses and the related runoffs are in phase for the same storage compartments, Eq.(15). For a negligible river network time constant ($\tau_R = 10^{-3}$) river runoff R_R is in phase with the catchment storage M_C.

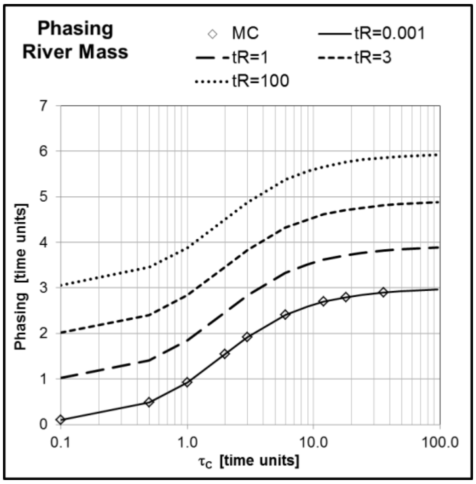


Fig.5: Phasing of river network mass w.r.t. recharge time series displayed versus τ_C for different τ_R

The functional form of the phasing ω_{MC} for the catchment mass M_C or the corresponding runoff R_C relative to recharge N(t) (Fig.5) can be empirically described by the monotonous function :

$$\omega_{MC}(\tau_C) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) \tag{24}$$

with the empirical parameters $\omega_{max} = 3$ and $\lambda = 3{,}2$ and an error $\varepsilon < 1\%$ relative to the maximum.

As the catchment runoff R_C with the phasing ω_{MC} serves as input into the river system, the phasing of the river system w.r.t. to catchment runoff R_C, which has the same functional form as Eq. (24), is added on top of it (Fig.5). The resulting phasing of the river network storage or river runoff is thus given by a superposition in the form:

$$\omega_{RR}(\tau_C, \tau_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1 - e^{-\frac{\tau_R}{\lambda}}\right) \tag{25}$$

for any combination (τ_C, τ_R) and with the same empirical parameters as in Eq. (24).

As total mass $M_T(t) = M_C(t) + M_R(t)$ is the superposition of the signals with the respective amplitudes and phasings, the phasing of total mass $M_T(t)$ is situated between catchment and the river system mass according to τ_R. This means that for non negligible river network mass ($\tau_R > 0$) a phase shift between total mass (GRACE) and observed runoff and thus between total mass and modelled catchment mass must occur. The phasing of total mass $M_T(t)$ for all combinations (τ_C, τ_R) Fig.6 shows the same functional form as ω_{MC} and ω_{MR}, Eq.(24), (25) if displayed versus the total time constant $\tau_T = \tau_C + \tau_R$.

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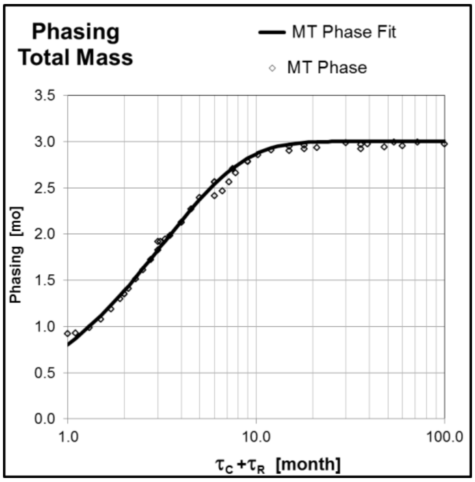

Fig.6: Phasing of Total mass versus total time constant $\tau_T = \tau_C + \tau_R$

It can be approximated by the fitting function M_T fit :

$$\omega_{MT}(\tau_C,\tau_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C+\tau_R}{\lambda}}\right) \tag{26}$$

The phase shift between GRACE total mass and river runoff is thus given by :

$$\Delta\omega(\tau_C,\tau_R) = \omega_{RR} - \omega_{MT} = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1 - e^{-\frac{\tau_R}{\lambda}}\right) - \omega_{\max}\left(1 - e^{-\frac{\tau_C+\tau_R}{\lambda}}\right) \tag{27}$$

Use of the total time constant $\tau_T = \tau_C + \tau_R$ and phase shift $\Delta\omega$ taken from a simple temporal adaption of measured GRACE and runoff in principle allows to determined τ_C and τ_R separately according to Eq. (27). However, errors introduced by the linear interpolation used for the adaption of the phase shift lead to a much lower accuracy than the parameter estimation via the time series.

3.4 Parameter estimation

The analytical solutions for synthetic recharge time series allow to evaluate the uniqueness and accuracy of the parameter optimization for given observables independent from limitations in the accuracy of numerical schemes and independent from noise in real world data sets. For given combinations (τ_C, τ_R) the analytical solutions are used as synthetic measurements and are fitted with the same algorithm in order to retrieve the fit parameters ($\underline{\tau}_C$, $\underline{\tau}_R$).

As the total mass M_T, Eq.(20), and the phasing, Eq.(25-27), are commutative in (τ_C, τ_R), either the data range $\tau_R < \tau_C$ or $\tau_R > \tau_C$ has to be used for a unique optimization. This is realized via an additional constraint in the optimization. For the discussion here the condition $\tau_R < \tau_C$ is used, which hydrologically reflects the more frequent situations that the inundation volume is smaller than the catchment storage but the results can also be applied to $\tau_R > \tau_C$, which might be the case in flat areas with a dense river network (such as the Amazon), which typically leads to temporarily inundated areas.

As absolute signal values are not relevant for the determination of the time constant from runoff or not available for GRACE data, the optimization versus the respective time series is based on signal amplitudes and the phasing. Thus, for a unique determination of ($\underline{\tau}_C$, $\underline{\tau}_R$) the following conditions have to be fulfilled:

a) Optimization versus runoff

$$\sigma_{RR} / \sigma_N (\hat{\tau}_C, \hat{\tau}_R) = \sigma_{RR} / \sigma_N (\tau_C + \tau_R) \tag{28}$$

$$\omega_{RR}(\hat{\tau}_C, \hat{\tau}_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1 - e^{-\frac{\tau_R}{\lambda}}\right) \tag{29}$$

b) Optimization versus mass anomalies

$$\sigma_{MT} / \sigma_N (\hat{\tau}_C, \hat{\tau}_R) = \sigma_{MT} / \sigma_N (\tau_C, \tau_R) \tag{30}$$

$$\hat{\omega}_{MT}(\hat{\tau}_C, \hat{\tau}_R) = \omega_{MT}(\tau_C + \tau_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C + \tau_R}{\lambda}}\right) \tag{31}$$

With the constraints $\tau_R < \tau_C$ or $\tau_R > \tau_C$ there is only one ($\underline{\tau}_C$, $\underline{\tau}_R$) fulfilling the respective conditions, thus leading to unique solutions. The optimization delivers RMSE errors for the time series in the range 10^{-8} - 10^{-7} and estimated time constants ($\underline{\tau}_C$, $\underline{\tau}_R$) with a relative error $\varepsilon(\underline{\tau}_X)/\tau_X$ which does not depend on absolute values of (τ_C, τ_R) but on their ratio τ_R / τ_C (Fig.7).

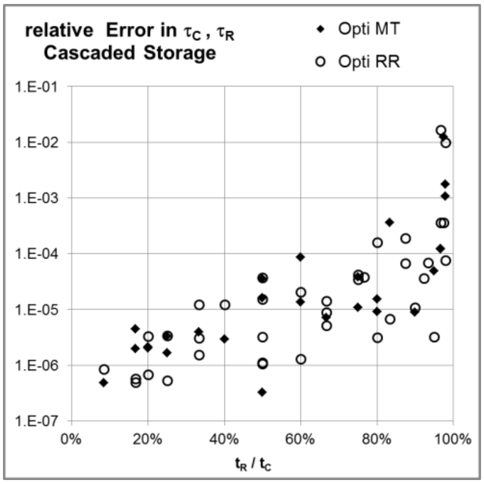

Fig.7: Relative error of the catchment and river time constants τ_C and τ_R for the Cascaded Storage approach and for optimizations versus Total mass M_T or river runoff R_R.

For the synthetic case relative errors $\varepsilon(\underline{\tau}_X)/\tau_X$ are very small ($\sim10^{-7}$ at $\tau_R/\tau_C\sim 0$) and show an exponential increase to a maximum of ~ 1% at $\tau_R\sim\tau_C$. The error for $\tau_R<\tau_C$ is analogous to $\tau_R>\tau_C$ and equal for an optimization versus runoff or mass deviation.

For catchments showing a phase shift between total mass and runoff the description of the system by a Single Storage approach ($\tau_R = 10^{-3}$) leads to a considerably higher relative error $\varepsilon(\underline{\tau}_X)/\tau_X$ in the estimated time constant $\underline{\tau}_C\sim(\tau_C+\tau_R)$ and thus also in Drainable Storage volume. It follows a power function and corresponds to $\varepsilon<10\%$ for $\underline{\tau}_C<3$ and $\varepsilon>40\%$ for $\underline{\tau}_C>6$. For this case the optimization versus river runoff or mass deviation leads to different total time constants (rel. Diff. $\varepsilon>7\%$ for $\underline{\tau}_C>5$). Even though this might look like an acceptable result for $\underline{\tau}_C<3$, there are still inevitable deviations in signal amplitudes (10-20%) and phasing between the modelled and measured signals for both total mass and river runoff time series.

It can be summarized that opposite to the Single Storage approach the Cascaded Storage approach allows to determine both time constants ($\underline{\tau}_C$, $\underline{\tau}_R$) independently in a unique, highly accurate way for optimizations with respect to either deviations in total mass or river runoff if recharge is given. However, it has to be mentioned that even though the theoretical error in time constants remains below 1% for $\tau_R\sim\tau_C$, the ambiguity for $\tau_R<\tau_C$ or $\tau_R>\tau_C$ cannot be solved without further information on the volume of the river network.

4 Application to the Amazon catchment

The R-S diagram of the Amazon catchment shows a hysteresis (Fig.1) corresponding to a phase shift, which is interpreted as the time lag of river runoff. The Amazon catchment upstream Obidos is situated in a fully humid tropic environment with permanent, yet variable recharge and is large enough (4704394km^2) for low noise levels in the signals of GRACE and moisture flux divergence. With permanent input contributions of overland flow and groundwater flow cannot be distinguished in the discharge curve. Also uncoupled storages (like soil water storage, open water bodies etc.) are not time dependent and thus do not appear as a hysteresis in the R-S diagram. So there is no need for a separation of hydraulically coupled and uncoupled storage components for this catchment. In general, any contributions from time dependent, non drainable i.e. uncoupled storage compartments or from processes like freezing, melting, evapotranspiration etc. can be recognized in the R-S diagram or by the respective deviations between the calculated and measured runoffs and storage volumes and have to be removed from total storage by means of by remote sensing or a conceptual description (Riegger and Tourian, 2014).

Generally recharge from different approaches and products can serve as input to the system such as :

1. $$N(t) = P(t) - ET(t) \tag{32}$$

   from the hydrometeorological products precipitation P and actual evapotranspiration ETa

2. $$N(t) = -\nabla \cdot \vec{Q} \tag{33}$$

   from atmospheric data, with the vertically integrated moisture flux divergence viMFD

3. $$N(t) = \frac{\partial}{\partial t} M(t) + R(t) \tag{34}$$

   from the terrestrial water balance with total mass M_T from GRACE measurements and runoff R_o from the measured river runoff of the catchment.

Here recharge [mm/month] is taken either from the water balance, Eq.(34), or from moisture flux divergence, Eq.(33), provided by ERA-INTERIM of ECMWF and processed by the Institute of Meteorology and Climate Research, Garmisch, Germany. For GRACE mass deviations data from GeoForschungsZentrum GFZ Potsdam Release 5 are used in mm equivalent water height. Both are handled as described in Riegger and Tourian, 2014. River discharge is taken from the ORE HYBAM project (http://ore-hybam.org) and converted to runoff [mm/month] by normalization with catchment area. For a comparison of the calculated river network storage with observationsfrom the "Global Inundation Extent from Multi-Satellite GIEMS (Prigent et al., 2001) flood area [km^2] is used. As GRACE mass deviation is most accurate for a monthly time resolution at present, the other data sets are aggregated to a monthly resolution as well. For the parameter optimization time series of river runoff and GRACE mass deviation are used for the time period from January 2004 until January 2009. Monthly runoff and the storage volume of the catchment and river network are calculated for Amazon based on different recharge products here and optimized either versus runoff or GRACE mass deviation. The results calculated with recharge from the terrestrial water balance optimized versus GRACE mass deviation are shown in Figures 8-10 for both (a) the monthly signal and (b) the monthly residual for January 2003-2009 .

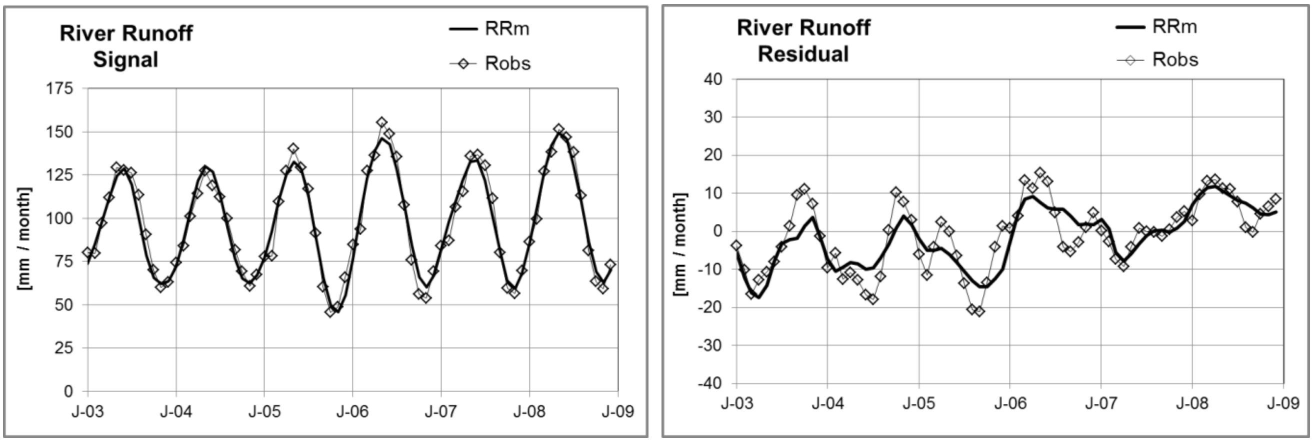

Fig.8: Time series of river runoff for the Amazon basin (a) for the signal (b) for the residual

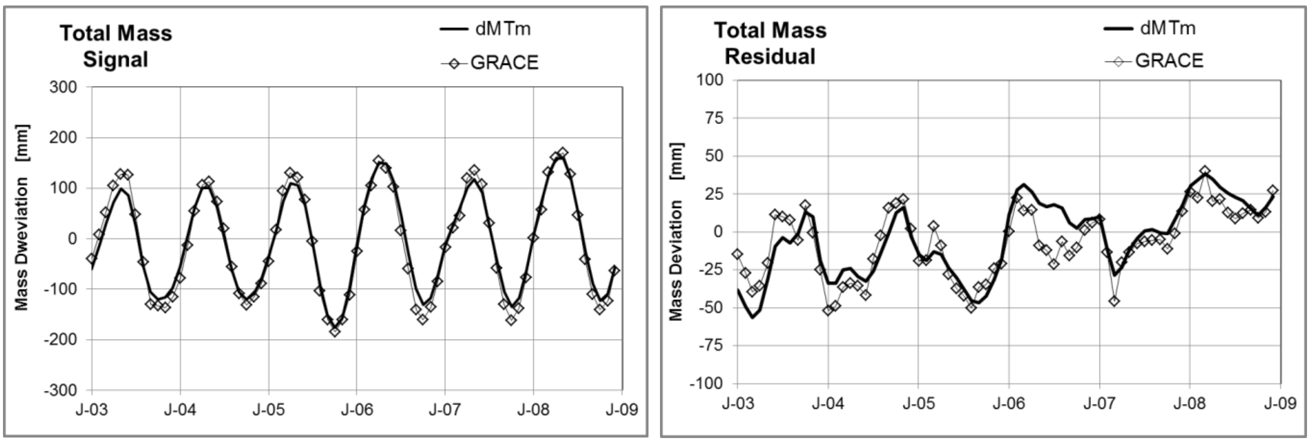

Fig.9: Time series of Total mass deviation for the Amazon basin (a) for the signal (b) for the residual

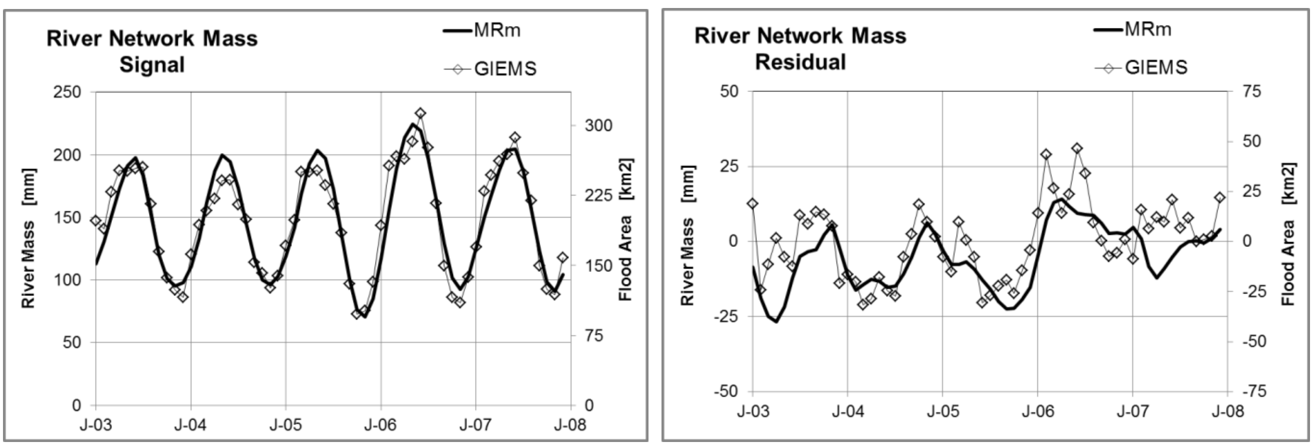

Fig.10: Time series of river network storage and inundated area from GIEMS (a) for the signal (b) for the residual

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The calculated river runoff R_R, total mass deviation dM_T and river network mass M_R fit very well with the measured river runoff, GRACE mass deviation and the flooded area from GIEMS both with respect to the signal and the deseasonalized monthly residual. The Cascaded Storage approach reproduces the phase shift between measured runoff R_o and total mass dM_T (or GRACE respectively). The calculated river network mass M_R of about 50% of the total mass M_T for Amazon is linear to observed runoff R_o without any phase shift (Fig.11)!

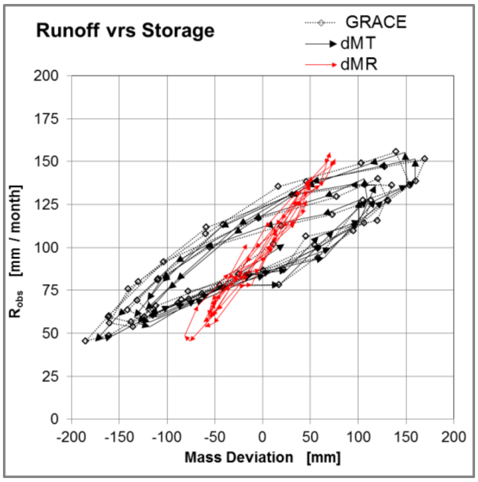

Fig.11: R-S relationships for observed runoff versus GRACE, calculated Total mass dM_T and river network mass dM_R

Calculated hydraulic time constants, mean values and signal amplitudes for the absolute storages volumes are provided in Table.2 for the Amazon basin. In addition the performance of optimizations either versus river runoff (Column a) or versus GRACE mass deviation (Column b) and for different recharge products (Column d-f) is displayed. In order to illustrate the benefits of the Cascaded versus a Single Storage approach even in the fitting quality, results for a fixed $\tau_R = 10^{-3}$, which correspond to a Single Storage, are shown (Column c, f) for different recharge products.

	a	b	c	d	e	f
Approach	**Cascaded**	**Cascaded**	**Single**	**Cascaded**	**Cascaded**	**Single**
Recharge	R+dM/dt	R+dM/dt	R+dM/dt	-divQ	-divQ	-divQ
Optimization	RR	dMT	dMT	RR	dMT	dMT
τ_C [month]	1.53	1.62	3.55	1.68	1.87	3.95
τ_R [month]	1.53	1.62	0.001	1.68	1.87	0.001
Avg M_T [mm]	304.81	321.77	353.29	333.00	370.93	392.02
Avg M_C [mm]	152.17	160.58	353.19	166.23	185.12	391.92
Avg M_R [mm]	152.64	161.18	0.10	166.77	185.81	0.10
Avg R_R [mm month^{-1}]	99.53	99.59	99.39	99.35	99.49	99.34
Avg N [mm month^{-1}]	98.80	98.80	98.80	99.07	99.07	99.07
Stdev M_T [mm]	98.46	100.38	84.09	101.73	105.34	87.83
Stdev M_C [mm]	58.49	60.40	84.06	61.22	65.05	87.80
Stdev M_R [mm]	45.48	46.02	0.02	46.70	47.55	0.02
RMSE R_R-R_o [mm month^{-1}]	5.76	6.08	12.13	11.99	12.57	18.08
RMSE M_T- GRACE [mm]	15.28	14.73	28.93	35.45	34.54	42.31
NS_S R_R-R_o	0.96	0.96	0.84	0.85	0.83	0.65
NS_R R_R-R_o	0.74	0.72	0.73	-0.09	-0.08	-0.10
$corr_S$ R_R-R_o	0.98	0.98	0.94	0.92	0.92	0.82
$corr_R$ R_R-R_o	0.86	0.85	0.87	0.48	0.46	0.41
NS_S dM_T- GRACE	0.98	0.98	0.92	0.89	0.89	0.84
NS_R dM_T- GRACE	0.74	0.72	0.71	-0.57	-0.81	-0.71
$corr_S$ dM_T- GRACE	0.99	0.99	0.98	0.94	0.94	0.93
$corr_R$ dM_T- GRACE	0.90	0.90	0.88	0.58	0.56	0.51
$corr_S$ dGIEMS- GRACE	0.92	0.92	0.92	0.92	0.92	0.92
$corr_S$ GIEMS-M_T	0.93	0.94	0.95	0.82	0.84	0.82
$corr_S$ GIEMS-M_R	0.96	0.95	0.95	0.88	0.86	0.82
$corr_R$ dGIEMS- GRACE	0.65	0.65	0.65	0.65	0.65	0.65
$corr_R$ GIEMS-M_R	0.76	0.75	0.78	0.04	-0.01	0.01

Table.2: The statistical characteristics are listed for river runoff R_R, total mass M_T, catchment mass M_C and river network mass M_R and observations of river runoff R_o, GRACE (mass deviations) and flood areas from GIEMS using: RMSE: Root-mean-square error of simulated versus measured, NS_S: Nash Sutcliffe coefficient of signal for simulated versus long-term mean of measured, NS_R: Nash Sutcliffe coefficient of monthly residual for simulated versus monthly mean of measured, $corr_S$: correlation of simulated versus measured signals, $corr_R$: correlation of simulated versus measured monthly residuals, Avg and Stdev are the long term mean and standard deviations, prefix “d” used for deviations from long term mean

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The Cascaded Storage approach with recharge from the water balance, Eq. (34), leads to high accuracy fits between calculated and measured and river runoff and total storage mass for the signal (NSS R_R-R_o=0.96, NSS dM_T- GRACE = 0.98) and for the residuals (NSR R_R-R_o = 0.74, NSR dM_T- GRACE = 0.74). With the Single Storage approach - beside the much worse fitting performance - the resulting time constant $\tau_T = \tau_C + \tau_R$ is overestimated (corresponding to the investigations in section 3) and the modelled signal amplitude is about 20% less than the measured from GRACE. In addition a non negligible phase shift remains between the modelled runoff and measured discharge.

The calculated river network mass M_R of the Amazon varies in the range of 40-65% of total mass M_T with an average ~50%, corresponding to the values found by Paiva et al., 2013 and Papa et al., 2013. The correlation versus the observed flood Area from GIEMS is higher for the calculated river network mass (0.96 for the signal and 0.76 for the monthly residual) than for GRACE (0.92 and 0.65 respectively). The conformance of calculated river network mass (and the corresponding observed river runoff $R_R = M_R \tau_R^{-1} = 0.742\ M_R$) with the flood areas is seen much clearer in the phasing (Fig.12), which shows a clear phase shift of GRACE versus GIEMS (see also Papa et al., 2008).

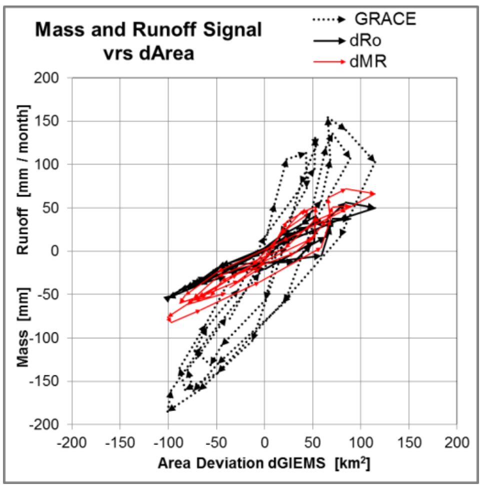

Fig.12: GRACE mass, calculated river network mass dM_R and observed river runoff Ro versus flood Area from GIEMS displayed as deviations (please consider $M_R = R_R \tau_R = 1.53\ R_R$)

The optimization results (Table.2) also show that there is hardly any difference in fitting performance for an optimization versus measured runoff and an optimization purely based on GRACE measurements (Columns a, b). The respective time constants and thus storage masses differ in a range of ~5%. The quality of the chosen recharge data significantly determines the quality of the results as can be seen, if moisture flux divergence, Eq.(33), is used as recharge (Columns d-f). Recharge from the water balance, Eq.(34), is preferable if river runoff is gauged as it is more accurate than moisture flux divergence (Riegger and Tourian, 2012). Here different recharge products are used in order to illustrate the performance of the approach depending on the quality of recharge data.

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5 Conclusions and discussion

The determination of the hydraulic time constants via the R-S relationship taken from measured runoff and/or GRACE mass deviation allows to quantify Drainable Storage Volumes in a catchment under the prerequisites that:

a. Uncoupled storage compartments are negligible or not time dependent i.e. the hysteresis is fully described by the phase shift.
b. Coupled and uncoupled storage compartments can be separated by a conceptual approach or by remote sensing if impacts from uncoupled storages occur.
c. The Hysteresis for this case can be fully described by the quantified contributions of the coupled and uncoupled storage compartments.

The concept of a sequence of cascaded catchment and river network storages provides an explanation for the observed phase shift between GRACE total mass and river runoff. Together with the piecewise analytical solutions it allows to describe the system behaviour i.e. the signal amplitudes and the phasing with high accuracy and to quantify the Drainable Storage volumes in the catchment *and* river network separately. Opposite to this, the description of a system (showing a phase shift) by a Single Storage approach i.e. without a sequence of storages leads to phasing differences between the calculated and measured runoff or storage and to considerable errors in the time constant of the total system.

The extensive evaluation of the parameter optimization process with synthetic recharge reveals the ambiguity for $\tau_R < \tau_C$ or $\tau_R > \tau_C$, and thus the related storage compartments. This problem can only be solved by reasonable assumptions or better by additional information on the volume of river network or flood areas, which can be taken from ground based observations or remote sensing. In principle, river network storage could be directly used for the parameter optimization. River network storage can be provided the GIEMS flood areas and water levels from altimetry which might also help to decide whether the catchment storage is clearly bigger than the river network storage, even if these data are not too accurate. Close to $\tau_R \sim \tau_C$, the generally high accuracy of the approach is limited as the separation of the storages depends on the quality of the respective additional information in this case.

Based on the Cascaded Storage approach the long term average of the Drainable Storage volumes can be determined via the hydraulic time constants (τ_C, τ_R) directly from long term averages of recharge N or observed runoff R_o according to Eq. (22a,b,c) and Eq. (23).

$$\overline{M}^C = \overline{N} \cdot \tau_C \qquad \overline{M}^R = \overline{N} \cdot \tau_R \qquad \overline{M}^T = \overline{N} \cdot (\tau_C + \tau_R) \qquad \text{with } \overline{R}^R(t) = \overline{R}^C(t) = \overline{N}$$

The related time series have to be calculated by the given piecewise analytical solutions leading to different phasing for non negligible τ_R. The time series of the total Drainable Storage volume can be calculated directly from GRACE measurements and long term average of recharge or runoff as the calculated total mass deviation corresponds to the GRACE signal:

$$M^T(t) = dM^T(t) + \overline{M}^T = GRACE(t) + \overline{M}^T = GRACE(t) + \overline{N} \cdot \tau_T = GRACE(t) + \overline{R} \cdot \tau_T \qquad \text{with } \tau_T = \tau_C + \tau_R \tag{35}$$

As the quality of the parameter optimization either versus river runoff data or versus GRACE mass deviations is comparable, the optimization versus measured GRACE data can be used to determine river runoff as well as catchment and river network storage for ungauged basins (provided that there is information on the relative contribution of river network to total mass). On the other hand recharge and measured runoff could be used to determine the catchment and river network storage volumes on large scales independent from GRACE measurements. For these cases the availability of sufficiently accurate recharge data limits the accuracy of runoff and storage calculations at present. However, for ungauged catchments it still provides quite acceptable results exclusively taken from remote sensing and atmospheric data if ground based measurements are not available. Further developments in hydrometeorological data products thus will improve the quantification of river runoff and Drainable water storage volumes from space.

6 Outlook

On a global scale discharge from Monsoon regions with seasonally dry periods plays an important role in the global water budget. For these regions high precipitation events limited to certain seasons lead to a distinguished surface runoff with a short time constant τ_S and a groundwater flow with a much longer time constant τ_{GW} as can be recognized in the discharge curves. The time dependent uncoupled storage compartments like soil or isolated surface water bodies, which do not contribute to discharge, have to be quantified by remote sensing (soil moisture and open water body altimetry from satellites) and subtracted from the total catchment mass measured by GRACE for an adequate description of the Drainable Storage Volumes. In this case both surface runoff and groundwater flow with their individual time constants τ_S and τ_{GW} have to be considered as parallel input into the river network storage and are both subject to the phase shift introduced by a non negligible time constant of the river system τ_R.

With respect to river network and flood volumes further investigations on the relationship between flood areas, volumes, river runoff and calculated river network mass can provide insights into river hydraulics i.e. routing times and the mass-area- and level- relationships of flooded areas.

As the spatial resolution of GRACE and the accuracy of moisture flux divergence is limiting applications to large scale catchments (>200000km^2) at the moment, any improvement in spatial / temporal resolution and accuracy will tremendously increase the number of catchments which can be described in their system behaviour by remote sensing exclusively.

Data availability

Calculations and data are provided in an EXCEL workbook for the synthetic case and for the Amazon catchment in the supplement.

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