Editor Decision: Reconsider after major revisions (further review by editor and referees) (10 Oct 2018) by Stan Schymanski
Comments to the Author:

Thank you for the detailed responses to the referee comments. All referees recommend major revisions before the manuscript can be reconsidered for publication, and I will follow this recommendation.

Thanks to the editor for the review. The comments of referees Mark Bierkens and Anonymous 3 are really helpful and certainly help to improve the paper. I will implement all issues addressed in the comments in the revised version.

While following up on the referee comments and your responses, I stumbled over a very recent paper by one of your former co-authors:

Tourian, M. J., Reager, J. T. and Sneeuw, N.: The Total Drainable Water Storage of the Amazon River Basin: A First Estimate Using GRACE, Water Resources Research, 54(5), 3290–3312, doi:10.1029/2017WR021674, 2018.

This a very relevant paper that uses the same data and gets to very similar results in terms of estimating the total drainable water storage of the Amazon basin. The paper goes even further and analyses water table data and sub-catchments within the Amazon basin. I am aware that Tourian et al. (2018) use the data in a different way to your work and that it was published after submission of your manuscript, but in order for the advantages of your approach to be understood, the manuscript will need to be put into relation to Tourian et al. (2018) and other recent papers, as pointed out by the referees.

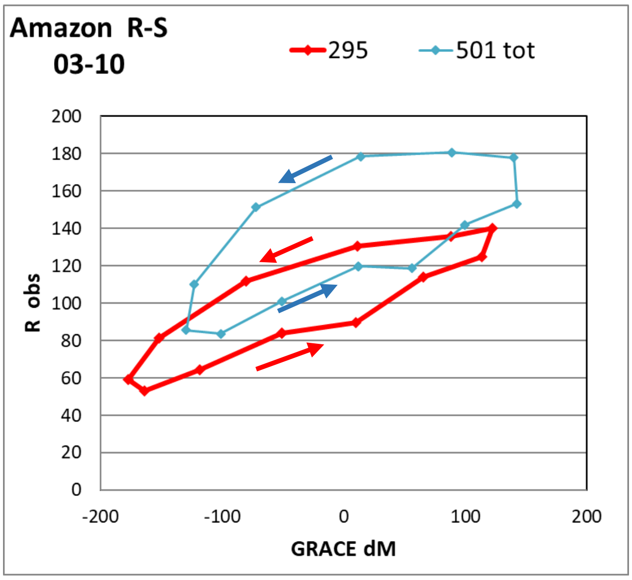
The editor states that this paper goes even further than my manuscript and that it describes analyses of Amazon sub-catchments.

There are two parts in the study of Tourian et al. (2018). The first is related to the determination of drainable storage making use of the R-S relationship, the second part determines the river network volume and compares it to GIEMS observations. Relevant for the discussion here is mainly the first part.

This describes nothing else but the application of the direct approach of Riegger and Tourian (2014) to the sub-catchments of the Amazon with a different phase shifting scheme than the one used in the original. However, the study does not consider the prerequisites of the approach described by Riegger and Tourian (2014), namely, that for a determination of the time constant from the R-S relationship the hysteresis must be fully described by a phase shift. This condition is fulfilled, if after the application of the phase shift, no systematic deviations from a linear relationship are observed. The consequence is, that before the scheme can be applied, it must be made sure that there is no impact from uncoupled storages and that there is no other impact on the hysteresis other than a pure phase shift.

In Tourian et al (2014), however, the scheme is applied without addressing the individual R-S diagrams of the Amazon sub-catchments in detail.

My own investigations of the Amazon sub-catchments reveal, that some of the sub-catchments (DBID 501) fulfill the prerequisites in the same way as the full Amazon catchment (DBID 295). The counter clockwise hysteresis corresponds to a positive phase shift and is explainable by a time lag (Fig 1)



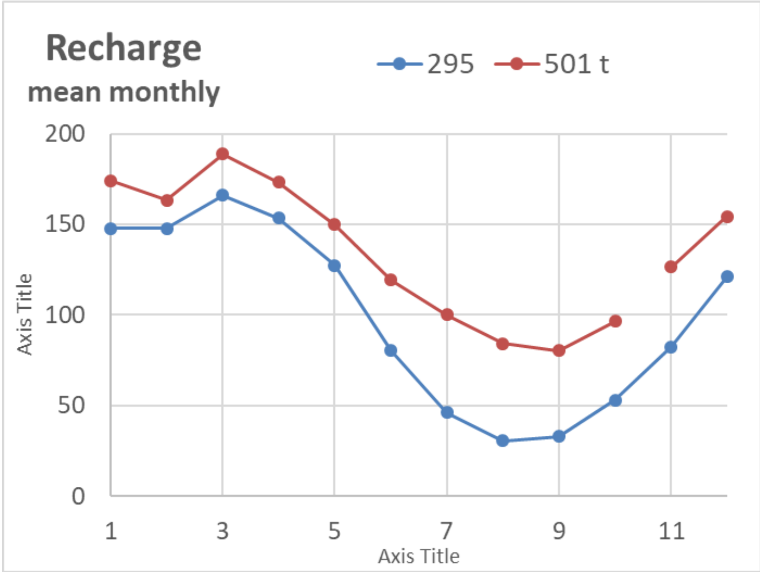
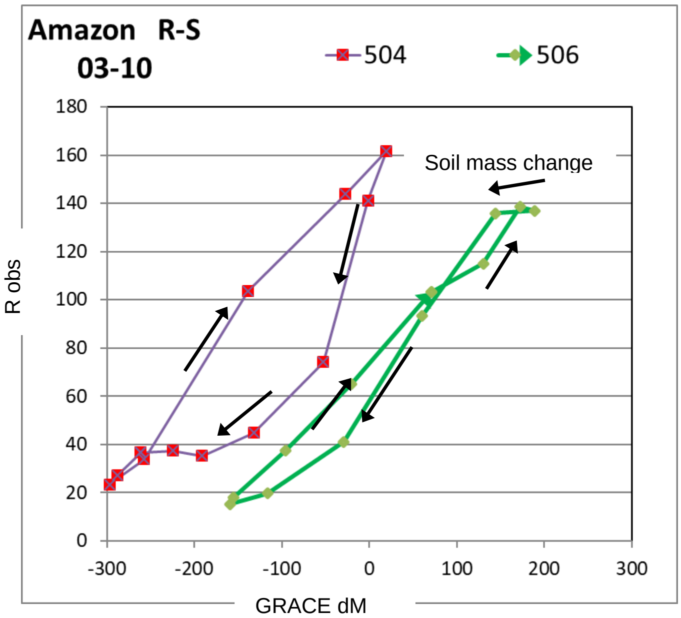


Fig.1 l: mean monthly values of observed runoff versus GRACE mass anomaly dM for catchments with counterclockwise hysteresis.caused by time lag

Fig.1 r : mean monthly values of recharge from water balance

Other catchments (DBID 504, 506, Fig.2) show a totally different behavior with a clockwise hysteresis (as other seasonally dry catchments like Niger etc (Riegger and Tourian (2014)) and a form which does not correspond to a time lag.



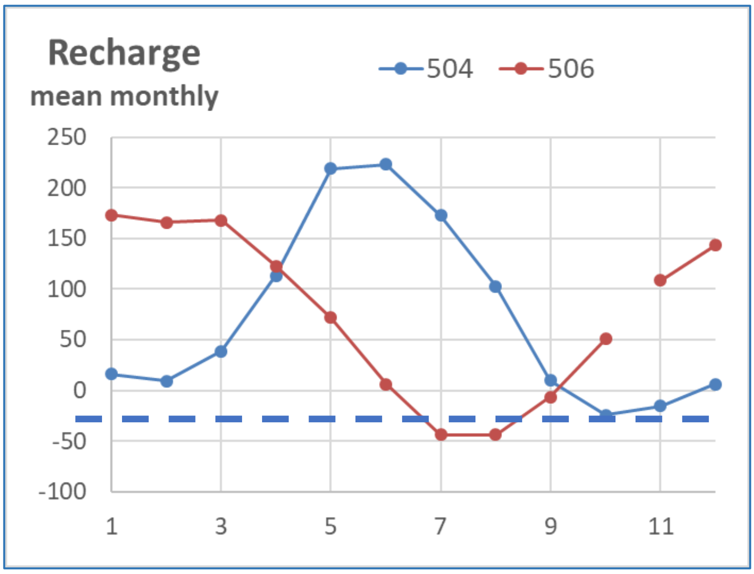
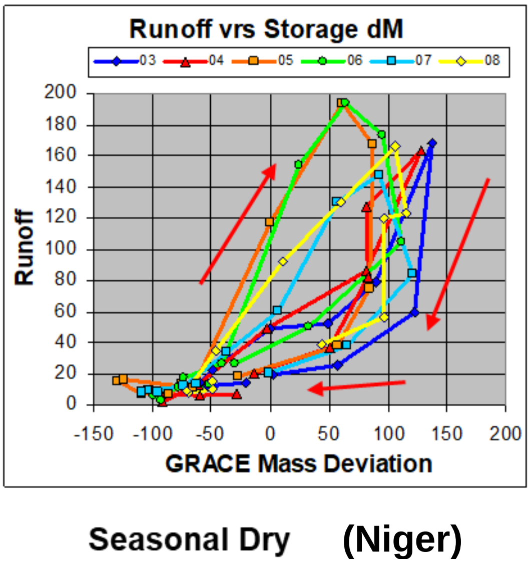


Fig.2 l: mean monthly values of observed runoff versus GRACE mass anomaly dM for catchments with clockwise hysteresis.

Fig.2 r : mean monthly values of recharge from water balance

They are typical examples for catchments with impacts from time variable soil water storage (506) and from superpositions of fast and slow runoff components (504) during periods of zero or negative recharge as for the Niger basin (Fig.3).



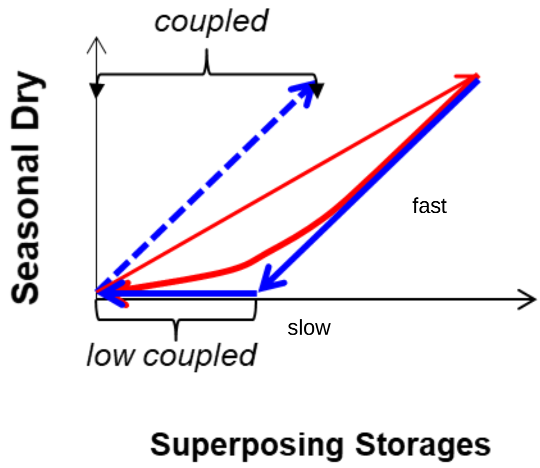


Fig.3 l: R-S graph typical for seasonally dry catchments with clockwise hysteresis

Fig.3 r : interpretation of negative hysteresis asas superposition of drainage contributions with different time constants

The superposition of drainage components with different time constants (which appears in the above form in the R-S graph) can be recognized and determined directly from the runoff time series for periods with no input (Fig.4).

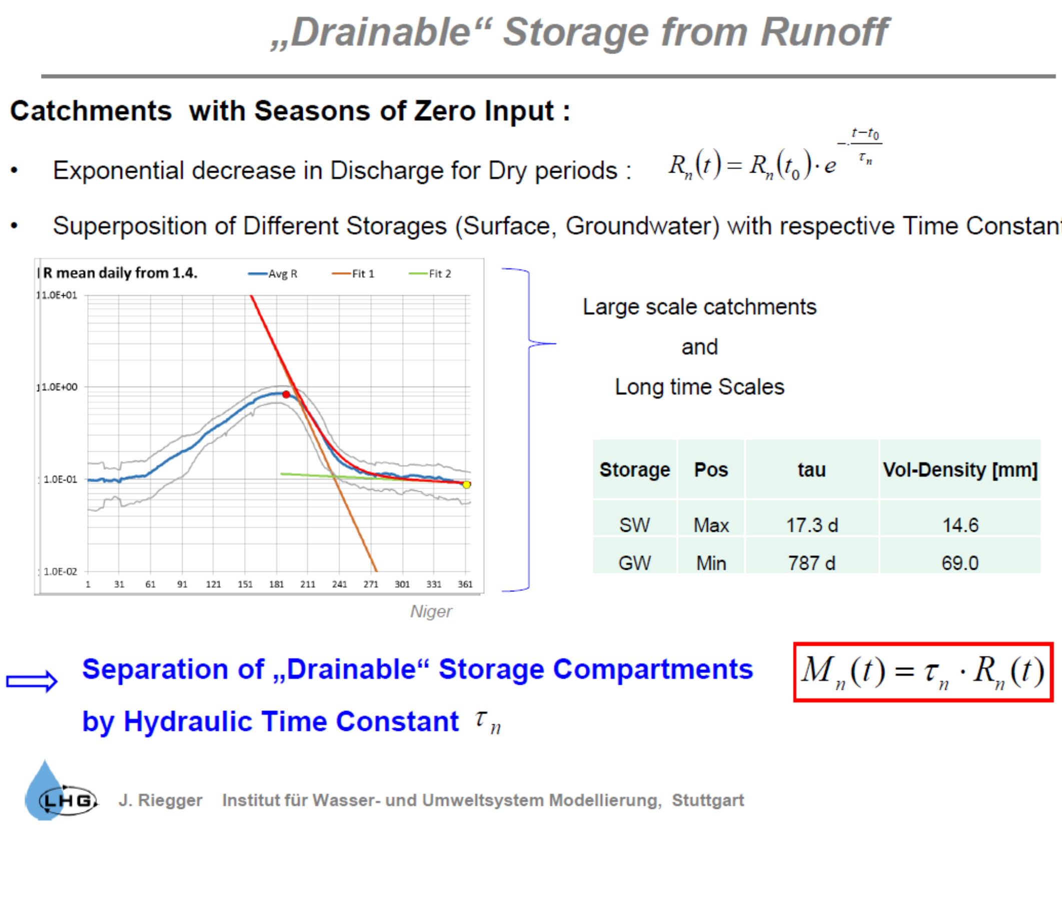


Fig.4 Separation of drainable storage volumes from contributions with different time constants during dry periods for Niger

Slide from (Riegger J., (2017) Geophysical Research Abstracts Vol. 19, EGU General Assembly 2017, "Global Scale Determination of Drainable Water Resources by GRACE and/or Runoff").

Only in the R-S diagrams the direction and form of the hysteresis is recognizable yet not from respective time series directly. The hysteresis in R-S must be counterclockwise and must disappear once the phase shift is applied in order to interpret the hysteresis as a pure time lag.
This is the reason why the prerequisites for an application of the scheme is emphasized at the beginning of the conclusions (P23 L2-9).

A negative phase shift as applied by Tourian et al. (2018) to sub-catchments like 504 or 506 does not address these effects and is not physical as it is not causal. There is no negative time lag. Thus, the direct approach of Riegger and Tourian (2014) may not be applied (as already mentioned in Riegger and Tourian (2014)) for some of the Amazon sub-catchments.

For these catchments the uncoupled storages and the different dynamics of the drainage via overland and groundwater flow must be implemented in the model description. Based on such a more detailed description of the drainage of the catchment by flows of different time constants as input into the river network the cascaded storage approach is expected to be able to describe the time lag for both flow types and the respective storages as well as the volume of river network storage appropriately.

The referees raised two general concerns related to the applicability of your approach, related to

(a) the representation of the catchment as 2 linear storage compartments in series

According to the HESS obligations for referees "the referees should explain and support their judgements adequately".
The mathematical framework is presented in detail in the manuscript. It is clearly showing the phase shift created by the cascade of storages. Thus, the referees have to formulate their concerns mathematically and provide contradictions.

and (b) the absence of significant uncoupled storages.

With the above given behavior of catchments with negligible and non negligible time variable soil / unsaturated zone storage (Fig.2, Fig.3) it should become apparent that non negligible effects of time variant soil water storage or other impacts on the R-S hysteresis can be recognized from the R-S graphs directly.

I agree with you that a model should be as simple as possible for a desired task and therefore, for the purpose of estimating drainable storage volume (DSV), fitting two time constants likely has advantages over fitting the parameters of a full-blown hydrological model to GRACE and streamflow data.
However, given that Tourian et al. (2018) have already demonstrated a way to estimate DSV from GRACE and streamflow data in the Amazon, your manuscript will need to illustrate more clearly how it advances current understanding beyond this.

Tourian et al. (2018) apply the direct approach described in Riegger and Tourian (2014) for determining mass offset, time constant and phase shift to Amazon sub-catchments without appropriately addressing their specific characteristics such as non negligible uncoupled storage or the temporal response from drainage contributions with different time constants. They determine the total drainable storage (i.e. catchment plus river network storage) by just fitting the phase shift phenomenologically even for sub-catchments with seemingly negative phase shift, which cannot be explained by a time lag, This implicit single storage approach leads to very questionable results and very inaccurate mass offsets and time constants (Table1 and P22 L3-6).

The Cascaded storage approach goes far beyond this approach as it explains the phase shift physically by the impact of a sequence of storages and permits to make use of the information which is contained in the phase shift. The piecewise analytical approach provides a very accurate and unique description of the absolute drainable storage both for the catchment AND for the river network. In contrast to a single storage approach for the total drainable storage (implicitly used in Tourian et al. (2018)) the cascaded storage approach provides modelled runoff and river network storage with much higher accuracy (Table.2) and with a very small phase shift with respect to the observations (Fig.12).

Referee #2 made a very valid point that water in the unsaturated zone would constitute time-varying uncoupled storage, which would violate your own criteria for applicability of your approach.

According to the above discussion on the impact of the uncoupled storage there is no evidence for any relevant contribution recognizable in R-S diagrams for the full Amazon catchment and some of the sub-catchments.

The referee #2 is asked to substantiate his statement that the unsaturated zone constitutes time-varying uncoupled storage with data. He is asked to prove that their temporal variation is non negligible and substantial compared to the storage amplitudes in the catchment and river network storage.

The editor repeats the arguments of referee #2 as “very valid” without taking reference to the comments to referee #2 already given by the author. He neither gives substantial evidence himself nor explains and supports his judgements adequately. Thus, it is up to the editor to provide intelligible reasons why he does not accept the arguments of the author. If no reasons or data are provided that substantiate the concerns of the referee and the editor, the argument of the author has to be accepted as sufficiently sound.

The reason why the model still performs well might be that the relative magnitudes of total DSV are much larger than the maximum variations of water in the unsaturated zone at the monthly time scale. This should be adequately discussed and analysed.

In my comments to A. Güntner I have alreadygiven three reasons why for the full Amazon basin a time variable water storage in soil or in the unsaturated zone is negligible. These shall be repeated here :

1. Average recharge N=P-ET is always positive for the full Amazon basin, so, on a spatial average no dry out and thus no storage variation occurs (even if this happens on some sub-catchments)

2. Non negligible time variable effects would appear in the R-S relationship as mass changes with no changes in runoff would occur.
3. Any non negligible effects not described by the model would appear in the scatter plots of modelled versus measured mass or runoff or flood area as systematic deviations, possibly with a phase shift. This is not the case for the Amazon catchment. Even the scatter plots of the residuals do not show any systematic effect.

In addition, as can be seen from Table.2 in the manuscript and from data and calculations in the supplement (where several performance metrices are presented for a comparison to other models including correlations and NS for the monthly residuals and not for signals only), the modelling performance of the cascaded storage approach outperforms all LSMs studied in Getirana et al. (2014), Fig. 14.
The editor as an expert in the field certainly has a better overview on the details of the description of soil moisture storage in the investigated LSMs (Getirana et al. (2014)) and the magnitude of the effect of storage CHANGES in soil and in the unsaturated zone. Yet, as the LSMs do not perform better for the full Amazon basin, it does not seem relevant if the respective storage changes are explicitly described in the model approaches or not.

It must be concluded that the assumption of a negligible soil or unsaturated storage CHANGE is an adequate description and need not be further analysed in more detail for a lumped global scale application.

Maybe a good way to address concerns about limited applicability of the appraoch would be to apply it to more than one basin and test whether the cascade storage model can be calibrated using GRACE and then used to predict variations in streamflow and inundated areas.
The Cascaded storage approach is data driven and makes use of both, GRACE mass anomalies and measured runoff. For the full Amazon catchment predicted streamflow (here river runoff), mass anomalies and river network mass are provided in Fig. 8-10 both for signals and residuals.

As several sub-catchments of the Amazon do not fulfill the prerequisites needed for an appropriate application of the cascaded storage approach a detailed description of the sub-catchments is needed. This would reveal the flaws in Tourian et al. (2018), which is not the intent of this manuscript.
Detailed investigations of the Amazon sub-catchments and the development of appropriate calculation schemes integrating drainage flows of different time constants are beyond the scope of this publication and will be addressed in a follow-on paper.

Language: I agree with the referees that the language and use of specific terms is confusing and needs significant improvements. Precipitation surplus more accurately expresses P-ET than recharge, The expression recharge N=P-Eta will be clearly defined in the text. I prefer it to precipitation surplus as the latter might be imagined as always positive while recharge can be negative for Eta > P.

and it would also be good to distinguish between drainage (or runoff) and streamflow, as suggested by one of the referees. I think that runoff may not be a good term, as it is often expected to include infiltration excess runoff, which would not be a function of storage. I would recommend "drainage" for subsurface flow into the channel network, and streamflow for the flow at the outlet.

As already described in the comments to referee 3 I would like to clearly distinguish between flows [Vol. / time] and fluxes [Vol. / time / area]. Instead of “drainage” the expression “runoff is preferred (as in WGHM and other models using surface/subsurface runoff). Here the expression catchment runoff will be introduced comprising surface/subsurface runoff and it is distinguished from the runoff from the river network, which corresponds to streamflow at the outlet devided by catchment area.
These expressions will be introduced properly.

The abstract has to be shortened substantially, as suggested by the reviewers, and made understandable without reading any other portions of the paper. I agree with the referees that its current form is more like a summary, which would be read after reading the paper, not before.

The abstract will be shortened.

For all the other referee comments, please consider them as typical thoughts readers might have and modify the revised manuscript in a way to not leave any of these questions open. I would actually propose to include a separate Discussion section for this purpose, followed by a short Conclusions section.

Concerns plead in the referees comments will be addressed in the manuscript for clarity and the Conclusions and Discussion chapter will be revised in that sense.

With your revised manuscript, please provide also a version with all changes highlighted and explain how and where the different referee comments were addressed in the revised manuscript.

RANDOM TECHNICAL COMMENTS:

All comment not specifically addressed will be implemented in the manuscript

- Title is misleading, as it implies that you can use the approach for the whole globe, not just for very large catchments.

As already explained extensively in my response to referee #3’s comments the expression global scales (emphasis on scales!) means a spatial limitation introduced by the resolution of GRACE and not a “Global coverage”. This is explained in the manuscript. It has been made very clear in the text that this approach is not limited to the Amazon basin or other fully humid climate zones. It has also been emphasized in the manuscript (P24 L14-22) that it has to be modified in order to account for the description of uncoupled storages - as has been already done in Riegger and Tourian (2014) - or for a description of runoff components with different time scales.

In fact, you only showed application in the Amazon, so unless you add more catchments to the analysis, generalisation may not be warranted.

The main point – namely the description of the phase shift by a storage cascade – can be generalized to all catchments if a reliable recharge P-ET is provided. The paper proves mathematically that there is a unique solution for the quantification of absolute catchment AND river network storage volumes. This is a novel approach. It offers a new way for the determination of river discharge by remote sensing (even for ungauged catchments) and of a an independent description of catchment and river network storage volumes in general.

Yet, I cannot be expected that in addition to the presentation of a new approach including a new mathematical formulation and extensive numerical tests the approach has to be applied to all global catchments before it can be published.

This is unusual and would be disproportional.

You may also consider emphasising the difference to Tourian et al. (2018) in the title.

I could imagine to change the title to :

Quantification of Drainable Water Storage Volumes in Catchments *And* in River Networks utilizing GRACE, River Runoff and their Phasing

- Please do not forget considering Mark Bierken's annotations.

- Please clarify the origins of the different equations, e.g. Eq. 1 is the result of a linear storage function, provide reference.

This is basic mathematics for an ordinary differential equation of first order with no input

- Please put each labelled equation on a separate line (e.g. P9).

- Please include more information in the figure captions. Figures and captions should be informative without having to read the main text, and contain all information needed to reproduce the figures (e.g. which data set, what model/equation, what parameter set).

- Fig 1: what does "phase-adapted" mean?

- P5L10-15: Why not just state that extrapolation of curve to 0 runoff gives the 0-point for drainable storage?

This is my understanding of proportionality in contrast to linearity.

- Table 1: Please describe a-f in the caption.

- P5L22: "Opposite to parallel storages..."

- Please include label for table with abbreviations

- P8L13: There is no RC in Eq. 10.

- Eq. 11: Please provide reference for this general solution.

This is basic mathematics for ordinary differential equations of first order with input

- Fig. 2: N is a flux, the others are state variables. Need two axes!

Please specify what parameter values were used to generate these plots.

described in the text P10 L18-20, “unit” means value=1

- Eq. 24: Is this an arbitrary function that can be fitted to the simulation results?

This an empirical function describing an ensemble of solutions for different time constants.

Then it would only be valid for these model runs, not for real conditions with non-sinusoidal forcing, right? Please clarify.

As any time series can be described by a Fourier representation real world time series can be described by a superposition of sinusoidal functions. The applicability to real world conditions has been tested for the Amazon basin.

The phasings of the catchment, river network and the total storage time series resulting from the optimizations of the time constants correspond to the respective empirical value of the phase shift (Eq.27) and also to the observed phase shift from measurements. Thus, this parameterization of the phase shift facilitates a simple and quite accurate determination of storage or runoff time series directly from measurements. The respective calculation schemes will be integrated in the conclusions chapter.

- P14L7: "lambda = 3.2"?

Lamda=2.7

- P15L9: Isn't this limited to sinusoidal forcing?

See above

Eq. 28: Where does this come from, what are the sigmas?

All calculations, results and graph are provided in the supplement

See supplement xls file table “results synthetic”

- Figs 8-10: How is separation between signal and residual performed?

Monthly residual = monthly signal – mean monthly signal (see Table “Amazon”)

- Fig. 9: Typo: mass deviation

- Fig. 11: Caption unclear. Mention symbols and colours in the caption.

How was everyhting scaled to 0, i.e. how was dMT and dMR computed?

dMT and dMR are anomalies calculated from monthly signals minus their long term average so that they are comparable to GRACE anomalies

What does the hysteresis mean and in what direction is it?

Fig.11 shows measured runoff versus measured GRACE anomaly,

The calculated total mass anomaly shows the same phase shift as GRACE anomaly

The calculated river network mass anomaly does not show a recognizable phase shift to measured runoff

The hysteresis is counterclockwise corresponding to a positive phase shift explained by a time lag.

For dMR no hysteresis with respect to R obs is recognizable.

-P20L10: This should be moved to figure caption.

- Table 1: Describe meaning of columns a-f in the caption.

Quantification of Drainable Water Storage Volumes in Both the Catchment and the River Network utilizing GRACE, River Discharge and their Phasing

~~Quantification of Drainable Water Storage Volumes in Catchments and in River Networks on Global Scales using the GRACE and / or River Runoff~~

Johannes Riegger [1]

[1] Institute for Modelling Hydraulic and Environmental Systems, University of Stuttgart, Germany

Correspondence to: Johannes Riegger (Johannes.Riegger@iws.uni-stuttgart.de)

Abstract.

The knowledge of water storage volumes in catchments and in river networks leading to river discharge is essential for the ~~management of water resources and for a comprehensive description of the environmental compartment Water in the context of climate change~~description of river ecology, the prediction of floods and specifically for a sustainable management of water resources in the context of climate change. ~~.~~ Measurements of water storage variations by the GRACE gravity satellite or by ground based observations of river or groundwater level variation do not ~~allow to determine~~ permit the determination of the respective ~~total~~ storage volumes, which could be considerably larger than the mass variations themselves. In addition mass variations measured by GRACE comprise all time variant storage compartments whether they are hydraulically coupled, contributing to river ~~runoff~~discharge, or uncoupled like soil moisture, isolated surface water or snow and ice.

The characterization of the Runoff-Storage relationship (Riegger and Tourian, 2014) revealed a linear relationship for hydraulically coupled storages with a phase shift. This allows to determine the hydraulic time constant and thus to quantify the total drainable storage directly from observed runoff, if the observed phase shift can be interpreted as a river system time lag. Global hydrological models describe a large number of different storages storage compartments yet the determination of the time lag and the related river network storage still means a huge effort. As a time lag can be described by a storage cascade a lumped conceptual model with cascaded storages for the catchment and river network is set up here with individual hydraulic time constants and mathematically solved by piecewise analytical solutions.

Tests of the scheme with synthetic recharge time series show that a parameter optimization either versus mass anomalies or runoff reproduces the time constants for both, the catchment and the river network τ_C and τ_R in a unique way, and hence permits an individual quantification of the respective storage volumes. The application to the full Amazon basin leads to a very good fitting performance for total mass, river runoff and their phasing (Nash-Sutcliffe for signals 0.96, for monthly residuals 0.72). The calculated river network mass highly correlates (0.96 for signals, 0.76 for monthly residuals) with the

Formatted: English (United States)

Formatted: Not Superscript/ Subscript

Formatted: Not Highlight

Formatted: Not Highlight

observed flood area from GIEMS and corresponds to observed flood volumes. This provides additional independent information for investigations on flood hydraulics.

The fitting performance versus GRACE permits to determine river runoff and Drainable storage volumes from recharge and GRACE exclusively i.e. even for ungauged catchments. An adjustment of the hydraulic time constants (τ_C, τ_R) on a training period facilitates a simple determination of Drainable storage volumes for other times directly from measured river discharge and/or GRACE and thus a closure of data gaps without the necessity of further model runs.

In the context of water resources management and climate change there is an ongoing discussion on how to assess available water resources i.e. the storage volumes which can be used for water supply in a dynamic way beyond the limitations of sustainable extraction rates. The maximum average extraction rate for a sustainable use of water resources is limited by the long-~~–~~term recharge of a catchment (Sophocleous, 1997, Bredehoeft, 1997), however, this rate based definition of groundwater stress only allows an assessment of water resources ~~w.r.t.~~with respect to ~~the ~~long-~~–~~term sustainability and does not allow ~~to ~~consideration of the volume of available water resources as a basis for short term groundwater management in order to satisfy specific demands. In addition, the knowledge of the storage volumes is essential for climate studies as it might lead to limitations in the water cycle.

Thus, the attempt was made by different authors to estimate available water resources by the volume of the respective groundwater storage (under the assumption that the contribution of surface water is comparably small). Korzun, 1978 and Nace, 1969 provide estimates of total storage volumes across the global land masses (except for Greenland and Antarctica) based on very coarse assumptions for aquifers with homogeneous thickness and porosity. As a consequence the uncertainties cover orders of magnitude as Shiklomanov, 1993 Alley, 2006 and Famiglietti, 2014 are warning. The revision of these estimates by the introduction of specific yield instead of porosity for dominant soil classes together with the assumption of different saturated thicknesses in order to ~~receive ~~obtain the “Extractable” storage (Richey et al., 2015a, Alley, 2006) does not solve the problem of missing information on the contributing soils or aquifers in general. Regional storage estimates for specific aquifers derived from groundwater models (Cao et al., 2013) or measured estimates of saturated thickness and porosity (Williamson et al.,1989) are considered to deliver more realistic estimates for storage volumes, yet are sparsely distributed over the globe. However, for semi-~~ / ~~arid ~~/~~and arid climate zones with very low recharge and/ or deep aquifer systems with fossil water resources they deliver the best possible estimation at present. Richey et al., 2015b try to bypass the huge uncertainties in the quantification of total storage by the introduction of a “Total Groundwater Stress” indicator, which is defined by the time needed for the depletion of groundwater storage to a ~~certain ~~specified extent. It is determined by the ratio of an estimated total storage of a (sufficiently large) catchment and the measured trend in the mass anomalies given by GRACE.

Very little attention ~~is given~~has so far been given to the storage volume of renewable water resources participating in the dynamic water cycle driven by precipitation P, actual evapotranspiration ET_a and river runoff R. The reason for this is seen

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: English (United States)

in the problem that observations of time variant groundwater or river levels only permit the estimation of volume changes yet no absolute storage volumes, which could be considerably bigger. Natural systems consist of many different storage components like canopy, snow/ice, surface, soil, unsaturated/saturated underground, drainage system etc. Direct measurements of storage volumes from water or pressure levels are problematic as they are based on assumptions and approximations. Ground based measurements of storages for example are based on point measurements and quite rare on large spatial scales compared to the heterogeneity scale of the respective compartments. This leads to large interpolation errors. In addition, the storage coefficients for porous media describing the relationship between the measurable groundwater heads or capillary pressure on the one hand, and storage volume or absolute soil saturation on the other hand, are insufficiently known on large scales. Remote sensing data have been limited to near surface water storage (open water bodies, soil) up until now and are thus of limited benefit for the quantification of water storage with respect to accuracy and coverage due to methodological constraints (Schlesinger, 2007).

~~Opposite~~ In contrast to discharge-less basins and/or arid areas, which are nearly exclusively driven by precipitation and evapotranspiration, the storage dynamics of catchments draining into a river system allows to address the hydraulically coupled storage compartments via their contributions to ~~runoff~~river discharge. These comprise groundwater, surface water, the river network and temporarily inundated areas. All storages draining into the river system by gravity are referred to as “Drainable” storage here. So aquifers or parts of them not draining into the river system without an energy input are not considered here.

For time periods with no recharge hydraulically coupled storage components with a linear runoff storage (R-S) relationship lead to an exponential decrease in river discharge or streamflow ~~for time periods with no recharge~~ depending on the related hydraulic time constant τ:

$$Q(t) = Q(t_0) \cdot e^{-\frac{t-t_0}{\tau}}$$

(1)

The corresponding total Drainable Storage in terms of mass density $M_{Storage}$ for any given time t_0 is then given by an infinite temporal integration over river discharge Q(t) from ~~a~~ the corresponding catchment area A - called~~catchment area A~~ river Runoff R(t) = Q(t)/A here - starting at time t_0 ~~starting at time t_0~~:

$$M_{storage}(t_0) = \frac{V_{tot}(t_0)}{A} = \frac{1}{A} \cdot \int_{t_0}^{\infty} Q(t)dt = \frac{Q(t_0)}{A} \cdot \int_{t_0}^{\infty} e^{-\frac{t-t_0}{\tau}} dt = \tau \cdot \frac{Q(t_0)}{A} = \tau \cdot R(t_0)$$

(2)

Contributions from several storage compartments (with individual time constants) superpose, if they drain in parallel and if there is no feedback from the river system. For this case, there is a wide range of time series analysis methods (Tallaksen,

Formatted: Indent: First line: 1,27 cm

Formatted: Subscript

Formatted: Indent: First line: 1,27 cm

1995), which allow to separate the flow components into fast, medium or slow and the corresponding surface, interflow or groundwater flow contributions according to their individual time constants. Thus, measurements of the different time constants allow to determine the Drainable Storage of the respective storage compartment and the corresponding mean Drainable Storage :

$$\overline{M}^{X} = \overline{R} \cdot \tau_X = \overline{N} \cdot \tau_X \quad . \qquad (3)$$

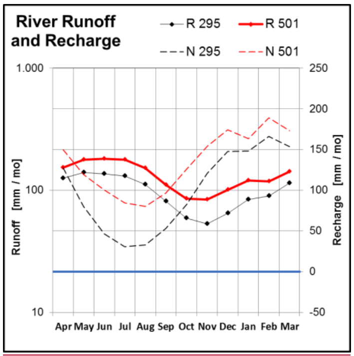
from mean runoff R or recharge N.

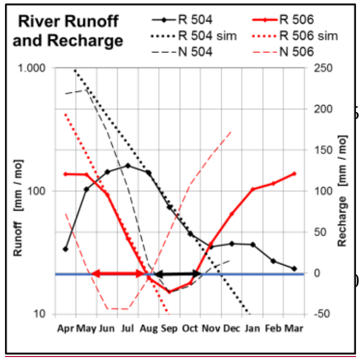
On global scales the absolute storage volume of the Drainable Storages can be determined from runoff time series directly, if there are distinct and long enough periods of negligible or even negative recharge (actual evapotranspiration > precipitation) ~~like~~ as it occurs in seasonally dry regions (Niger, ~~Tocantins, .etc.~~Mekong, some Amazon sub-catchments) ~~long enough for a sufficient fit~~ etc.). From the purely exponential decrease in river distcharge the time constant can be determined directly from a curve fit as shown in Fig.1b for Amazon sub-catchments. If the dry period is long enough ~~For some catchments~~ the sequence of different time constants taken from the discharge curve even ~~allows~~ permits a ~~a~~ discrimination ~~of~~ between the fast response by ~~surface runoff~~overland flow and the slow~~er~~ response ~~via the~~by the groundwater system. ~~.~~Catchments with permanent input i.e. no periods of negligible recharge, however, do not show an exponential behaviour for discharge (Fig.1a). For these cases the hydraulic time constant cannot be taken from ~~runoff~~ discharge dynamics directly, but has to be estimated either from the ~~runoff – storage (~~R-S~~)~~ relationship or by hydrological models. ~~Numerical models based on climatic data in principle allow to simulatefacilitate the simulation of the time dependent flow components and the resulting non-exponential form of runoff~~ via an adaption to measured runoff, or ~~–~~ at least ~~in principle~~theoretically - to ~~observations of~~ observed storage.

Since 2013 GRACE observations of the time-variable gravity field provide monthly ~~mass~~ distributions of mass ~~for~~ density on large spatial scales >~ 200000km^2 (Tapley et al., 2004).~~.~~ However, as the water storage in different compartments (snow, ice, vegetation, soil, surface-, ground- water etc.) superposes with all other terrestrial (geophysical) masses, only ~~only~~ the time variant part of the GRACE signal can be used to quantify~~for the quantification of~~ the Terrestrial Water storage (TWS) anomalies (monthly mass signals minus long term average)~~not in the form of absolute storage volumes but instead by monthly deviations from the long term mean i.e. mass anomalies~~, but not the related absolute storage volumes. ~~This~~ Nevertheless, this for the first time ~~for the first time allows~~ permits a direct comparison of measured TWS and river runoff R_o and an investigation of the ~~runoff – storage~~R-S relationships ~~(R-S)~~ on large spatial and monthly time scales. ~~which~~ The R-S diagrams show ~~a~~ ~~hystereses~~hysteresis curves~~is~~ of ~~characteristic~~ distinct form and extent, which are characteristic for different climatic ~~zones~~ conditions like fully humid, seasonally dry or boreal (Riegger and Tourian, 2014).

Formatted: Indent: First line: 1,27 cm
Formatted: Not Highlight
Formatted: Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight

Catchments in fully humid conditions (like the full Amazon basin upstream Obidos (295) and some of its sub-catchments like upstream Manacapuru (501)) with a permanent input i.e. only positive recharge (Fig.1a) show a counterclockwise hysteresis (Fig.1c), which can be interpreted as a phase shift caused by a time lag, as it is causal. River runoff and storage then behave like a Linear Time Invariant (LTI) system (Riegger and Tourian, 2014) i.e. the R-S relationship is linear, if the phase shift is adapted as shown in Fig.1c. For this case the slope in the R-S diagram corresponds to the hydraulic time constant via τ^{-1}. The time constant and the reasonable assumption of a proportional R-S relationship (no runoff for empty storage) then facilitates the quantification of the Drainable Storage, Eq. (3), i.e. the volume related to the hydraulically coupled storage compartment, which drains by gravity.





In contrast to this, catchments with distinct periods of zero or negative recharge (like Niger, Mekong or Rio Branco (504), Rio Jurua (506) in the Amazon basin (Fig.1b)) show a clockwise hysteresis in the R-S diagram (Fig.1d) and a form, which is determined by an increase in mass and runoff during wet periods, a decrease in mass and runoff with different slopes corresponding to different time constants and a possible mass loss without a related runoff (possibly from the soil zone by evapotranspiraton) during dry periods. This hysteresis cannot be explained by a time lag as it is not causal.

Formatted Table

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Fig.1a: Mean monthly Runoff R and Recharge N for fully humid catchments (Log scale for R)

Fig.1b: Mean monthly Runoff R and Recharge N for seasonally dry catchments (Log scale for R) incl. exp. fittings for Runoff

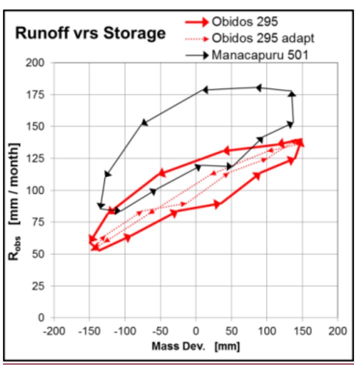


Fig.1c: R-S diagram with counter clockwise hysteresis for mean monthly runoff R vrs GRACE dM for fully humid catchments incl. a phase adaption for Amazon upstream Obidos

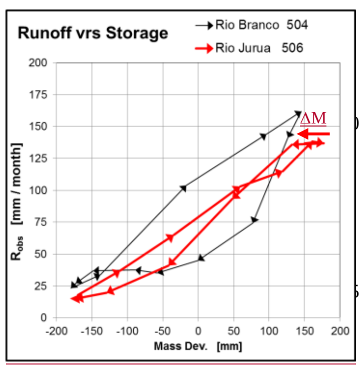


Fig.1d R-S diagram with clockwise hysteresis for mean monthly runoff R vrs GRACE anomaly dM for seasonally dry catchments

~~For Amazon the hysteresis in the R-S relationship (Fig.1) can be fully explained and described by a phase shift or time lag. For this case river runoff and storage behave like a Linear Time Invariant (LTI) system i.e. the R-S relationship is linear, if the phase shift is adapted as shown in Fig.1.~~

Formatted: English (United Kingdom)

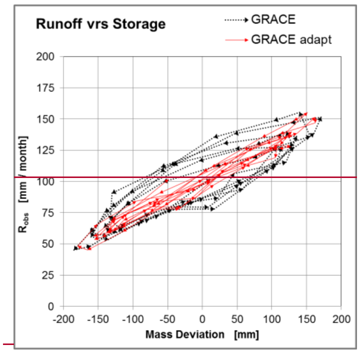
Formatted: Font: Symbol, 9 pt, Font color: Red

Formatted: Font: 9 pt, Font color: Red

Formatted: Font: 10 pt

Formatted Table

Formatted: Highlight



~~Even though global hydrological models comprise a number of storages like soil, surface water, groundwater etc. some of them show considerable phase shifts between the calculated and measured runoff and an underestimation of the signal~~ Recent developments on river routing schemes with hydrodynamic modelling of the flow in the river network system have successfully dealt with the description of phase shifts generated by the time lag in the river network (Paiva et al., 2013, Luo et al., 2017, Siqueira et al., 2018). Getirana et al., 2017a emphasize the importance of integrating an adequate river routing schemes not only for an improved phase agreement with observed river discharge but also for a better fit of the total mass amplitude with GRACE by the inclusion of the corresponding river network storage. Yet a hydrodynamic modelling of a complete river network system for the determination of the river network time lag and storage means a huge modelling effort (Getirana et al., 2017b).

A far more simple method is the Direct approach (Riegger and Tourian, 2014), in which the phase shift between the observations of GRACE and river runoff is adapted numerically (Eq.38 & 39) as shown in Fig.1c. This permits to determine the time constant of the whole system comprising catchment and river network storage from the slope in the R-S diagram without the need for modelling. Prerequisite for this method is that time dependent uncoupled storages are negligible, i.e. the hysteresis can be purely described by a time lag i.e. a positive phase shift. Tourian et al. (2018) apply an adaption of the phase shift using a Hilbert transform in order to determine the hydraulic time constants and the total Drainable water storage for the sub catchments of the Amazon basin. As shown in Fig1 this leads to reasonable results for the sub catchments with permanent input (Fig.1 a, c) for which the time dependent uncoupled storage is negligible. For Rio Branco (504) or Rio Jurua (506) this condition is not fulfilled as the hysteresis is determined by mass changes in the uncoupled storage and by

Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight

runoff with different time constants (Fig.1b, d). For these catchments the exclusive adjustment of the phase shift leads to negative time lags (which are not physical) and to misleading time constants. This leads to considerable errors in the determination of Drainable storage volumes.

~~A consideration of the phase shift between measured runoff and GRACE, however, either in the R-S relationship or in numerical models (Riegger and Tourian, 2014) can lead to a description of the system behaviour with high accuracy (Nash Sutcliffe 0.97 for Amazon), even though the reason for its occurrence is not understood in detail so far.~~

Another disadvantage of the above Direct approach is that it does not permit to quantify the individual Drainable storage volumes of both, the catchment and the river network separately, but only the total volume of the system. The information contained in the phase shift or time lag is not used for a quantification of the river network storage volume. Yet, as observations of inundated areas in river networks such as from the GIEMS “Global Inundation Extent from Multi-Satellites” project (Prigent et al, 2007, Papa et al., 2008, Papa et al., 2013) and hydrodynamic models of the river network (Paiva et al., 2013, Getirana et al., 2017b, Siqueira et al., 2018) indicate a considerable contribution of river network storage corresponding to a non negligible time lag, the river network storage should be considered in the integration of the total catchment water balance. As a sequence of storages (cascaded storages) leads to a time lag i.e. a phase shift (Nash, 1957), and storages draining in parallel (as for overland and groundwater flow) just lead to a superposition (with no time lag), a storage cascade is considered as an appropriate description to account for a time lag.

This paper explores the accuracy and uniqueness of a lumped top down approach called “Cascaded storage approach” based on the integration of given recharge in the water balance utilizing a cascade of a catchment and a river network storage for a simple description of the observed time lag and the individual storage volumes. This permits to describe the system with a minimum number of macroscopic observation data and an adaption of only two parameters, the hydraulic time constants of the catchment and the river network. These time constants then could be used for nowcasts or even forecasts (within the time lag) of river discharge and/or Drainable storage volumes directly from measurements without the need for further modelling.

The paper is structured as follows: ~~in s~~Section 2 presents the mathematical framework of piecewise analytical solutions of the water balance equation for a cascade of catchment and river network storages~~ is presented~~. It also contains the description of observables, which ~~allow to compare~~ permit the comparison of calculated and measured values. The Single Storage approach is handled as the specific case for a negligible river network time constant. In section 3, the properties of the Cascaded Storage approach and its impact on the performance of the parameter optimization are described for synthetic recharge data and compared to the “Single Storage” approach.~~.~~ In section 4 the approach is applied to data from the Amazon basin and evaluated versus measurements of GRACE mass, river runoff and flood area from GIEMS. Conclusions are drawn and discussed in section 5. In section 6 a purely data drive determination of Drainable storage volumes from observations of GRACE and river discharge is presented and an outlook on future investigations and possibilities is given in section ~~6~~7.

Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight

2 Mathematical framework

In order to investigate the impact of a ~~possible~~ non negligible river water storage ~~corresponding to a non negligible hydraulic time constant~~on the time lag in ~~for~~ the river system, the water balance of the total system comprising both the catchment and river network storage has to be considered. A conceptual model corresponding to a Nash cascade (Nash, 1957), called "Cascaded Storage" approach here, is set up with individual time constants for the different storages and with the following properties:

- ~~For simplicity the s~~Surface and the groundwater ~~systems~~ storages both fed by recharge are summarized ~~in a first approach~~ to one catchment storage M_C with time constant τ_C draining into the river network as overland and groundwater flow summarized as catchment runoff.~~. (This is not necessarily appropriate for catchments in other than fully humid climate zones like seasonally dry or boreal regions)~~
- Temporal variations of uncoupled storage compartments like soil or open water bodies are considered as negligible.
- The river network storage M_R with time constant τ_R is assumed to be instantaneously distributed within the river network system. Internal routing effects, which might lead to an additional delay in ~~runoff~~ streamflow response, are not considered ~~here~~.
-

These conditions are chosen for conceptual and mathematical simplicity in this first approach here regardless of the necessity to address several coupled storages with different time constants and different uncoupled storage compartments for the general applicability with a global coverage. Thus, applications of this first approach are limited to catchments for which the hysteresis can be fully described by a time lag.

The following abbreviations are used in the mathematical description (Table.1):

Abbreviation	**Description**	**Units:** general / for application
N	recharge = (precipitation - actual evapotranspiration)	volume area^{-1} time^{-1} [mm month^{-1}]
M_C	Storage mass catchment	mass density in equivalent water height [mm]
τ_C	Time constant catchment	time unit [month]
R_C	Runoff catchment	volume area^{-1} time^{-1} [mm month^{-1}]
ω_{MC}	Phasing catchment mass	time unit [month]
M_R	Storage mass river network	mass density in equivalent water height [mm]
τ_R	Time constant river network	time unit [month]
R_R	Runoff river network	volume area^{-1} time^{-1} [mm month^{-1}]

Formatted: English (United States)

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Bulleted + Level: 1 + Aligned at: 0,5 cm + Indent at: 1,14 cm

ω_{RR}	Phasing river network mass	time unit [month]
M_T	Storage mass total system	mass density in equivalent water height [mm]
τ_T	Time constant total system	time unit [month]
ω_{MT}	Phasing mass total system	time unit [month]
Ro	Observed river runoff	volume area^{-1} time^{-1} [mm month^{-1}]
GRACE	GRACE mass ~~deviation~~anomaly	mass density in equivalent water height [mm]
GIEMS	Flood area	area [km^2]
Prefix "d"	indicates signal ~~deviations~~ anomalies from long term mean (anomalies)	
Suffix "m"	indicates mean values on the intervals	

Table.1: Abbreviations in the mathematical descriptions:

The total system behaviour is described by two balance equations :

1. ~~1.~~ catchment storage $\frac{\partial}{\partial t} M^C(t) = N(t) - R^C(t) = N(t) - \frac{1}{\tau_C} \cdot M^C(t)$ (4)

with $R^C(t) = \frac{1}{\tau_C} \cdot M^C(t)$ ~~(4)~~ (5)

2. ~~2.~~ river storage $\frac{\partial}{\partial t} M^R(t) = R^C(t) - R^R(t) = R^C(t) - \frac{1}{\tau_R} \cdot M^R(t)$ (6)

with $R^R(t) = \frac{1}{\tau_R} \cdot M^R(t)$ ~~(6)~~ (7)

with a proportional R-S relationship for hydraulically coupled storages. N denotes the recharge as input, R_C the catchment runoff from the catchment storage M_C, which cannot be measured directly on large spatial scales, and R_R the river runoff from the river network storage M_R which can be measured at discharge gauging stations.

The water balance equation, Eq.(4), for the catchment is generally solved by:

Formatted: Numbered + Level: 1 + Numbering Style: 1, 2, 3, … + Start at: 1 + Alignment: Left + Aligned at: 0,63 cm + Indent at: 1,9 cm

Formatted: Indent: Left: 10,79 cm, First line: 0,63 cm

Formatted: Numbered + Level: 1 + Numbering Style: 1, 2, 3, … + Start at: 1 + Alignment: Left + Aligned at: 0,63 cm + Indent at: 1,9 cm

Formatted: Indent: Left: 10,79 cm, First line: 0,63 cm

$$M^{C}(t-t_0)=M^{C}(t_0)\cdot e^{-\frac{t-t_0}{\tau_C}}+\int_{t_0}^{t}N(w)\cdot e^{\frac{w-t}{\tau_C}}\cdot dw$$

(8)

where $M_C(t_0)$ is the initial condition and N(w) the time dependent recharge.

For recharge N(t) being given with a certain temporal resolution in time units or by periods of piecewise constant values and arbitrary length (stress periods) the recharge time series can be described as :

$$N(t)=\sum_{i=0}^{n-1}N_{i+1}\cdot\gamma_{i+1}(t)\quad\text{with}\quad\gamma_{i+1}(t)=\begin{cases}1\\0\end{cases}\ for\ \begin{cases}t\in[t_i,t_{i+1}]\\t\notin[t_i,t_{i+1}]\end{cases}\quad\text{for each interval }\lfloor t_i,t_{i+1}\rfloor$$

(9)

For calculation convenience Eq. (8) can be solved successively for each stress period using the values at the end of the last period as starting value, which leads to the piecewise analytical solution for catchment mass for a time $t\in\lfloor t_i,t_{i+1}\rfloor$ in stress period $_{i+1}$:

$$M_{i+1}^{C}(t-t_i)=M_i^{C}(t_i)\cdot e^{-\frac{t-t_i}{\tau_C}}+N_{i+1}\cdot\tau_C\cdot\left(1-e^{-\frac{t-t_i}{\tau_C}}\right)\ : \qquad (10)$$

The respective catchment runoff R_C ~~from~~ based on Eq. (5) and M_C from Eq. (10) is used as input for the river network water balance, Eq.(6), and leads to the general solution for the river network storage M_R :

$$M^{R}(t-t_0)=M^{R}(t_0)\cdot e^{-\frac{t-t_0}{\tau_R}}+\int_{t_0}^{t}R^{C}(u)\cdot e^{\frac{u-t}{\tau_R}}\cdot du$$

(11)

and the iterative solutions for time $t\in\lfloor t_i,t_{i+1}\rfloor$ in stress period $_{i+1}$:

$$M_{i+1}^{R}(t-t_i)=M_i^{R}(t_i)\cdot e^{-\frac{t-t_i}{\tau_R}}+\ N_{i+1}\cdot\tau_R\cdot\left(1-e^{-\frac{t-t_i}{\tau_R}}\right)\ +\ \left[M_i^{C}(t_i)-N_{i+1}\cdot\tau_C\right]\cdot\frac{\tau_R}{\tau_C-\tau_R}\cdot\left(e^{-\frac{t-t_i}{\tau_C}}-e^{-\frac{t-t_i}{\tau_R}}\right)$$

(12)

The total mass M_T is then given by : $M_i^{T}=M_i^{C}+M_i^{R}$.

(13)

Formatted: Indent: First line: 1,27 cm

Formatted: Indent: First line: 1,27 cm

Formatted: Indent: First line: 1,27 cm

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Subscript, Not Highlight

Formatted: Not Highlight

Formatted: Indent: First line: 1,27 cm

The mixed term in Eq. (12) and thus the total mass are commutative in (τ_C, τ_R) and show a singularity at $\tau_C = \tau_R$ with an asymptotic value. For $\tau_R > \tau_C$ solutions also exist with analogous values in total mass M_T for $M_R > M_C$.

It has to be emphasized here, that the piecewise analytical solutions for time periods of constant recharge provide a mathematical solution for an arbitrary temporal resolution. Finite Difference solutions are limited by stability criteria $(t_{i+1}-t_i)<\tau$ and accuracy criteria $(t_{i+1}-t_i)<\tau/10$ for the smallest τ. Analytical solutions ~~allow to calculate~~facilitate the calculation of the response of the river network on the time interval of constant recharge (though the time constant of the river network is much shorter than the time constant of the catchment), and thus avoid the very high temporal discretization, which otherwise would be needed for a Finite Difference scheme.

The observables ~~based on~~related to measurements by GRACE and discharge from gauging stations are the total mass ~~deviation~~anomaly dM_T and the river runoff R_R. GRACE observations with acceptable error are still limited to monthly ~~values~~resolution. Discharge as well as some of the meteorological inputs like precipitation, evapotranspiration or moisture flux divergence are often measured in daily values, some of the products in monthly values. For an optimal adaption to the monthly resolution of GRACE products, the approach presented here is based on monthly values but could also be applied to daily data without problems.

Formatted: Subscript

Formatted: Subscript

The mass values used in the calculations here are assigned to the interval boundaries while the values for monthly recharge and measured runoff are constant over the interval and temporally assigned to the centre of the interval. Thus, for a comparison of the calculated mass and runoff values versus the observed monthly values of GRACE and discharge the calculated values have to be averaged over the interval. As the dynamics follow an exponential behaviour the mean values cannot be taken from arithmetic averages at the interval boundaries but instead from an integral average over the interval.

The mean storage mass for M_X is given for each interval $\lfloor t_i, t_{i+1} \rfloor$ by :

$$\overline{M}^X_{i+1} = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} M^X_{i+1}(t-t_i)\cdot dt \qquad (14)$$

leading to mean runoff $\overline{R}^X(t) = \frac{1}{\tau_X}\cdot \overline{M}^X(t)$

~~(14)~~ (15)

Formatted: Indent: Left: 6,35 cm, First line: 1,27 cm

i.e. mean catchment mass and runoff :

$$\overline{M}^C_{i+1} = \left(M^C_i - N_{i+1}\cdot\tau_C\right)\cdot\frac{\tau_C}{(t_{i+1}-t_i)}\left(1-e^{-\frac{t_{i+1}-t_i}{\tau_C}}\right)+N_{i+1}\cdot\tau_C \qquad (16)$$

and $$\overline{R}_i^C = \frac{1}{\tau_C} \cdot \overline{M}_i^C$$

~~(16)~~ (17)

Formatted: Indent: Left: 8,89 cm, First line: 1,27 cm

and mean river mass and runoff :

$$\overline{M}_{i+1}^R = \left(M_i^R - N_{i+1} \cdot \tau_R\right) \cdot \frac{\tau_R}{\left(t_{i+1} - t_i\right)} \cdot \left(1 - e^{-\frac{t_{i+1}-t_i}{\tau_R}}\right) + N_{i+1} \cdot \tau_R$$
$$+ \frac{\left[M_i^C - N_{i+1} \cdot \tau_C\right]}{\left(t_{i+1} - t_i\right)} \cdot \frac{\tau_R}{\tau_C - \tau_R} \cdot \left(\tau_R \cdot e^{-\frac{t_{i+1}-t_i}{\tau_R}} - \tau_C \cdot e^{-\frac{t_{i+1}-t_i}{\tau_C}} + \left(\tau_C - \tau_R\right)\right)$$

(18)

Formatted: Indent: First line: 1,27 cm

The Observables, which allow a comparison to measured data are :

- average river runoff $\overline{R}_i^R = \frac{1}{\tau_R} \cdot \overline{M}_i^R$ corresponding to measured monthly runoff (19)
- average total mass $\overline{M}_i^T = \overline{M}_i^C + \overline{M}_i^R$ corresponding to monthly GRACE data (20)

The equations Eq. (10) - Eq. (20) are self-consistent, i.e. the corresponding balance equations are fulfilled with :

$$\frac{M_{i+1}^T\left(t - t_i\right) - M_i^T}{\left(t - t_i\right)} + \overline{R}_{i+1}^R\left(t - t_i\right) = N_{i+1}$$

(21)

Formatted: Indent: First line: 1,27 cm

For the Single Storage approach the above piecewise analytical solutions of the Cascaded Storage approach, Eq. (8) - Eq. (21), are used for $\tau_R << \tau_C$ (here $\tau_R = 10^{-3}$ months)~~)~~. For this case the river network mass is negligible compared to the catchment mass.

3 Properties and optimization performance

Formatted: English (United States)

For the evaluation of the parameter optimization performance of the Cascaded Storage approach an example with synthetic recharge as input is investigated. This permits the quantification of the uniqueness and accuracy of the parameter estimation undisturbed by noise. It also facilitates the discrimination of errors in the calculation scheme itself and impacts arising from undescribed processes when compared to real world data. For an application to GRACE measurements the main question is if and why the time constants τ_C and τ_R can be determined independently by an optimization versus ~~deviations~~ anomalies in total mass and/or river runoff. Thus, in order to understand the optimization results with respect to uniqueness the general properties of the approach are presented and discussed first. For the synthetic case a recharge time series of sinusoidal form with a period of 12 arbitrary time units and length units with an ~~unit~~ amplitude and mean value of one is used as the driving force and the calculation is run until equilibrium is reached. The example in Fig.2 shows the effect of a non negligible river network time constant τ_R =2.5 time units for a catchment time constant τ_C = 3 time units which leads to an increase in total mass $M_T(t) = M_C(t)+M_R(t)$ ~~w.r.t.~~with respect to the average level and signal amplitude and to a phase shift between total mass M_T and river mass M_R i.e. the corresponding river runoff R_R.

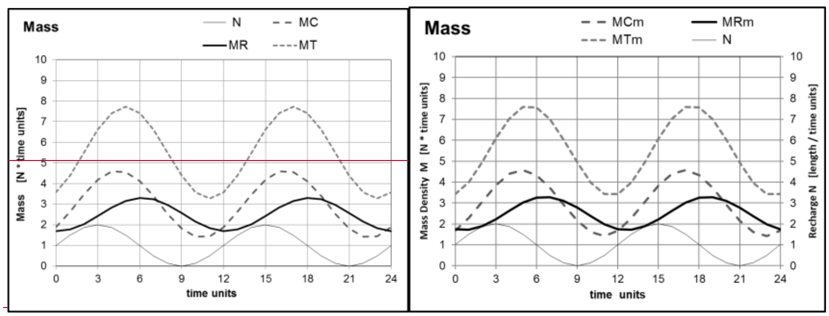


Fig.2: Time series of Recharge N, catchment mass M_C, river network mass M_R, and Total masses M_T for the synthetic case at equilibrium

In order to describe the general behaviour of the mass and runoff time series and their dependence on τ_C and τ_R, their properties are summarized here in the form of statistical values for the synthetic case with the sinusoidal recharge in equilibrium. This helps to understand why unique values for the time constants are achieved in the parameter optimization process. The values of time constants τ_C and τ_R used for the statistical description cover a wide range from 0.1 to 100 time units and are combined independently.

3.1 Catchment and river mass

Based on the mean mass values, Eq.(14), (16), (18), of each stress period the long term averages for the storage compartments are given by :

$$\overline{M}^C = \overline{N}\cdot\tau_C \qquad \overline{M}^R = \overline{N}\cdot\tau_R \qquad \overline{M}^T = \overline{N}\cdot(\tau_C+\tau_R) \qquad (22a,b,c)$$

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: English (United States)

Formatted: Indent: First line: 1,27 cm

For $\tau_R << \tau_C$ (here $\tau_R = 10^{-3}$) the river network mass is negligible and the solution corresponds to a Single Storage approach. For a non negligible river network storage the given average values for total mass M_T mean that the effective "total" time constant is given by the sum of the catchment and river time constants $\tau_T = \tau_C + \tau_R$, which means that the total mass M_T observed by GRACE is bigger than the mass M_C calculated for the catchments alone. However, Equation (22c) cannot be used for the determination of $\tau_T = \tau_C + \tau_R$ from GRACE measurements directly as GRACE only provides mass anomalies.

The relative signal amplitudes (standard deviations normalized with those of thee respective input) of both the catchment mass M_C or river mass M_R show the same functional form $\sigma_{MC} / \sigma_N \sim \sigma_{MR} / \sigma_{RC} = stdev(M_C) / N$ for the respective time constants τ_C or τ_R (Fig.3, $\tau_R = 10^{-3}$) with a monotonous increase to an asymptotic value $\sigma_{MC} / \sigma_N \sim \sigma_{MR} / \sigma_{RC} = 2$ which is reached at about one full period of the input. The superposition of the signal amplitudes for the observable total mass $M_T(t) = M_C(t) + M_R(t)$ leads to a complex behaviour for $\sigma_{MT} / \sigma_N (\tau_C, \tau_R)$ (Fig.3), if the river time constant τ_R is not negligible ($\tau_R = 10^{-3}$) and especially if it gets close to τ_C.

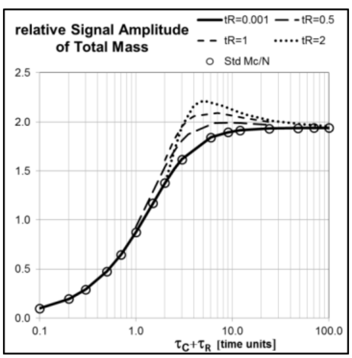


Fig.3: ~~Relative~~ Signal Amplitudes of Total mass normalized by recharge: σ_{MT} / σ_N versus total mass time constant $\tau_T = \tau_C + \tau_R$ for different river time constants τ_R

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

3.2 Catchment and river runoff

Formatted: English (United States)

The calculated long term averages of the runoff contributions R_C and R_R correspond to the ones of the water balance equations, Eq.(4), (6), given by the mean recharge and thus are not dependent on the time constants.

$$\overline{R}^R(t) = \overline{R}^C(t) = \overline{N} \quad (23)$$

Thus, an observed long term average of runoff does not permit the determination of the time constant and hence the storage volume, Eq. (22).

The relative signal amplitudes of both, catchment and river runoff (normalized with the respective input σ_{RC} / σ_N and $\sigma_{RR} / \sigma_{RC}$ show the same functional form corresponding to a Single Storage approach (Fig.4, $\tau_R = 10^{-3}$) and decrease monotonously with the respective time constants τ_C and τ_R to an asymptotic zero. However, the signal amplitude of the observable river runoff σ_{RR} / σ_N ($\tau_C + \tau_R$), normalized with recharge N, shows a ~~2D dependence~~deviations for different ~~combinations in~~ τ_C and τ_R with the same $\tau_C+\tau_R$ (Fig.4).

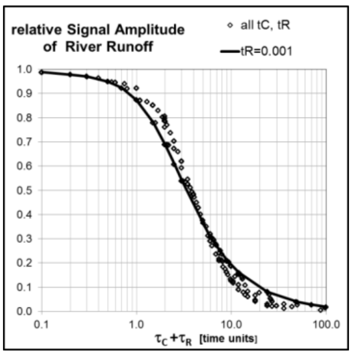


Fig.4: ~~Relative~~ Signal Amplitudes for river runoff normalized by recharge: σ_{RR} / σ_N versus total mass time constant $\tau_T = \tau_C + \tau_R$ for combinations in (τ_C, τ_R)

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Both observables, total mass and river runoff, show a non unique behaviour with respect to combinations in (τ_C, τ_R) for the same $\tau_T = \tau_C + \tau_R$ and considerable deviations from the Single Storage approach ($\tau_R = 10^{-3}$). Measurements of the signal amplitudes thus only provide coarse estimates of the total time constant τ_T, yet do not ~~allow to distinguish~~perrmit distinction between τ_R ~~versus~~ and τ_C and ~~to separate~~betweeen catchment and river network storage.

However, so far, only the signal amplitudes are examined, ~~yet~~ but not the specific properties of the time series, i.e. the dynamic response to input signals in form and phase. The convolution in the solution of the balance equation, Eq.(8) and (11), leads to a different phasing ~~w.r.t.~~with respect to the input N(t), which can be utilized for a separation of the respective time constants.

3.3 Phasing

For the synthetic example with a sinusoidal recharge time series N(t) as input the phasing ω of the different response signals is determined by the fit of a sinusoidal function (Fig.5). This ~~allows to easily determine~~~~permits~~facilitates the easy determination of the phasing and thus the relative phase shift $\Delta\omega$ between the signals. Masses and the related runoffs are in phase for the same storage compartments, Eq.(15). For a negligible river network time constant ($\tau_R = 10^{-3}$) river runoff R_R is in phase with the catchment storage M_C.

Fig.5: Phasing of river network mass ~~w.r.t.~~with respect to recharge time series displayed versus τ_C for different τ_R

The functional form of the phasing ω_{MC} for the catchment mass M_C or the corresponding runoff R_C relative to recharge N(t) (Fig.5) can be empirically described by the monotonous function :

$$\omega_{MC}(\tau_C) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) \tag{24}$$

with the empirical parameters $\omega_{\max} = 2,\!$~~95~~8 and $\lambda =$ ~~2,7~~2.7 and an error $\varepsilon <$ ~~1~~2% relative to the maximum.

As the catchment runoff R_C with the phasing ω_{MC} serves as input into the river system, the phasing of the river system ~~w.r.t.~~with respect to to catchment runoff R_C, which has the same functional form as Eq. (24), is added on top of it (Fig.5). The resulting phasing of the river network storage or river runoff is thus given by a superposition in the form:

$$\omega_{RR}(\tau_C, \tau_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1 - e^{-\frac{\tau_R}{\lambda}}\right) \tag{25}$$

for any combination (τ_C, τ_R) and with the same empirical parameters as in Eq. (24).

As total mass $M_T(t) = M_C(t) + M_R(t)$ is the superposition of the signals with the respective amplitudes and phasing~~s~~, the phasing of total mass $M_T(t)$ is situated between catchment and the river system mass according to τ_R. This means that for non negligible river network mass ($\tau_R > 0$) a phase shift between total mass (GRACE) and observed ~~runoff~~ river discharge and ~~thus~~ also between total mass and modelled catchment mass must occur. The phasing of total mass $M_T(t)$ for all combinations (τ_C, τ_R) Fig.6 shows the same functional form as ω_{MC} and ω_{MR}, Eq.(24), (25) if displayed versus the total time constant $\tau_T = \tau_C + \tau_R$.

Formatted: English (United States)

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Indent: First line: 1,27 cm

Formatted: Not Highlight

Formatted: Not Highlight

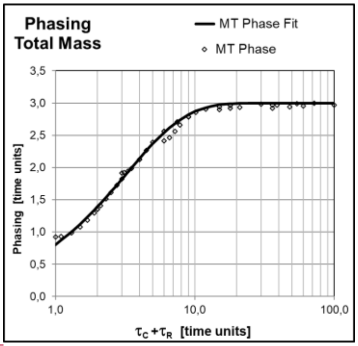


Fig.6: Phasing of Total mass versus total time constant $\tau_T = \tau_C + \tau_R$

It can be approximated by the fitting function M_T fit :

$$\omega_{MT}(\tau_C,\tau_R) = \omega_{\max}\left(1-e^{-\frac{\tau_C+\tau_R}{\lambda}}\right) \tag{26}$$

with the empirical parameters ω_{max} = 2,95 and λ_T = 3,2~~15~~

The phase shift between GRACE total mass and river runoff is thus given by :

$$\Delta\omega(\tau_C,\tau_R) = \omega_{RR} - \omega_{MT} = \omega_{\max}\left(1-e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1-e^{-\frac{\tau_R}{\lambda}}\right) - \omega_{\max}\left(1-e^{-\frac{\tau_C+\tau_R}{\lambda}}\right) \tag{27}$$

The empirical phase shift $\Delta\omega$ from Eq.27 corresponds to the one determined by a phase adaption $\Delta\omega_{adapt}$ (Eq. 38 & 39) of total mass and runoff within <~5% (see supplement). This in principle allows for a determination of τ_C and τ_R separately from the adapted phase shift $\Delta\omega_{adapt}$ and the total mass time constant $\tau_T = \tau_C + \tau_R$ according to Eq. (27).

3.4 Parameter estimation

The analytical solutions for synthetic recharge time series ~~allow to evaluate~~ permit the evaluation of the uniqueness and accuracy of the parameter optimization for given observables independent from limitations in the accuracy of numerical

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Indent: First line: 1,27 cm

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Subscript

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: English (United States)

schemes and independent from noise in real world data sets. For given combinations (τ_C, τ_R) the analytical solutions are used as synthetic measurements and are fitted with the same algorithm in order to retrieve the fit parameters ($\underline{\tau}_C$, $\underline{\tau}_R$).

As the total mass M_T, Eq.(20), and the phasing, Eq.(25-27), are commutative in (τ_C, τ_R), either the data range $\tau_R < \tau_C$ or $\tau_R > \tau_C$ has to be used for a unique optimization. This is realized via an additional constraint in the optimization. For the discussion here the condition $\tau_R < \tau_C$ is used, which hydrologically reflects the more frequent situations that the inundation volume is smaller than the catchment storage but the results can also be applied to $\tau_R > \tau_C$, which might be the case in flat areas with a dense river network (such as the Amazon), which typically leads to temporarily inundated areas.

As absolute signal values are not relevant for the determination of the time constant from runoff or not available for GRACE data, the optimization versus the respective time series is based on signal amplitudes and the phasing. Thus, for a unique determination of ($\underline{\tau}_C$, $\underline{\tau}_R$) the following conditions have to be fulfilled:

a) Optimization versus runoff

$$\sigma_{RR} / \sigma_N (\hat{\tau}_C, \hat{\tau}_R) = \sigma_{RR} / \sigma_N (\tau_C + \tau_R) \tag{28}$$

$$\omega_{RR}(\hat{\tau}_C, \hat{\tau}_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C}{\lambda}}\right) + \omega_{\max}\left(1 - e^{-\frac{\tau_R}{\lambda}}\right) \tag{29}$$

b) Optimization versus mass anomalies

$$\sigma_{MT} / \sigma_N (\hat{\tau}_C, \hat{\tau}_R) = \sigma_{MT} / \sigma_N (\tau_C, \tau_R) \tag{30}$$

$$\hat{\omega}_{MT}(\hat{\tau}_C, \hat{\tau}_R) = \omega_{MT}(\tau_C + \tau_R) = \omega_{\max}\left(1 - e^{-\frac{\tau_C + \tau_R}{\lambda}}\right) \tag{31}$$

With the constraints $\tau_R < \tau_C$ or $\tau_R > \tau_C$ there is only one ($\underline{\tau}_C$, $\underline{\tau}_R$) fulfilling the respective conditions, thus leading to unique solutions. The optimization delivers RMSE errors for the time series in the range 10^{-8} - 10^{-7} and estimated time constants ($\underline{\tau}_C$, $\underline{\tau}_R$) with a relative error $\varepsilon(\underline{\tau}_X)/\tau_X$ which does not depend on absolute values of (τ_C, τ_R) but on their ratio τ_R / τ_C (Fig.7).

Formatted: Indent: First line: 0,75 cm

Formatted: Indent: First line: 0,75 cm

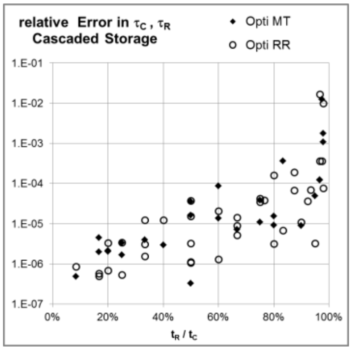


Fig.7: Relative error $\varepsilon(\tau_X) / \tau_X$ of the ~~catchment and river~~ time constants τ_C and τ_R for the Cascaded Storage approach ~~and for~~with an optimizations versus Total mass M_T or versus river runoff R_R.

For the synthetic case relative errors $\varepsilon(\tau_X) / \tau_X$ are very small (~10^{-7} at $\tau_R / \tau_C \sim 0$) and show an exponential increase to a maximum of ~ 1% at $\tau_R \sim \tau_C$. The error for $\tau_R < \tau_C$ is analogous to $\tau_R > \tau_C$ and equal for an optimization versus runoff or mass anomalies ~~deviation~~.

For catchments showing a phase shift between total mass and runoff the description of the system by a Single Storage approach ($\tau_R = 10^{-3}$) leads to a considerably higher relative error $\varepsilon(\underline{\tau}_X) / \tau_X$ in the estimated time constant $\underline{\tau}_C \sim (\tau_C + \tau_R)$ and thus also in Drainable Storage volume. It follows a power function and corresponds to $\varepsilon < 10\%$ for $\underline{\tau}_C < 3$ and $\varepsilon > 40\%$ for $\underline{\tau}_C > 6$. For this case the optimization versus river runoff or mass anomalies ~~deviation~~ leads to different total time constants (rel. Diff. $\varepsilon > 7\%$ for $\underline{\tau}_C > 5$). Even though this might look like an acceptable result for $\underline{\tau}_C < 3$, there are still inevitable deviations in signal amplitudes (10-20%) and phasing between the modelled and measured signals for both total mass and river runoff time series.

It can be summarized that ~~opposite~~ in contrast to the Single Storage approach the Cascaded Storage approach ~~allows to determine~~permits the determination of both time constants (τ_C, τ_R) independently in a unique, highly accurate way for optimizations with respect to either ~~deviations in~~ total mass anomalies or river runoff ~~if recharge is given~~. However, it has to be mentioned that even though the theoretical error in time constants remains below 1% for $\tau_R \sim \tau_C$, the ambiguity for $\tau_R < \tau_C$ or $\tau_R > \tau_C$ cannot be solved without further information on the volume of the river network.

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Highlight

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Highlight

Formatted: Font: Not Bold

4 Application to the Amazon catchment

The R-S diagram of the full Amazon ~~catchment~~ basin shows a hysteresis (Fig.1b, d) corresponding to a phase shift, which ~~is~~ can be interpreted as the time lag of river ~~runoff~~discharge. The Amazon ~~catchment~~ basin upstream Obidos is situated in a fully humid tropic environment with permanent, yet variable recharge and is large enough (4704394km^2) for low noise levels in the signals of GRACE and moisture flux divergence. With permanent ~~input~~ recharge flow contributions ~~of~~ from overland flow and groundwater ~~flow~~ cannot be distinguished in the discharge curve. Also, on a spatial average over the full Amazon basin, with permanent recharge ~~Also~~ the uncoupled storages (like soil water storage, open water bodies etc.) are not time ~~dependent~~ variant, i.e. there is no dry out effect. Any contribution from time dependent, uncoupled storages could be recognized in the R-S diagram as it ~~and thus do not~~would appear as a hysteresis ~~in the R-S diagram~~, which does not correspond to a time lag. ~~.~~ This is not the case.

~~So there is no need for a separation of hydraulically coupled and uncoupled storage components for this catchment. In general, any contributions from time dependent, non drainable i.e. uncoupled storage compartments or from processes like freezing, melting, evapotranspiration etc. can be recognized in the R-S diagram or by the respective deviations between the calculated and measured runoffs and storage volumes and have to be removed from total storage by means of by remote sensing or a conceptual description (Riegger and Tourian, 2014).~~

1. $$N(t) = P(t) - ET(t) \tag{32}$$

   from the hydrometeorological products precipitation P and actual evapotranspiration ETa

2. $$N(t) = -\nabla \cdot \vec{Q} \tag{33}$$

   from atmospheric data, with ~~the~~ monthly vertically integrated moisture flux divergence viMFD

3. $$N(t) = \frac{\partial}{\partial t} M(t) + R(t) \tag{34}$$

   from the terrestrial water balance with monthly ~~total mass M_t, from~~temporal derivatives of GRACE measurements and ~~runoff R_o from the~~ measured river runoff R_o of the catchment.

Here recharge [mm/month] is taken either from the water balance, Eq.(34), or from moisture flux divergence, Eq.(33), provided by ERA-INTERIM of ECMWF and processed by the Institute of Meteorology and Climate Research, Garmisch, Germany. For GRACE mass anomalies ~~deviations~~ data from GeoForschungsZentrum GFZ Potsdam Release 5 are used in mm equivalent water height. Both are handled as described in detail in Riegger and Tourian, 2014. Their spatial resolution limits the application of the approach to global scales >>200000km^2. River discharge is taken from the ORE HYBAM project (http://ore-hybam.org) and converted to runoff [mm/month] by normalization with the catchment area. For a comparison of the calculated river network storage with observations from the "Global Inundation Extent from Multi-Satellite GIEMS (Prigent et al., 2001) flood area [km^2] is used. As GRACE mass anomalies ~~deviation is~~ are most accurate for a monthly time resolution at present, the other data sets are aggregated to a monthly resolution as well. For the parameter optimization time series of river runoff and GRACE mass anomalies~~deviation~~ are used for the time period from January

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Font: 8 pt, Highlight

Formatted: Font: 8 pt

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Superscript

Field Code Changed

2004 until January 2009. Monthly runoff and the storage volume of the catchment and river network are calculated for Amazon based on different recharge products here and optimized either versus runoff or GRACE mass anomalies . The results calculated with recharge from the terrestrial water balance optimized versus GRACE are shown in Figures 8-10 for both (a) the monthly signal and (b) the monthly residual (monthly value minus mean monthly value) for January 2003-2009 .

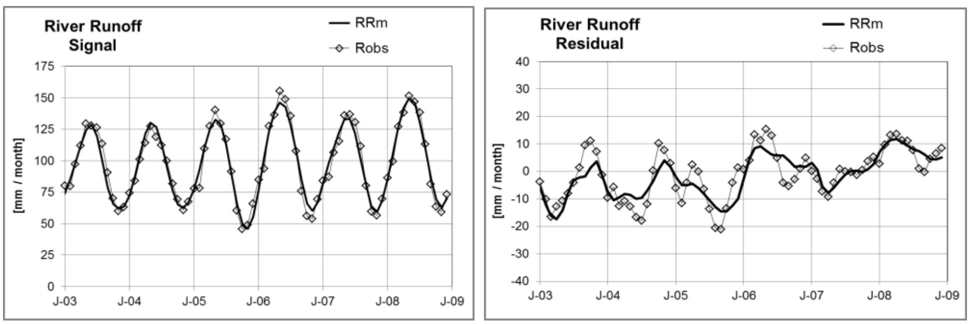


Fig.8: Time series of river runoff for the Amazon basin (Obidos) and optimization versus GRACE (a) for the signal (b) for the residual

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

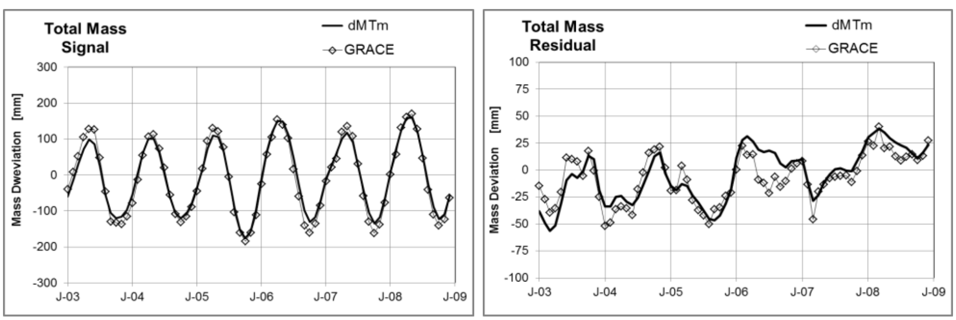


Fig.9: Time series of Total mass anomalies for the Amazon and optimization versus GRACE (a) for the signal (b) for the residual

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

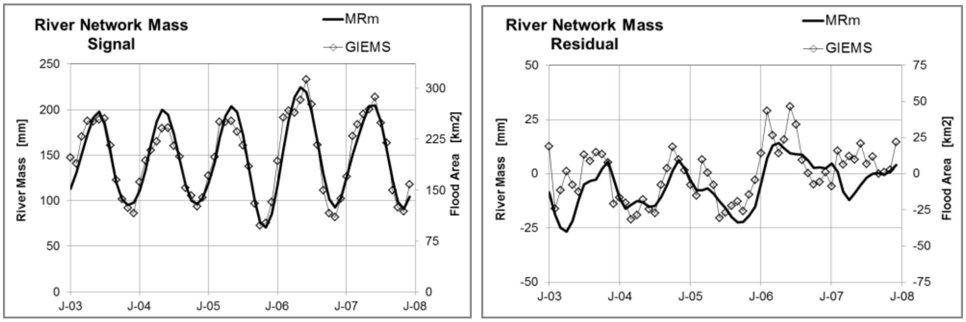


Fig.10: Time series of ~~river~~ River network storage and inundated area from GIEMS for the Amazon (a) for the signal (b) for the residual

The calculated river runoff R_R, total mass ~~deviation~~ anomaly dM_T and river network mass M_R fit very well with the measured river runoff, GRACE ~~mass deviation~~ and the flooded area from GIEMS both with respect to the signal and the de-seasonalized monthly residual.

The Cascaded Storage approach reproduces the phase shift between measured runoff R_o and total mass dM_T (or GRACE respectively). The calculated river network mass M_R of about 50% of the total mass M_T for Amazon is ~~linear~~ proportional to observed runoff R_o without any phase shift (Fig.11)!

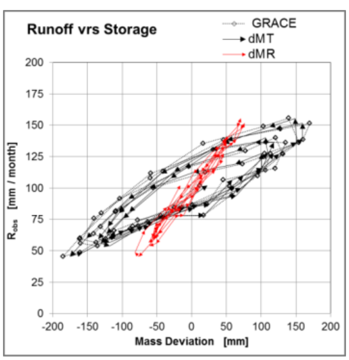


Fig.11: R-S relationships for observed runoff versus the mass anomalies of GRACE, calculated Total mass dM_T and river network mass dM_R

Calculated hydraulic time constants, mean values and signal amplitudes for the absolute storage~~s~~ volumes are provided in Table.2 for the full Amazon basin upstream Obidos.~~.~~ In ad~~d~~dition the performance of optimizations either versus river runoff (Column ~~a~~A) or versus GRACE ~~mass deviation~~ (Column ~~b~~B) and for different recharge products (Column ~~d~~ ~~f~~D, E) is

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: 9 pt

Formatted: Font: Not Bold

displayed. This shows that the optimization versus different references leads to a very similar results while the fitting performance for the two recharge products (Columns A, B and D, E) is quite different. For recharge from water balance, Eq.(34), the resulting time constants and thus the storage masses differ in a range of ~5% for the different references while they vary ~10% for recharge from moisture flux divergence.

In order to illustrate the benefits of the Cascaded versus a Single Storage approach even in the fitting quality, results for a fixed $\tau_R = 10^{-3}$, which correspond to a Single Storage, are shown (Column ~~c~~C, ~~f~~F) for different recharge products. With the Single Storage approach - beside the much worse fitting performance - the resulting time constant $\tau_T = \tau_C + \tau_R$ is overestimated (corresponding to the investigations in section 3) and the modelled signal amplitude is about 20% less than that measured from GRACE. In addition a non negligible phase shift remains between the modelled runoff and measured discharge.

Formatted: Not Highlight

Formatted: Not Highlight

	~~a~~A	~~b~~B	~~c~~C	~~d~~D	~~e~~E	~~f~~F
Approach	**Cascaded**	**Cascaded**	**Single**	**Cascaded**	**Cascaded**	**Single**
Recharge	R+dM/dt	R+dM/dt	R+dM/dt	-divQ	-divQ	-divQ
Optimization	RR	dMT	dMT	RR	dMT	dMT
τ_C [month]	1.53	1.62	3.55	1.68	1.87	3.95
τ_R [month]	1.53	1.62	0.001	1.68	1.87	0.001
Avg M_T [mm]	304.81	321.77	353.29	333.00	370.93	392.02
Avg M_C [mm]	152.17	160.58	353.19	166.23	185.12	391.92
Avg M_R [mm]	152.64	161.18	0.10	166.77	185.81	0.10
Avg R_R [mm month^{-1}]	99.53	99.59	99.39	99.35	99.49	99.34
Avg N [mm month^{-1}]	98.80	98.80	98.80	99.07	99.07	99.07
Stdev M_T [mm]	98.46	100.38	84.09	101.73	105.34	87.83
Stdev M_C [mm]	58.49	60.40	84.06	61.22	65.05	87.80
Stdev M_R [mm]	45.48	46.02	0.02	46.70	47.55	0.02
RMSE R_R-R_o [mm month^{-1}]	5.76	6.08	12.13	11.99	12.57	18.08
RMSE M_T- GRACE [mm]	15.28	14.73	28.93	35.45	34.54	42.31
NS_S R_R-R_o	0.96	0.96	0.84	0.85	0.83	0.65
NS_R R_R-R_o	0.74	0.72	0.73	-0.09	-0.08	-0.10
$corr_S$ R_R-R_o	0.98	0.98	0.94	0.92	0.92	0.82
$corr_R$ R_R-R_o	0.86	0.85	0.87	0.48	0.46	0.41
NS_S dM_T- GRACE	0.98	0.98	0.92	0.89	0.89	0.84
NS_R dM_T- GRACE	0.74	0.72	0.71	-0.57	-0.81	-0.71
$corr_S$ dM_T- GRACE	0.99	0.99	0.98	0.94	0.94	0.93
$corr_R$ dM_T- GRACE	0.90	0.90	0.88	0.58	0.56	0.51
$corr_S$ dGIEMS- GRACE	0.92	0.92	0.92	0.92	0.92	0.92
$corr_S$ GIEMS-M_T	0.93	0.94	0.95	0.82	0.84	0.82
$corr_S$ GIEMS-M_R	0.96	0.95	0.95	0.88	0.86	0.82
$corr_R$ dGIEMS- GRACE	0.65	0.65	0.65	0.65	0.65	0.65
$corr_R$ GIEMS-M_R	0.76	0.75	0.78	0.04	-0.01	0.01

Table.2: The statistical characteristics are listed for calculated river runoff R_R, total mass M_T, catchment mass M_C and river network mass M_R and ~~observations of~~observed river runoff R_o, GRACE ~~(~~mass anomalies ~~deviations)~~ and flood areas from GIEMS using: RMSE: Root-mean-square error of simulated versus measured, NS_S: Nash Sutcliffe coefficient of the signal ~~(for~~ simulated values versus long-term mean of measured~~)~~, NS_R: Nash Sutcliffe coefficient of monthly residuals ~~(for~~ simulated ~~versus~~ values versus monthly mean of measured~~)~~, $corr_S$: correlation of simulated versus measured signals, $corr_R$: correlation of simulated versus measured monthly residuals, Avg and Stdev are the long term mean and standard deviations, prefix "d" used for ~~deviations~~ anomalies ~~from~~ related to the long term mean.

Formatted: Font: Bold

Formatted: Space After: 0 pt

Results are compared for the different optimization references runoff R_o or GRACE ~~different~~for recharge ~~products (~~from water balance R+dM/dt (A, B) and for atmospheric input -divQ (D, E)~~)~~. The Cascaded storage approach is compared to Single storage approach in (C, F).~~as well as for optimizations versus observed runoff R_o or GRACE mass anomalies.~~

This is mainly seen as the result of the quality of recharge data taken from the water balance using GRACE and river runoff, as the use of moisture flux divergence for this purpose leads to much worse results.

~~With the Single Storage approach – beside the much worse fitting performance – the resulting time constant $\tau_T = \tau_C + \tau_R$ is overestimated (corresponding to the investigations in section 3) and the modelled signal amplitude is about 20% less than the that measured from GRACE. In addition a non negligible phase shift remains between the modelled runoff and measured discharge.~~

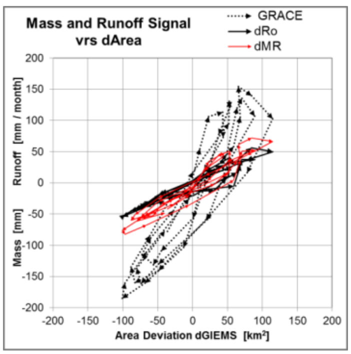


Fig.12: GRACE mass, calculated river network mass dM_R and observed river runoff dR_o versus ~~flood~~ Flood Area d~~from~~ GIEMS; all displayed as anomalies ~~deviations~~ (please consider $dM_R = dR_R \tau_R = 1.53\ dR_R$)

6 Conclusions and Discussion

The test of the Cascaded storage approach with synthetic recharge data has shown that the parameter optimization either versus mass d anomalies or runoff reproduces the time constants (τ_C, τ_R) for both, the catchment and the river network in a unique way with high accuracy, yet with an ambiguity for $\tau_R \le \tau_C$ or $\tau_R \ge \tau_C$, and thus in the related storage volumes. This problem can only be solved by reasonable assumptions or better by additional information on the volume of river network or flood areas, which can be taken from ground based observations or remote sensing. In principle, river network storage could be directly used for the parameter optimization. It could be provided by GIEMS flood areas and water levels from altimetry. This might also help to decide whether the catchment storage is clearly bigger than the river network storage, even if these data are not too accurate. Close to $\tau_R \sim \tau_C$, the generally high accuracy of ~1% RMSE is limited as the separation of the storages depends on the quality of the respective additional information on the volume of the river network. In contrast to this, the description of a system (showing a phase shift) by a Single Storage approach can only address the total Drainable

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Subscript

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: Font: Not Bold

Formatted: English (United States)

Formatted: Not Highlight

Formatted: Not Highlight

storage and leads to phasing differences between the calculated and measured runoff or storage and to considerable errors in the time constant of the total system.

The application to the full Amazon catchment shows that the system behaviour including the time lag can be described by a simple conceptual model with a catchment and a river network storage in sequence and an adjustment of only two parameters, the time constants. The storage amplitudes for the total Drainable water storage and the time lag to runoff are described with high precision. Calculated river network volume and the observed flood area are in phase with river discharge.

The accuracy of the Drainable storage volume mainly depends on the quality of recharge data. The use of recharge from water balance with data from observed river discharge and GRACE is preferable to the use of moisture flux divergence, yet, it limits the approach to a lumped description of basins on global scales (>> 200000km^2) due to the resolution of GRACE. This means that the simplicity and accuracy of this approach is payed by the lack of the spatial information, regardless of the difficulty to evaluate storages volumes locally.

The Cascaded storage approach, if it is based on one catchment and one river network storage only, is limited to climatic and physiographic conditions for which the hysteresis is completely explained by a time lag. This prerequisite is fulfilled if the R-S diagram shows a counter clockwise hysteresis and if a phase shift adaption leads to positive values and a linear relationship. If any contributions from time dependent, not drainable i.e. uncoupled storage compartments occur, which could result from processes like freezing, melting, evapotranspiration etc., these can be recognized in the R-S diagram (Fig.1d) or by the respective deviations in the scatter plots of calculated versus measured runoff or storage volumes (see supplement). For this case both, the time dependent coupled and uncoupled storage components have to be addressed explicitly in the lumped model description of the system. The uncoupled storage components then have to be quantified either with their absolute storage volume or by their relative contribution to total storage.

As Riegger and Tourian, 2014, have shown for boreal catchments, this can be done by means of remote sensing and a conceptional description. Boreal catchments are temporarily dominated by snow leading to a huge hysteresis due to a superposition of masses from fully coupled (liquid) and uncoupled (solid) storage compartments. Remote sensing of the catchments snow coverage by MODIS facilitates the separation of the coupled liquid storage (proportional to river runoff) and the uncoupled frozen part. The coupled liquid storage determined in this way actually constitutes a LTI system, i.e. the hysteresis can be fully explained by a phase shift. This fulfils the prerequisites for the Cascaded storage approach and thus permits an application to boreal catchments as well. In consequence, the principle of the Cascaded storage approach is not limited to fully humid climatic conditions. It permits an application to other climatic regions as well provided that the coupled and uncoupled storage can be quantified, which of course is a major task.

~~The determination of the hydraulic time constants (τ_{GR}) via the R-S relationship taken from measured runoff and GRACE mass variations allows to quantify the quantification of Drainable Storage Volumes in catchments under the prerequisite that~~

As the optimization performance is comparable for either reference, the observed river runoff or GRACE anomalies, a calculation with given recharge and an optimization versus measured GRACE data can be used to determine both, the river discharge as well as the Drainable storage volumes even for ungauged basins. For these cases the availability of accurate recharge data limits the accuracy of runoff and storage calculations at present. However, for ungauged basins the use of moisture flux divergence still provides quite acceptable results based on remote sensing and atmospheric data exclusively.

Where river discharge is available the Cascaded storage approach facilitates a simple determination of the Drainable water storage volumes both for the catchment and for the river network directly from observations without the necessity of new model runs.

~~The related time series have to be calculated by the given piecewise analytical solutions leading to different phasing for non negligible τ_R. The time series of the total Drainable Storage volume can be calculated directly from GRACE measurements and long term average of recharge or runoff as the calculated total mass deviation corresponds to the GRACE signal:~~

These can be determined by the following calculations :~~With the hydraulic time constants (τ_C, τ_R) determined by the Cascaded Storage approach for a given optimization time period following data and time series can be calculated :~~

~~according to Eq. (22a,b,c) and Eq. (23) :~~
according to Eq. (22a,b,c) and Eq. (23) :

2. Time series of Drainable storage volumes ~~without a phase shift directly~~ from GRACE and observed runoff R_o without the need for a phase adaption:

$$M_{sim}^{R}(t) = \tau_R \cdot R_o(t) \qquad (35)$$

(NSS 0.961, NSR 0.576, corrR 0.859 vs M_R, from Eq.18)

$$M_{sim}^{T}(t)=dM^{T}(t)+\overline{M}^{T}=GRACE(t)+\overline{M}^{T}=GRACE(t)+\tau_T\cdot\overline{N}\cdot=GRACE(t)+\tau_T\cdot\overline{R}_0 \qquad (36)$$

(NSS 0~~,~~.973, NSR 0.751, corrR 0.901 vs M_T~~,~~ from Eq.20)

$$M_{sim}^{C}(t)=M^{T}(t)-M^{R}(t)=GRACE(t)+\tau_T\cdot\overline{R}_o\ -\ \tau_R\cdot R_o(t) \qquad (3~~6~~7)$$

(—NSS 0~~,~~.906, NSR -0.065, corrR 0.607 vs M_C~~,~~ from Eq.16)

The simplified calculations directly from observations lead to accurate equivalences to the fully calculated time series of total and the river network storage volumes M_T and M_R and to a reasonable description of the catchment storage volumes M_C.

3. Time series of total Drainable storage volumes M_T, from observed runoff R_0 or simulated river runoff R^R_{sim} from GRACE with a numerical phase adaption of Δω.

Use of the phase shift Δω between GRACE and observed river runoff for a linear temporal interpolation (Riegger and Tourian, 2014) permits a simple description of river runoff directly from GRACE (Eq.38) or of total Drainable water storage MT directly from observed runoff (Eq.39). The phase shift $\Delta\omega_{adapt}$ □□□□also □□□adapted □irectly (Eq.38, Eq.39) and corresponds to Δω from Eq.27 within <~10%. Both lead to very similar fitting performances. ~~with a phase shift with a simulation of the phase shift applied on GRACE and observed runoff R_o~~

$$R_{sim}^{R}(t_i)\ =\ \frac{1}{\tau_T}\cdot\left[\ (1-\Delta\omega)\cdot GRACE(t_i)\ +\ \Delta\omega\cdot GRACE(t_{i-1})\ +\ \tau_T\cdot\overline{R}_o\right] \qquad (38)$$

(NSS 0~~,~~.94~~8~~3, NSR 0.698, corrR 0.864 vs measured R_o.)

$$M_{sim}^{T}(t_i)=\tau_T\cdot\left\lfloor\ (1-\Delta\omega)\cdot R_o(t_i)\ +\ \Delta\omega\cdot R_o(t_{i+1})\ \right\rfloor \qquad (3~~8~~9)$$

(NSS 0~~,~~.9~~52~~46, NSR 0.483, corrR 0.859 —vs ~~measured M_T, Eq.35~~GRACE))

This facilitates a simple ~~and quite accurate~~ determination of Drainable storage volume ~~or~~or runoff time series directly from measurements and thus a closure of data gaps with high accuracy comparable to the best performances in the LSM test by Getirana et al. (2014). The related calculations are accessible in the xls-workbook provided in the supplement.

Field Code Changed

Formatted: Indent: Left: 6,35 cm

Field Code Changed

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Indent: Left: 0 cm

Formatted: Not Highlight

Formatted: Indent: Left: 1,27 cm, No bullets or numbering

Formatted: Superscript

Formatted: Subscript

Formatted: Font: Times New Roman

Formatted: Font: Times New Roman

Formatted: Font: Times New Roman

Formatted: Highlight

Formatted: Indent: Left: 1,9 cm

Field Code Changed

Formatted: Indent: First line: 1,27 cm

Field Code Changed

Formatted: Indent: Left: 6,35 cm

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

~~and thus permits a closure of data gaps in drainable mass or runoff observations by simple calculations without the necessity of additional model runs.~~

The lumped description of the water storages in catchment systems by a cascade of catchment storages and a river network storage facilitates the quantification of their individual Drainable storage volumes and their phasing directly from large scale observations of GRACE, remote sensing and river discharge. No detailed information on vegetation, soil etc., complex flow processes nor hydrodynamic modelling with detailed hydraulic information on river roughness, cross section, gradient or backwater effects is needed. An optimization versus GRACE measurements permits a determination of river runoff and Drainable storage volumes from recharge and GRACE exclusively, and thus provides reasonable results for ungauged catchments. The piecewise analytical solutions for time periods of constant recharge provide accurate calculations even for the much shorter time constants of the river network without numerical limitations.

The fact, that river network and flood volumes can be quantified by this approach independently, permits an investigation of the relationship between flood areas, flood volumes, river runoff and calculated river network with additional information and might provide insights into river hydraulics i.e. routing times and the mass- area- and level- relationships of flooded areas.

The individual adjustment of the hydraulic time constants for the catchment and the river network (τ_C, τ_R) on a training period facilitates a determination of Drainable storage volumes M_C, M_R and M_T at other times directly from measurements of river discharge and GRACE without the necessity of further model runs. River runoff can also be determined directly from GRACE and vice versa by an adaption of the time lag. This permits to close data gaps in river discharge or GRACE time series and even provides the possibility for operational forecasts within the period of the time lag.

For a global coverage the Cascaded storage approach has to be extended for an implementation of the uncoupled storage components according to the regional climatic and physiographic conditions. For boreal catchment MODIS snow coverage can be used for a quantification or coupled and uncoupled storage components. The description of monsoonal regions, which play an important role in the global water budget, remains a major challenge.

~~In this case both surface runoff and groundwater flow with their individual time constants τ_S and τ_{GW} have to be considered as parallel input into the river network storage and are both subject to the phase shift introduced by a non negligible time constant of the river system τ_R.~~

As the spatial resolution of GRACE and the accuracy of moisture flux divergence is limiting ~~applications~~ the Cascaded storage approach to ~~large~~ global scale catchments ~~(>200000km^2)~~ at the moment, any improvement in the spatial / temporal resolution and accuracy of GRACE and hydrometeorological data products will tremendously increase the number of catchments which can be described ~~in their system behaviour by remote sensing exclusively~~by this approach in future.

Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Subscript, Not Highlight
Formatted: Not Highlight
Formatted: Subscript, Not Highlight
Formatted: Not Highlight
Formatted: Subscript, Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: Not Highlight
Formatted: English (United States)
Formatted: Not Highlight

In the supplement ~~C~~calculations and data are provided in an EXCEL workbook for the synthetic case and for the Amazon catchment. ~~in the supplement.~~

Acknowledgements

The author would like to thank Nico Sneeuw and Mohammad Tourian from the Institute of Geodesy, Stuttgart, for the handling of GRACE data , Harald Kunstmann and Christof Lorenz from the Institute of Meteorology and Climate Research, Garmisch, for the provision of moisture flux data and Catherine Prigent, Observatoire de Paris, for the provision of Flood areas from the GIEMS project.

References

Alley, W. M.: Tracking U.S. groundwater reserves for the future?, Environment, 48(3), 10–25, 2006.

Bredehoeft, J.: Safe yield and the water budget myth, Ground Water, 35(6), 929–929, doi:10.1111/j.1745-6584.1997.tb00162.x, 1997

Cao, G., Zheng, C., Scanlon, B. R., Liu, J., and Li W.: Use of flow modeling to assess sustainability of groundwater resources in the North China Plain, Water Resour. Res., 49, doi:10.1029/2012WR011899, 2013

~~Chen, J. L.; Wilson, C. R.; Famiglietti, J. S. et al.: Attenuation effect on seasonal basin-scale water storage changes from GRACE time-variable gravity, .Journal of Geodesy (4) 237-245, 2007~~

Frappart, F., Papa, F., da Silva, .,J.S., Ramillien, G., Prigent, C., Seyler, F., Calmant, S.: Surface freshwater storage and dynamics in the Amazon basin during the 2005 exceptional drought, Environ. Res. Lett. 7, doi: 10.1088/1748-9326/7/044010, 2012

Getirana, A. et al.: Water Balance in the Amazon Basin from a Land surface Model Ensemble, Journal of Hydrometeorolgy, 2586-2614,doi:10.1175/JHM-D-14-0068.1, 2014~~Güntner, A., Stuck, J., Werth, S., Döll, P., Verzano, K., and Merz, B.: A global analysis of temporal and spatial variations in continental water storage, Water Resour. Res. 43, W05416, doi:10.1029/2006WR005247, 2007~~

Getrirana, A., Kumar, S., Girotto, M., & Rodell, M.: River and Floodplains as Key Components of Global Terrestrial Water Storage Variability, Geophysical Research Letters, 44-50, doi: 10.1002/2017GL074684, 2017a

~~Güntner, A.: Improvement of global hydrological models using GRACE data, Surv. Geophys., 29, 375–397, 2009~~

Korzun, V. I.: World Water Balance and Water Resources of the Earth [in Russian], 638 pp., Hydrometeoizdat, Leningrad, 1974

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Formatted: English (United States), Not Highlight

Formatted: Not Highlight

Formatted: Not Highlight

Luo, X., Li, H.-Y., Leung, L. R., Tesfa, T. K., Getirana, A., Papa, F., and Hess, L. L.: Modeling surface water dynamics in the Amazon Basin using MOSART-Inundation v1.0: impacts of geomorphological parameters and river flow representation, Geosci. Model Dev., 10, 1233-1259, https://doi.org/10.5194/gmd-10-1233-2017, 2017.

Nace, R. L.: World water inventory and control, in Water, Earth, and Man, edited by R. J. Chorley, pp. 31–42, Methuen, London, U. K., 1969

Nash, J.E.: The form of the instantaneous unit hydrograph, Proc. I.A.S.H. Assem. Gen., Toronto, Ont., 3: 114-131, 1957

Paiva, R.C., Buarque, D.C., Collischonn, W., Bonnet, M. P., Frappart, F., Calmant, S., Mendes, C.A.B.: Large scale hydrologic and hydrodynamic modelling or the Amazon river Basin, Water Resour. Res., 49, doi: 10.1002/wrcr.20067, 2013

Papa, F., Güntner, A., Frappart, F., Prigent, C., Rossow, W.B.: Variations of surface water extent and water storage in large river basins: A comparison of different global sources, Geophysical Research Letters, 35, doi: 10.1029/2008GL033857, 2008

Papa, F., Frappart, F., Güntner, A., Prigent, Aires, F., Getirana, A.C.V., Maurer, R.: Surface freshwater storage and variability in the Amazon basin from multi-satellite observation, 1993-2007, J.~~ournal~~ ~~of~~ Geophys.~~ical~~ Res.~~earch~~: Atmos.~~pheres~~, *118*, 11,951-11,965, doi: 10.1002/2013JD020500, 2013

Prigent, C., E. Matthews F. Papa, F. Aires, Rossow, W. B.: Remote Sensing of global wetland Dynamics with multiple satellite data sets, Geo. Res. Lett., 28, 4631-4634, 2001

Prigent, C., F. Papa, F. Aires, F., Rossow, W. B. , and Matthews E.: Global inundation dynamics inferred from multiple satellite observations, 1993 – 2000, J. Geophys. Res., 112, D12107, doi:10.1029/ 2006JD007847, 2007

Richey, A. S., Thomas, B. F., Lo, M.-H., Famiglietti, J. S., Swenson, S., and Rodell M.: Uncertainty in global groundwater storage estimates in a Total Groundwater Stress framework, Water Resour. Res., 51, 5198–5216, doi:10.1002/2015WR017351, 2015a

Richey, A. S., Thomas, B. F., Lo, M.-H., Reager, J. T., Famiglietti, J. S., Voss, K., Swenson, S., and Rodell M.: Quantifying renewable groundwater stress with GRACE, Water Resour. Res., 51, 5217–5238, doi:10.1002/ 2015WR017349, 2015b

Riegger, J., Tourian, M., Devaraju, B., Sneeuw, N., Analysis of grace uncertainties by hydrological and hydro-meteorological observations, Journal Geodyn., 5960, 16-27, doi:10.1016/j.jog.2012.02.001, 2012

Riegger, J., and Tourian, M. J.: Characterization of runoff-storage relationships by satellite gravimetry and remote sensing. Water Resour. Res., 50, doi:10.1002/2013WR013847, 2014

Schlesinger, M.E.: Human-induced climate change : an interdisciplinary assessment, xviii, Cambridge University Press, 2007

Tapley, B. D., Bettadpur, S., Ries, J. C., Thompson, P. F., and Watkins M. M.: GRACE measurements of mass variability in the Earth system, Sciences N. Y., 305(5683), 503–505, doi:10.1126/science.1099192, 2004

Tourian, M. J., Reager, J. T. and Sneeuw, N.: The Total Drainable Water Storage of the Amazon River Basin: A First Estimate Using GRACE, Water Resources Research, 54(5), 3290–3312, doi:10.1029/2017WR021674, 2018.

Formatted: English (United States)

Formatted: Font: Italic

Williamson, A. K., Prudic, D. E., and Swain L. A.: Ground-water flow in the Central Valley, California, U.S. Geol. Surv. Prof. Pap. 1401-D, 225 pp, 1989

~~Schmidt, R., Petrovic, S., Güntner, A., Barthelmes, F., Wünsch, J., and Kusche, J.: Periodic components of water storage changes from GRACE and global hydrology models. J. Geophys. Res., 113, B08419, oi:10.1029/2007JB005363, 2008~~

Siqueira, V. A., Paiva, R. C. D., Fleischmann, A. S., Fan, F. M., Ruhoff, A. L., Pontes, P. R. M., Paris, A., Calmant, S., and Collischonn, W.: Toward continental hydrologic–hydrodynamic modeling in South America, Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess- 2018-225, in review, 2018

Sophocleous, M.: Managing water resources systems: Why ''safe yield'' is not sustainable, Ground Water, 35(4), 561–561, doi:10.1111/ j.1745-6584.1997.tb00116.x, 1997

Tallaksen, L.M.: A review of baseflow recession analysis, Journal of Hydrology, No. 165, S. 349-370, 1995

~~Werth, S., Güntner, A., Schmidt, R., and Kusche, J.: Evaluation of GRACE filter tools from a hydrological perspective. Geophys. J. Int., 179, 1499–1515, doi: 10.1111/j.1365-46X.2009.04355.x, 2009~~