Supplement

This is a supplement accompanying the paper:

Redressing the balance: quantifying net intercatchment groundwater flows in the Meuse basin

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The supplement provides details on the water balance equations, constitutive functions and the model parameters (including prior and posterior distributions). Additionally, the supplement contains an analysis of the inter-annual variability of net intercatchment groundwater flow processes.

1 Model equations

Symbols used to define the different fluxes and stores in the model schematizations (see Figure 4 of the paper) are detailed in Table 1 and Table 2, definitions of the symbols used for the parameters are provided in Section 2 of the Supplement. Water balance and constitutive equations of the zero, constant, preferential and overflow intercatchment groundwater flows models are provided in Table 3 and in Table 4.

Table 1: Definitions of the symbols used to denote the different fluxes in the models.

Fluxes (mm hr^{-1})	Definition
P	Precipitation
E_I	Evaporation from interception
E_U	Evaporation from the root zone storage
R_{IU}	Effective precipitation
R_{US}	Recharge to the slow reservoir
R_{UF}	Recharge to the fast reservoir
R_P	Percolation
Q_F	Fast runoff
Q_S	Slow runoff
Q_{River}	Discharge which ends up in the river
$Q_{IGF,constant}$	Net constant intercatchment groundwater flows
$Q_{IGF,pref.}$	Net preferential intercatchment groundwater flows
Q_{IGF}	Net intercatchment groundwater flows from overflow model

Table 2: Definitions of the symbols used to denote the different stores in the models.

Stores (mm)	Definition
S_I	Interception storage
S_U	Root zone storage
S_F	Fast reservoir store
S_S	Slow reservoir store

Table 3: Water balance equations. The ✓indicates for which model(s) the water balance equations apply.

Water balance equation	Zero	Constant	Preferential	Overflow
$\frac{dS_I}{dt} = P - E_I - R_{IU}$	✓	✓	✓	✓
$\frac{dS_U}{dt} = R_{IU} - E_U - R_P - R_{US} - R_{UF}$	✓	✓	✓	
$\frac{dS_U}{dt} = R_{IU} - E_U - R_{US}$				✓
$\frac{dS_F}{dt} = R_{UF} - Q_F$	✓	✓	✓	
$\frac{dS_F}{dt} = R_{SF} - Q_{River}$				✓
$\frac{dS_S}{dt} = R_{US} + R_P - Q_S$	✓		✓	
$\frac{dS_S}{dt} = R_{US} + R_P - Q_S - Q_{IGF}$		✓		
$\frac{dS_S}{dt} = R_{US} - R_{SF} - Q_{IGF}$				✓
$Q_{River} = Q_S + Q_F$	✓	✓	✓	
$Q_{Tot} = Q_{River} + Q_{IGF}$	✓	✓	✓	✓

Table 4: Constitutive functions. The ✓indicates for which model(s) the constitutive functions apply. The following values are fixed for the smoothing parameters $m_1 = m_2 = m_4 = 0.005$ and $m_3 = 0.05$ (σ value of the error function).

Constitutive functions	Zero	Constant	Preferential	Overflow
$\overline{S_I} = \frac{S_I}{I_{max}}$	✓	✓	✓	✓
$\overline{S_U} = \frac{S_U}{S_{U,max}}$	✓	✓	✓	✓
$\overline{S_S} = \frac{S_S}{S_{S,max}}$				✓
$E_I = E_P \cdot \frac{\overline{S_I} \cdot (1+m_1)}{\overline{S_I}+m_1}$	✓	✓	✓	✓
$R_{IU} = P \cdot (1 - \frac{(1-\overline{S_I})(1+m_2)}{1-\overline{S_I}+m_2})$	✓	✓	✓	✓
$E_U = (E_P - E_I) \cdot \frac{\overline{S_U}}{L_p}$	✓	✓	✓	✓
$R_U = R_{US} + R_{UF}$	✓	✓	✓	
$R_U = P \cdot \overline{S_U}^{\beta}$	✓	✓	✓	
$R_{US} = R_U \cdot d$	✓	✓	✓	
$R_{UF} = R_U \cdot (1 - d)$	✓	✓	✓	
$R_P = P_{max} \cdot \overline{S_U}$	✓	✓	✓	
$Q_F = K_F^{-1} \cdot S_F^{\alpha}$	✓	✓	✓	
$Q_S = K_S^{-1} \cdot S_S$	✓	✓	✓	
$Q_{IGF} = C_{IGF}$		✓		
$Q_{IGF} = erf(R_{US}, \mu, m_3) \cdot P_{erc} \cdot R_{US}$			✓	
$Q_{IGF} = K_{IGF}^{-1} \cdot S_S$				✓
$R_{US} = P \cdot \overline{S_U}^{\beta}$				✓
$R_{SF} = R_{US} \cdot \frac{\overline{S_S} \cdot (1+m_4)}{\overline{S_S}+m_4}$				✓
$Q_{River} = K_{River}^{-1} \cdot S_F$				✓

2 Prior and posterior parameter distributions

A description of model parameters, units and prior range is provided in Table 5. Posterior parameter ranges for the zero, constant and preferential models are given in Table 6. For the Aroffe catchment, the posterior parameter distributions of the overflow model are shown in Table 7.

Table 5: Model parameters, units and prior range (*MRC denotes the value determined with a master recession curve ± 10 days). The ✓indicates for which model(s) the parameters apply.

Parameter	unit	Definition	Range	Zero	Constant	Preferential	Overflow
I_{max}	mm	Maximum interception capacity	1 - 3	✓	✓	✓	✓
$S_{U,max}$	mm	Root zone storage capacity	50 - 350	✓	✓	✓	✓
β	-	Shape parameter of storage capacity distribution	1 - 5	✓	✓	✓	✓
L_p	-	Reduction parameter for potential evaporation	0 - 1	✓	✓	✓	✓
K_F	h	Characteristic time scale of the fast recession	2 - 960	✓	✓	✓	
K_S	h	Characteristic time scale of the slow recession	MRC*	✓	✓	✓	
T_F	h	Time lag	1 - 20	✓	✓	✓	
d	-	Fraction to slow reservoir	0 - 1	✓	✓	✓	
P_{max}	mm h^{-1}	Maximum percolation rate	0 - 0.05	✓	✓	✓	
α	-	Non linear coefficient of the fast reservoir	1 - 2	✓	✓	✓	
C_{IGF}	mm h^{-1}	Constant net intercatchment groundwater flow (IGF_{net})	-0.01 - 0.02		✓		
μ	mm h^{-1}	Threshold of the recharge above which IGF_{net} occurs	0.005 - 0.9			✓	
P_{erc}	-	Fraction of the recharge to IGF_{net}	-0.5 - 1			✓	
K_{IGF}	h	Characteristic time scale of the IGF_{net}	5 - 600				✓
K_{River}	h	Characteristic time scale of the river flow	5 - 600				✓
$S_{S,max}$	mm	Maximum capacity of underground stores	1 - 60				✓
d_{IGF}	-	Fraction to IGF_{net} reservoir	0.5 - 1				✓

Table 6: Posterior parameter range (5-95 percentiles) for the zero, constant and preferential models for a selection of parameters

Parameter	$S_{u,max}$	L_p	K_F	d	P_{max}	α	C_{IGF}	P_{erc}
Unit	mm	-	h	-	mm h^{-1}		mm h^{-1}	-
Prior	50 - 350	0 - 1	2 - 960	0 - 1	0 - 0.05	1 - 2	-0.01 - 0.02	-0.5 - 1
Sainte-Marie - Zero	224 - 324	0.1 - 0.5	108 - 908	0.02 - 0.16	0.007 - 0.020	1.5 - 2.0		
Sainte-Marie - Constant	220 - 309	0.5 - 0.9	83 - 747	0.00 - 0.15	0.016 - 0.038	1.3 - 2.0	0.004 - 0.016	
Sainte-Marie - Pref.	64 - 273	0.0 - 0.8	68 - 542	0.17 - 0.42	0.003 - 0.020	1.1 - 1.9		0.7 - 1.0
Straimont - Zero	162 - 226	0.1 - 0.5	316 - 904	0.07 - 0.18	0.001 - 0.013	1.2 - 1.6		
Straimont - Constant	102 - 292	0.0 - 0.9	234 - 934	0.06 - 0.20	0.001 - 0.020	1.1 - 1.5	-0.003 - 0.006	
Straimont - Pref.	152 - 291	0.1 - 0.7	258 - 886	0.03 - 0.16	0.001 - 0.010	1.1 - 1.5		-0.3 - 0.6
Tintigny - Zero	144 - 318	0.3 - 0.8	177 - 931	0.02 - 0.12	0.002 - 0.012	1.2 - 1.6		
Tintigny - Constant	125 - 248	0.3 - 0.8	151 - 896	0.02 - 0.13	0.002 - 0.028	1.2 - 1.7	-0.004 - 0.008	
Tintigny - Pref.	152 - 303	0.4 - 0.8	108 - 876	0.03 - 0.20	0.001 - 0.011	1.1 - 1.6		-0.4 - 0.9
Chiny - Zero	166 - 283	0.2 - 0.8	203 - 948	0.01 - 0.14	0.003 - 0.016	1.2 - 1.6		
Chiny - Constant	140 - 314	0.0 - 0.8	182 - 901	0.01 - 0.13	0.003 - 0.025	1.1 - 1.6	-0.002 - 0.008	
Chiny - Pref.	111 - 268	0.2 - 0.7	122 - 865	0.03 - 0.18	0.004 - 0.016	1.1 - 1.6		-0.4 - 0.9
Membre-Pont - Zero	114 - 232	0.1 - 0.6	307 - 884	0.01 - 0.16	0.004 - 0.022	1.2 - 1.5		
Membre-Pont - Constant	107 - 237	0.0 - 0.7	258 - 922	0.01 - 0.14	0.002 - 0.031	1.2 - 1.5	-0.005 - 0.003	
Membre-Pont - Pref.	129 - 260	0.1 - 0.7	195 - 919	0.02 - 0.15	0.004 - 0.020	1.1 - 1.5		-0.4 - 0.9
Huccorgne Pref.	146 - 316	0.3 - 0.8	86 - 837	0.07 - 0.52	0.006 - 0.015	1.3 - 1.8		-0.5 - 1.0
Yvoir Pref.	110 - 250	0.4 - 0.9	179 - 908	0.39 - 0.65	0.009 - 0.024	1.4 - 1.9		0.7 - 0.9
Sormonne Pref.	119 - 299	0.0 - 0.8	70 - 818	0.41 - 0.58	0.000 - 0.017	1.3 - 2.0		0.6 - 1.0
Crusnes Pref.	112 - 295	0.2 - 0.7	271 - 782	0.36 - 0.58	0.001 - 0.017	1.0 - 1.4		0.4 - 0.9

Table 7: Posterior parameter range (5-95 percentiles) for overflow model used in the Aroffe catchment at Vannes-le-Châtel

Parameter	I_{max}	$S_{u,max}$	β	L_p	K_{River}	K_{IGF}	$S_{S,max}$	d_{IGF}
Unit	mm	mm	-	-	h	h	mm	-
Prior	1 - 3	50 - 350	1 - 5	0 - 1	5 - 600	5 - 600	1 - 60	0.5 - 1
Aroffe - Overflow	1.0 - 2.9	88 - 210	2.0 - 4.9	0.2 - 0.8	87 - 226	212.7 - 594.1	30.9 - 52.0	0.75 - 0.83

3 Inter-annual variability of net intercatchment groundwater flows

The constant net intercatchment groundwater flow (IGF$_{net}$) model implies the same magnitude of IGF$_{net}$ each year, while in the preferential model, IGF$_{net}$ may vary inter-annually depending on meteorological conditions. Although meant for long term averages, the Budyko framework is shown for hydrological years in the Semois catchment at Sainte-Marie in Figure 1. Years with the highest precipitation amounts (2007 and 2012) plot beyond the energy limit, perhaps an indication that certain thresholds are exceeded and that more underground losses towards neighboring catchments occur in these years. Although the observed yearly variability may also be caused by changes in storage in the catchment, we consider that there may be years with more IGF$_{net}$ and we show that the preferential IGF$_{net}$ model is able to reproduce this behavior. Indeed, modeled mean annual net losses using the preferential model increase as the distance to the energy limit of each hydrological year decreases, as shown in Figure 1.

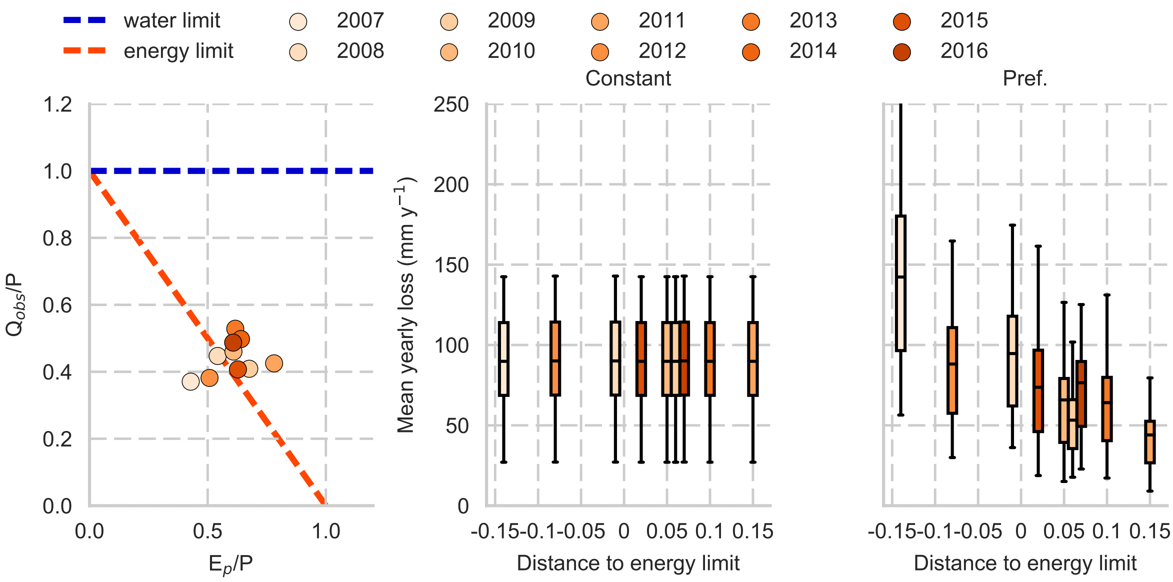

Figure 1: Left: dimensionless representation of the runoff coefficient $(Qobs/P)$ as a function of the dryness index (E_p/P), referred to as the Budyko framework, for hydrological years in the Semois catchment at Sainte-Marie. The blue line shows the water limit and the red line is the energy limit. Middle: mean yearly modeled net loss using the feasible realizations of the constant model as a function of the distance of each year to the energy line. Right: mean yearly modeled net loss using the feasible realizations of the preferential model as a function of the distance of each year to the energy line. The relation between the magnitude of the net loss and the distance to the energy line for each hydrological year shows that the preferential model is able to reproduce the observed inter-annual variability of the water balance by generating higher losses in years that plot beyond or close to the energy limit.