Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

Hydrology and Earth System Sciences
Discussions

Stochastic reconstruction of spatio-temporal rainfall pattern by inverse hydrologic modelling

Jens Grundmann[1], Sebastian Hörning[2], and András Bárdossy[3]

[1]Technische Universität Dresden, Institute of Hydrology and Meteorology, Dresden, Germany
[2]University of Queensland, School of Earth and Environmental Sciences, Brisbane, Australia
[3]Universität Stuttgart, Institute for Modelling Hydraulic and Environmental Systems, Stuttgart, Germany

Correspondence: (jens.grundmann@tu-dresden.de)

Abstract. Knowledge about the spatio-temporal rainfall pattern is required as input for distributed hydrologic models to perform several tasks in hydrology like flood runoff estimation and modelling. Normally, these pattern are generated from point observations on the ground using spatial interpolation methods. However, such methods fail in reproducing the true spatio-temporal rainfall pattern especially in data scarce regions with poorly gauged catchments or for highly dynamic, small scaled rainstorms which are not well recorded by existing monitoring networks. Consequently, uncertainties are associated with poorly identified spatio-temporal rainfall pattern in distributed rainfall-runoff modelling since the amount of rainfall received by a catchment as well as the dynamics of the runoff generation of flood waves are underestimated. For addressing this problem we propose an inverse hydrologic modelling approach for stochastic reconstruction of spatio-temporal rainfall pattern. The methodology combines the stochastic random field simulator Random Mixing and a distributed rainfall-runoff model in a Monte-Carlo framework. The simulated spatio-temporal rainfall pattern are conditioned on point rainfall data from ground monitoring networks as well as the observed hydrograph at catchment outlet and aims to explain measured data at best. Since we conclude from an integral catchment response on a three-dimensional input variable, several candidates of spatio-temporal rainfall pattern are possible which also describe their uncertainty. The methodology is testet on a synthetic rainfall-runoff event on subdaily timesteps and spatial resolution of 1km² for a catchment covered by rainfall partly. Results show that a set of plausible spatio-temporal rainfall pattern can be obtained by applying the inverse approach. Furthermore, results of a real world study for a flash flood event in a mountainious arid region are presented. They underline that knowledge about the spatio-temporal rainfall pattern is crucial for flash flood modelling even in small catchments and arid and semiarid environments.

1 Motivation

The importance of spatio-temporal rainfall pattern for rainfall runoff estimation and modelling is well known in hydrology and widely addressed by several simulation studies especially since distributed hydrological models have become available. Those studies showing either the effect of resulting runoff responses for different spatial rainfall pattern (Beven and Hornberger, 1982; Obled et al., 1994; Morin et al., 2006; Nicotina et al., 2008), or addressing the errors in runoff prediction and the difficulties in parameterisation and calibration of hydrologic models if the spatial distribution of rainfall is not well known (Troutman, 1983; Lopes, 1996; Chaubey et al., 1999; Andreassian et al., 2001), or investigating the required spatial configuration and temporal resolution of rainfall monitoring networks on the ground in order to monitor spatio-temporal rainfall pattern adequatly (Faures et al., 1995).

In generall, rainfall monitoring networks based on point observations on the ground (station data) require interpolation methods to obtain spatio-temporal rainfall fields usable for distributed hydrological modelling. However, those interpolation methods fail in reproducing the true spatio-temporal rainfall pattern especially for (i) data scarce regions with poorly gauged catchments and low network density, (ii) highly dynamic, small scaled rainstorms which are not well recorded by existing monitoring networks, and (iii) catchments which are covered by rainfall partly. Consequently, uncertainties are associated with poorly identified spatio-temporal rainfall pattern in distributed rainfall-runoff-modelling since the amount of rainfall received by a catchment as well as the dynamics of runoff generation processes are underestimated.

The effects of poorly estimated spatio-temporal rainfall fields are visible in particular for semiarid and arid regions, where rainstorms showing a great variability in space and time and the density of ground monitoring networks is sparsely compared to other regions (Pilgrim et al., 1988). Based on an analysis of 36 events in a mountainiuos region of Oman, McIntyre et al. (2007) show a wide range of event-based runoff coefficients, which underlines that achieving reliable runoff predictions by using hydrological models in those regions is extremely challenging. This is supported by several simulation studies (Al-Qurashi et al., 2008; Bahat et al., 2009), who address the uncertainties in model parametrisation due to uncertain rainfall input. In this context Gunkel and Lange (2012) report that reliable model parameter estimation was only possible by using rainfall rader information. However, those information are not available everywhere.

For adressing the inherent uncertainties described above, stochastic rainfall generators are used intensively to create spatio-temporal rainfall inputs for distributed hydrological models to transform rainfall into runoff. A large amount of literature exists describing different approaches for space-time simulation of rainfall fields, among them multisite temporal simulation frameworks (Wilks, 1998), approaches based on the theory of random fields (Bell, 1987; Pegram and Clothier, 2001) or approaches based on the theory of point processes and its generalization, which includes the popular Turning bands method (Mantoglou and Wilson, 1982). Enhancements were made in order to portray different rain storm pattern and distinct properties of rainfall fields, like spatial covariance structure, space-time anomalie, and intermittancy (see Leblois and Creutin 2013; Paschalis et al. 2013).

Applications of spatio-temporal rainfall simulations together with hydrological models are of straightforward, Monte-Carlo type, where a hugh amount of rainfall fields is generated driven by stochastic properties of observed rainstorms or longer timeseries. Those fields are inputs for distributed hydrologic model simulations to investigate the impact on resulting simulated catchment response for certain aspects of rainfall like uncertainty in measured rain depth, spatial variability, etc. Rainfall simulation applications are performed in unconditional mode (reproducing rainfield statistics only) or conditional mode, where observations (e.g from rain gauges) are reproduced too. The latter are commonly used for investigating the effect of spatial variability using fixed total precipitation and variations in spatial pattern (Krajewski et al., 1991; Shah et al., 1996; Casper et al., 2009; Paschalis et al., 2014).

With respect to the outlined problem in the second paragraph above, stochastic rainfall simulations in combination with hydrologic modeling might be a solution to reconstruct unknown spatio-temporal rainfall pattern. However, stochastic rainfall

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

simulations together with hydrologic modelling can be computationally demanding and can fail if rainfall fields are conditioned on rainfall point observations only. Therefore, our approach aims to include the observed runoff into the conditioning process. This implies that spatio-temporal rainfall pattern are conditioned on hydrologic model output in addition, why we call this an inverse modelling approach. The methodology of the inverse hydrologic modelling approach combines the stochastic random field simulator Random Mixing and a distributed rainfall-runoff model in a Monte-Carlo framework. Until now, Random Mixing, developed by Bárdossy and Hörning (2016b) for solving inverse groundwater modeling problems, has been used by Haese et al. (2017) for reconstruction and interpolation of precipitation fields using different data sources for rainfall. Our goal here is an eventbased reconstruction of possible realisations of spatio-temporal rainfall pattern which are able to explain measured point rainfall data and catchment runoff response at best. For that we are looking for potential candidates of three-dimensional rainfall fields for subdaily timesteps and spatial resolution of 1km² which, to our knowledge hasn't been done so far.

After this introduction the methods are described in chapter 2. It gives an overview of the approach and further details for the applied rainfall runoff model, the random mixing and its application for rainfall fields. Chapter 3 aims to test the methodology. A synthetic test site is introduced which is used to demonstrate and discuss the limits of common hydrologic modeling approaches (using rainfall interpolation) as well as conditional rainfall simulations only. In contrast, the functionality of the inverse hydrologic modelling approach is illustrated and discussed. In chapter 4, the inverse hydrologic modelling approach is applied for real world data by an example of an arid mountainious catchment in Oman. The test site is introduced and results are shown and discussed. Finally, summary and conclusions are given in chapter 5.

2 Methods

2.1 General approach

The methodology described here can be characterized as an inverse hydrological modelling approach. It aims to conclude on potential candidates for the unknown spatio-temporal rainfall pattern based on runoff observations at the catchment outlet, known parametrization of the rainfall-runoff model and rain gauge observations. The approach combines a grid-based spatially distributed rainfall-runoff model and a conditional random field simulation technique called Random Mixing (Bárdossy and Hörning, 2016a, b). Random Mixing is used to simulate a conditional precipitation field which honors the observed rainfall values as well as their spatial and temporal variability. In order to additionally condition the rainfall field on the observed runoff it is iteratively updated. Therefore the initial field is used as input to the rainfall-runoff model. The deviation between the simulated runoff and the observed runoff is evaluated based on the model efficiency (NSE) defined by Nash and Sutcliffe (1970). To minimize this deviation the rainfall field is *mixed* with another random field which exhibits certain properties such that the mixture honors the observed rainfall values and their spatio-temporal variability. This procedure is repeated until a satisfying solution, i.e. a conditional rainfall field that achieves a reasonable NSE is found. To enable a reasonable uncertainty estimation the procedure is repeated until a predefined number of potential candidates has been found.

2.2 Rainfall runoff model

A simple spatially distributed rainfall-runoff (RR) model is used as transfer function to portray the nonlinear transformation of spatially distributed rainfall into runoff at catchment outlets. The model is dedicated to describe rainfall-runoff processes in arid mountainous regions, working on regular cells in event-based mode. The model is parsimonious in number of parameters considering transmission losses but having no base flow component. Pre-state information at the beginning of an event is neglected since runoff processes starting under dry conditions mostly.

An initial and constant rate loss model is applied on each grid cell which is affected by rainfall. The calculated effective rainfall is transferred to the next river channel section considering translation and attenuation processes. Translation is accounted for with a grid-based travel-time model incorporating the effects of surface slope and roughness, and attenuation is accounted for with a single linear storage unit. The properties of several landscape units are addressed by different parameter sets following the concept of hydrogeological response units (Gerner, 2013) (since hydrological processes are mostly driven by hydrogeology in these regions). Runoff is routed to the catchment outlet by a simple lag model in combination with a constant rate loss model to portray transmission losses. The RR-model is applied for hourly time step on regular grids cells of 1km by 1km. Parameters are assumed to be known and fixed during the inverse modelling step.

2.3 Random Mixing for inverse hydrological modelling

Random Mixing is a geostatistical simulation approach first presented in Bárdossy and Hörning (2016a) and Bárdossy and Hörning (2016b) where the authors have applied it to inverse groundwater modeling problems. It uses copulas as spatial random functions (Bárdossy, 2006) and represents an extension to the gradual deformation approach (Hu, 2000). In the following a brief description of the Random Mixing algorithm is presented. A detailed explanation can be found in Hörning (2016).

The goal of the inverse hydrological modelling approach presented herein is to find a conditional rainfall field $P(x,t)$ with location $x \in D$ and time $t \in T$ which reproduces the observed spatial and temporal variability and marginal distribution of P. This field should also honor precipitation observations at locations x_j and times t_i:

$$P(x_j,t_i) = p_{j,i} \quad \text{for} \quad j = 1,\ldots,J \quad \text{and} \quad i = 1,\ldots,I \tag{1}$$

Furthermore, the solution of a rainfall-runoff model using the field P as input variable should honor the observed runoff:

$$Q_n(P) = q_n \quad \text{for} \quad n = 1,\ldots,N \tag{2}$$

where Q_n denotes the rainfall-runoff model and q_n-s represent the observed runoff values. Note that $Q_n(P)$ represents a non-linear function of the field P.

In order to find such a rainfall field P which fulfills the conditions given in Eq. (1) and Eq. (2) Random Mixing can be applied. Figure 1 shows a flowchart of the corresponding procedure.

After identifying the observations $p_{j,i}$ a marginal distribution $G(P(x,t))$ has to be fitted to them. For the applications presented herein the selected marginal distribution consists of two parts: the discrete probability of zero rainfall and an exponential

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

Hydrology and Earth System Sciences
Open Access
Discussions

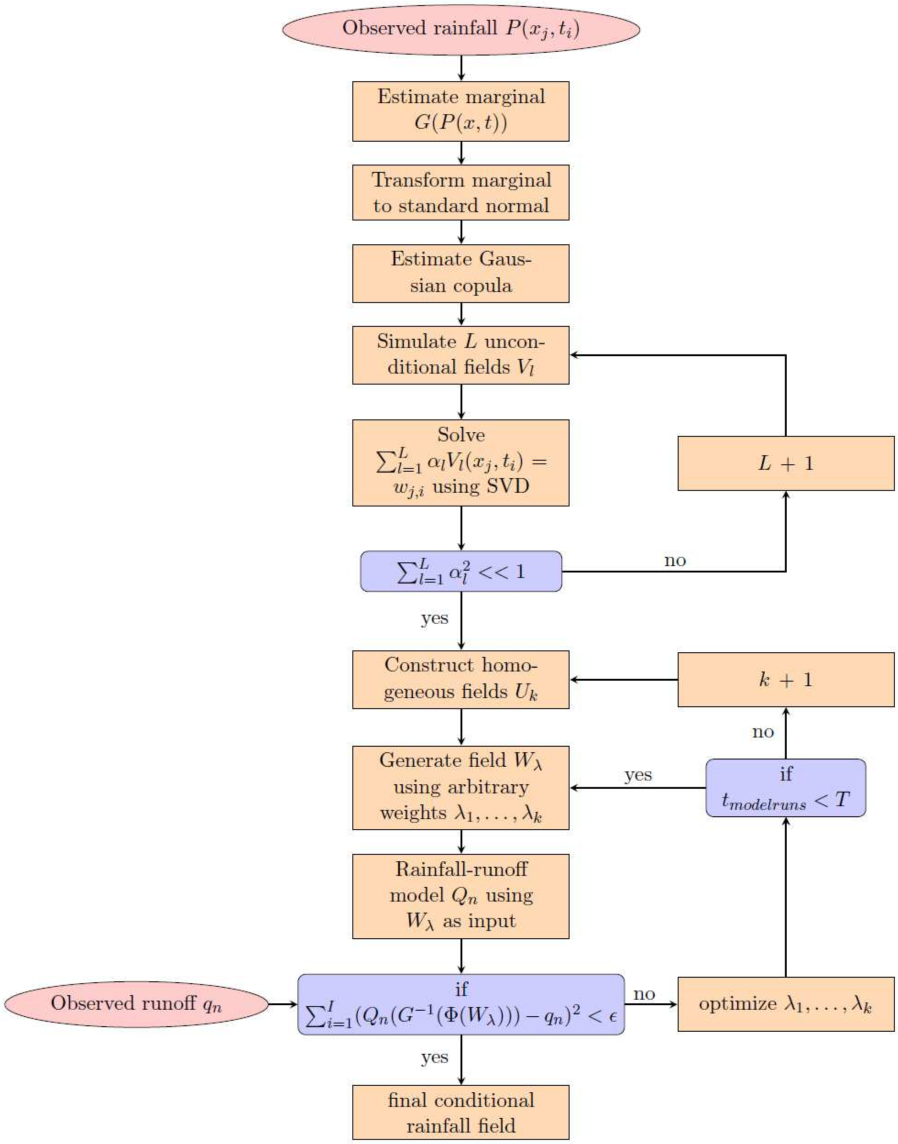

Figure 1. Flowchart of the Random Mixing algorithm for inverse hydrological modelling.

distribution for the wet rainfall observations. It is defined as:

$$G(P(x,t)) = \begin{cases} p_0 & \text{if } p = 0 \\ p_0 + (1 - \exp(-\lambda p)) & \text{otherwise} \end{cases} \tag{3}$$

with p denoting rainfall values, p_0 is the discrete probability of zero rainfall and λ denotes the parameter of the exponential distribution. Then the observed precipitation values are transformed to standard normal:

$$w = \begin{cases} < \Phi^{-1}(p_0) & \text{if } p = 0 \\ \Phi^{-1}(p_0 + (1 - \exp(-\lambda p))) & \text{otherwise} \end{cases} \tag{4}$$

where Φ^{-1} denotes the inverse standard normal distribution. Note that zero rainfall observations are not transformed to a certain value, but they are considered as inequality constraints as described in Hörning (2016). Further note that the transformation of the marginal distribution described in Eq.4 can be reversed via:

$$P(x,t) = G^{-1}(\Phi(W(x,t))) \tag{5}$$

where G^{-1} denotes the inverse marginal distribution of P and Φ denotes the standard normal distribution.

In the next step, a Gaussian copula is fitted to the observations according to the approach described in Li (2010). This copula describes the spatio-temporal dependence structure of the observations.

As a next step, unconditional standard normal random fields V_l with $l = 1,\ldots,L$ are simulated such that they all share the same spatial structure. Such fields can for example be simulated using Fast Fourier Transformation for regular grids (Wood and Chan, 1994; Wood, 1995; Ravalec et al., 2000) or Turning band simulation (Journel, 1974). Using the fields V_l, the system of linear equations:

$$\sum_{l=1}^{L} \alpha_l V_l(x_j,t_i) = w_{j,i} \tag{6}$$

is set up and solved using singular value decomposition (SVD) (Golub and Kahan, 1965). If no solution with a L^2 norm much smaller than one is found, an additional field V_{L+1} is created, added to the system of linear equation and the system is solved again. Once a solution with an acceptable L^2 norm i.e. $\sum \alpha_l^2 \ll 1$ is found the resulting field is defined as:

$$W^* = \sum_{l=1}^{L} \alpha_l V_l \tag{7}$$

and the algorithm moves on. Note that W^* fulfills the conditions defined in Eq. 1.

The next step is to simulate fields U_k with $k = 1,\ldots,K$ which fulfill the homogeneous conditions, i.e. $U_k(x_j,t_i) = 0$. These can be generated in a similar way as W^* (see Hörning (2016) for details). The advantage of these fields U_k is that they form a vector space (they are closed for multiplication and addition), thus:

$$W_\lambda = W^* + k(\lambda)(\lambda_1 U_1 + \ldots + \lambda_k U_k) \tag{8}$$

where λ_k-s denote arbitrary weights and $k(\lambda)$ denotes a scaling factor results in a field W_λ which fulfills the conditions prescribed in Eq. 1 . The scaling factor $k(\lambda)$ ensures that W_λ exhibits the correct covariance matrix Γ. Thus, transforming W_λ back to P using Eq. 5 will result in a precipitation field which has the correct spatio-temporal variability, marginal distribution and honors the rainfall observations.

To also honor the observed runoff defined in Eq. 2 an optimization problem can be formulated:

$$\mathcal{O}(\lambda) = \sum_{i=1}^{I} (Q_n(G^{-1}(\Phi(W_\lambda))) - q_n)^2 \tag{9}$$

which minimizes the difference between the modeled and observed runoff by optimizing the weights λ_k. As these weights are arbitrary they can be changed without violating any of the already fulfilled conditions, thus they can be optimized without any further constraints. If for a given set of fields and weights and after a certain number of iterations T no suitable solution is found, the number K of fields U_k can be increased and the optimization is repeated. If a suitable solution is found the whole procedure can be restarted using new random fields V_l. Thus multiple solutions can be obtained enabling uncertainty quantification

3 Test of the methodology

3.1 Synthetic test site

To test the ability of the methodology a synthetic example is designed consisting of a synthetic catchment partly covered by rainfall. The synthetic catchment has a size of 211 km² and elevations range between 100 to 1100 m.a.s.l. with homogeneous landscape properties (Figure 2). A synthetic rainfall event of 6 hours duration with hourly time step and a spatial extension of 118 km² is used. Rainfall amounts above 20mm/event covers an area of 25 km² with maximum rainfall of 36 mm/event and maximum intensity of 12mm/h (Figure 3). Based on this known spatio-temporal rainfall input pattern and RR-model parameterisation the catchment response at surface outlet is simulated and dedicated to be the known “observed” runoff q_n (see Figure 6, blue graph).

Furthermore, ten different cells are selected from the spatio-temporal rainfall pattern for representing virtual monitoring stations of rainfall. They are chosen in a way that the centre of the event is not recorded. They are dedicated to be the known “observed” rainfall $P(x_j, t_i)$ at J monitoring stations for T time steps and form the data basis for interpolation, conditional simulation, and inverse modelling of spatio-temporal rainfall pattern. Figure 4 shows their course in time. Note that virtual monitoring stations 2, 5, 9 and 10 measure 0mm/h rainfall only.

3.2 Results and discussion

3.2.1 Common hydrologic modelling approach

At first, hourly rainfall data from virtual monitoring stations are used to interpolate the spatio-temporal rainfall pattern on a regular grid of 1km by 1km cellsize by using the inverse distance method which is quite common in hydrological modelling.

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

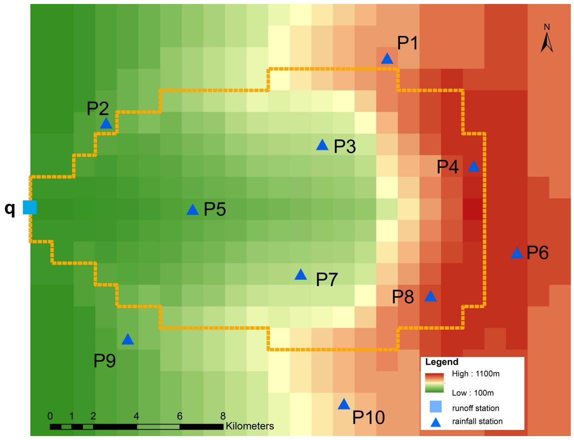

Figure 2. Topography, watershed and observation network of the synthetic chatchment

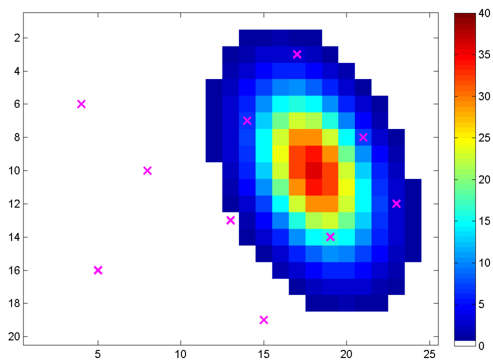

Figure 3. Eventbased rainfall amounts of the synthetic rainfall event. Virtual monitoring stations are marked by crosses.

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

Hydrology and Earth System Sciences
Open Access
Discussions

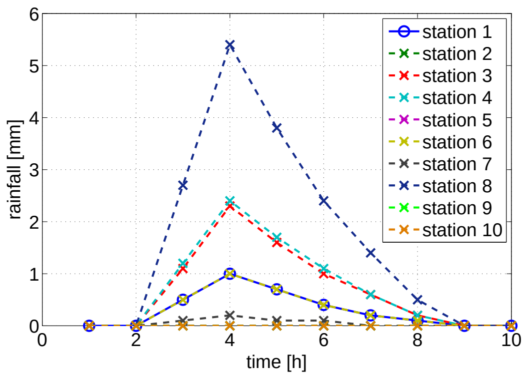

Figure 4. Time series of rainfall amounts at virtual monitoring stations

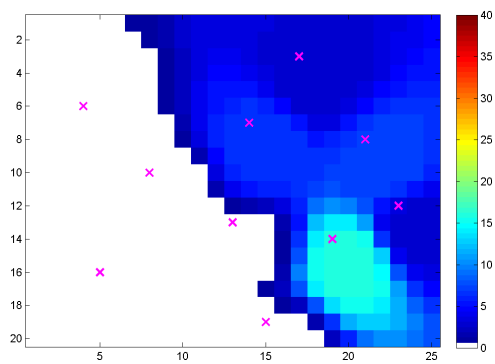

Figure 5. Interpolated rainfall pattern per event by using data of virtual monitoring stations

Afterwards, the response of the synthetic catchment is calculated. Figure 5 shows the interpolated pattern of the eventbased rainfall amounts as the sum over single timesteps. The pattern looks quite smooth and has only minor similarities with the true pattern in Figure 3. Maximum of rainfall amount per event is equal to the maximum of the observation at virtual station number 8 with 16,2mm/event. Therefore, the extension of a rainfall centre over 20mm/event cannot be estimated. Due to low rainfall intensities the simulated response of the RR-model shows a significant underestimation of the observed runoff with NSE value of -0.28 (see Figure 6, green graph).

3.2.2 Performance of conditional rainfall simulations

The random mixing approach was used to simulate 200 different spatio-temporal rainfall pattern which are conditioned on the virtual rainfall monitoring stations only. Resulting runoff simulations are displayed in Figure 6 showing a wide range of

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

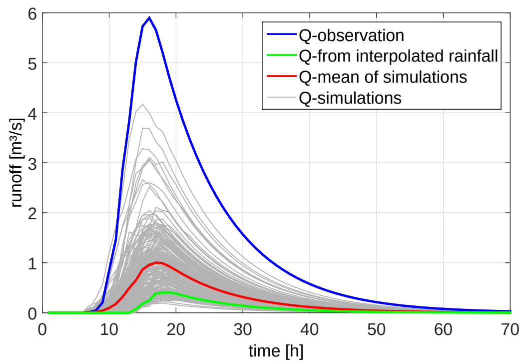

Figure 6. Runoff simulations based on simulated spatio-temporal rainfall pattern conditioned at rainfall point observations only (grey graphs) compared to its mean (red graph), runoff observation (blue graph), and simulation based on interpolated rainfall pattern (green graph)

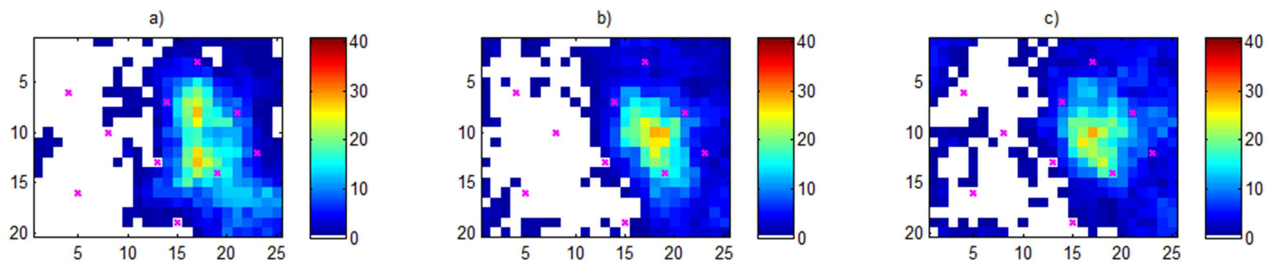

Figure 7. Eventbased rainfall pattern conditioned at rainfall point observations only for the top three runoff simulations in Figure 6

hydrographs with peak values between 0,19m³/s to 4,17m³/s and NSE values between -0,37 to 0,89. Compared to the runoff observation, the timing of peaks is acceptable, but the peak values are underestimated. Only four hydrographs have NSE values higher than 0,7. The corresponding spatial eventbased rainfall amounts for the top three runoff simulations regarding the NSE values ((a) 0,89 (b) 0,78 (c) 0,73) is shown in Figure 7. Their eventbased rainfall amounts ranging between 27,8 to 28,7mm/event with a spatial extent of 9 to 11km² of rainfall above 20mm/event and a maximum intensity 10,5 to 15,1mm/h. Compared to the observation (Figure 3), the spatial pattern look similar, at least regarding the spatial location of the event, and cover the maximum intensity. But the eventbased rainfall amounts as well as their spatial extent is too low. As a consequence, none of the simulated spatio-temporal rainfall pattern conditioned at the virtual rainfall monitoring stations only is able to match the observed peak value in resulting runoff.

3.2.3 Inverse hydrologic modelling approach

The inverse modelling approach was used to simulate 107 different spatio-temporal rainfall pattern which are conditioned on the virtual rainfall and runoff monitoring stations and runoff simulation results better than NSE values of 0,7. Afterwards a refinement have been carried out by selecting only those simulations with nearly identical runoff simulation results compared to observation. These simulations are characterized by NSE values larger than 0,995. Figure 8 shows the performance of the 20 selected events by grey graphs having only minor deviations during the flood peak range compared to the observation (blue graph). Associated rainfall pattern are displayed in Figure 9 for six selected events by their spatial eventbased rainfall amounts. Compared to the true spatial pattern (see Figure 3) none of them reproducing the true pattern exactly but all of them locate the centre of the event in the same region like the true pattern. This shows, that due to the incooperation of catchments' drainage characteristic represented by the RR-model, and the runoff observation into the rainfall simulation procedure, a localisation in terms of a reconstruction of the rainfall pattern inclusive its course in time is possible. However, the inference of a three dimensional input variable by using an integral output response results in a set or ensemble of different solutions. This ensemble can be used to describe the uncertainty in estimating spatio-temporal rainfall pattern. Rainfall amounts of the selected 20 realisations above 20mm/event cover an area of 13 to 25 km² with maximum rainfall of 26,7 to 40,4mm/event and maximum intensities of 10,7 to 17,1mm/h. The eventbased areal precipitation ranges between 98,2% - 114,7% of the observation (see Figure 3). Figure 9 presents spatial eventbased rainfall amounts for a) the realisation with the smallest area above 20mm/event and smallest intensity, b) the realisation with the largest area above 20mm/event c) the realisation with the highest intensity and rainfall amount per event, d) the realisation with the best NSE-value in resulting runoff, e) and f) realisations with similar event statistics like the true spatio-temporal rainfall pattern. Compared to the observed pattern (see Figure 3), the different realisations match the spatial location as well as the shape of the observed pattern very good. However, the spatial pattern of the realisations are not such smooth and symmetric like the constructed synthetic observation. Furthermore, the realisations show some scattered low rainfall amounts, which are not of importance for the hydrograph simulation since they are addressed by the initial and constant rate losses of the RR-model. Last but not least, data of the virtual monitoring stations have been always reproduced and are equal for each rainfall simulation.

Deriving an average rainfall pattern by calculating the cell wise mean value over all realisations of the ensemble for each time step will result in a smoother pattern more similar to the true one but with smaller intensities. Using this mean ensemble pattern for calculating the runoff response lead to an underestimation of the observed hydrograph like shown by the black hydrograph in Figure 8. Therefore, the ensemble mean of the hydrographs (red line in Figure 8) is a better representative for the sample than the mean ensemble rainfall pattern.

4 Application for real world data

4.1 Arid catchment test site

The real world example is taken from the upper Wadi Bani Kharus in the northern part of the Sultanate of Oman. The head-water catchment under consideration is the surface runoff gauging station of Al Awabi with an area of 257km², located in the Hadjar mountain range with heights ranging from 600m.a.s.l. to more than 2500m.a.s.l. The geology of the area is dominated by the Hadjar group consisting of limestone and dolomite. The steep terrain consists of rocks mainly. Soils are negligible. However, larger units of alluvial depositions in the valleys are important for hydrologic processes which is addressed by spatial differences in RR-model parameters. Vegetation is sparely and mostly cultivated in mountain oasis. Annual rainfall can reach

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

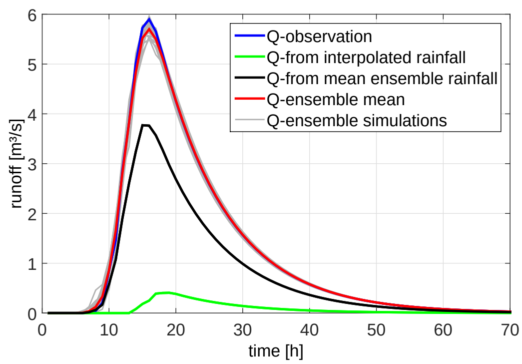

Figure 8. Comparison of hydrographs for the synthetic catchment shown by the observed runoff (blue) and rainfall-runoff simulation results based on: interpolated rainfall pattern (green), simulated ensemble of spatio-temporal rainfall pattern conditioned at rainfall and runoff observations (grey) and their mean value (red), as well as mean ensemble rainfall pattern (black)

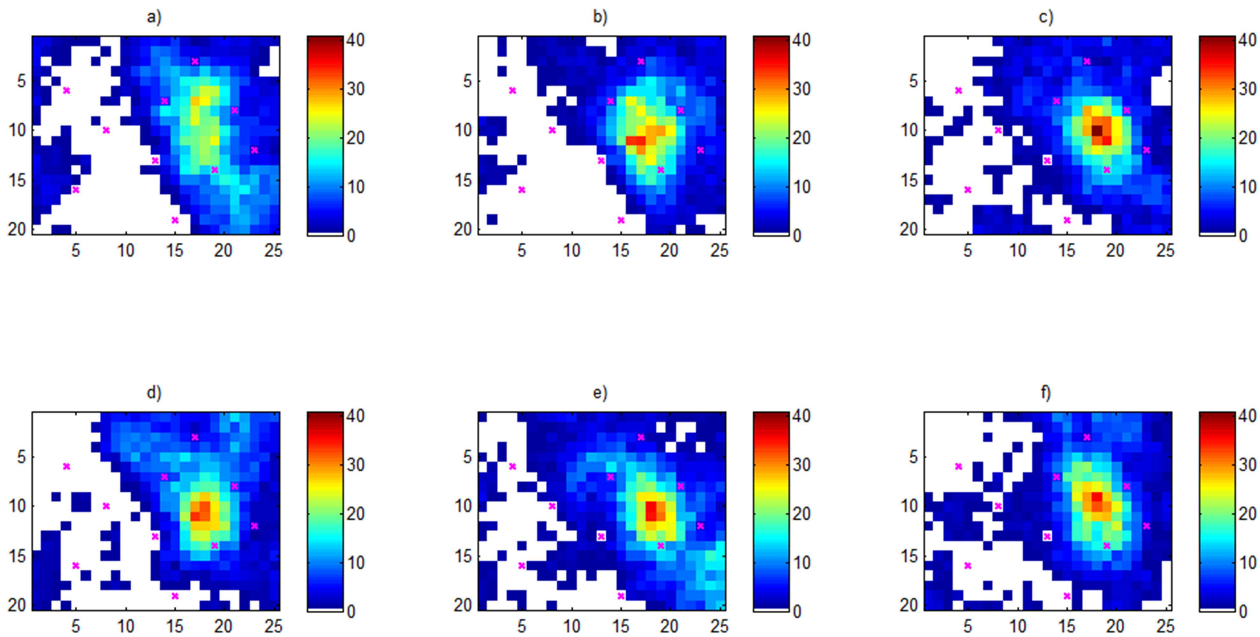

Figure 9. Selected realisations of spatial eventbased rainfall amounts with similar performance in resulting runoff obtained by the inverse modelling approch for simulating spatio-temporal rainfall pattern: a) realisation with the smallest area above 20mm/event and smallest intensity, b) realisation with the largest area above 20mm/event c) realisation with the highest intensity and rainfall amount per event, d) realisation with the best NSE-value in resulting runoff, e) and f) realisations with similar event statistics like the true spatio-temporal rainfall pattern

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

Hydrology and Earth System Sciences
Open Access
Discussions

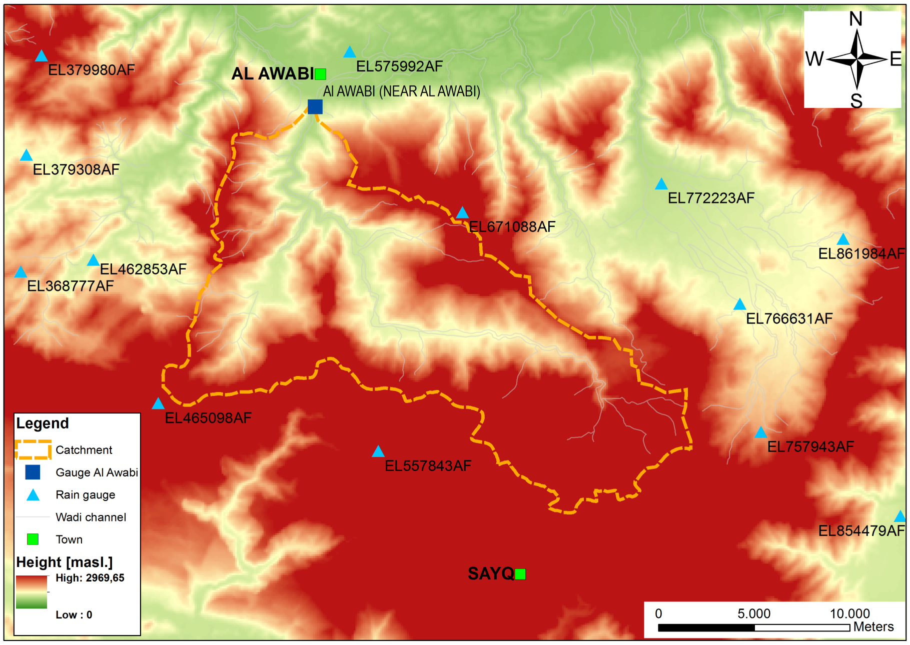

Figure 10. real world case study: catchment of gauge Al Awabi and subdaily monitoring network for runoff and rainfall

more than 300mm/yr showing a huge variability between consecutive years. Analysis of measured runoff data over a period of 24 years shows that runoff occurred in average only on 18days/yr. Figure 10 shows the available monitoring network for subdaily data. Runoff is measured in 5 to 10 minutes temporal resolution. Rainfall measurements vary from 1 minute to 1 hour. Therefore, a temporal resolution of 1 hour is chosen for the event under investigation in this study. Rainfall interpolation was performed by inverse distance method.

4.2 Results and discussion

The real world data example is performed for the runoff event from 12.2.1999 with an effective rainfall duration of three hours. The simulated runoff for the interpolated rainfall pattern shows an underestimation of the peak discharge as well as a time shift of the peak arrival time compared to the observation (Figure 11). Applying the inverse approach by conditioning spatio-temporal rainfall pattern on rainfall and runoff observations, an ensemble of 58 different hydrographs is simulated having NSE values larger than 0.9. As shown in Figure 11, all of these hydrographs (grey graphs) represent the observation quite good and overcome the timeshift. To explain this behaviour, differential maps are calculated which show the difference between a simulated and the regionalized rainfall pattern for each timestep (Figure 12). It is easy to see that the inverse approach allows for a shift of the centre of the rainfall event from time step 1 to time step 2 and towards the catchment outlet. This results in a faster response of the catchment by its runoff compared to the interpolated rainfall pattern. In general, the obtained ensemble of spatio-temporal rainfall pattern is able to explain the observed runoff without discrepancy in rainfall measurements. Similar

Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2018-350
Manuscript under review for journal Hydrol. Earth Syst. Sci.
Discussion started: 2 July 2018

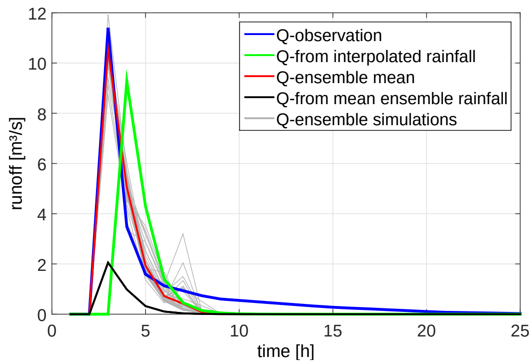

Figure 11. Comparison of hydrographs for the real world catchment shown by the observed runoff (blue) and rainfall-runoff simulation results based: on interpolated rainfall pattern (green), simulated ensemble of spatio-temporal rainfall pattern conditioned at rainfall and runoff observations (grey) and their mean value (red), as well as mean ensemble rainfall pattern (black)

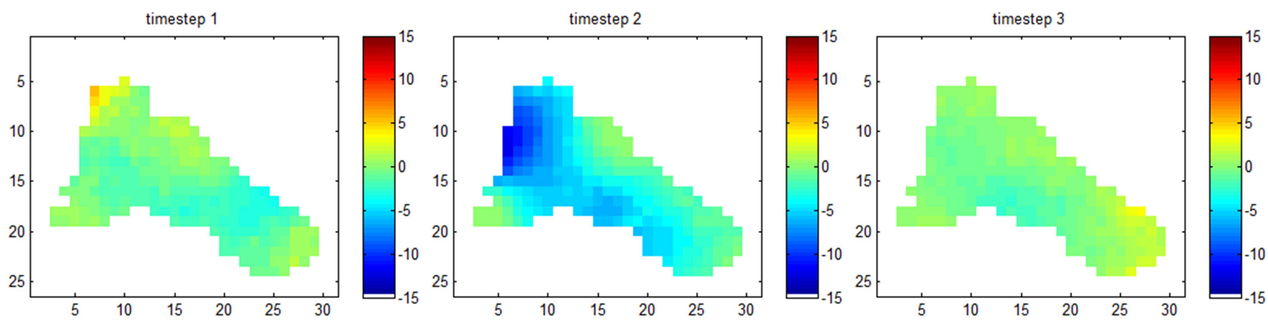

Figure 12. Differential maps of spatio-temporal rainfall pattern for three consecutive timesteps (simulation – interpolation)

to the synthetic example, the ensemble mean hydrograph (Figure 11, red graph) is a better representative for the sample than the hydrograph based on the mean ensemble rainfall spatio-temporal pattern (black graph).

5 Summary and conclusions

An inverse hydrologic modeling approach for simulating spatio-temporal rainfall pattern is presented in this paper. The approach combines the conditional random field simulator Random Mixing and a spatial distributed RR-model in a joint framework. It allows for obtaining resonable spatio-temporal rainfall pattern which are conditioned on point rainfall and runoff observations. This has been demonstrated by a synthetic data example for a catchment which is covered by rainfall partly.

Compared to other methods, like rainfall interpolation or conditional rainfall simulation, the inverse approach showed that a reconstruction of the eventbased spatio-temporal rainfall pattern has been possible, esspecially if runoff generation processes are driven by topography. As shown by the synthetic example, the pattern obtained by interpolation is too smooth. The method might be appropriate for long time intervalls, but in terms of rainfall data scarcity and high spatio-temporal rainfall variability a “good” interpolation result in least square sense is not a solution of the problem. Furthermore, conditional rainfall simulation only shows the pure spatio-temporal rainfall uncertainty. If rainfall pattern are conditioned on discharge too, appropriate candidates of spatio-temporal rainfall pattern can be identified more reliable with reduced uncertainty.

The inference of a three dimensional input variable by using an integral output response results in a set of possible solutions in terms of spatio-temporal rainfall pattern. This ensemble can be considered as a descriptor of the uncertainty in the spatio-temporal rainfall pattern estimates and described regarding rainfall amounts, intensities and spatial extend of the event. It allows also for deeper insights in model and catchment behavior and gives valuable information for the reanalysis of rainfall-runoff events. Like shown in the example, operaiting with an ensemble mean is less successful to match the runoff observation compared to an application of the whole ensemble due to smoothing effects.

The approach is also working under data scarce situation like a real world data example showed. Here, the flexibility of the approach becomes visible, since simulated rainfall pattern are also allow for overcoming a shift in timing of runoff. Therefore, the approach can be considered as a reanalysis tool for rainfall-runoff events especially in regions where runoff generation and formation based on surface flow processes and catchments with wide ranges in arrival times at catchment outlet e.g. mountainious regions or with distinct drainage structures e.g urban and periurban regions.

Nevertheless, there are still some weak points which require further research and investigation. Examples presented here based on hourly resolution in time and 1 km² grids. Especially for fast responding, small catchments finer resolutions are required and the limits in number of timesteps and gridcells as well as input data quality need to be explored. An other point is the spatial and temporal dependence structure. It controls the simulation of rainfall pattern and is determined based on observations. In these examples gaussian copulas are used which might be not a good estimator for the spatial dependency in any case. Finally, our assumption that hydrologic model parameters are known and fixed during model application might be valid only for catchments with longterm observations and modeling experience, where modellers are interested to explain the extraordinary rainfall-runoff events. In generall, incorporation of further sources of uncertainties (e.g. model parameters, observations, ...) is required for contributing to the solution of the hydrologic modeling uncertainty puzzle.

References

Al-Qurashi, A., McIntyre, N., Wheater, H., and Unkrich, C.: Application of the Kineros2 rainfall-runoff model to an arid catchment in Oman, Journal of Hydrology, 355, 91–105, https://doi.org/10.1016/j.jhydrol.2008.03.022, 2008.

Andreassian, V., Perrin, C., Michel, C., Usart-Sanchez, I., and Lavabre, J.: Impact of imperfect rainfall knowledge on the efficiency and the parameters of watershed models, Journal of Hydrology, 250, 206–223, https://doi.org/10.1016/S0022-1694(01)00437-1, 2001.

Bahat, Y., Grodek, T., Lekach, J., and Morin, E.: Rainfall-runoff modeling in a small hyper-arid catchment, Journal of Hydrology, 373, 204–217, https://doi.org/10.1016/j.jhydrol.2009.04.026, 2009.

Bárdossy, A.: Copula-based geostatistical models for groundwater quality parameters., Water Resources Research, 42(W11416), doi:10.1029/2005WR004 754, 2006.

Bárdossy, A. and Hörning, S.: Random Mixing: An Approach to Inverse Modeling for Groundwater Flow and Transport Problems, Transport in Porous Media, 114, 241–259, https://doi.org/10.1007/s11242-015-0608-4, http://dx.doi.org/10.1007/s11242-015-0608-4, 2016a.

Bárdossy, A. and Hörning, S.: Gaussian and non-Gaussian inverse modeling of groundwater flow using copulas and random mixing, Water Resources Research, 52, 4504–4526, https://doi.org/10.1002/2014WR016820, http://dx.doi.org/10.1002/2014WR016820, 2016b.

Bell, T. L.: A space-time stochastic model of rainfall for satellite remote-sensing studies, Journal of Geophysical Research: Atmospheres, 92, 9631–9643, https://doi.org/10.1029/JD092iD08p09631, https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JD092iD08p09631, 1987.

Beven, K. and Hornberger, G.: Assessing the effect of spatial pattern of precipitation in modeling stream-flow hydrographs, Water Resources Bulletin, 18, 823–829, 1982.

Casper, M. C., Herbst, M., Grundmann, J., Buchholz, O., and Bliefernicht, J.: Influence of rainfall variability on the simulation of extreme runoff in small catchments, HYDROLOGIE UND WASSERBEWIRTSCHAFTUNG, 53, 134–139, 2009.

Chaubey, I., Haan, C., Grunwald, S., and Salisbury, J.: Uncertainty in the model parameters due to spatial variability of rainfall, Journal of Hydrology, 220, 48–61, https://doi.org/10.1016/S0022-1694(99)00063-3, 1999.

Faures, J., Goodrich, D., Woolhiser, D., and Sorooshian, S.: Impact of small-scale spatial rainfall variability on runoff modeling, Journal of Hydrology, 173, 309–326, https://doi.org/10.1016/0022-1694(95)02704-S, 1995.

Gerner, A.: A novel strategy for estimating groundwater recharge in arid mountain regions and its application to parts of the Jebel Akhdar Mountains (Sultanate of Oman), Ph.D. thesis, Technische Universitresden, 2013.

Golub, G. and Kahan, W.: Calculating the Singular Values and Pseudo-Inverse of a Matrix, Journal of the Society for Industrial and Applied Mathematics Series B Numerical Analysis, 2, 205–224, 1965.

Gunkel, A. and Lange, J.: New Insights Into The Natural Variability of Water Resources in The Lower Jordan River Basin, Water Resources Management, 26, 963–980, https://doi.org/10.1007/s11269-011-9903-1, 2012.

Haese, B., Horning, S., Chwala, C., Bardossy, A., Schalge, B., and Kunstmann, H.: Stochastic Reconstruction and Interpolation of Precipitation Fields Using Combined Information of Commercial Microwave Links and Rain Gauges, Water Resources Research, 53, 10 740–10 756, 2017.

Hörning, S.: Process-oriented modeling of spatial random fields using copulas, Eigenverlag des Instituts für Wasser- und Umweltsystemmodellierung der Universität Stuttgart, 2016.

Hu, L.: Gradual deformation and iterative calibration of Gaussian-related stochastic models., Math Geol, 32, 87–108, 2000.

Journel, A.: Geostatistics for conditional simulation of ore bodies., Economic Geology, 69, 673–687, 1974.

Krajewski, W. F., Lakshmi, V., Georgakakos, K. P., and Jain, S. C.: A Monte Carlo Study of rainfall sampling effect on a distributed catchment model, Water Resources Research, 27, 119–128, https://doi.org/10.1029/90WR01977, http://dx.doi.org/10.1029/90WR01977, 1991.

Leblois, E. and Creutin, J.-D.: Space-time simulation of intermittent rainfall with prescribed advection field: Adaptation of the turning band method, Water Resources Research, 49, 3375–3387, https://doi.org/10.1002/wrcr.20190, https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/wrcr.20190, 2013.

Li, J.: Application of Copulas as a New Geostatistical Tool, Eigenverlag des Instituts Wasserbau, 2010.

Lopes, V.: On the effect of uncertainty in spatial distribution of rainfall on catchment modelling, CATENA, 28, 107–119, https://doi.org/10.1016/S0341-8162(96)00030-6, 1996.

Mantoglou, A. and Wilson, J.: The Turning Bands Method for simulation of random fields using line generation by a spectral method, Water Resources Research, 18, 1379–1394, https://doi.org/10.1029/WR018i005p01379, 1982.

McIntyre, N., Al-Qurashi, A., and Wheater, H.: Regression analysis of rainfall-runoff data from an arid catchment in Oman, HYDROLOGICAL SCIENCES JOURNAL-JOURNAL DES SCIENCES HYDROLOGIQUES, 52, 1103–1118, https://doi.org/10.1623/hysj.52.6.1103, International Conference on Future of Drylands, Tunis, TUNISIA, JUN, 2006, 2007.

Morin, E., Goodrich, D., Maddox, R., Gao, X., Gupta, H., and Sorooshian, S.: Spatial patterns in thunderstorm rainfall events and their coupling with watershed hydrological response, ADVANCES IN WATER RESOURCES, 29, 843–860, https://doi.org/10.1016/j.advwatres.2005.07.014, 2006.

Nash, J. and Sutcliffe, J.: River flow forecasting through conceptual models part I — A discussion of principles, Journal of Hydrology, 10, 282 – 290, https://doi.org/https://doi.org/10.1016/0022-1694(70)90255-6, http://www.sciencedirect.com/science/article/pii/0022169470902556, 1970.

Nicotina, L., Celegon, E. A., Rinaldo, A., and Marani, M.: On the impact of rainfall patterns on the hydrologic response, Water Resources Research, 44, https://doi.org/10.1029/2007WR006654, 2008.

Obled, C., Wendling, J., and Beven, K.: The sensitivity of hydrological models to spatial rainfall patterns - an evaluation using observed data, Journal of Hydrology, 159, 305–333, https://doi.org/10.1016/0022-1694(94)90263-1, 1994.

Paschalis, A., Molnar, P., Fatichi, S., and Burlando, P.: A stochastic model for high-resolution space-time precipitation simulation, Water Resources Research, 49, 8400–8417, https://doi.org/10.1002/2013WR014437, http://dx.doi.org/10.1002/2013WR014437, 2013.

Paschalis, A., Fatichi, S., Molnar, P., Rimkus, S., and Burlando, P.: On the effects of small scale space-time variability of rainfall on basin flood response, Journal of Hydrology, 514, 313–327, https://doi.org/10.1016/j.jhydrol.2014.04.014, 2014.

Pegram, G. and Clothier, A.: High resolution space-time modelling of rainfall: the "String of Beads" model, Journal of Hydrology, 241, 26–41, https://doi.org/10.1016/S0022-1694(00)00373-5, 2001.

Pilgrim, D., Chapman, T., and Doran, D.: Problems of rainfall-runoff modeling in arid and semiarid regions, HYDROLOGICAL SCIENCES JOURNAL-JOURNAL DES SCIENCES HYDROLOGIQUES, 33, 379–400, https://doi.org/10.1080/02626668809491261, 1988.

Ravalec, M. L., Noetinger, B., and Hu, L. Y.: The FFT Moving Average (FFT-MA) Generator: An Efficient Numerical Method for Generating and Conditioning Gaussian Simulations, Mathematical Geology, 32, 2000.

Shah, S., O'Connell, P., and Hosking, J.: Modelling the effects of spatial variability in rainfall on catchment response. 2. Experiments with distributed and lumped models, Journal of Hydrology, 175, 89 – 111, https://doi.org/https://doi.org/10.1016/S0022-1694(96)80007-2, http://www.sciencedirect.com/science/article/pii/S0022169496800072, 1996.

Troutman, B.: Runoff prediction errors and bias in parameter-estimation induced by spatial variability of precipitation, Water Resources Research, 19, 791–810, https://doi.org/10.1029/WR019i003p00791, 1983.

Wilks, D.: Multisite generalization of a daily stochastic precipitation generation model, Journal of Hydrology, 210, 178–191, https://doi.org/10.1016/S0022-1694(98)00186-3, 1998.

Wood, A.: When is a truncated covariance function on the line a covariance function on the circle?, Statistics & Probability Letters, 24, 157–164, 1995.

Wood, A. and Chan, G.: Simulation of stationary Gaussian process in $[0,1]^d$, Journal Computational and Graphical Statistics, 3, 409–432, 1994.