Author's response on referee's comments

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Title: Stochastic reconstruction of spatio-temporal rainfall pattern by inverse hydrologic modelling
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We would like to thank the referees for their time to review the manuscript. Our reply is organized as follows: (1) comments from Referees are in black color, (2) author's response is marked in blue color and placed within referee comments whenever it's needed, (3) author's changes in manuscript based on comments of both referees are summarized at the end of this document followed by a marked-up manuscript with tracked changes.

REFEREE 1:

My comments are in the order I read the paper:

p2l3 - spelling mistake - 'generall'
Thanks, the spelling mistakes are fixed in the revised manuscript.

p3l3 - some grammar issue - please check
Grammar is improved throughout the whole paper. In general spelling and grammar is double checked by a native speaker prior to resubmission.

p6 - in my understanding, the approach is roughly along the following lines - first transformed empirical CDFs are ascertained and used to create equivalent Normal observed rainfalls donated as w.
We are not quite sure what you mean by transformed empirical cdfs are ascertained. What is done is that a cdf (and in general this cdf can be any type of cdf, i.e. parametric or non-parametric or a combination of both) is fitted to the observed precipitation values. The distribution used in this work is described in Eq. 3. It is a combination of a discrete probability for zero precipitation values and an exponential distribution for values greater than zero. Thus the parameters that need to be estimated are p_0 (the discrete probability of zero precipitation) and λ (the parameter of the exponential distribution).Subsequently, using this fitted cdf the observed precipitation values are transformed to standard normal values according to Eq. 4.

line 11 says Gaussian copula is fitted to describe spatio-temporal dependence structure - a few lines of what this entails should be provided for completeness
We added more information on copulas in general, the Gaussian copula and the fitting process to the revised manuscript.

- in my reading this step seems independent of what is described next in the paper but if I am wrong this should be corrected. I am guessing the $w_{j,i}$ is not from this copula but from equation 4 for each site and time step. So you have L sequences and the aim is to fine α_l such that there is some minimal deviation with the transformed normal rainfall at each location and time step. So I guess the idea here is to keep generating fields until they match the observed rainfalls transformed to Normal. If that happens then you will have $L = J$ and all the alpha's being equal to $1/L$. And since there is spatial dependence, you would kind of expect $L < J$ if this works fine. Am I correct? May be good to spell this out a bit more.
Yes, to some extent but there also seems to be some misunderstanding. The $w_{j,i}$-s (which represent the transformed precipitation observations at locations x and time steps t) are derived by Eq. 4 (see your third comment). The random fields V_l (which are independent standard normal spatial random fields) are simulated such that they all have the same spatial structure which is described by this Gaussian copula (this information was missing in the paper and added now). Eq. 6 says that we want to find a linear combination of these independent standard normal random fields V_l such that this linear combination results in the values $w_{j,i}$ at locations x and time steps t. Thus Eq. 6 describes a linear equation system with the weights α_l being the unknowns (the values $V_l(x_j, t_i)$ are known). This equation system can be solved for $L \geq J$, the bigger L is the smaller the $\sum \alpha_l^2$ sum gets - if the solution is calculated using SVD.

P6l23 - what are homogenous conditions? I didn't understand what is meant by $U_k(x_j, t_i) = 0$ - please clarify what this is and why is it needed.
The homogeneous conditions are $U_k(x_j, t_i) = 0$ (a system of linear equations with the right hand side being all zeros is called a homogeneous equation system). This means that now we want to find a linear combination of random fields which fulfills $U_k(x_j, t_i) = 0$, i.e. a linear combination that results in zeros at locations x and time steps t. This is done the same way as constructing the field W^*, i.e. by setting up an equation system using independent standard normal random fields $\sum_{k=1}^{K} \beta_k V_k(x_j, t_i) = 0$ (we didn't put this equation in the paper as it's basically the same as Eq. 6 with the right-hand side being zero). The explanation why this is needed is actually given in the following sentences. Line 24: 'The advantage of these fields U_k is that they form a vector space (they are closed for multiplication and addition)...' This means when adding such a field U_k (or k of them) to W^*, the resulting field W_λ will exhibit the correct values $w_{j,i}$ at locations x and times t because the zeros in the field U_k do not affect these values. However, the rest of the field is affected (as fields U_k are conditional random fields) which enables modifying the final field W_λ without changing the conditioning values. By changing the arbitrary weights λ one can modify the field such that it represents the observed runoff (and therefore the procedure needs to be coupled with the rainfall runoff model) to a certain degree.

p7l2 - this is starting to become confusing now. Where did the covariance matrix come from? covariance of what?
This goes back to the missing information that the fields V_l all have the same spatial structure which is described by the fitted Gaussian copula. The covariance we are referring to is the spatio-temporal covariance of the observations to which we have fitted the Gaussian copula. We changed the wording (as it wasn't consistent) and added more information to the revised manuscript. As the field that fulfills the homogeneous conditions can be combined using arbitrary weights λ the scaling factor $k(\lambda)$ can be used to scale the final field such that the resulting field exhibits the spatio-temporal correlation/covariance of that copula. This is the case when the L^2 norm of the weights of the linear combination is equal to 1. As $\sum \alpha_l^2 << 1$ the weights λ need to be scaled (using the scaling factor $k(\lambda)$) such that $\sum \alpha_l^2 + \sum \lambda_k^2 = 1$. It's also worth mentioning that in this case the covariance is equal to the correlation as we are working in standard normal space (mean is zero and unit variance).

if $W*$ represents more or less the transformed observed precipitation field (from what I could gather), is this W_λ some randomised representation of that? If you are adding positive random

values to this, arent you changing the probability distribution of W_λ from uniform to something shifted/tending to Gaussian?
W^* is already a random representation of a precipitation field that is conditioned on the available point observations. Due to the additional constraint $\sum \alpha_l^2 << 1$ it however is a very smooth field (like an interpolated field), i.e. it does not represent the observed spatial variability of the precipitation. By adding the fields $U_1, \ldots, U_k$ to W^* one can easily scale these fields to have the correct spatial dependence without modifying the observed values at the observation locations (because the weights λ are arbitrary as the fields $U_1, \ldots, U_k$ fulfill the homogeneous conditions) such that the final field W_λ exhibits the correct spatial variability. Further, each realization of W_λ (e.g. by taking different λ) is a conditional random field, i.e. a possible representation of the precipitation field. We are not adding positive random values and we are also not working with a uniform distribution but with a standard normal distribution (Eq. 4). This standard normal distribution doesn't change due to the linear combinations (zero mean will always remain zero mean in this case and the unit variance is ensured due to the scaling of the weights λ. Precipitation fields are obtained via back-transformation of these fields.

P7l7 - I presume this is a minimisation being performed which I think should attain a minimum value if the W* is representing the observed precipitation field and the scaling weight $k(\lambda)$ equals zero. I am unclear about this approach - this is attempting to create the observed rainfall sequence instead of doing a stochastic generation as far as I can figure this out.
There seems to be another misunderstanding. The described approach is a stochastic procedure as all fields used are random fields. We do not try to create the observed rainfall sequence except that we want to represent point observations as well as the observed runoff. Thus we are working with conditional random fields. As described above the weights λ are arbitrary if we only intend to reproduce the observed precipitation at the observation locations and the spatial variability. From these λ weights we identify those which also reproduce the discharge. Thus the optimization described here is a function of these λ (and because it is an unconstrained optimization it is straight forward). In simple words, the field W_λ (which is already conditioned on precipitation observations) is modified such that the resulting simulated runoff (by the RR-model) is close to the observed runoff.

P7l12 - the authors are saying multiple sequences are created by generating new random fields V_l and enabling something called uncertainty quantification - please explain what this means. I am very curious how different the sequences end up being - and when they are really different, whether their probability distributions are consistent with the observed series that was used. Also - am I correct in stating that the timing of these sequences will be fairly similar to the observed sequence - hence the final sequences will be representing uncertainty about each observed value more than representing a stochastic system that is generating equally plausible sequences (a bit like a weather generator does conditional to exogenous inputs, compared to a stochastic generator where no two sequences have any exogenous binding variable).
Yes this sentence should explained a bit more. It is mentioned in P3L7 that "... Our goal here is an event based reconstruction of possible realizations of spatio-temporal rainfall patterns which are conform with the measured point rainfall data and catchment runoff response at best. For that we are looking for potential candidates of three-dimensional (space-time) rainfall fields for sub daily time steps and spatial resolution of 1km^2 ... ". This means that each candidate (or sequence) reproduce the point observation of rainfall without any uncertainty (or deviation). Only the grid points between the observation differs within the 3D rainfall field and contain the stochasticity given by simulations conditioned on the observed values.

p7l15 - Am I correct in interpreting that the rainfall is generated known the marginal distribution at each pixel of the 118km2 catchment? Or is it based on the 6 hours of rainfall at the 10 monitoring stations alone? If it is the latter, assumptions must have been made to spatially interpolate/extrapolate the rainfall to other pixels. Please state these. If it is the former, this is a limitation

I believe as you need to be sure about the spatio-temporal structure of your storm to help refine it further using the flows.
We are not quite sure what you mean with assumptions must have been made? Do you mean assumption must have been made to generate the synthetic reality? Or assumptions must have been made to generate possible realizations based on the 6 hours of rainfall at the 10 monitoring stations? If it is the latter then the assumptions that we made are that we can fit a marginal distribution and a spatial copula to these observations. Therefore only the values at the rainfall monitoring stations are used for the fitting etc. in order to make the synthetic test case a realistic scenario. But since this a synthetic test case all values at each pixel are known which enables comparison of the simulated results with the synthetic reality. We assume that the 6h precipitation distribution for the whole area is the same as the precipitation distribution derived from the observations (corresponding to the observation location).

P7l26 - some mention of the number of time steps in the observed record for rainfall and flows should be provided - there is a mention of 6 hours but I wasnt sure if that is the time step of the duration.
It is already mentioned in the manuscript nine lines above (P7L17: "A synthetic rainfall event of 6 hours duration with hourly time step ..")

P9fig6 - I see all hydrographs are having roughly the same timing of the peak. So what I suspected about the time sequences of the rainfall is most likely correct. The differences across the storms would not be significant in terms of the spatial or the temporal pattern uncertainty that exists in real cases. I think this could be a limitation if the approach were being pitched as a stochastic generator - but could form an interesting way to generate alternate realisations of a storm sampled at specified point locations alone.
The goal of the work is an stochastic "reconstruction" of spatio-temporal rainfall pattern ... (see title) which seams to be similar to what you called "to generate alternate realisations of a storm sampled at specified point locations alone". We are not interested in exploring overall spatio-temporal pattern uncertainty (e.g. by performing unconditional stochastic simulations and considering measurement uncertainty too) since this was already done in research and has no benefit for the focus of this paper. Fig 6 shows the results of 200 simulated spatio-temporal rainfall pattern conditioned at rainfall point observations only, but containing the spatial uncertainty for the unobserved points. The hydrographs have to look a bit similar since all simulations used the same rainfall values at observation points transformed into runoff by the same hydrologic model (representing the hydrologic properties of the catchment).

And the need for having an accurate hydrologic model is a big limitation too as the uncertainty that arises from this can be significant.
Of course hydrologic model uncertainty plays an important role, but instead of changing the model to fit the observed discharge we estimate rainfall fields which fit the model and the discharge. As such plausible rainfall fields can be identified, the corresponding model and the rainfall field is plausible.

On the whole, I am unclear how I would use an approach such as this for my modelling application. I will need to have a fairly good idea of the spatio-temporal nature of the storm system to put this into use - along with having point rainfalls and modelled flow time series to help ascertain which sequences are good.
Some hints are given in the the summary section. (see P15L13 "... a reanalysis tool for rainfall-runoff events especially in regions where runoff generation and formation based on surface flow processes and catchments with wide ranges in arrival times at catchment outlet ..." or P15L22 "... where modelers are interested to explain the extraordinary rainfall-runoff events ...".) However, this section is discussed more detailed in the revised manuscript.

I think the authors need to add more examples of this in their revision to establish a clear scenario how users will put their method into use. And some details of the tolerences etc that are used to make this stochastic should be added as I think they are not stated in the paper very clearly. Some indication of how this might perform over long storms/large catchments/very few point locations etc will really help readers
We are not sure, what you mean by “tolerences”. In general, the manuscript aims to present a new method and to show that it can deal with real world data. However, it is basic research and we are also very curious to explore the method further (see outlook P15L16). But this requires further developments (e.g. common interfaces for data, models, other types of copulas) which are not manageable within the next months. Among others we intended to show that models may be good even without any strong modification if we take the uncertainty of the precipitation into account. Thus models may help to improve precipitation estimation and one could consider model calibration under consideration of precipitation uncertainty.

REFEREE 2:

The paper ‘Stochastic reconstruction of spatio-temporal rainfall pattern by inverse hydrological modelling’ by Grundmann J., Hörning, S. and Bárdossy, A. proposes a method to estimate high resolution space-time rain fields from sparse rain gauges observations complemented by streamflow measurements. I find the idea of incorporating streamflow measurements and inverse hydrological modelling to reconstruct rain fields very interesting. And to my knowledge it is the first time that it is proposed to apply this idea to the reconstruction of high resolution space-time rain fields. In that respect I find this paper original. In addition the topic is relevant for the readers of HESS.
However, I feel that in the present version of the manuscript, the authors do not provide enough information (and of sufficient quality) to be able to assess the proposed framework. In addition I have the impression that even if interesting, the proposed approach cannot reach all the targets stated by the authors.
To sum up, I have the feeling that this paper addresses an interesting idea, but the current version is very preliminary (too much in my opinion) and does not allow to capitalize on the framework developed by the authors. I start by listing the points I would need to know in order to fully understand and assess the proposed method. After that, I will detail some concerns I have about the method itself. Afterwards I finish my review by few minor comments.

Possible improvements to better explain the method:

First of all, the written English must be improved. The present version of the manuscript is full of errors that shocked me even though I am not a native English speaker. At a minimum, a spell checker must be used. When I applied mine to the present manuscript I obtained dozens of errors and typos... In addition, some sentences are grammatically incorrect or difficult to understand. For instance: p1L20-24, p3L3-4, p11L5-8.
We tried to fix all typos and improved the grammar in the revised manuscript. The revised manuscript was double checked by a native speaker.

Regarding the introduction and the context of this study, I acknowledge that the application of inverse modelling to the reconstruction of space-time rain fields is new. However the idea of inverse hydrology in general (i.e. without space-time application) has already been proposed by several authors, as well as the idea of using streamflow data to improve rainfall input estimation. Unfortunately, none of these works are mentioned in the introduction. I find it quite unfair. I really would like to see more background about previous studies addressing similar ideas in order to better contextualize the present study. I can suggest for instance the following papers (I didn’t participate to these works):
- Kirchner J.W. (2009): Catchments as simple dynamical systems: Catchment char-acterization, rainfall-runoff modeling, and doing hydrology backward, Water Resources Research, 45, W02429, doi:10.1029/2008WR006912.

- Kretzschmar A. et al (2014): Reversing hydrology: Estimation of sub-hourly rainfall time-series from streamflow, Environmental Modelling and software, 60, 291-301.
-Del Giudice, D. et al (2016): Describing the catchment-averaged precipitation as a stochastic process improves parameter and input estimation, Water Resources Research, 52, 3162-3186, doi:10.1002/2015WR017871.
Thank you very much for these references. We improved the introduction and broaden the literature review and discussion.

Regarding the description of the rainfall-runoff model, very few information is provided. What is specified is basically that it is a distributed model, no more. For instance I don't know the name of the model, there is no reference about this model, and no equation to explain how it works. However I am sure that the hydrological model used for the inversion of the streamflow to reconstruct rainfall has a significant impact on the final result. By the way, the impact of the choice of the hydrological model (e.g. distributed vs semi-lumped) should be discussed somewhere in the paper.
We added additional information in the revised manuscript. Up to now, the model has only a working title. It uses only simple approaches known from hydrologic textbooks for the simulation of single events (no long-term water balance). It focuses on Hortonion runoff and considers spatial distributed travel times for surface runoff. You are right, the choice of the model has impact on its results. We enhanced the discussion of this issue in the last section.

Regarding the description of the Random Mixing approach, I really lack information about the underlying statistical model and the inference of its parameters. To be honest I had to read the paper of Haese et al (2017) to be able to understand the application of Random Mixing to rainfall modelling. Therefore I think that not enough efforts have been made to explain the Random Mixing method in the present paper. In particular I would be interested to know:

- Which spatio-temporal copula is used? Does it need to be a valid covariance function (or is it irrelevant in the context of copulas)?
We have used a Gaussian copula. And yes it needs to be a valid covariance function. We added more information on copulas in general and the Gaussian copula in the revised manuscript.

- How are the parameters of the model (i.e. the marginal transform function and the copulas) inferred in practice? In particular how do you deal with dry measurements (i.e. rain intensity=0) in the inference process? (I think it is important here since rain intermittency can be significant in semi-arid and arid regions). Ok there is a reference to Li (2010), but more information within this paper would be a plus for the reader.
We added a bit more information (and more references) on the inference process however we do not want to go into great detail as this is not the main focus of this work.

- Which simulation method is used in practice to generate the unconditional simulations? You cite several methods but I would like to know the one you are actually using.
We have actually used the spectral representation method: Shinozuka, M., and G. Deodatis (1996), Simulation of multi-dimensional gaussian stochastic fields by spectral representation, Appl Mech Rev, 49(1). It was not in the references list yet so we added it and mentioned it in the revised manuscript.

Regarding the synthetic case study, it is not clear to me if the parameters of the statistical rainfall model used in the random mixing are inferred from the synthetic data. I suppose that it is the case, but it should be clearly mentioned. If it is the case, it would be interesting to show the results of the fitting procedure. For instance: which copula (with which parameters) has been fitted? And also which marginal distribution? And how do the estimated values of the model parameters compare with the true ones (in this case you know the true values because it is a synthetic case)? In fact I suspect that the inferred statistical rainfall model cannot capture properly the true statistics of rainfall because the center of the rain cell is not observed. This can explain why conditional simulations

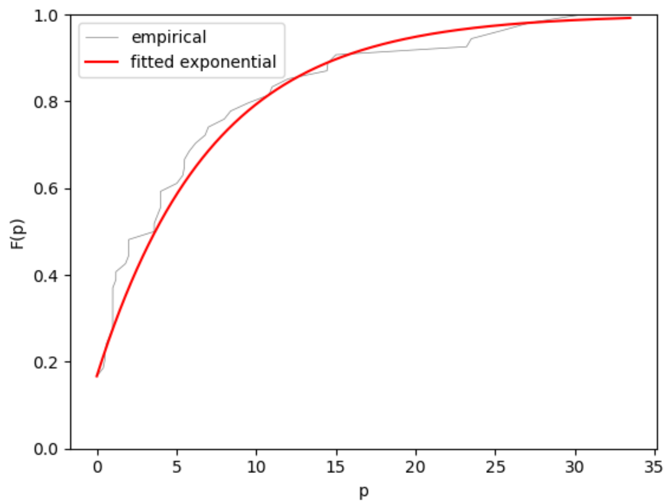

Figure 1: Marginal empirical and fitted exponential distribution of rainfall for the real test case

(without streamflow constraints) cannot reproduce the observed hydrograph. I will come back to this point in my concerns about the method.

Yes the parameters are inferred from the synthetic data (only from the 'observations' though). Thus you are right, the inferred statistical model cannot capture properly the true statistics as for example the center of the rain cell is not observed. And this of course also leads to the fact that conditional simulations (without conditioning on runoff data) are not able to reproduce the observed hydrograph (but that is a general problem of course). The marginal distribution throughout the whole paper is the mixed distribution described in Eq.3 with a discrete probability of zeros (p_0) and an exponential distribution for all values > 0. Based on the available observations the fitted parameters for the synthetic test site are: $p_0 = 0.36$ and $\lambda = 0.48$. The fitted copula is a Gaussian copula with an exponential correlation function with a range of 2.5 km in space and a range of 1.5 $hours$ in time. Parameters for the real test case are: $p_0 = 0.17$ and $\lambda = 0.14$, and a Gaussian copula with an exponential correlation function with a range of 10 km in space and a range of 1 $hours$ in time. We added these information in the revised manuscript. Following figure shows a comparison of empirical and fitted exponential distribution of observed rainfall for the real test case. The result is acceptable.

Regarding the real world application. I would have been more convinced if you have shown an example with cross-validation. For instance the reconstruction of space-time rainfall for a well instrumented catchment (with many rain gauges). In this case you can select some stations for the inference of the mixing model parameters and the estimation of rain fields, and keep other stations to cross-validate the rain estimations. In addition, in the real world application, the altitude of the catchment ranges from 600m to 2500m; in this case one can expect some non-stationarities in rainfall statistics. Could you please discuss a bit this potential issue?
The presented real world application in this manuscript is more or less the initiator for this research. It is based on our multiyear research on hydrologic processes in this arid region under data scarcity and small scale rainstorms. We understand your wish for "... an example with cross-validation." and we acknowledge this idea. However, in this case data quality and situation is bad and scarce. The walnut gulch catchment in US might be more appropriate for an investigation with cross-validation, but not manageable now. We will consider this in our future research. Thank you for this hint.

Regarding the non-stationarities in rainfall, in this case the application shows a reconstruction of a single rainstorm which doesn't consider rainfall non-stationarities. The Figure 2 below shows the measurements of the rainfall gauging stations for this event and their altitudes. Most of the rain is recorded on stations with lower altitudes located in the north-west and south-eastern part of the catchment. We added this figure and information in the revised manuscript. Obviously much more research is needed to fully exploit the advantages and limits of this procedure but we thought that we are at a level so that results can be communicated to the advantage of the possible readers of the journal.

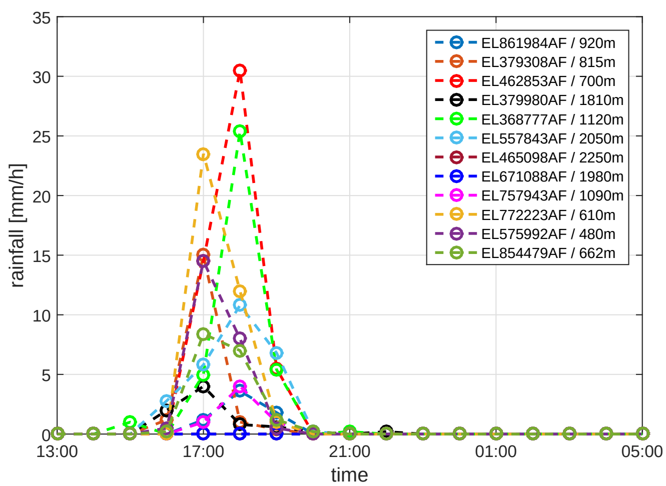

Figure 2: Rainfall amounts and altitudes of rainfall gauging stations for the case study area

Concerns about the method itself:

In the proposed method, the parameters of the hydrological model are supposed to be known and fixed. But at the same time the goal is to infer high resolution space-time rain fields to... improve hydrological modelling. This seems a bit circular reasoning. I see two options to break the circle:
- Either clearly acknowledge that the proposed framework is a first step that only aims at reconstructing space-time rain fields from rain gauge and streamflow measurements. Basically a proof of concept with strong assumptions (incl. known hydrological model), that will be relaxed only in future work. And in this case do not claim that the goal is to improve hydrological modelling, but just to show that doing reverse hydrology to reconstruct space-time rain fields is somehow feasible. In my opinion this is already a very nice contribution.
- Or improve the proposed framework to jointly reconstruct space-time rain fields and calibrate the hydrological model. This can be seen as the extension of the work of Del Giudice et al (2016) (see ref above) to the case of space-time rain fields. But I suspect that this will require a lot of developments... and I am not sure it will work in many configurations... But if it works it would be an even nicer contribution.
It is definitely option one and as you argued correctly, option two would require lots of developments and is not manageable within this manuscript. We had in mind that an improved estimate of the model input also improves the hydrologic modeling results. But you are right, this can be misunderstood and we formulated our arguments more carefully in the revised manuscript.

In the synthetic case study, runoff simulations based on simulated rain fields conditioned to point observation only do not encompass the ones based on conditioning to rain gauge observations and

streamflow observations. This is clearly visible by comparing figures 6 and 8. I am very surprised about it. Indeed, in my understanding, the second case (adding conditioning to streamflow) should just add constraints to the first case. Therefore it should only select the rain fields obtained by simulation conditional to rain gauge only that are compatible with streamflow observations. But it is clearly not the case here... Therefore either I am missing something, and in this case I believe the reasons why this result arises must be explained in much more details by the authors; or there is some issue. One possible reason I could suggest (but I am not sure) is that the actual rain field that generates the observed hydrograph is kind of an extreme of the multivariate statistical distribution that underlies the random mixing model (after fitting model parameters). And therefore this extreme is not sampled by the 200 realizations performed in Figure 6.
Yes, your assumption is right and we improved the discussion in the revised manuscript. Most probably, if we would sample more than 1000000 conditioned rainfall fields we would find a realisation which matches the runoff observation too, since the amount of possible conditioned rainfall fields is very much higher than the amount of rainfall fields matching point observation and runoff. Due to additional conditioning we find these realisation faster.

Regarding the assessment of uncertainty, I would be more cautious before stating "This ensemble can be used to describe the uncertainty in estimating spatio-temporal rainfall patterns" (p11, L9). In my opinion, the ensemble of realizations that is obtained is only a very partial descriptor of the total uncertainty. Indeed, in the proposed framework, both the statistical rainfall model and the runoff generation model have fixed parameters. Therefore the uncertainty originating from these two components is neglected. In the end, only the uncertainty related to the scarcity of the rain gauge measurement network is accounted for. I think this should be more clearly explained to the reader.
You are right. It is a partial descriptor of the total uncertainty. It describes the remaining uncertainty of spatio-temporal rainfall fields if all available data are exploited (under the assumption of known hydrologic model parameter, error-free measurements, and reliable statistical rainfall models). We believe that we can reduce the uncertainty of precipitation this way. We formulated our arguments more carefully in the revised manuscript.

Minor comments:
-P2L22: In my knowledge, the turning band method is linked to the theory of random fields rather than to point processes.
To our knowledge, the turning band method has been introduced in its general form by Matheron (1973) and popularized for 2-D applications in hydrology by Mantoglou and Wilson (1982). So, it starts from 1-D point processes and was generalized to generate random fields.

-P2L34: "with respect to the outlined problem in the second paragraph above" - not clear what you are referring to.
We improved this.

-Modeling vs modelling: you have to choose one spelling.
Thanks we use "modeling" in the revised manuscript.

-Eq 2: why qn and not q(t)?
You are right, $q(t)$ would make more sense. We changed this in the revised manuscript.

-P4L20: It would be more clear if you say that P(x,t) is precipitation instead of rainfall. Or maybe call the variable R?
That's also right, we changed it in the revised manuscript.

-Eq 3: Don't mix P and p.
This is actually correct but I admit that it is rather confusing. P represents a field, while p is a single

value at a specific location within that field. However, the equations are improved in the revised manuscript.

-Eq 4: Don't mix W and w.
Same as for P and p.

-Eq 6: You should mention that conditioning is made at this step.
Yes you are right we pointed this out more clearly in the revised manuscript.

-Figure 3, 5, 7, 9: Please add units to the X and Y axes as well as to the color bar. You could also add the limits of the watershed.
You are right. We improved the figures.

-Real case study: you should show the observation dataset.
We added the figure shown here in the comments.

author's changes in manuscript

Some hints regarding author's changes in manuscript have been already given in the comments section. Here, a summary of author's changes in manuscript based on comments of both referees is given.

- chapter 1 "Motivation": adding and discussion of literature regarding inverse hydrologic modeling, improved reasoning
- chapter 2.2 "Rainfall runoff model": description was improved
- chapter 2.3 "Random Mixing for inverse hydrologic modeling": description was improved
- chapter 3.1 "Synthetic test site": Information about parameters for rainfall simulation was added.
- chapter 3.2 "Results and discussion": discussion of the synthetic example was enhanced and performed more precise.
- chapter 4.1 "Arid catchment test site": figure of rain gauge measurements and additional information was added. Information about parameters for rainfall simulation was added.
- chapter 5 "Summary and conclusion" reasoning was enhanced and performed more precise
- revision of Figures 1,3,5,7,9,13. (Please note, revised figures are not included in the marked-up manuscript)
- all typos were fixed and grammar was improved

Stochastic reconstruction of spatio-temporal rainfall pattern by inverse hydrologic ~~modelling~~modeling

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Abstract. Knowledge ~~about the~~ of spatio-temporal rainfall ~~pattern~~ patterns is required as input for distributed hydrologic models ~~to perform several tasks in hydrology like~~ used for tasks such as flood runoff estimation and ~~modelling~~modeling. Normally, these ~~pattern~~ patterns are generated from point observations on the ground using spatial interpolation methods. However, such methods fail in reproducing the true spatio-temporal rainfall pattern, especially in data scarce regions with poorly gauged catchments, or for highly dynamic, small scaled rainstorms which are not well recorded by existing monitoring networks. Consequently, uncertainties ~~are associated with~~ arise in distributed rainfall-runoff modeling if poorly identified spatio-temporal rainfall pattern ~~in distributed rainfall-runoff modelling~~ are used, since the amount of rainfall received by a catchment as well as the dynamics of the runoff generation of flood waves are underestimated. ~~For addressing~~ To address this problem we propose an inverse hydrologic ~~modelling~~ modeling approach for stochastic reconstruction of spatio-temporal rainfall pattern. The methodology combines the stochastic random field simulator Random Mixing and a distributed rainfall-runoff model in a Monte-Carlo framework. The simulated spatio-temporal rainfall ~~pattern~~ patterns are conditioned on point rainfall data from ground monitoring networks ~~as well as~~ and the observed hydrograph at the catchment outlet and ~~aims~~ aim to explain measured data at best. Since we ~~conclude~~ infer from an integral catchment response on a three-dimensional input variable, several candidates of spatio-temporal rainfall pattern are ~~possible which also describe~~ feasible and allow for an analysis of their uncertainty. The methodology is ~~testet~~ tested on a synthetic rainfall-runoff event on ~~subdaily timesteps~~ sub-daily time steps and spatial resolution of 1km² for a catchment covered by rainfall partly. ~~Results show that a~~ A set of plausible spatio-temporal rainfall ~~pattern~~ patterns can be obtained by applying ~~the~~ this inverse approach. Furthermore, results of a real world study for a flash flood event in a ~~mountainious~~ mountainous arid region are presented. They underline that knowledge about the spatio-temporal rainfall pattern is crucial for flash flood ~~modelling~~ modeling even in small catchments and arid and semiarid environments.

1 Motivation

The importance of spatio-temporal rainfall pattern for ~~rainfall runoff estimation and modelling~~ rainfall-runoff estimation and modeling is well known in hydrology~~and widely~~, and has been addressed by several simulation studies, especially since distributed ~~hydrological~~ hydrologic models have become available. ~~Those studies showing either~~ Many of those studies demonstrated the effect of resulting runoff responses for different spatial rainfall pattern (Beven and Hornberger, 1982; Obled et al., 1994; Morin et al., 2006; Nicotina et al., 2008), or ~~addressing~~ addressed the errors in runoff prediction and the difficulties in parameterisation and calibration of hydrologic models if the spatial distribution of rainfall is not well known (Troutman, 1983; Lopes, 1996; Chaubey et al., 1999; Andreassian et al., 2001)~~, or investigating the required spatial configuration and temporal resolution~~ . As a consequence, studies were performed to investigate configurations of rainfall monitoring networks ~~on the ground in order to monitor spatio-temporal rainfall pattern adequatly (Faures et al., 1995)~~(Faures et al., 1995), and rainfall errors and uncertainties for hydrologic modeling (McMillan et al., 2011; Renard et al., 2011).

In ~~generall~~general, rainfall monitoring networks based on point observations on the ground (station data) require interpolation methods to obtain spatio-temporal rainfall fields usable for distributed ~~hydrological modelling. However, those~~ hydrologic modeling. Traditional interpolation methods fail in reproducing the true spatio-temporal rainfall pattern especially for: (i) data scarce regions with poorly gauged catchments and low network density, (ii) highly dynamic, small scaled rainstorms which are not well recorded by existing monitoring networks, and (iii) catchments which are covered by rainfall partly. Consequently, uncertainties are associated with poorly identified spatio-temporal rainfall pattern in distributed ~~rainfall-runoff-modelling~~ rainfall-runoff-modeling since the amount of rainfall received by a catchment as well as the dynamics of runoff generation processes are ~~underestimated~~typically underestimated by current methods.

The effects of poorly estimated spatio-temporal rainfall fields are visible in particular for semiarid and arid regions, where rainstorms ~~showing~~ show a great variability in space and time and the density of ground monitoring networks is sparsely compared to other regions (Pilgrim et al., 1988). Based on an analysis of 36 events in a ~~mountainiuos~~ mountainous region of Oman, McIntyre et al. (2007) show a wide range of event-based runoff coefficients, which underlines that achieving reliable runoff predictions by using ~~hydrological~~ hydrologic models in those regions is extremely challenging. This is supported by several simulation studies (Al-Qurashi et al., 2008; Bahat et al., 2009), who address the uncertainties in model ~~parametrisation~~ parameterisation due to uncertain rainfall input. In this context Gunkel and Lange (2012) report that reliable model parameter estimation was only possible by using rainfall rader information. However, ~~those information are~~ this information is not available everywhere.

~~For adressing~~ To address the inherent uncertainties described above, stochastic rainfall generators are used intensively to create spatio-temporal rainfall inputs for distributed ~~hydrological~~ hydrologic models to transform rainfall into runoff. A large amount of literature exists describing different approaches for space-time simulation of rainfall fields~~, among them multisite~~ ; among them multi-site temporal simulation frameworks (Wilks, 1998), approaches based on the theory of random fields (Bell, 1987; Pegram and Clothier, 2001) or approaches based on the theory of point processes and its generalization, which includes the popular Turning bands method (Mantoglou and Wilson, 1982). Enhancements were made in order to portray different rain storm pattern and distinct properties of rainfall fields, like spatial covariance structure, space-time ~~anomalie, and intermittancy~~ anomaly, and intermittency (see Leblois and Creutin 2013; Paschalis et al. 2013).

Applications of spatio-temporal rainfall simulations together with ~~hydrological~~ hydrologic models are of straightforward, Monte-Carlo type, where a ~~hugh amount of rainfall fields is~~ large number of potential rainfall fields are generated driven by stochastic properties of observed rainstorms or longer ~~timeseries. Those fields are~~ time series. These fields are used as inputs for distributed hydrologic model simulations to investigate the impact ~~on resulting simulated catchment response for~~ of certain aspects of rainfall like uncertainty in measured rain depth, spatial variability, etc., on simulated catchment responses. Rainfall simulation applications are performed in unconditional mode (reproducing ~~rainfield~~ rain field statistics only) or conditional mode, where observations (e.g from rain gauges) are reproduced too. The latter are commonly used for investigating the effect of spatial variability using fixed total precipitation and variations in spatial pattern (Krajewski et al., 1991; Shah et al., 1996; Casper et al., 2009; Paschalis et al., 2014).

~~With respect to the outlined problem in the second paragraph above~~However, stochastic rainfall simulations in combination with ~~hydrologic modeling might be a solution to reconstruct unknown spatio-temporal rainfall pattern. However, stochastic rainfall simulations together with hydrologic modelling~~ distributed hydrologic modeling can be computationally demanding and can fail at matching the observed stream flow if rainfall fields are conditioned on rainfall point observations only. ~~Therefore, our approach aims to include the observed runoff into the conditioning process. This implies that~~

On the other hand, inverse hydrologic modeling approaches have been developed to estimate rainfall time series based on observed stream flow data. Those approaches require either an inverting of the underlying mathematical equations for the nonlinear transfer function (Kirchner, 2009; Kretzschmar et al., 2014) or an application of the hydrologic model in a Bayesian inference scheme (Kavetski et al., 2006; Del Giudice et al., 2016). Up to now, both approaches delivers time series of catchment-averaged rainfall only, which gives no idea about the spatial extent and distribution of rainfall. This is particularly important when considering events such as localised rainstorms, which might be underestimated and not accurately portrayed.

The goal here is an event based reconstruction of spatio-temporal rainfall pattern ~~are conditioned on hydrologic model output in addition, why we call this an inverse modelling approach~~which explain measured point rainfall data and catchment runoff response at best. For that we are looking for potential candidates of three-dimensional rainfall fields for sub daily time steps and spatial resolution of 1km² which, to our knowledge hasn't been done so far. To achieve this task, we combine stochastic rainfall simulations and distributed hydrologic modeling in an inverse modeling approach, where spatio-temporal rainfall pattern are conditioned on rainfall point observations and observed runoff. The methodology of the inverse hydrologic ~~modelling approach combines~~ modeling approach consists of the stochastic random field simulator Random Mixing and a distributed rainfall-runoff model in a Monte-Carlo framework. Until now, Random Mixing, developed by Bárdossy and Hörning (2016b) for solving inverse groundwater modeling problems, has been used by Haese et al. (2017) for reconstruction and interpolation of precipitation fields using different data sources for rainfall. ~~Our goal here is an eventbased reconstruction of possible realisations of spatio-temporal rainfall pattern which are able to explain measured point rainfall data and catchment runoff response at best. For that we are looking for potential candidates of three-dimensional rainfall fields for subdaily timesteps and spatial resolution of 1km² which, to our knowledge hasn't been done so far.~~

After this introduction the methods are described in chapter 2. It gives an overview of the ~~approach~~ methodology and further details for the applied ~~rainfall runoff~~ rainfall-runoff model, the ~~random mixing~~ Random Mixing and its application for rainfall fields. Chapter 3 aims to test the methodology. A synthetic test site is introduced which is used to demonstrate and discuss the limits of common hydrologic modeling approaches (using rainfall interpolation) as well as conditional rainfall simulations only. In contrast, the functionality of the inverse hydrologic ~~modelling~~ modeling approach is illustrated and discussed. In chapter

4, the inverse hydrologic ~~modelling~~ modeling approach is applied for real world data by an example of an arid ~~mountainious~~ mountainous catchment in Oman. The test site is introduced and results are shown and discussed. Finally, summary and conclusions are given in chapter 5.

2 Methods

2.1 General approach

The methodology described here can be characterized as an inverse ~~hydrological modelling~~ hydrologic modeling approach. It aims to conclude on potential candidates for the unknown spatio-temporal rainfall pattern based on runoff observations at the catchment outlet, known ~~parametrization~~ parameterisation of the rainfall-runoff model and rain gauge observations. The approach combines a grid-based spatially distributed rainfall-runoff model and a conditional random field simulation technique called Random Mixing (Bárdossy and Hörning, 2016a, b). Random Mixing is used to simulate a conditional precipitation field which honors the observed rainfall values as well as their spatial and temporal variability. In order to additionally condition the rainfall field on the observed runoff it is iteratively updated. Therefore, the initial field is used as input to the rainfall-runoff model. The deviation between the simulated runoff and the observed runoff is evaluated based on the model efficiency (NSE) defined by Nash and Sutcliffe (1970). To minimize this deviation the rainfall field is *mixed* with another random field which exhibits certain properties such that the mixture honors the observed rainfall values and their spatio-temporal variability. This procedure is repeated until a satisfying solution, i.e. a conditional rainfall field that achieves a reasonable NSE, is found. To enable a reasonable uncertainty estimation the procedure is repeated until a predefined number of potential candidates has been found. In the following, rainfall is used interchangeably with precipitation.

2.2 Rainfall runoff model

A simple spatially distributed rainfall-runoff (RR) model is used as transfer function to portray the nonlinear transformation of spatially distributed rainfall into runoff at catchment outlets. The model is dedicated to describe rainfall-runoff processes in arid mountainous regions, which are mostly based on infiltration excess and Hortonian overland flow. The model is working on regular grid cells in event-based mode. ~~The model~~ It is parsimonious in number of parameters considering transmission losses but having no base flow component. Pre-state information at the beginning of an event is neglected since runoff processes starting under dry conditions mostly (Pilgrim et al., 1988).

~~An~~ More specifically, only simple approaches known from hydrologic textbooks for the simulation of single rainfall-runoff events (no long-term water balance) are used (Dyck and Peschke, 1983). Effective precipitation $Pe(x,t)$ is calculated by an initial and constant rate loss model ~~is~~ applied on each grid cell which is affected by rainfall. The ~~calculated effective rainfall~~ initial loss I_a represents interception and depression storage. If the accumulated precipitation exceeds I_a surface runoff may occur, which is reduce by the constant rate f_c throughout an event to consider infiltration. The calculated effective precipitation (respectively surface runoff) is transferred to the next river channel section considering translation and attenuation processes.

Translation is accounted for with a grid-based travel-time ~~model incorporating~~ function to include the effects of surface slope and roughness~~, and attenuation~~. Attenuation is accounted for with a single linear storage unit with recession constant f_r. Both approaches are applied on grid cells affected by effective precipitation only to fully support spatial distributed calculations corresponding to the spatial extent of the rain field. The properties of several landscape units are addressed by different parameter sets (for I_a, f_c, f_r) following the concept of hydrogeological response units (Gerner, 2013) (since ~~hydrological~~ hydrologic processes are mostly driven by hydrogeology in these regions). Runoff is routed to the catchment outlet by a simple lag model in combination with a constant rate (f_t) loss model to portray transmission losses along the stream channel. The RR-model is applied for hourly time step on regular grids cells of ~~1km by 1km~~1 km by 1 km. Parameters are assumed to be known and fixed during the inverse ~~modelling step~~modeling procedure. Up to now, the RR-model is linked to Random Mixing directly and named with working title NAMarid.

2.3 Random Mixing for inverse ~~hydrological modelling~~hydrologic modeling

Random Mixing is a geostatistical simulation approach first presented in Bárdossy and Hörning (2016a) and Bárdossy and Hörning (2016b) where the authors have applied it to inverse groundwater modeling problems. It uses copulas as spatial random functions (Bárdossy, 2006) and represents an extension to the gradual deformation approach (Hu, 2000). In the following a brief description of the Random Mixing algorithm is presented. A detailed explanation can be found in Hörning (2016).

The goal of the inverse ~~hydrological modelling~~ hydrologic modeling approach presented herein is to find a conditional ~~rainfall~~ precipitation field $P(x,t)$ with location $x \in D$ and time $t \in T$ which reproduces the observed spatial and temporal variability and marginal distribution of P. This field should also honor precipitation observations at locations x_j and times t_i:

$$P(x_j,t_i) = p_{j,i} \quad \text{for} \quad j = 1,\ldots,J \quad \text{and} \quad i = 1,\ldots,I \tag{1}$$

Note that P denotes a spatial field and p denotes a precipitation value within that field. Furthermore, the solution of a rainfall-runoff model using the field P as input variable should approximately honor the observed runoff:

$$Q_{\cancel{n}t}(P) \cancel{=} \approx q_{\cancel{n}t} \quad \text{for} \quad \cancel{n}t = 1,\ldots,\cancel{N}T \tag{2}$$

where ~~Q_n~~ Q_t denotes the rainfall-runoff model and ~~q_n~~ q_t-s represent the observed runoff values at time step t. Note that ~~$Q_n(P)$~~ $Q_t(P)$ represents a non-linear function of the field P.

In order to find such a ~~rainfall~~ precipitation field P which fulfills the conditions given in Eq. (1) and Eq. (2) Random Mixing can be applied. Figure 1 shows a flowchart of the corresponding procedure.

After identifying the observations $p_{j,i}$, a marginal distribution ~~$G(P(x,t))$~~ $G(p)$ has to be fitted to them. Note that in general any type of distribution function (e.g. parametric, non-parametric and combinations of distributions) can be used. For the applications presented herein the selected marginal distribution consists of two parts: the discrete probability of zero ~~rainfall~~ precipitation and an exponential distribution for the wet ~~rainfall~~ precipitation observations. It is defined as:

$$G(\cancel{P(x,t)}p) = \begin{cases} p_0 & \text{if } p = 0 \\ p_0 + p_0(1 - \exp(-\lambda p)) & \text{otherwise} \end{cases} \tag{3}$$

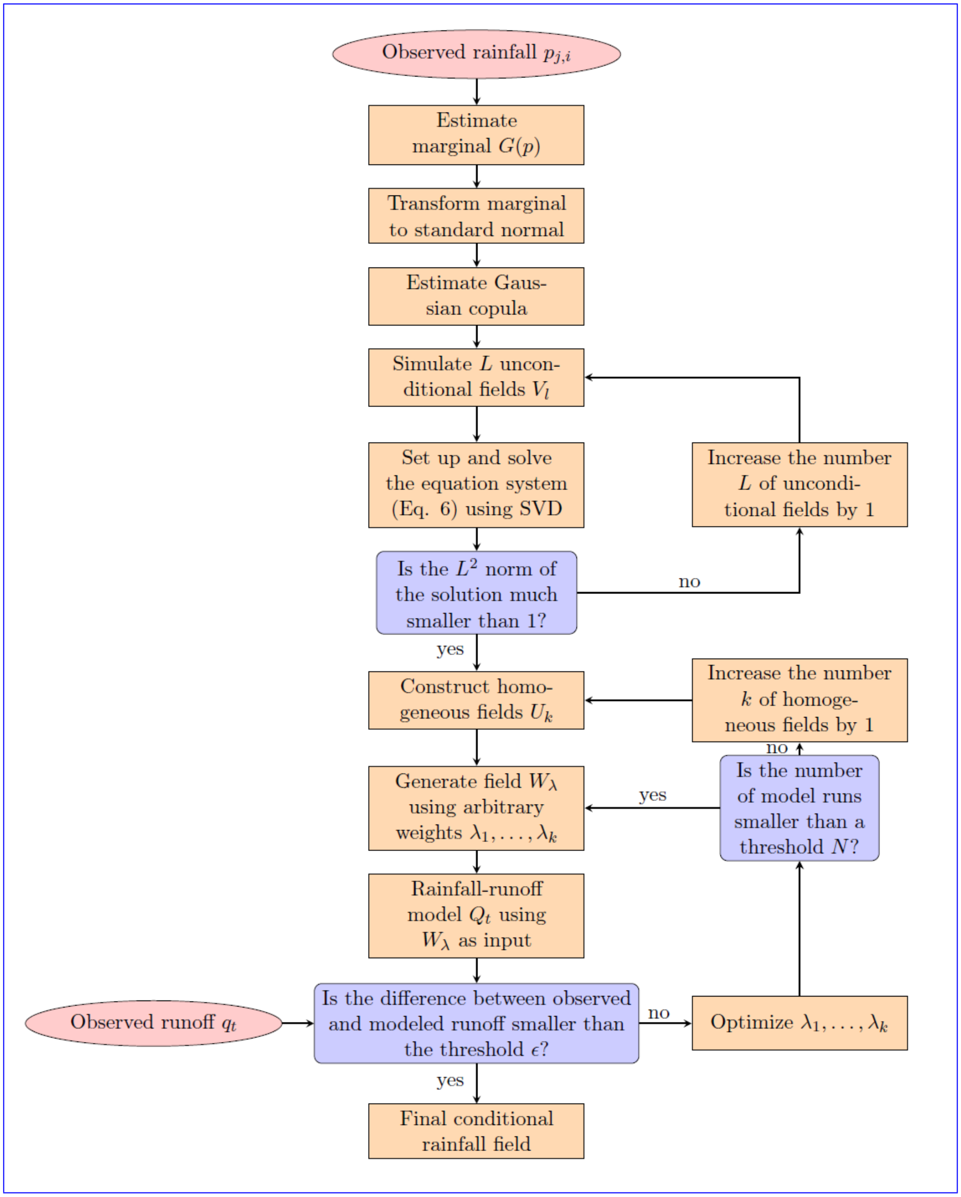

Figure 1. Flowchart of the Random Mixing algorithm for inverse ~~hydrological modelling~~hydrologic modeling.

with p denoting ~~rainfall~~ precipitation values, p_0 is the discrete probability of zero ~~rainfall~~ precipitation and λ denotes the parameter of the exponential distribution. ~~Then~~ Thus the parameters that need to be estimated are p_0 and λ. Then, using the fitted marginal distribution the observed precipitation values are transformed to standard normal:

$$w = \begin{cases} < \Phi^{-1}(p_0) & \text{if } p = 0 \\ \Phi^{-1}(p_0 + p_0(1 - \exp(-\lambda p))) & \text{otherwise} \end{cases} \tag{4}$$

where Φ^{-1} denotes the univariate inverse standard normal distribution. Note that zero ~~rainfall~~ precipitation observations are not transformed to ~~a certain~~ the same value, but they are considered as inequality constraints as described in Eq. (4). Thus the spatio-temporal dependence structure of the variable is taken into account as described in Hörning (2016). Further note that the transformation of the marginal distribution described in Eq.4 can be reversed via:

$$P(x,t) = G^{-1}(\Phi(W(x,t))) \tag{5}$$

where G^{-1} denotes the inverse marginal distribution of P and Φ denotes the univariate standard normal distribution. Also note that W denotes the transformed spatial field while w denotes a transformed observed value within that field. Note that in this approach we assume that the precipitation distribution is the same for each location x and each time-step t. One could use a location and/or time specific distribution to take spatial or temporal non-stationarity into account, however this requires a relatively large amount of precipitation observations and/or additional information.

~~In the next step, a Gaussian copula is fitted to the observations according to the approach~~ As a next step we assume that the field W is normal, thus its spatio-temporal dependence is described by the normal copula with correlation matrix Γ_c. In general copulas are multivariate distribution functions defined on the unit hypercube with uniform univariate marginals. They are used to describe the dependence between random variables independently of their marginal distributions. The normal copula can be derived from a multivariate standard normal distribution (see Bárdossy and Hörning (2016b) for details). It enables modeling a Gaussian spatio-temporal dependence structure with arbitrary marginal distribution. Note that its correlation matrix Γ_c has to be assessed from the available observations. If no zero observations are present the maximum likelihood estimation procedure described in Li (2010) ~~. This copula describes the spatio-temporal dependence structure of the observations.~~ can be applied to estimate the copula parameters. If zero values are present a modified maximum likelihood approach has to be used (Bárdossy, 2011). It uses a combination of three different cases (wet-wet pairs, wet-dry pairs, dry-dry pairs of observations) for the estimation of the copula parameters.

As a next step, unconditional standard normal random fields V_l with $l = 1, \ldots, L$ are simulated such that they all share the same ~~spatial structure~~ spatio-temporal dependence structure which is described by Γ_c of the fitted normal copula. Such fields can for example be simulated using Fast Fourier Transformation for regular grids (Wood and Chan, 1994; Wood, 1995; Ravalec et al., 2000) or Turning band simulation (Journel, 1974). Here we used the spectral representation method introduced by Shinozuka and Deodatis (1991, 1996). Using the fields V_l, the system of linear equations:

$$\sum_{l=1}^{L} \alpha_l V_l(x_j, t_i) = w_{j,i} \quad \text{for} \quad i = 1, \ldots, I \quad j = 1, \ldots, J \quad \text{with} \quad L > N = I \cdot J \tag{6}$$

is set up~~and solved using .~~ . Note that α_l denotes the weights of the linear combination, $w_{j,i} = \Phi^{-1}(G(p_{i,j}))$ are the transformed precipitation values and $V_l(x_j,t_i)$ are the values of the random fields at the observation locations. Using singular value decomposition (SVD) (Golub and Kahan, 1965) ~~. If no solutionwith~~ to solve this equation system leads to a minimum L^2 norm solution. In order to obtain a smooth, low variance field a L^2 norm ~~much smaller than one~~ $\sum \alpha_l^2 \ll 1$ is required. If no such solution is found, an additional field V_{L+1} is created, added to the system of linear equation and the system is solved again. Note that with increasing degrees of freedom (i.e. more fields) the L^2 norm of the solution decreases.

Once a solution with an acceptable L^2 norm i.e. $\sum \alpha_l^2 \ll 1$ is found the resulting field is defined as:

$$W^* = \sum_{l=1}^{\cancel{L}\,L+M} \alpha_l V_l \tag{7}$$

~~and the algorithm moves on~~where M denotes the number of additional fields added to the equation system. Note that W^* fulfills the conditions defined in Eq. ~~1~~(1) however it does not fulfill Eq. (2) and it does not represent the correct spatio-temporal dependence structure.

The next step is to simulate fields U_k with $k = 1,\ldots,K$ which fulfill the homogeneous conditions, i.e. $U_k(x_j,t_i) = 0$. ~~These~~ Further all fields U_k need to share the same spatio-temporal dependence structure, again described by Γ_c. Such fields can be generated in a similar way as W^* (see Hörning (2016) for details). The advantage of these fields U_k is that they form a vector space (they are closed for multiplication and addition), thus:

$$W_\lambda = W^* + k(\lambda)(\lambda_1 U_1 + \ldots + \lambda_k U_k) \tag{8}$$

where λ_k-s denote arbitrary weights and $k(\lambda)$ denotes a scaling factor results in a field W_λ which also fulfills the conditions prescribed in Eq. ~~1~~ (1). The scaling factor ~~$k(\lambda)$~~ is defined as:

$$k(\lambda) = \pm\sqrt{\frac{1-\sum \alpha_l^2}{\sum \lambda_k^2}} \tag{9}$$

It ensures that W_λ exhibits the correct ~~covariance matrix Γ~~spatio-temporal dependence structure. Thus, transforming W_λ back to P using Eq. 5 will result in a precipitation field which has the correct spatio-temporal ~~variability~~dependence structure, marginal distribution and honors the ~~rainfall~~ precipitation observations.

To also honor the observed runoff defined in Eq. 2 an optimization problem can be formulated:

$$\mathcal{O}(\lambda) = \sum_{i=1}^{I} (Q_{nt}(G^{-1}(\Phi(W_\lambda))) - q_{nt})^2 \tag{10}$$

which minimizes the difference between the modeled and observed runoff by optimizing the weights λ_k. As these weights are arbitrary they can be changed without violating any of the already fulfilled conditions, thus they can be optimized without any further constraints. If for a given set of fields and weights and after a certain number of iterations ~~T~~ N no suitable solution is found, the number K of fields U_k can be increased and the optimization is repeated. A suitable solution is found when the deviation between simulated and observed runoff is smaller than the criterion of acceptance ε (here, $1-NSE$ is used). If a suitable solution is found the whole procedure can be restarted using new random fields V_l. Thus multiple solutions can be obtained enabling uncertainty quantification of spatio-temporal rainfall fields.

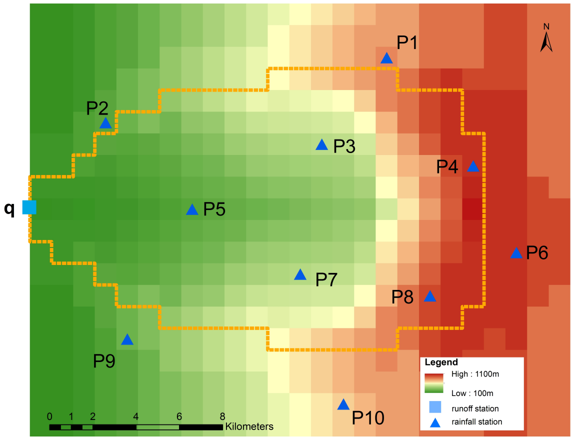

Figure 2. Topography, watershed and observation network of the synthetic ~~chatchment~~catchment

3 Test of the methodology

3.1 Synthetic test site

To test the ability of the methodology a synthetic example ~~is designedconsisting~~ was designed. The example consists of a synthetic catchment partly covered by rainfall. The synthetic catchment has a size of 211~~km² and~~ km² with elevations range between 100 to 1100 m.a.s.l. ~~with~~ and homogeneous landscape properties (Figure 2). A synthetic rainfall event of 6 hours duration with hourly time step and a spatial extension of 118~~km²~~ km² on a regular grid of 1 km by 1 km cell size is used. Rainfall amounts above ~~20mm~~20 mm/event covers an area of 25 km² with maximum rainfall of 36 mm/event and maximum intensity of ~~12mm~~12 mm/h (Figure 3). Based on this known spatio-temporal rainfall input pattern and RR-model parameterisation the catchment response at surface outlet ~~is~~ was simulated and dedicated to be the known “observed” runoff $\cancel{q_n}$ q_t (see Figure 6, blue graph).

Furthermore, ten different cells ~~are~~ were selected from the spatio-temporal rainfall pattern ~~for representing~~ to represent virtual monitoring stations of rainfall. They ~~are~~ were chosen in a way that the centre of the event is not recorded. They are dedicated to be the known “observed” rainfall $P(x_j, t_i)$ at J monitoring stations for $\cancel{T}$ T time steps and ~~form~~ provide the data basis for interpolation, conditional simulation, and inverse ~~modelling~~ modeling of spatio-temporal rainfall pattern. Figure 4 shows their course in time. Note that virtual monitoring stations 2, 5, 9 and 10 measure ~~0mm~~0 mm/h rainfall only. Based on these observations the fitted parameters for the marginal distribution (Eq.3) are: $p_0 = 0,36$ and $\lambda = 0,48$. The fitted copula for the dependency structure in space and time is a Gaussian copula with an exponential correlation function with a range of 2,5 km in space and a range of 1,5 km in time.

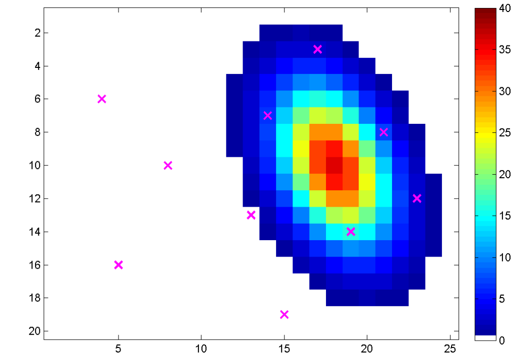

Figure 3. ~~Eventbased rainfall~~ Rainfall amounts of the synthetic rainfall event. Virtual monitoring stations are marked by crosses.

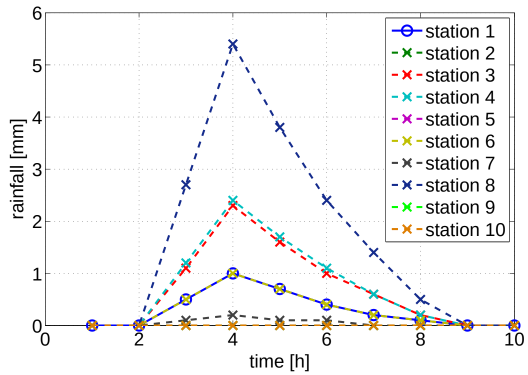

Figure 4. Time series of rainfall ~~amounts~~ intensities at virtual monitoring stations

3.2 Results and discussion

3.2.1 Common hydrologic ~~modelling~~ modeling approach

At first, hourly rainfall data from virtual monitoring stations ~~are~~ were used to interpolate the spatio-temporal rainfall pattern on a regular grid of ~~1km by 1km cellsize~~ 1 km by 1 km cell size by using the inverse distance method which is quite common in ~~hydrological modelling~~hydrologic modeling. Afterwards, the response of the synthetic catchment ~~is calculated~~ was calculated by the RR-model. Figure 5 shows the interpolated pattern of the ~~eventbased~~ event based rainfall amounts as the sum over single ~~timesteps~~time steps. The pattern looks quite smooth and has only minor similarities with the true pattern in Figure 3. Maximum of rainfall amount per event is equal to the maximum of the observation at virtual station number 8 with 16,~~2mm~~2 mm/event. Therefore, the extension of a rainfall centre over ~~20mm~~20 mm/event cannot be estimated. Due to low rainfall intensities, the

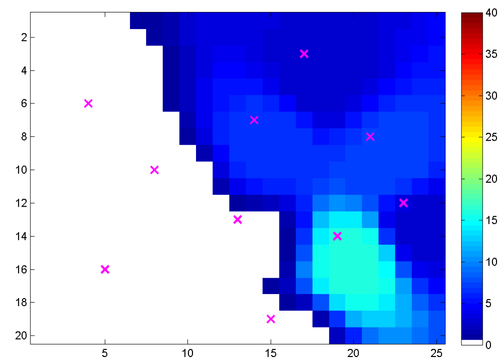

Figure 5. Interpolated rainfall ~~pattern~~ amounts per event by using data of virtual monitoring stations

simulated response of the RR-model shows a significant underestimation of the observed runoff with NSE value of -0.28 (see Figure 6, green graph).

3.2.2 Performance of conditional rainfall simulations

The random mixing approach was used to simulate 200 different spatio-temporal rainfall pattern ~~which are~~ conditioned on the virtual rainfall monitoring stations only. Resulting runoff simulations are displayed in Figure 6~~showing~~. They show a wide range of hydrographs with peak values between 0,~~19m~~19 m³/s to 4,~~17m~~17 m³/s and NSE values between -0,37 to 0,89. Compared to the runoff observation, the timing of peaks is acceptable, but the peak values are underestimated. Only four hydrographs have NSE values higher than 0,7. The corresponding spatial ~~eventbased~~ event based rainfall amounts for the top three runoff simulations regarding the NSE values ((a) 0,89 (b) 0,78 (c) 0,73) is shown in Figure 7. Their ~~eventbased~~ rainfall amounts ranging between 27,8 to 28,~~7mm~~7 mm/event with a spatial extent of 9 to ~~11km~~11 km² of rainfall above ~~20mm~~20 mm/event and a maximum intensity 10,5 to 15,~~1mm~~1 mm/h. Compared to the observation (Figure 3), the spatial pattern look similar, at least regarding the spatial location of the event, and cover the maximum intensity. But the ~~eventbased rainfall amounts~~ rainfall amounts per event as well as their spatial extent is too low. As a consequence, none of the simulated spatio-temporal rainfall ~~pattern~~ fields conditioned at the virtual rainfall monitoring stations only ~~is~~ are able to match the observed peak value in resulting runoff.

3.2.3 Inverse hydrologic ~~modelling~~ modeling approach

The inverse ~~modelling~~ modeling approach was used to simulate 107 different spatio-temporal rainfall pattern which are conditioned on the virtual rainfall and runoff monitoring stations, and runoff simulation results better than NSE values of 0,7. Afterwards a refinement have been carried out by selecting only those simulations with nearly identical runoff simulation

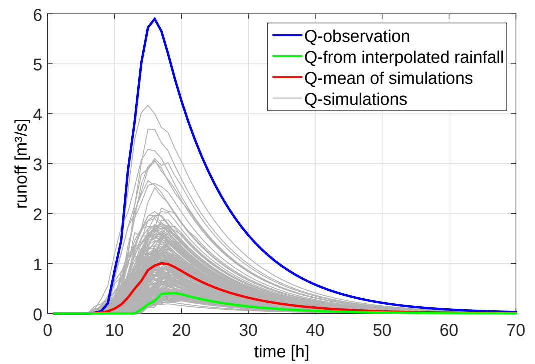

Figure 6. Runoff simulations based on simulated spatio-temporal rainfall pattern conditioned at rainfall point observations only (grey graphs) compared to its mean (red graph), runoff observation (blue graph), and simulation based on interpolated rainfall pattern (green graph)

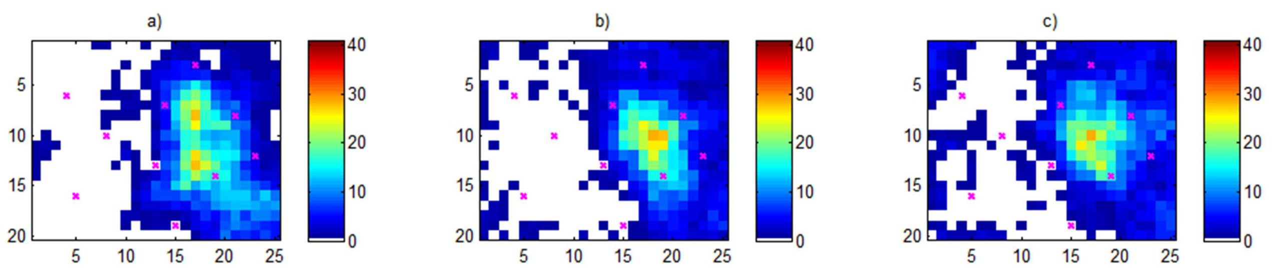

Figure 7. ~~Eventbased~~ Event based rainfall pattern conditioned at rainfall point observations only for the top three runoff simulations in Figure 6

results compared to observation. These simulations are characterized by NSE values larger than 0,995. Figure 8 shows the performance of the 20 selected ~~events~~ realisations by grey graphs having only minor deviations during the flood peak range compared to the observation (blue graph). Associated rainfall ~~pattern~~ patterns are displayed in Figure 9 for six selected ~~events~~ realisations by their spatial ~~eventbased rainfall amounts~~ rainfall amounts per event. Compared to the true spatial pattern (see Figure 3) none of them ~~reproducing~~ reproduce the true pattern exactly, but all of them locate the centre of the event in the same region ~~like~~ as the true pattern. This shows, that ~~due to the incooperation of catchments'~~ by additional conditioning of spatio-temporal rainfall pattern on runoff observation and consideration of catchment's drainage characteristic represented by the RR-model, ~~and the runoff observation into the rainfall simulation procedure, a localisation in terms of a reconstruction of the rainfall pattern inclusive~~ the rainfall event can be localised and reconstructed in its spatial extent as well as in its course in time~~is possible.~~. Most probably, if we would sample a large number of rainfall fields conditioned on rainfall observation only, we would find a realisation which matches the runoff observation too. Due to additional conditioning on runoff we find these realisation faster.

However, the inference of a three dimensional input variable by using an integral output response results in a set or ensemble of different solutions. ~~This ensemble can be used to describe the uncertainty in estimating spatio-temporal rainfall pattern.~~ Rainfall amounts of the selected 20 realisations above ~~20mm~~20 mm/event cover an area of 13 to 25 km^2 with maximum rainfall of 26,7 to 40,~~4mm~~4 mm/event and maximum intensities of 10,7 to 17,~~1mm~~1 mm/h. The ~~eventbased areal precipitation~~ event based areal precipitation of the catchment ranges between 98,2 % - 114,7 % of the observation (see Figure 3). Figure 9 presents spatial ~~eventbased rainfall amounts for~~ rainfall amounts per event for:a) the realisation with the smallest area above ~~20mm~~20 mm/event and smallest intensity, b) the realisation with the largest area above ~~20mm~~20 mm/event c) the realisation with the highest intensity and rainfall amount per event, d) the realisation with the best ~~NSE-value~~ NSE value in resulting runoff, e) and f) realisations with similar event statistics like the true spatio-temporal rainfall pattern. Compared to the observed pattern (see Figure 3), the different realisations match the spatial location as well as the shape of the observed pattern very good. However, the spatial ~~pattern~~ patterns of the realisations are not such smooth and symmetric like the constructed synthetic observation. Furthermore, the realisations show some scattered low rainfall amounts, which are not of importance for the hydrograph simulation, since they are addressed by the initial and constant rate losses of the RR-model. ~~Last but not least,data of the virtual monitoring stations have been always reproduced and are equal for each rainfall simulation.~~

Deriving an average rainfall pattern by ~~calculating~~ calculation of the cell wise mean value over all realisations of the ensemble for each time step~~will result in~~ , a smoother pattern is obtained, which looks more similar to the true one but ~~with smaller~~ has smaller rainfall intensities. Using this mean ensemble pattern for calculating the runoff response, lead to an underestimation of the observed hydrograph ~~like~~ as shown by the black hydrograph in Figure 8. Therefore, the ensemble mean of the hydrographs (red line in Figure 8) is a better representative for the sample than the mean ensemble rainfall pattern.

In addition, data of the virtual monitoring stations (the observation) have been always reproduced and are equal for each rainfall simulation.This means, that each realisation reproduce the point observation of rainfall without any uncertainty. Only the grid points between the observation differs within the three-dimensional rainfall field and contain the stochasticity given by rainfall simulations conditioned on the observed values. In this context, the ensemble can be used as a partial descriptor of the total uncertainty. It describes the remaining uncertainty of precipitation if all available data are exploited under the assumption of error-free measurements, reliable statistical rainfall models, and known hydrologic model parameters. .

4 Application for real world data

4.1 Arid catchment test site

The real world example is taken from the upper Wadi Bani Kharus in the northern part of the Sultanate of Oman. The headwater ~~catchment~~ under consideration is the ~~surface runoff~~ catchment of the stream flow gauging station of Al Awabi with an area of ~~257km~~257 km^2, located in the Hadjar mountain range with heights ranging from ~~600m~~600 m.a.s.l. to more than ~~2500m~~2500 m.a.s.l. The geology of the area is dominated by the Hadjar group consisting of limestone and dolomite. The steep terrain consists of rocks mainly. Soils are negligible. However, larger units of alluvial depositions in the valleys are important for hydrologic processes which is addressed by spatial differences in RR-model parameters. Vegetation is sparely and mostly

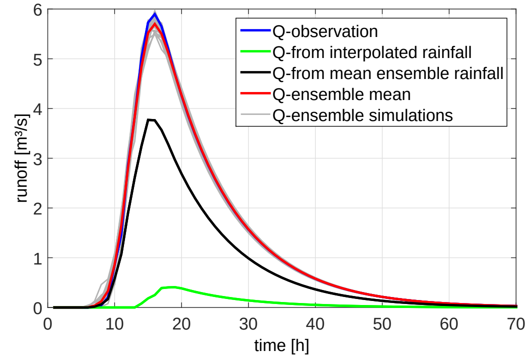

Figure 8. Comparison of hydrographs for the synthetic catchment shown by the observed runoff (blue) and rainfall-runoff simulation results based on: interpolated rainfall pattern (green), simulated ensemble of spatio-temporal rainfall pattern conditioned at rainfall and runoff observations (grey) and their mean value (red), as well as mean ensemble rainfall pattern (black)

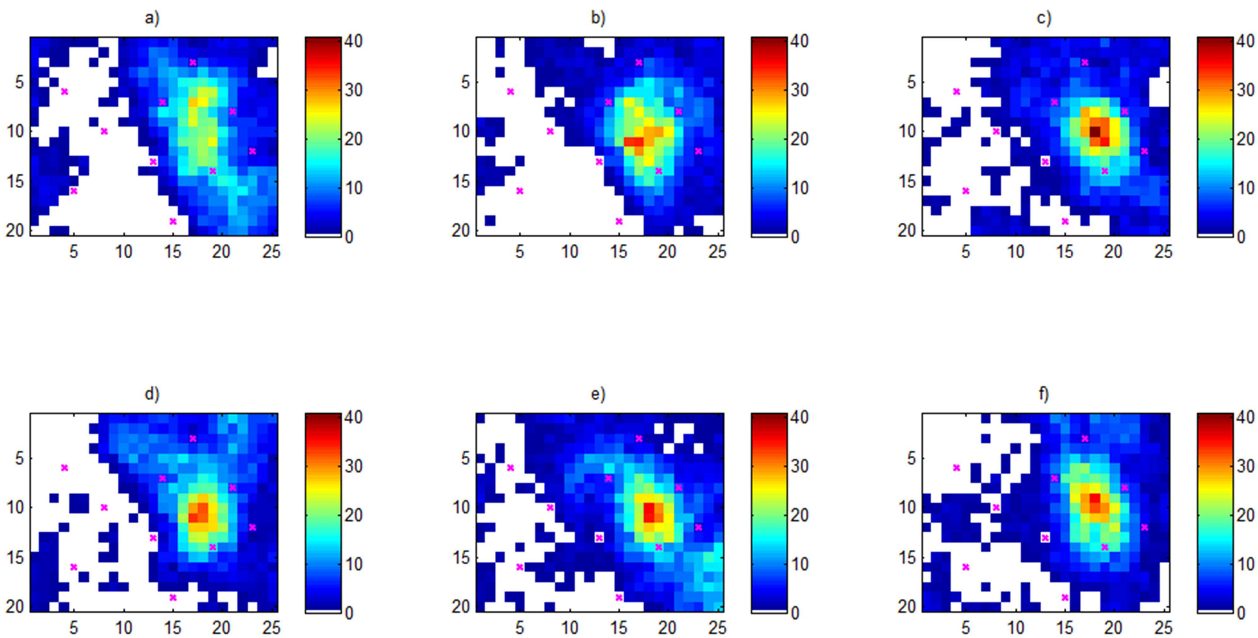

Figure 9. Selected realisations of spatial ~~eventbased~~ rainfall amounts per event with similar performance in resulting runoff obtained by the inverse ~~modelling~~ modeling approch for simulating spatio-temporal rainfall pattern: a) realisation with the smallest area above ~~20mm~~20 mm/event and smallest intensity, b) realisation with the largest area above ~~20mm~~20 mm/event c) realisation with the highest intensity and rainfall amount per event, d) realisation with the best ~~NSE-value~~ NSE value in resulting runoff, e) and f) realisations with similar event statistics like the true spatio-temporal rainfall pattern

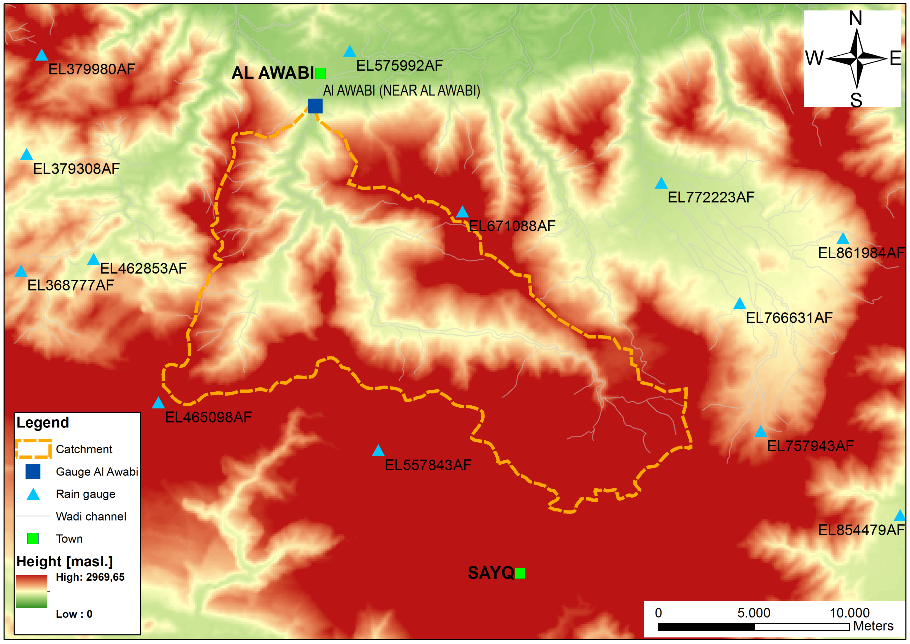

Figure 10. real world case study: catchment of gauge Al Awabi and ~~subdaily~~ sub daily monitoring network for runoff and rainfall

cultivated in mountain oasis. Annual rainfall can reach more than ~~300mm~~300 mm/~~yr~~ year showing a huge variability between consecutive years. Analysis of measured runoff data over a period of 24 years shows that runoff occurred in average only on ~~18days~~18 days/~~yr~~year. Figure 10 ~~shows~~ displays the available monitoring network for ~~subdaily~~ sub daily data. Runoff is measured in 5 to 10 minutes temporal resolution. Rainfall measurements vary from 1 minute to 1 hour. Therefore, a temporal resolution of 1 hour ~~is~~ was chosen for the event under investigation in this study. Figure 11 shows the measurements of the rainfall gauging stations and their altitudes for the rainstorm from 12 February 1999. Most of the rain was recorded on stations with lower altitudes located in the north-west and south-eastern part of the catchment. Rainfall interpolation was performed by inverse distance method, since there was no dependency of rainfall from altitude identifiable for this single heavy rainfall event. Parameters for the inverse modeling approach are: $p_0 = 0,17$ and $\lambda = 0,14$ for the marginal distribution (Eq.3). The fitted copula for the dependency structure in space and time is a Gaussian copula with an exponential correlation function with a range of 10 km in space and a range of 1 km in time.

4.2 Results and discussion

The real world data example ~~is~~ was performed for the runoff event from ~~12.2.1999~~ 12 February 1999 with an effective rainfall duration of three hours. The simulated runoff for the interpolated rainfall pattern shows an underestimation of the peak discharge as well as a time shift of the peak arrival time compared to the observation (Figure 12). Applying the inverse approach by conditioning spatio-temporal rainfall pattern on rainfall and runoff observations, an ensemble of 58 different hydrographs is

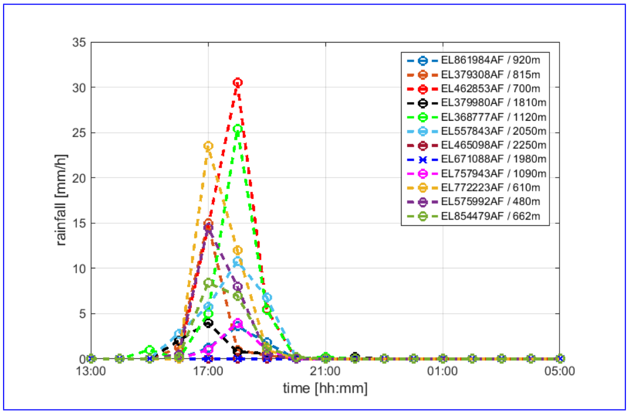

Figure 11. Rainfall amounts and altitudes of rainfall gauging stations from 12 February1999

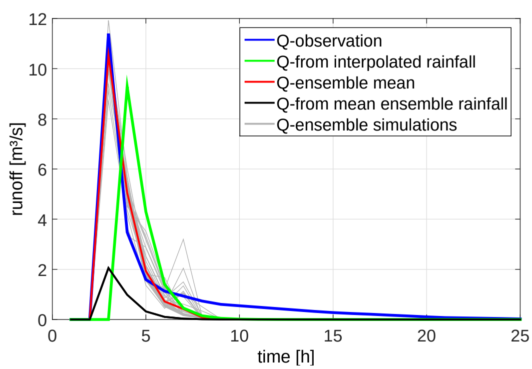

Figure 12. Comparison of hydrographs for the real world catchment shown by the observed runoff (blue) and rainfall-runoff simulation results based: on interpolated rainfall pattern (green), simulated ensemble of spatio-temporal rainfall pattern conditioned at rainfall and runoff observations (grey) and their mean value (red), as well as mean ensemble rainfall pattern (black)

simulated having NSE values larger than ~~0.9.~~ 0,9. As shown in Figure 12, all of these hydrographs (grey graphs) represent the observation ~~quite good~~ well and overcome the ~~timeshift~~time shift. To explain this behaviour, differential maps are calculated which show the difference between a simulated and the ~~regionalized~~ interpolated rainfall pattern for each ~~timestep~~ time step (Figure 13). It is easy to see that the inverse approach allows for a shift of the centre of the rainfall event from time step 1 to time step 2 and towards the catchment outlet. This results in a faster response of the catchment by its runoff compared to the interpolated rainfall pattern. In general, the obtained ensemble of spatio-temporal rainfall pattern is able to explain the observed runoff without discrepancy in rainfall measurements. Similar to the synthetic example, the ensemble mean hydrograph (Figure 12, red graph) is a better representative for the sample than the hydrograph based on the mean ensemble rainfall spatio-temporal pattern (black graph).

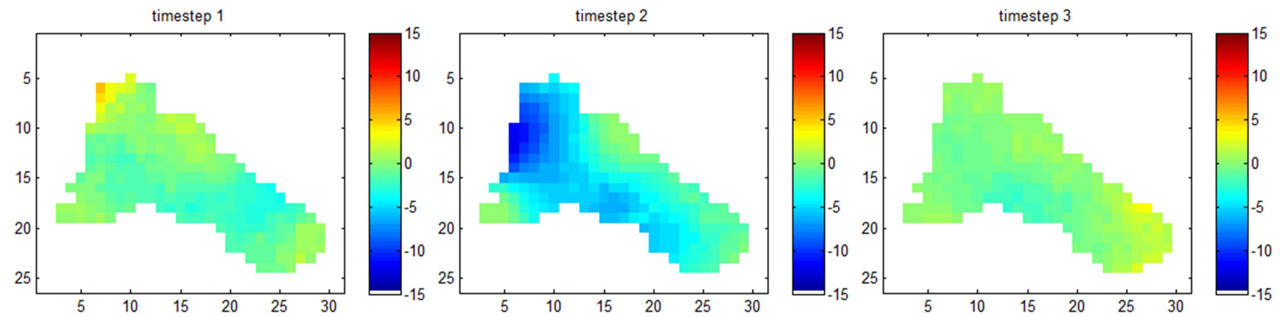

Figure 13. Differential maps of spatio-temporal rainfall pattern for three consecutive ~~timesteps~~ time steps (simulation – interpolation)

5 Summary and conclusions

An inverse hydrologic modeling approach for simulating spatio-temporal rainfall pattern is presented in this paper. The approach combines the conditional random field simulator Random Mixing and a spatial distributed RR-model in a joint Monte-Carlo framework. It allows for obtaining ~~resonable~~ reasonable spatio-temporal rainfall ~~pattern which are~~ patterns conditioned on point rainfall and runoff observations. This has been demonstrated by a synthetic data example ~~for a catchment which is~~ as well as a real world data example for single rainstorms and catchments which are covered by rainfall partly.

~~Compared to other methods , like rainfall interpolation or~~ The proposed framework was compared to the methods of rainfall interpolation and conditional rainfall simulation~~, the inverse approach showed that a reconstruction of the eventbased~~ . Reconstruction of event based spatio-temporal rainfall pattern has been ~~possible, esspecially if runoff generation processes are driven by topography~~feasible by the inverse approach, if runoff observation and catchment's spatial drainage characteristic represented by the RR-model with spatial distributed travel times of overland flow are considered. As shown by the synthetic example, the rainfall pattern obtained by interpolation ~~is too smooth. The method might be appropriate for long time intervalls, but in terms of rainfall data scarcity and high spatio-temporal rainfall variability a "good"~~ didn't match the observed rainfall field and runoff. If rain gauge observations don't portray the rain field adequately, a "good" interpolation result in least square sense is not a solution of the problem. ~~Furthermore, conditional rainfall simulation only shows the pure~~ This is the case in particular for small scale rainstorms with high spatio-temporal rainfall ~~uncertainty. If rainfall pattern are conditioned~~ variability and/or rainfall data scarcity due to insufficient monitoring network density. By rainfall simulations conditioned on rain gauge observation only, reasonable spatio-temporal rainfall fields are obtained, but with a wide spread in resulting runoff hydrographs. A large number of simulated rainfall fields is required to find those realisations which match the observed runoff, since the amount of possible conditioned rainfall fields is very much higher than the amount of rainfall fields matching point observation and runoff. By conditioning of rainfall pattern on discharge too, appropriate candidates of spatio-temporal rainfall pattern can be identified more reliable, faster, and with reduced uncertainty.

The inference of a three dimensional input variable by using an integral output response results in a set of possible solutions in terms of spatio-temporal rainfall pattern. This ensemble can be considered as a descriptor of the ~~uncertainty in the~~

partial uncertainty resulting from spatio-temporal rainfall pattern estimates ~~and described regarding~~ (under the assumption of error-free measurements, reliable statistical rainfall models, and known hydrologic model parameters). Realisations of the ensemble vary in rainfall amounts, intensities, and spatial extend of the event~~. It allows also~~ , but they reproduce the point rainfall observation exactly and yield to similar runoff hydrographs. This allows for deeper insights in hydrologic model and catchment behavior and gives valuable information for the reanalysis of rainfall-runoff events~~. Like~~ , since rainstorm configurations leading to similar flood responses become visible. As shown in the example, ~~operaiting~~ operating with an ensemble mean is less successful to match the runoff observation compared to an application of the whole ensemble due to smoothing effects.

The approach is also ~~working~~ applicable under data scarce situation ~~like~~ as demonstrated by a real world data example~~showed~~. Here, the flexibility of the approach becomes visible, since simulated rainfall pattern are also allow for overcoming a shift in timing of runoff. Therefore, the approach can be considered as a reanalysis tool for rainfall-runoff events especially in regions where runoff generation and formation are based on surface flow processes (Hortonian runoff), and catchments with wide ranges in arrival times at catchment outlet e.g. ~~mountainious regions or with~~ mountainous regions or distinct drainage structures e.g urban and ~~periurban~~ peri-urban regions.

Nevertheless, ~~there are still some weak points which require~~ further research and ~~investigation~~investigations are required. Examples presented ~~here~~ in this paper are based on hourly resolution in time and 1~~km² grids~~ km² grid size in space. Especially for rainstorms in fast responding, small catchments finer resolutions ~~are required and the limits~~ in space and time are required. Here the limits of the approach in number of ~~timesteps and gridcells as well as input data quality~~ time steps and grid cells need to be explored. An other point is the ~~spatial and temporal dependence structure. It controls~~ required amount and quality of observation data as well as statistical model selection to obtain space-time rain fields. Both impact the simulation of rainfall ~~pattern and is determined based on observations~~amounts and of pattern by the derived spatial and temporal dependence structure. In these examples ~~gaussian~~ Gaussian copulas are used which might be not a good estimator for the spatial dependency in any case ~~. Finally, our assumption that hydrologic model parameters are known and fixed during model application might be valid only for catchments with longterm observations and modeling experience, where modellers are interested to explain the~~ of heavy rainfall.

The proposed framework is a first step that only aims at reconstructing spatio-temporal rainfall pattern under the assumption of fixed hydrologic model structure and parameters. Certainly, hydrologic model uncertainty is of importance. But instead of changing the model to fit the observed discharge, we estimate rainfall fields which fit the model and the discharge by doing reverse hydrology. As such plausible rainfall fields can be identified, the corresponding model and the rainfall field is plausible. Thus, the framework can be applied to proof hypothesis about hydrologic model selection or to explain extraordinary rainfall-runoff events ~~. In generall, incorporation of~~ by using a well calibrated, spatial distributed hydrologic model for the catchment of interest. In this context, further research is dedicated to provide a common interface within the Monte-Carlo framework to exchange the hydrologic model and allow for broader use within the community. Also, further sources of uncertainties (e.g. model parameters, observations, ...) ~~is required for contributing to~~ need to be considered to contribute for the solution of the hydrologic modeling uncertainty puzzle.

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