

**Impact of skin effect on single-well push-pull tests with the presence of regional
groundwater flow**

Xu Li^a, Zhang Wen^{a*}, Hongbin Zhan^{a,b}, Qi Zhu^a

Manuscript submitted to *Hydrology and Earth System Sciences*

^aSchool of Environmental Studies, China University of Geosciences, Wuhan, 430074, China

^bDepartment of Geology and Geophysics, Texas A & M University, College Station, TX
77843-3115, USA, Email: zhan@geo.tamu.edu

*Corresponding author: Zhang Wen, Ph.D. Professor. Affiliation: School of Environmental
Studies, China University of Geosciences, Wuhan, Hubei, 430074, P. R. China. Email:
wenz@cug.edu.cn. Tel: 86-27-67883159. Fax: 86-27-87436235.

Abstract

Single-well push-pull (SWPP) test is one of the most important ways to estimate aquifer transport parameters, e.g. porosity, dispersivity, regional groundwater velocity. The wellbore is surrounded by a finite-thickness skin, such as a gravel pack usually, thus the aquifer can be regarded as a radial two-zone system. If the hydraulic conductivity of the skin is smaller than that of the aquifer formation zone, the skin is defined as a positive one. Otherwise, it is a negative skin. In this study, a numerical model of SWPP test considering skin effects was established using the finite-element COMSOL Multiphysics software to estimate aquifer transport parameters. Several important results were obtained. Firstly, different regional groundwater velocity affects the types of breakthrough curves (BTCs). Secondly, a positive (or negative) skin leads to a slower (or faster) tracer transport process, and a smaller ratio between the hydraulic conductivity of the skin and that of the aquifer formation results in greater solute plume retardation in the skin zone. Thirdly, a larger thickness of the positive skin leads to a higher tracer concentration around the well. The opposite is true if the skin is negative. Besides, different hydraulic conductivities and skin thickness can result in different ratios of tracer mass recovered during the pumping phase. The general conclusion is that the skin effects on SWPP test are significant and should be considered.

Keywords: push-pull test, regional groundwater velocity, solute transport, skin effect

1. Introduction

The single-well push-pull (SWPP) tests have been commonly employed to estimate aquifer parameters, e.g. porosity, dispersivity, biogeochemical reaction rate (Gelhar and Collins, 1971; Hall et al., 1991; Schroth and Istok, 2006; Johnsen et al., 2009). The process of this test can be summarized as follows: A tracer is injected into a target aquifer (injection phase), then the mixed solution is pumped out from the same location (pumping phase). Groundwater samples are taken at regular time intervals at the test well during the pumping process, and parameters can be obtained by fitting the observed breakthrough curves (BTCs) using a proper mathematical model. Conservative or reactive tracers can be utilized, depending on the purpose of the test. In general, conservative tracers have been widely used to estimate regional groundwater flow velocity, porosity, and dispersivity, etc. (Leap and Kaplan, 1988; Haggerty et al., 2001; Hebig-Schubert, 2014). Similarly, one can obtain the information of sorption, cation exchange, microbial processes by applying reactive tracers (Trudell et al., 1986; Field et al., 2000; Tong et al., 2016). For instance, Tong et al. (2016) used SWPP tests to validate the abundant production of hydroxyl radicals due to the oxidization of subsurface sediments.

To interpret the SWPP test results, a proper mathematical model considering the fundamental physical and biogeochemical processes of the test is indispensable (Haggerty et al., 2001; Kleikemper et al., 2002; Schroth and Istok, 2006). From a transport perspective, many existing models are governed by the conventional advection-dispersion equation (ADE), assuming the validity of Fick's law in the SWPP tests (Schroth et al., 2000; Huang et al., 2010). Subsequently, many analytical and numerical solutions of various single-well

models have been developed. For instance, Huang et al. (2010) obtained an exact analytical solution of SWPP test by using Fick's law, considering a partially penetrating well in the aquifer. Besides ADE, a number of non-Fickian transport models of SWPP tests have also been developed in recent years to recognize the influence of media heterogeneity, especially in fractured aquifers (Chen et al., 2017). Such models include multi-rate mass transfer models (Haggerty et al., 2001, Hansen et al., 2016), continuous time random walk (CTRW) (Le Borgne and Gouze, 2008), and fractional advection-dispersion equation (FADE) models (Benson et al., 2004; Chen et al., 2017), to name a few. For instance, Chen et al. (2017) developed a fractional model of multistage SWPP test to simulate non-Fickian behavior for a fractured aquifer. In addition, Schroth et al. (2005) obtained an approximate analytical solution of SWPP test for spherical-flow conditions. Wang et al. (2017) investigated the impacts of transient flow and wellbore storage on SWPP test under transient Forchheimer flow using a finite-difference method.

As mentioned above, SWPP test is a powerful tool for aquifer characterization, but its application for determining the regional groundwater velocity has rarely been discussed in previous studies. The groundwater flow velocity may be measured directly using a two-well tracer test conducted under nature gradient condition, but this requires a monitoring well that is located directly down-gradient at a convenient distance from the test well, which is unlikely in most field applications (as one may not be aware of the hydraulic gradient and groundwater flow direction before the installation of monitoring wells). In fact, in most cases, the hydraulic gradient is determined using a three-well triangle and the groundwater flow velocity (including its magnitude and direction) may be obtained if the hydraulic conductivity

and effective porosity are also known. If the hydraulic conductivity and effective porosity are unavailable, one may rely on the BTCs obtained from such a three-well system in a natural gradient tracer test as an alternative to determine the regional flow velocity and longitudinal and transverse dispersivities as well. This is can be done using the following procedures. First, the direction of hydraulic gradient can be determined based on the hydraulic head measurements in three monitoring wells, and the groundwater flow direction is directly opposite of the hydraulic gradient direction in a horizontally isotropic media (which is usually true for most field applications). Second, after determining the direction of groundwater flow, one has three more parameters to determine: the magnitude of the groundwater flow velocity and longitudinal and transverse dispersivities. Such three unknown parameters can be obtained using the concentrations measured in above three (or more) wells by conducting the natural gradient tracer tests rather than the SWPP tests (Pickens et al., 1981; Michie, 1996; Zimmermann and Huenges, 1999).

However, the natural gradient tracer tests usually take much longer time to complete. This is particularly troublesome when the medium is less permeable, and the regional groundwater flow velocity is relatively small (Schubert et al., 2011). The natural gradient tracer test method is also very costly to implement in deep aquifers as the installation of multiple wells in deep aquifers can be formidably expensive. In contrast, the SWPP test only needs a single well, thus can substantially reduce the cost of test, and becomes a nice alternative for the determination of regional groundwater flow velocity (Leap and Kaplan, 1988; Butler et al., 2009). To serve such a purpose, the SWPP test usually consist of three phases: tracer injection, drift, and pumping, is different from the traditional SWPP test model. The drift

phase allows the injected tracer to drift with regional groundwater velocity, thus it is a key phase to include. Leap and Kaplan (1988) obtained an equation for push-pull test to determine regional groundwater flow velocity in a confined aquifer, and a series of laboratory tests were conducted to verify the accuracy of the model, and they described a “velocity shadow” that exists for some distance downgradient of the well. Hall et al. (1991) considered that if the tracer was not drifted over the “velocity shadow” before pumping, the calculated results of the regional flow velocity would produce a small error with the comparison to the actual value. After that, Monkmeyer and Netzer (1993) in their comment on the paper of Leap and Kaplan (1998), firstly considered a stagnation point (where groundwater velocity is zero) during a SWPP pumping phase, and they considered that the tracer can be recovered, if the tracer was not drifted over stagnation point at the edge of the capture zone. Recently, Charles et al. (2018) conducted a SWPP test in unconfined aquifer, and estimated effective porosity and regional groundwater flow velocity, by using modified equations of Leap and Kaplan (1988).

In addition, the impacts of skin near a pumping well are usually neglected for a SWPP test, which might bring about great errors for the estimation of aquifer parameters and regional groundwater flow velocity. During the process of well implementation, the intrusion of drilling mud into the aquifer in the vicinity of well is inevitable, which can result in the change of the porosity and permeability surrounding the well screen (Hurst et al., 1969). This phenomenon can be regarded as the skin effect (Chen and Chang, 2002; Wang et al., 2012). The thickness of skin usually ranges from a few millimeters to several meters (Novakowski, 1989). The skins can be classified into positive and negative types according to the hydraulic

conductivity contrast between the skin and the aquifer formation zone. If the hydraulic conductivity of the skin is smaller than that of the aquifer formation zone, the skin is defined as a positive one. Otherwise, it is a negative skin (Park and Zhan, 2002; Yeh et al., 2003; Wen et al., 2011). Such a skin, regardless of positive or negative, will inevitably alter the flow field near the test well, thus its effect must be taken into consideration for interpreting the SWPP test. For instance, the streamlines of skin zone can converge toward the well in the case of a negative skin, but the opposite is true for a positive skin (Drost et al., 1968; Schubert et al., 2011).

In summary, the skin effect is a very important issue from the perspective of SWPP test interpretation. Through a careful check on the literature, we notice that the model of SWPP for estimating groundwater flow velocity needs further investigation, besides, the impacts of skin effects on SWPP tests for estimating groundwater flow velocity have rarely been studied, which will be the purpose of this study. To accomplish the objective, we will investigate a SWPP test containing three phases of injection, rest and pumping using a fully penetrating well. We will use the finite-element COMSOL Multiphysics to numerically simulate the steady-state, two-dimensional (2D) horizontal flow, with specific attention paid to the skin effect.

2. Mathematical model of the SWPP test

To illustrate the problem, we will use a conservative tracer. A confined aquifer is assumed to be unbounded laterally with a uniform regional groundwater flow presented over the entire duration of test. The aquifer is assumed to be homogeneous and horizontally isotropic. A fully penetrating well is used so only the horizontal flow is of concern here. Flow

is assumed to be Darcian and transport is assumed to be Fickian. The test well radius (r_w) is assumed to be sufficiently small so that the wellbore storage is not a concern. The tracer is injected with a constant rate and a constant concentration at the injection phase and is pumped with a constant rate (which could be different from the injection rate) at the pumping phase, after a certain period of drift phase to allow the injected tracer drifting with the regional groundwater flow. The coordinate system is established as follows with the origin at the center of the test well and the x -axis pointing to the direction of regional groundwater flow. A 2D schematic diagram investigated here is depicted in Fig.1. The confined aquifer consists of two zones in the radial direction. The first zone, known as the wellbore skin zone, is located around the injection well and has a thickness of $r_s - r_w$, where r_s is skin radius. The second zone is known as the aquifer formation zone.

2.1 Mathematical model of groundwater flow

Flow of the SWPP test is assumed to be steady-state, thus the groundwater flow velocity of the skin zone can be expressed as the superposition of the flow component generated by the pumping well and the regional flow:

$$\vec{v}_1 = \vec{v}_{r1} + \vec{v}_{x1} \quad (1)$$

$$\vec{v}_{r1} = v_{r1} \vec{e}_r = Q / (2\pi\theta_s Br) \vec{e}_r \quad (2)$$

$$\vec{v}_{x1} = v_{x1} \vec{e}_x = (-K_s J_s / \theta_s) \vec{e}_x = (v_{d1} / \theta) \vec{e}_x \quad (3)$$

$$r = \sqrt{x^2 + y^2} \quad (4)$$

where the arrow over a symbol represents a vector hereinafter; \vec{v}_{r1} is the average radial pore velocity vector of the skin zone generated by the injection (or pumping well) with a magnitude of v_{r1} [L/T] and \vec{e}_r is a unit vector along the radial direction; \vec{v}_{x1} is the regional

168 groundwater pore flow velocity vector of the skin zone with a magnitude of v_{x1} [L/T] and v_{d1}
 169 is the regional groundwater Darcy flow velocity of the skin zone, and \vec{e}_x is a unit vector
 170 long the x -axis; \vec{v}_1 is the lumped groundwater flow velocity in the skin zone [L/T]; B is the
 171 aquifer thickness [L]; Q is the injection or pumping rate [L³/T], which is positive for the
 172 injection and negative for pumping, and Q is 0 during the drift phase; K_s is hydraulic
 173 conductivity of the skin zone [L/T]; J_s is the hydraulic gradient of regional flow in the skin
 174 zone[L/L]; θ_s is the effective porosity of the skin zone [dimensionless], which is assumed to
 175 be the same as the total porosity of the aquifer when all the pore spaces are well-connected
 176 with negligible immobile porosity; r is the radial distance [L] from the well and x and y are
 177 two horizontal coordinates [L], parallel and perpendicular to the regional groundwater flow
 178 direction, respectively.

179 In addition, the groundwater flow velocity in the aquifer formation zone can be expressed
 180 as:

$$181 \quad \vec{v}_2 = \vec{v}_{r2} + \vec{v}_{x2} \quad (5)$$

$$182 \quad \vec{v}_{r2} = v_{r2} \vec{e}_r = Q / (2\pi\theta Br) \vec{e}_r \quad (6)$$

$$183 \quad \vec{v}_{x2} = v_{x2} \vec{e}_x = (-KJ / \theta) \vec{e}_x = (v_{d2} / \theta) \vec{e}_x \quad (7)$$

184 where \vec{v}_{r2} is the average radial pore velocity vector of the aquifer formation zone generated
 185 by the injection (or pumping well) with a magnitude of v_{r2} [L/T]; \vec{v}_{x2} is the regional
 186 groundwater pore flow velocity vector of the aquifer formation zone with a magnitude of v_{x2}
 187 [L/T] and v_{d2} is the regional groundwater Darcy flow velocity of the aquifer formation zone;
 188 \vec{v}_2 is the lumped groundwater flow velocity in the aquifer formation zone [L/T]; K is the

189 aquifer hydraulic conductivity [L/T]; J is the hydraulic gradient of regional flow in the
 190 aquifer formation zone [L/L]; θ is the aquifer effective porosity [dimensionless].

191 **2.2 Mathematical model of solute transport**

192 The solute transport is dominated by advection and dispersion in the two-zone system.

193 The ADE of a conservative solute in the skin zone without source/sink can be written as:

$$194 \quad \frac{\partial C_1}{\partial t} = \nabla \cdot (D \nabla C_1) - \nabla \cdot (\vec{v}_1 C_1), \quad r_w < r \leq r_s, t > 0 \quad (8)$$

195 and in the aquifer formation zone as:

$$196 \quad \frac{\partial C_2}{\partial t} = \nabla \cdot (D \nabla C_2) - \nabla \cdot (\vec{v}_2 C_2), \quad r_s < r < \infty \quad (9)$$

197 where C_1 and C_2 are solute concentrations in the skin and aquifer formation zones,
 198 respectively [M/L³]; t is the transport time [T]; D_I is the hydrodynamic dispersion [L²/T]; $\nabla \cdot$
 199 and ∇ are the divergence operator and the gradient operator respectively; the hydrodynamic
 200 dispersion is a velocity-dependent tensor depicted as:

$$201 \quad D_{xx} = \frac{\alpha_L v_x^2}{|\vec{v}|} + \frac{\alpha_T v_y^2}{|\vec{v}|} + D_{diff} \quad (10)$$

$$202 \quad D_{yy} = \frac{\alpha_L v_y^2}{|\vec{v}|} + \frac{\alpha_T v_x^2}{|\vec{v}|} + D_{diff} \quad (11)$$

$$203 \quad D_{xy} = D_{yx} = (\alpha_L - \alpha_T) \frac{v_x v_y}{|\vec{v}|} \quad (12)$$

$$204 \quad v = \sqrt{v_x^2 + v_y^2} \quad (13)$$

205 where D_{xx} , D_{xy} , and D_{yy} are components of the hydrodynamic dispersion coefficient tensor
 206 [L²/T]; D_{diff} is the molecular diffusion coefficient [L²/T]; α_L is the longitudinal dispersivity
 207 [L]; α_T is the transverse dispersivity [L]; the transverse dispersion effect is much smaller, thus

one usually assumes $\alpha_L = 10\alpha_T$ (Guvanasen and Guvanasen, 1987; Chen et al., 1999; Chen et al., 2006); v_x is pore velocity in the x direction; v_y is pore velocity in the y direction.

3. Numerical solution of the SWPP test

In this study, a steady-state flow model of 2D horizontal plane was developed based on COMSOL Multiphysics, as shown in Fig. 1. The model region was set to be $40 \text{ m} \times 40 \text{ m}$, and the well has a radius of 0.1 m . In addition, $B=10 \text{ m}$, $K=8.0 \text{ m/d}$, $\theta=0.3$. According to Fig. 1, the boundary conditions for the domain of concern can be expressed as:

$$H(x, y)|_{s_1} = H_1, \quad H(x, y)|_{s_2} = H_2 \quad (14)$$

$$K \frac{\partial H}{\partial n}|_{s_3} = 0, \quad K \frac{\partial H}{\partial n}|_{s_4} = 0 \quad (15)$$

where s_1, s_2, s_3 and s_4 are the boundaries of the model; s_1 and s_2 are constant-head boundaries with prescribed total heads of H_1 and H_2 , respectively; both s_3 and s_4 are no-flux boundaries; n is the normal vector of the boundary (an outward pointing vector perpendicular to the boundary). Therefore, a constant regional flow field can be generated and one can obtain different values of v_{x2} by changing the head differences of H_1 and H_2 . In this model, constant-head boundaries were prescribed, and the value of H_2 was set as constant 15 m , according to Eq. (7), one can obtain different values of v_{x2} by changing the value of H_1 . In the model, a continuous mass flux of injection or pumping rate was assigned at $r=r_w$, which can be expressed as:

$$N_0 = \frac{Q\rho}{2\pi r_w B} \quad (16)$$

where, N_0 is the mass flux per unit thickness $[\text{M/L}^2/\text{T}]$; ρ is the density of groundwater $[\text{M/L}^3]$.

A uniform skin near the well was considered in a confined aquifer, and the well skin thickness of r_s-r_w the well skin was assumed to be constant in this model. The skin hydraulic conductivity and effective porosity were set as K_s and θ_s respectively. The default values of the parameters were shown in Table 1.

For the solute model, the initial condition in both the skin and aquifer formation zones are:

$$C_1(r,0) = C_2(r,0) = 0, \quad r \geq r_w, \quad (17)$$

During the injection phase, the boundary condition at $r=r_w$ can be described as:

$$C_1(r_w, t) = C_0, \quad r = r_w, 0 < t < t_{inj} \quad (18)$$

where C_0 represents the concentration of the injection phase [M/L³]; t_{inj} is the duration of the injection phase [T]. The third-type boundary condition may also be used to replace the first-type boundary condition of Eq. (20). However, our numerical exercises indicate that both conditions yield nearly the same results except for a very short period of time since the start of injection. Therefore, without loss of generality, we use the first-type boundary condition here as an example to illustrate the methodology.

The concentration and the flux at the interface between the skin and aquifer formation zones are continuous, can be expressed as:

$$C_1(r_s, t) = C_2(r_s, t), \quad t > 0 \quad (19)$$

and

$$D \frac{\partial C_1(r_s, t)}{\partial r} = D \frac{\partial C_2(r_s, t)}{\partial r}, \quad t > 0 \quad (20)$$

During the pumping phase, the time-dependent concentration is measured in the borehole. The main target of the test is to obtain several parameters by fitting the observed

BTCs with corresponding theoretical BTCs obtained from a proper analytical or numerical solution. When solute transport through well screen, the boundary condition at the well screen is (Wang et al., 2017):

$$\left. \frac{\partial C}{\partial r} \right|_{r \rightarrow r_w^-} = 0 \quad (21)$$

Because the values of velocity and concentration are different around the perimeter of the borehole, it is necessary to integrate the concentration around the borehole with the velocity as the weight to obtain the accurate value of concentration at the well, thus, the flux-averaged concentrations can be expressed as:

$$\bar{C}_{pump} = \frac{\oint_{l_w} v_w C_w}{\oint_{l_w} v_w}, \quad r = r_w, t_{res} < t < t_{pump} \quad (22)$$

where v_w represents the velocity around the borehole during the pumping [L/T]; C_w represents the concentration around the well perimeter [M/L³]; \bar{C} is the concentration inside the well [M/L³]; l_w is the perimeter of the wellbore [L]. The SWPP test was divided into three phases, the simulation results at the end of each phase, including the hydraulic head and the solute concentration, were set as the initial values for the simulation in the next phase.

The model domain was discretized into 21688 elements, and the mesh size was progressively refined near the well, and the well perimeter was discretized into 28 elements. When the number of element is doubled, the peak solute concentration for the pumping phase varied about 0.17%. Therefore the selected mesh is regarded as sufficiently fine for the problem investigated here. To check the accuracy of the numerical model further, the numerical solution for a special case (without skin) was used to compare with the analytical solution of Huang et al. (2010), who investigated a steady-state flow SWPP model with

injection and extraction phases, without the regional groundwater flow, as shown in Fig.2.

The simulated time span of tracer injection and pumping were 0.5 and 1 day, respectively.

The other parameters were given as: $Q_{inj}= 50 \text{ m}^3/\text{d}$, $Q_{pump}=-50 \text{ m}^3/\text{d}$, $B=10 \text{ m}$, $\alpha_L=0.1 \text{ m}$, 0.5 m, 1 m, and $\theta=0.3$. C_0 at $r=r_w$ was set as 1.0 mol/m^3 . The results showed that our numerical solution agreed perfectly with the analytical solution. For the following analysis, the default values of the parameters are listed in Table 1.

4. Results and discussions

4.1. Types of breakthrough curves for different regional groundwater velocity

The main target of this SWPP test is to estimate the unknown regional groundwater velocity. In order to interpret the BTCs, a parametric study over a wide range of regional groundwater velocities was conducted. Fig. 3 shows the BTCs for the pumping phase with different regional groundwater velocities, like $5 \times 10^{-7} \text{ m/s}$, $1 \times 10^{-6} \text{ m/s}$, $1.5 \times 10^{-6} \text{ m/s}$, $2 \times 10^{-6} \text{ m/s}$, 2.5×10^{-6} and $3 \times 10^{-6} \text{ m/s}$. Besides, $\alpha_L = 0.1 \text{ m}$, $Q_{inj}= 30 \text{ m}^3/\text{d}$, $Q_{pump}=-15 \text{ m}^3/\text{d}$, and the other parameters are the same as those used in Table 1. It is found that different regional groundwater velocities have great impacts on BTCs, and such impacts depend on the value of the regional groundwater velocities. It is found that the tracer concentration is smaller at early stage with a greater regional groundwater velocity. Additionally, it is notable that a larger regional groundwater velocity will result in a longer tailing.

Fig. 4 shows the superposition of flow components generated by the pumping well and the regional flow. A minor point to note is that Figure 4 only shows a flow pattern for a small area nearby the well, not for the entire domain, thus the streamlines there appear not orthogonal to the upper and lower boundaries. For the pumping phase, one can see that there

294 is a stagnation point (Sp), where the groundwater velocity (v_2) is zero, located at the dividing
 295 streamline (Ds) or envelope of the capture zone as shown in Fig.4. The Ds divides the flow
 296 into the capture zone and the non-capture zone, while the Sp represents the uppermost
 297 location downgradient from the pumping well inside a capture zone. If the tracer does not
 298 drift beyond the stagnation point of the capture zone during the drift phase, it can be extracted
 299 from the aquifer, which acts like a fishing net to collect all the products together. Therefore,
 300 different convergence situations of tracer due to variable dividing streamlines result in a
 301 series of BTC types. In order to interpret this behavior explicitly, the concentration
 302 distributions in a 2D horizontal plane at $t_{pump}=0$ hr with different regional groundwater
 303 velocities are shown in Fig. 5. One can see that a certain amount of tracer mass may be
 304 retained near the symmetry axis (x -axis) and around the well when the regional groundwater
 305 velocity is relatively low, such as $v_d=1 \times 10^{-6}$ m/s and 1.5×10^{-6} m/s, as shown in Figs.5a-5b,
 306 resulting in relatively high concentrations in the wellbore at early stage. On the contrary, a
 307 larger regional groundwater velocity leads to a faster tracer transport process, causing the
 308 tracer mass drifting away from the well, as shown in Figs.5c-5d. Besides, one can also see
 309 that a larger regional groundwater velocity leads to a smaller distance from Sp to well,
 310 resulting in a smaller portion of tracer mass that can be extracted during the pumping phase.
 311 The opposite is true for the case of a smaller regional groundwater velocity. For instance, as
 312 for the tracer mass on the left side of the dividing streamline, they can be extracted by smaller
 313 velocities such as $v_d=5 \times 10^{-7}$ m/s and 1×10^{-6} m/s, as shown in Figs.5a-5b, but very limited
 314 tracer can be captured with larger velocities like $v_d=3 \times 10^{-6}$ m/s. And this further confirms
 315 the reasonability of the BTC types in Fig. 3. In addition, the effects of the duration of the rest

phase, porosity and dispersivity on BTCs have been analyzed in a supplementary material as references.

4.2. The effects of skin hydraulic conductivity on BTCs in the SWPP test

As mentioned above, the well skin includes two general types, i.e., a positive skin or a negative skin. Denoting the hydraulic conductivities of skin and aquifer (or aquifer formation zone) respectively as K_s and K , one can use a new parameter $\delta=K_s/K$ to reflect the skin impact, where δ is a parameter reflecting the type of the skin and called the “skin index” hereinafter. Specifically, $\delta<1$ represents a positive skin, while $\delta>1$ represents a negative skin. Note that the case of $\delta=1$ represents the case without a skin. In this section, we will provide a detailed analysis on the impact of the skin hydraulic conductivity on SWPP test.

Fig. 6 shows the effect of different skin indexes on BTCs during the pumping phase. The parameters are given as: $v_d=3\times 10^{-6}$ m/s, $Q_{inj}=30$ m³/d, $Q_{pump}=-30$ m³/d, $\alpha_L=0.1$ m, $r_s=0.7$ m, $\delta=0.25, 0.5, 1, 1.5$ and 2 , respectively. The results indicate that the concentration gets higher at early stage of pumping when the skin index is lower, as shown in Fig. 6. This may be explained as follows. A skin with a lower δ value (or a lower permeability value in respect to that of the formation) essentially serves as a somewhat “shield” around the test well that can make the spreading of the tracer mass out of the test well more difficult during the injection phase. Consequently, more tracer mass will be retained near the test well either in the skin or near the skin. Therefore, during the pumping phase of the test, more tracer mass can be extracted during the early stage of the pumping phase, leading to higher concentration during that stage. The facts are just the opposite for a larger δ value, this is because a negative skin ($\delta>1$) is somewhat like a “high conductance zone” rather than a “shield”, and can

facilitate the spreading of tracer mass away from the test well during the injection phase. Therefore, less tracer mass will be retained near the test well for a higher skin index, thus less concentration will be seen during the early stage of pumping in the wellbore.

To further explicitly interpret this behavior, the concentration distributions in a 2D horizontal plane at $t_{pump}=0$ hr with different δ values are shown in Fig. 7. As can be seen in Fig. 7, a lower δ value (or a lower skin hydraulic conductivity) leads to more tracer accumulation in the skin zone after the drift phase. On the contrary, it is quite obvious to see that a larger δ value (or a larger skin hydraulic conductivity) leads to a less tracer concentration in the skin zone at the early stage of the pumping phase. For instance, we can see that tracer accumulation in the skin zone for the cases of $\delta=2$ is clearly less than the cases of $\delta=0.5$ and 0.25.

Fig. 8 displays the curves of concentration versus distance at $t_{pump}=0$ hr with $\delta=0.25, 0.5, 1, 1.5$ and 2, respectively. We can see that the concentration gets higher in the skin zone when the skin index δ is lower, as shown in Fig. 8. The results indicate that a lower skin hydraulic conductivity leads to more tracer accumulation in the skin zone after the drift phase. The opposite is true if the skin index δ is larger. In addition, we can see that the peak of solute transport shorter distances at corresponding times with a lower δ value, results in the concentration gets higher at early stage of pumping as shown in Fig. 6. Therefore, the presence of the skin zone will lead to the variations of the concentration curves versus distance in the aquifer system.

4.3. Effects of skin thickness on BTCs in the SWPP test

In this section, we will analyze the impacts of the skin thickness on BTCs. The parameters are given as: $v_d=3 \times 10^{-6}$ m/s, $Q_{inj}= 30$ m³/d, $Q_{ext}=-30$ m³/d, $\delta= 0.5$ (positive skin), $\delta= 2$ (negative skin), $\alpha_L=0.1$ m, and $r_s= 0, 0.3$, and 0.7 m, respectively. For the case of $\delta= 0.5$ (positive skin), one can see that the concentration gets higher at early stage with the increase of r_s in Fig. 9. The explanation is similar to that for Fig. 6, as a thicker positive skin means a thicker “shield” surrounding the test well, preventing the injected tracer mass from spreading further away from the test well, thus leading to higher concentrations during the early stage of the extraction phase. While for the case of $\delta= 2$ (negative skin), it can be found that the concentration shows a decreasing trend with the increase of r_s , as a thicker negative skin means a thicker high conductance zone surrounding the test well, which will facilitate the spreading of injected tracer mass further away from the test well.

To further explicitly interpret this behavior, the concentration distributions in a 2D horizontal plane at $t_{pump}=0$ hr with different positive (or negative) skin thickness are shown in Fig. 10. It is evident that a larger positive skin thickness leads to more tracer accumulation in the skin zone after the drift phase. While for the case of negative skin, one can see that a greater portion of tracer mass migrates away from the test well after the cease of injection with a greater r_s in Fig. 10. For instance, one can see that tracer accumulation in the skin zone for the cases of $r_s=0.7$ and $\delta=2$ is clearly less than the case of $r_s=0.7$ and $\delta=0.5$.

Fig. 11 displays the curves of concentration versus distance at $t_{pump}=0$ hr with $\delta= 0.5$ (positive skin), $\delta= 2$ (negative skin), $r_s= 0, 0.3$, and 0.7 m, respectively. For the case of $\delta= 0.5$ (positive skin), one can see that the concentration gets higher in the skin zone when r_s is larger, as shown in Fig. 11. The results indicate that a thicker skin leads to more tracer

accumulation in the skin zone after the drift phase for a lower skin hydraulic conductivity. The facts are just the opposite for the case of negative skin. In addition, one can see that the peak of solute for a positive skin transport shorter distances at corresponding times with a thicker skin, results in the concentration gets higher at early stage of pumping as shown in Fig. 9. While for the case of a negative skin, the peak of solute transport longer distances with a thicker skin. Therefore, different skin properties (skin type and thickness) will result in different shapes of BTCs in Fig.9.

4.4 Recovered tracer mass estimations

To allow for a more quantitative comparison between simulations, we have computed the ratio of tracer mass recovered at the end of each SWPP test for different skin properties (skin type and thickness). Such a ratio can be written as:

$$\mu = \frac{m_{pump}}{m_{inj}} = \frac{\int_0^{t_{pump}} Q_{pump} \bar{C}_{pump} dt}{Q_{inj} t_{inj} C_0} \quad (26)$$

where, m_{inj} is the tracer mass injected during the injecting phase [M]; m_{pump} is the tracer mass recovered during the pumping phase [M]; μ is the percentage of recovered tracer mass in the injecting tracer mass [dimensionless].

According to Fig. 9, the value the tracer concentration recovered was integrated from $t_{pump}=0$ to 48 hr, then we can obtain the tracer mass recovered for different skin properties, e.g. $\delta=0.5$ (positive skin), $\delta=2$ (negative skin), and $r_s=0, 0.2$, and 0.6 m, respectively. Table 2 shows the percentage of recovered tracer mass in the injecting tracer mass for different skin properties. The results indicate that a lower δ value (or a lower permeability value in respect to that of the aquifer formation) leads to a larger ratio of tracer mass recovery. In addition, for the case of a positive skin, a larger r_s leads to larger ratio of tracer mass recovery, while the

opposite is true for the case of a negative skin. In short, the skin properties (skin type and thickness) can result in different tracer mass recovery ratios.

4.5. Parameter estimations

As discussed in the above sections, the difference between the BTCs of the SWPP test under a skin and a non-skin is obvious. From a practical standpoint, hydrogeologists are interested in the accuracy of parameter estimation based on SWPP models, however, the phenomenon of skin effect is inevitable. Therefore, in the following, we will analyze how the skin effect affects the estimation of aquifer properties such as dispersivity, porosity and regional groundwater flow velocity, and the SWPP model with skin effect is chosen as the reference.

Now an interesting question is: what is the consequence if one adopts the non-skin SWPP model to interpret BTCs obtained from the model with skin? In another word, how different are the estimated parameters (e.g. dispersivity, porosity and regional groundwater velocity) applying the non-skin SWPP model from their “actual” values based on the SWPP model with skin? If the differences between estimated and “actual” parameter values are not negligible, one can conclude that the skin effect should be taken into consideration when using the SWPP test model to estimate parameters. Fig. 12 shows BTCs at the well fitted by the non-skin model for different skin types (a positive skin or a negative skin). The “actual” parameters used in the SWPP model with skin are set as: $v_d=3 \times 10^{-6}$ m/s, $Q_{inj}= 30$ m³/d, $Q_{ext}=-30$ m³/d, $\delta= 0.5$ (positive skin), $\delta= 2$ (negative skin), $\alpha_L=0.1$ m, $\theta=0.3$ and $r_s= 0.7$ m, respectively. The circle lines are created by the numerical solution of the SWPP test with skin in three phases. The dashed lines are created by the non-skin SWPP test model. Using the

trial-and-error method, we find that it is difficult to fit BTCs at the early stage of the SWPP test for a positive skin, but the fitness is good for a negative skin as shown in Fig. 12. Table 3 shows the parameter estimated by the non-skin model. One can see that for the case of a positive skin, both groundwater flow velocity and porosity estimated are smaller than their real values, and longitudinal dispersivity is larger than the real value based on the model with skin. In addition, for the case of a negative skin, groundwater flow velocity, porosity and longitudinal dispersivity estimated are larger than their real values based on the model with skin. In summary, the parameters estimated by the non-skin model are considerable different from the real values, results in larger errors in parameter estimation if the non-skin model is mistakenly used when the skin presents.

5. Conclusions

In this study, a numerical model for a SWPP test with the presence of a regional groundwater flow field, considering both the positive and negative skin effects was investigated. There is a strong interaction between regional groundwater flow and well flow, and various types of BTCs were analyzed for different regional groundwater velocity during pumping phase. Besides, the numerical model of SWPP test can offer a way to estimate unknown parameters: i.e., regional groundwater velocity, effective porosity, and dispersivity by fitting to the observed BTCs. The effects of both the hydraulic conductivities and thickness of the skin zone on BTCs had also been considered. The following conclusions can be drawn:

1. Regional groundwater velocity has a significant effect on the shape of BTCs, a lower regional groundwater velocity means that more tracer can be accumulated near the

symmetry plane around the well. The opposite is true for a case of a larger regional flow velocity, resulting in a longer tailing of BTCs obtained during the extraction phase. In addition, the pattern and location of the dividing streamline determine the quantity of tracer mass extracted during the pumping phase.

2. A larger skin index results in a lower concentration for BTCs at early stage of pumping. On the contrary, a smaller skin index means a higher concentration for BTCs. A larger thickness of a positive skin leads to a larger concentration of tracer near the symmetry plane around the well, and the opposite is true for the case of a negative skin. In addition, a smaller skin index means that solute plume can accumulate more in the skin zone, otherwise, a larger skin index results in a solute plume drifting further away from the skin zone after the cease of the injection phase.
3. The non-skin SWPP test model is used to interpret BTCs obtained from the model with skin, and the estimated parameters are very different from the “actual” values, results in larger errors in parameter estimation if the non-skin model is mistakenly used when the skin presents. Therefore, the impact of skin effect near a pumping well should not be neglected for SWPP tests.

Acknowledgements

This research was partially supported by the National Natural Science Foundation of China (Grant Numbers: 41772259, 41372253, 41521001), the Natural Science Foundation of Hubei Province, China (2018CFA085, 2018CFA028), the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan). We appreciate the

468 comments raised by three anonymous reviewers and the Editor, which help us improve the
469 quality of the paper substantially.
470

References

- Benson, D. A., Tadjeran, C., Meerschaert, M. M., Farnham, I., and Pohl, G.: Radial fractional-order dispersion through fractured rock, *Water Resour. Res.*, 40(12), 87-87, 2004.
- Butler, A.P., Mathias, S.A., Gallagher, A.J., Peach, D.W., and Williams, A.T.: Analysis of flow processes in fractured chalk under pumped and ambient conditions (UK), *Hydrogeol J.*, 17(8), 1849-1858, 2009.
- Chen, C. S., and Chang, C. C.: Use of cumulative volume of constant-head injection test to estimate aquifer parameters with skin effects: Field experiment and data analysis, *Water Resour. Res.*, 38(5), 189-195, 2002.
- Chen, J. S., Chen, C. S., Gau, H. S., and Liu, C. W.: A two-well method to evaluate transverse dispersivity for tracer tests in a radially convergent flow field, *J. Hydrol.*, 223(3-4), 175-197, 1999.
- Chen, K.W., Zhan, H.B., and Yang, Q: Fractional models simulating non-fickian behavior in four-stage single-well push-pull tests, *Water Resour. Res.*, 53(11), 9528-9545, 2017.
- Drost, W., Klotz, D., Koch, A., Moser, H., Neumaier, F., and Rauert, W: Point dilution methods of investigating ground water flow by means of radioisotopes, *Water Resour. Res.*, 4(1), 125-146, 1968.
- Field, J. A., Sawyer, T. E., Schroth, M. H., Humphrey, M. D., and Istok, J. D.: Effect of cation exchange on surfactant-enhanced solubilization of trichloroethene, *J. Contam. Hydrol.*, 46(1-2), 131-149, 2000.

492 Gelhar, L. W., and Collins, M. A.: General analysis of longitudinal dispersion in nonuniform
 493 flow. *Water Resour. Res.*, 7(6), 1511-1521, 1971.

494 Guvanasen V., and Guvanasen V M.: An approximate semianalytical solution for tracer
 495 injection tests in a confined aquifer with a radially converging flow field and finite
 496 volume of tracer and chase fluid, *Water Resour. Res.*, 23(8), 1607-1619, 1987.

497 Haggerty, R., Fleming, S. W., Meigs, L. C., and McKenna, S.A.: Tracer tests in a fractured
 498 dolomite: 2. Analysis of mass transfer in single-well injection-withdrawal tests, *Water*
 499 *Resour. Res.*, 37(5), 1129-1142, 2001.

500 Hall, S.H., Luttrell, S.P., and Cronin, W.E.: A method for estimating effective porosity and
 501 ground-water velocity, *Ground Water.*, 29(2), 171-174, 1991.

502 Hansen, S. K., Berkowitz, B., Vesselinov, V. V., O'Malley, D., and Karra, S. Push-pull tracer
 503 tests: their information content and use for characterizing non-fickian, mobile-immobile
 504 behavior, *Water Resour. Res.*, 52(12), 9565-9585, 2016.

505 Hebig-Schubert, K.: Deep groundwater flow systems and their characterization in single-well
 506 settings by "push-pull" tracer tests, *Int. J. Fatigue.*, 15(5), 441, 2014.

507 Huang, J. Q., Christ, J. A., and Goltz, M. N.: Analytical solutions for efficient interpretation
 508 of single-well push-pull tracer tests, *Water Resour. Res.*, 46(8), 863-863, 2010.

509 Hurst, W., Clark, J. D., and Brauer, E. B.: Skin effect in producing wells, *J. Petrol. Technol.*,
 510 21(11), 1483-1489, 1969.

511 Kleikemper, J., Pelz, O., Schroth, M. H., and Zeyer, J.: Sulfate-reducing bacterial community
 512 response to carbon source amendments in contaminated aquifer microcosms, *Fems*
 513 *Microbiol Ecol.*, 42(1), 109-118, 2002.

514 Johnsen, S. G., Whitson, C. H.: Analytical treatment of a push-pull “echo” test. *Transp.*
 515 *Porous Media*, 77(3), 399. 2009.

516 Leap, D. I., Kaplan, P. G.: A single-well tracing method for estimating regional advective
 517 velocity in a confined aquifer: Theory and preliminary laboratory verification. *Water*
 518 *Resour. Res.*, 24(7), 993-998, 1988.

519 Le Borgne, T., and Gouze, P.: Non-Fickian dispersion in porous media: 2. Model validation
 520 from measurements at different scales, *Water Resour. Res.*, 44(6), 2389-2393, 2008.

521 Michie, U.: The geological framework of the sellafield area and its relationship to
 522 hydrogeology, *Q. J. Eng. Geol., Hydroge.* 29(Supplement_1), S13-S27, 1996.

523 Novakowski, K. S.: A composite analytical model for analysis of pumping tests affected by
 524 well bore storage and finite thickness skin, *Water Resour. Res.*, 25(9), 1937-1946, 1989.

525 Park, E. and Zhan, H. B.: Hydraulics of a finite-diameter horizontal well with wellbore
 526 storage and skin effect, *Adv. Water Resour.*, 25(4), 389-400, 2002.

527 Pickens, J. F., Jackson, R. E., Inch, K. J., and Merritt, W. F.: Measurement of distribution
 528 coefficients using a radial injection dual-tracer test, *Water Resour. Res.*, 17(3), 529-544,
 529 1981.

530 Paradis, C. J., Mckay, L. D., Perfect, E., Istok, J. D., and Hazen, T. C.: Push-pull tests for
 531 estimating effective porosity: expanded analytical solution and in situ application.
 532 *Hydrogeol J* 26(3), 1-13, 2018. Schroth, M. H., Istok, J. D., and Haggerty, R.: In situ
 533 evaluation of solute retardation using ingle-well push-pull tests, *Adv. Water Resour.*,
 534 24(1), 105-117, 2001.

535 Schroth, M. H. and Istok, J. D.: Models to determine first-order rate coefficients from single-
536 well push-pull tests, *Ground Water.*, 44(2), 275-283, 2006.

537 Schroth, M. H., and Istok, J. D.: Approximate solution for solute transport during spherical-
538 flow push-pull tests, *Ground Water.*, 43(2), 280-284, 2005.

539 Schubert, M., Brueggemann, L., Knoeller, K., and Schirmer, M.: Using radon as an
540 environmental tracer for estimating groundwater flow velocities in single-well tests,
541 *Water Resour. Res.*, 47(3), 944-956, 2011.

542 Tong, M., Yuan, S., Ma, S., Jin, M., Liu, D., Cheng, D., and Wang, Y.: Production of
543 abundant hydroxyl radicals from oxygenation of subsurface sediments, *Environ. Sci.*
544 *Technol.*, 50(1), 214-221, 2016.

545 Trudell, M. R., Gillham, R. W., and Cherry, J. A.: An in-situ study of the occurrence and rate
546 of denitrification in a shallow unconfined sand aquifer, *J. Hydrol.*, 83(3), 251-268, 1986.

547 Wang, C. T., Yeh, H. D., and Tsai, C. S.: Transient drawdown solution for a constant
548 pumping test in finite two-zone confined aquifers, *Hydrol. Earth Syst. Sci. Discuss.*,
549 16(2), 441-449, 2012.

550 Wang, Q., Zhan, H., and Wang, Y.: Single-well push-pull test in transient forchheimer flow
551 field, *J. Hydrol.*, 549, 125-132, 2017.

552 Wen, Z., Zhan, H. B., Huang, G. H., and Jin, M. G.: Constant-head test in a leaky aquifer
553 with a finite-thickness skin, *J. Hydrol.*, 399(3-4), 326-334, 2011.

554 Yeh, H. D., Yang, S. Y., and Peng, H. Y.: A new closed-form solution for a radial two-layer
555 drawdown equation for groundwater under constant-flux pumping in a finite-radius well,
556 *Adv. Water Resour.*, 26(7), 747-757, 2003.

557 Zimmermann, G., and Huenges, E.: Rock permeability and fluid pressure at the KTB.
558 implications from laboratory-and drill hole-measurements, Oil. Gas. Sci. Technol.,
559 54(6), 689-694, 1999.

560 **Figure Captions:**

561 Fig.1 The schematic diagram of the flow system. s_1 , s_2 , s_3 and s_4 are the boundaries of the
562 model, and s_1 and s_2 are constant-head boundaries; both s_3 and s_4 are no-flux boundaries;

563 Fig.2 Comparison between the numerical solutions of this study and the analytical solutions
564 of Huang et al. (2010).

565 Fig.3 BTCs for different values of v_d at the well during the pumping phase.

566 Fig.4 The schematic diagram of the flow system nearby well for the pumping phase.

567 Fig.5 Concentration distributions in a 2D horizontal plane at $t_{pump}=0$ hr. a) $v_d = 1 \times 10^{-6}$ m/s,
568 b) $v_d = 1.5 \times 10^{-6}$ m/s; c) $v_d = 2 \times 10^{-6}$ m/s; d) $v_d = 3 \times 10^{-6}$ m/s.

569 Fig.6 BTCs for different skin hydraulic conductivities at the well during the pumping with
570 $r_s=0.7$ m, and $\delta=0.25, 0.5, 1, 1.5$ and 2 .

571 Fig.7 2D horizontal plane distributions of concentration with $r_s=0.7$ m at $t_{pump}=0$ hr. a) $\delta=$
572 0.25 ; b) $\delta=0.5$; c) $\delta=1$; d) $\delta=1.5$; e) $\delta=2$.

573 Fig.8 Curves of concentration versus distance at $t_{pump}=0$ hr with $\delta=0.25, 0.5, 1, 1.5$ and 2 .

574 Fig. 9 BTCs for different skin indexes δ at the well during the pumping phase with $r_s=0$ m,
575 0.3 m, and 0.7 m.

576 Fig.10 2D horizontal plane distributions of concentration for different skin indexes δ at
577 $t_{pump}=0$ hr. a) $r_s=0.7$ m, $\delta=0.5$; b) $r_s=0.3$ m, $\delta=0.5$; c) $r_s=0$ m; d) $r_s=0.3$ m, $\delta=2$ m; e) r_s
578 $=0.7$ m, $\delta=2$.

579 Fig.11 Curves of concentration versus distance for different skin indexes δ at $t_{pump}=0$ hr with
580 $r_s=0$ m, 0.3 m, and 0.7 m.

581 Fig.12 BTCs at the well fitted by the non-skin model for different skin indexes δ with $r_s=0.3$
582 m, $\delta=0.25$ and 2 ,

583 Table 1. The parameter values used in this study

Parameter name	Symbols	Values
Aquifer thickness (m)	B	10
Radius of well screen (m)	r_w	0.1
Density of groundwater(kg/m ³)	ρ	1000
Effective porosity of aquifer	θ	0.3
Hydraulic conductivity of aquifer (m/d)	K	8
Constant heads of S_1 (m)	H_1	15.22, 15.44, 15.69, 15.65, ,15.87, 16.08, 16.30
Constant head of S_2 (m)	H_2	15.0
Regional groundwater Darcy velocities (m/s)	v_d	5×10^{-7} , 1×10^{-6} , 1.5×10^{-6} , 2×10^{-6} , 2.5 $\times 10^{-6}$, 3×10^{-6}
Longitudinal dispersivities of aquifer (m)	α_L	0.1
Hydraulic conductivity of skin zone (m/d)	K_s	2, 4, 12, 16
Effective porosity of skin zone	θ_s	,0.24,0.27,0.33,0.36
Skin radius (m)	r_s	0.3, 0.7
Injection or pumping rate (m ³ /d)	Q	15, 30
Mass flux per unit area (kg/(m ² • s))	N_0	0.02765, 0.5529

Injection time (hr)	t_{inj}	6
Rest time (hr)	t_{res}	24
Pumping time (hr)	t_{pump}	48

584

585

586 Table 2. The relative tracer mass recovered by the end of each SWPP test

types	positive skin	positive skin	non-skin	negative skin	negative skin
	$\delta=0.5, r_s=0.7$	$\delta=0.5, r_s=0.3$		$\delta=2, r_s=0.3$	$\delta=2, r_s=0.7$
μ (%)	0.95	0.85	0.83	0.82	0.74

587

588

589

590 Table 3. Parameter estimated by SWPP test model without skin

Real values used in the model with skin		Estimated by the model without skin	
		positive skin($\delta=0.5$)	negative skin($\delta=2$)
α_L (m)	$\alpha_L=0.1$	$14\alpha_L$	$17\alpha_L$
θ (dimensionless)	$\theta=0.3$	0.5θ	1.53θ
v_d (m/s)	$v_d=3.0\times10^{-6}$	$0.6v_d$	$1.47v_d$

591

592

593

594

595

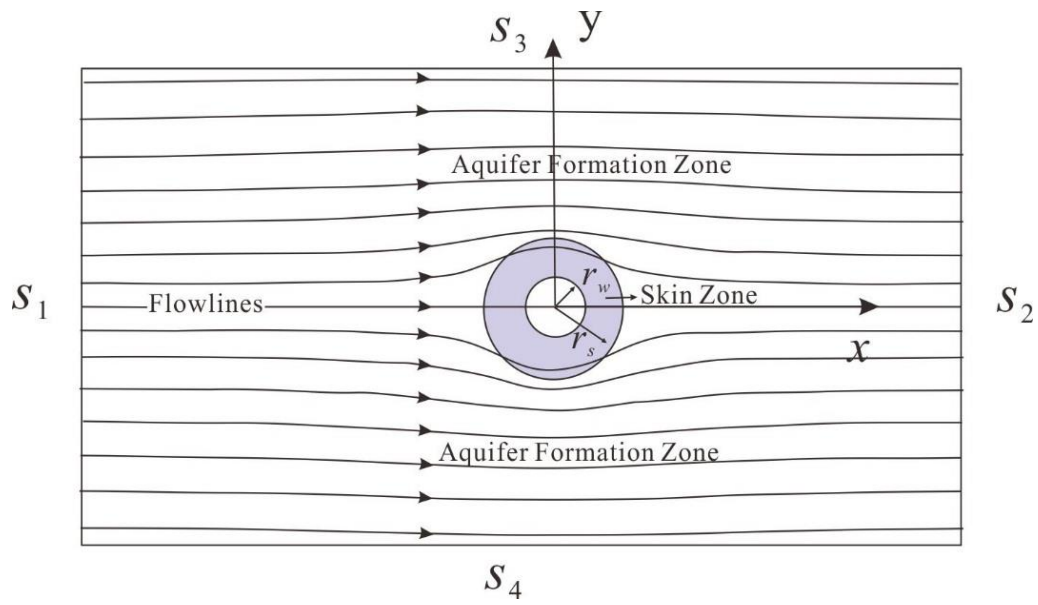


Fig.1

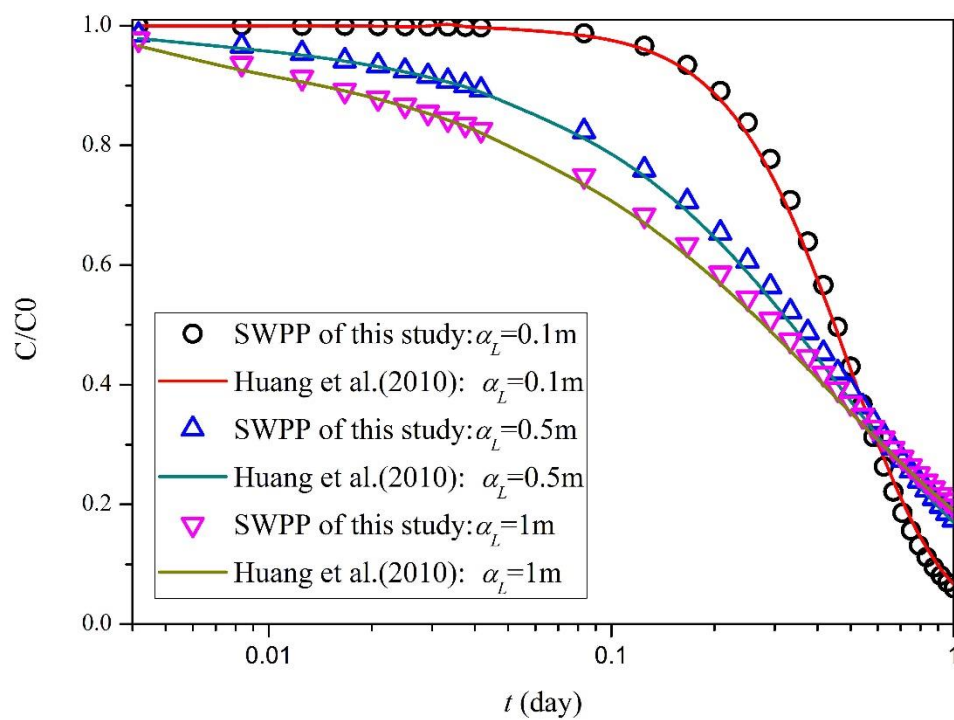


Fig.2

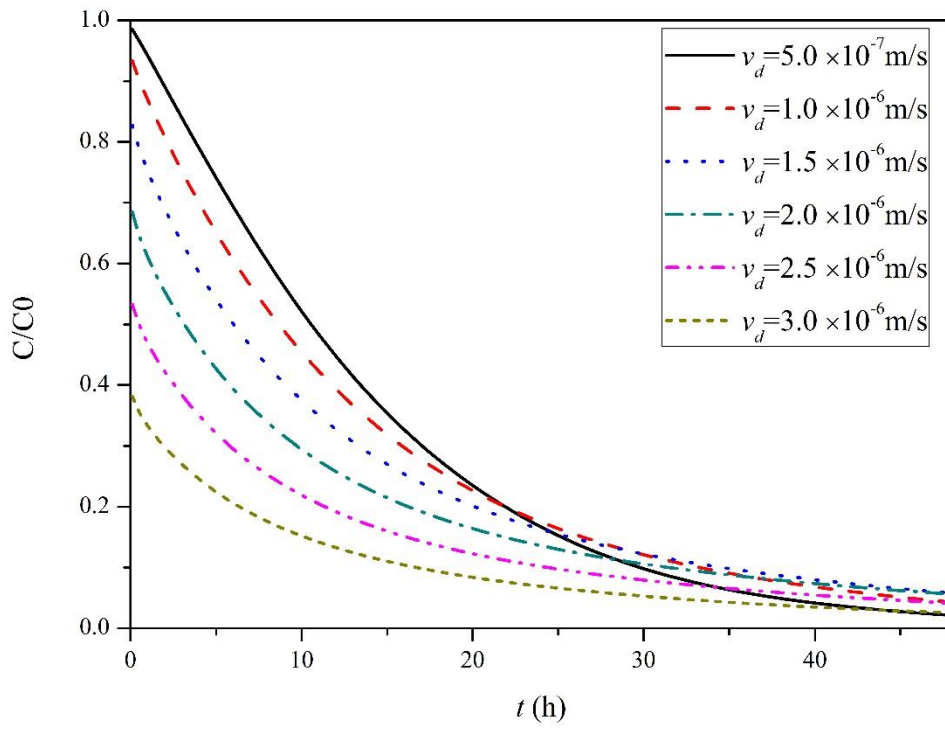
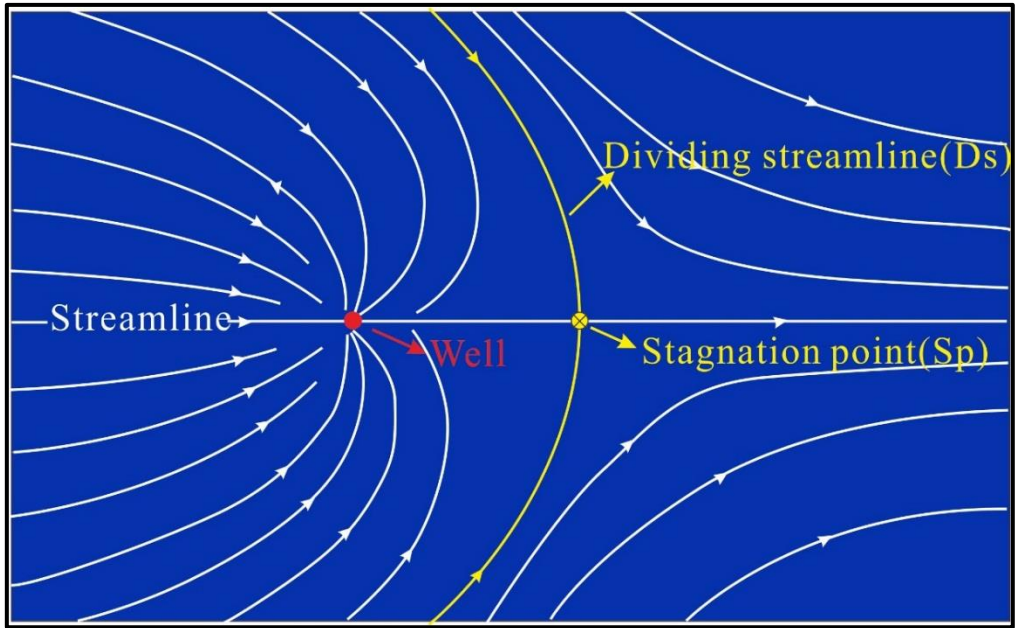


Fig.3



612

613

614

615

616

617

618 Fig.4

619

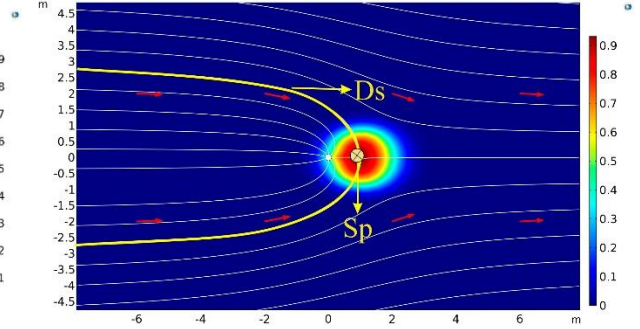
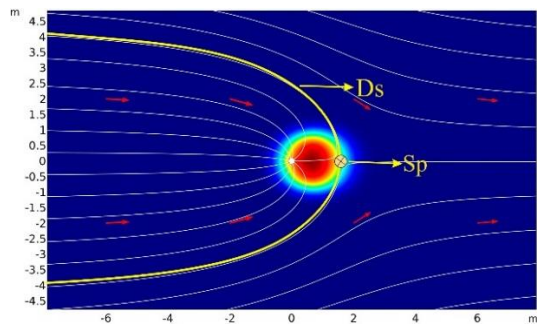
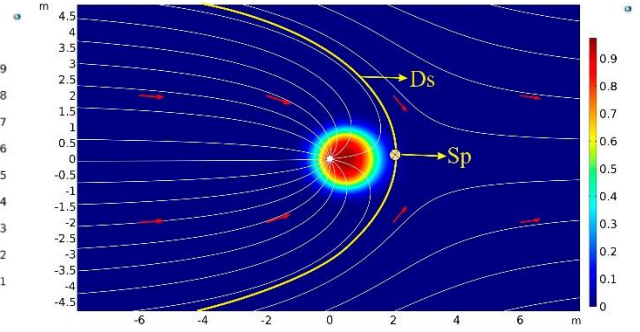
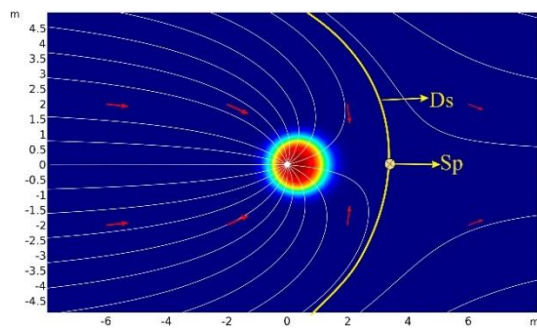


Fig.5

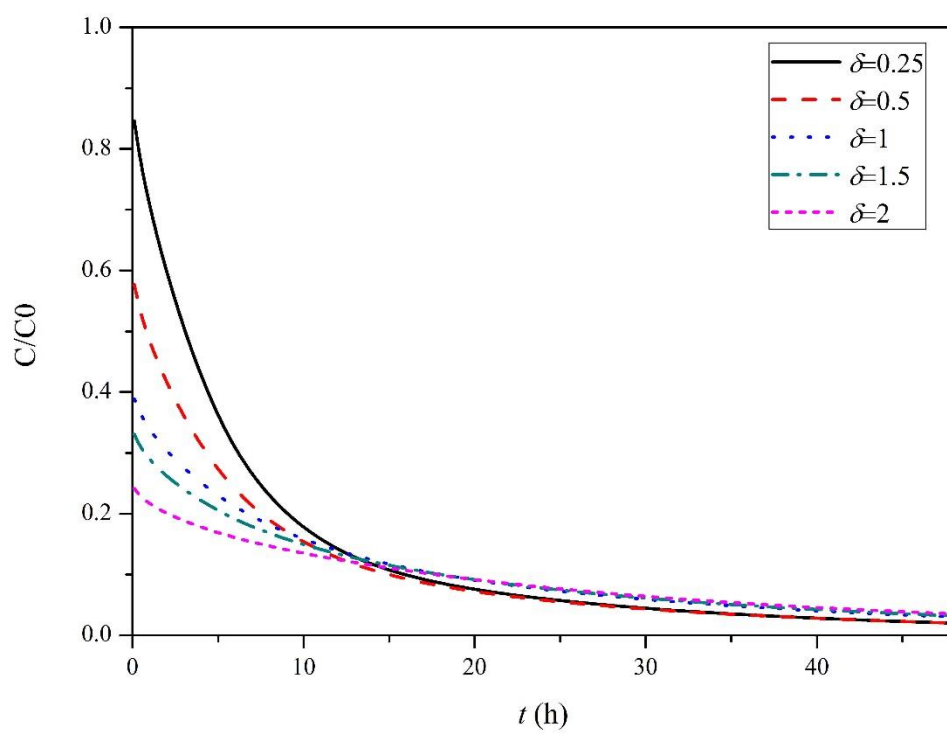
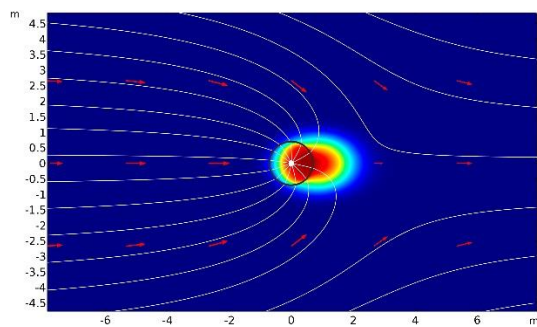
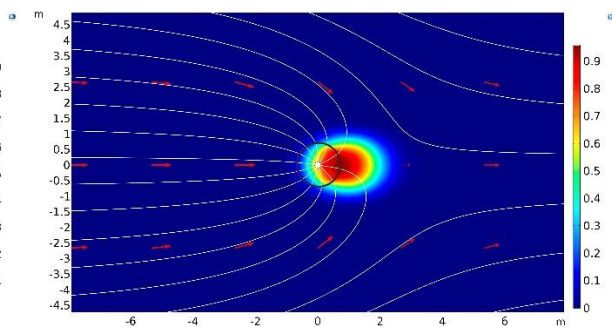


Fig.6



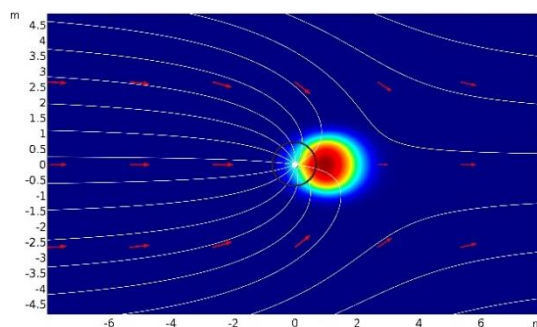
631



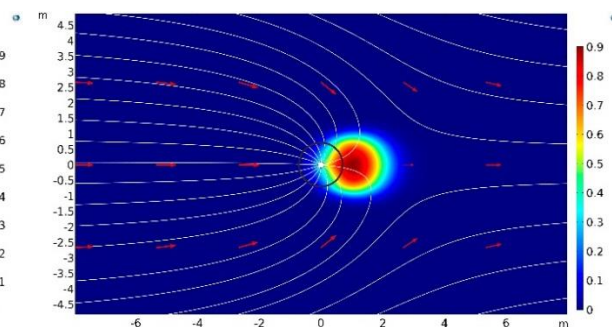
632

a. $\delta=0.25$

b. $\delta=0.5$



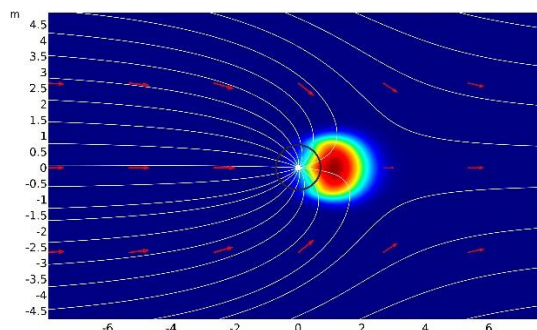
633



634

c. $\delta=1$

d. $\delta=1.5$



635

636

e. $\delta=2$

637

638

Fig.7

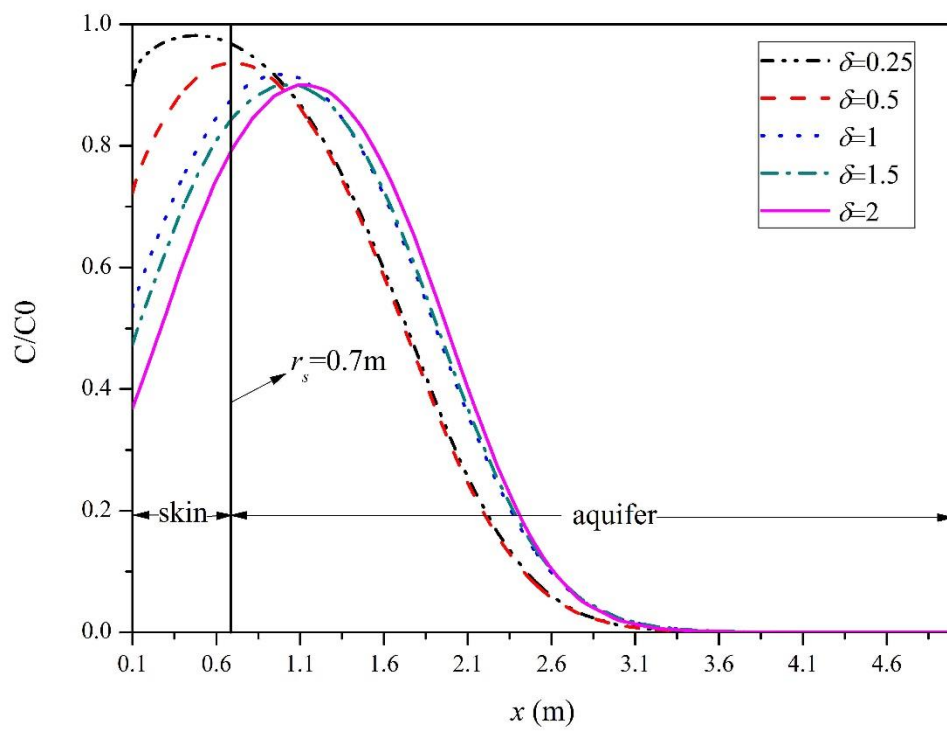


Fig.8

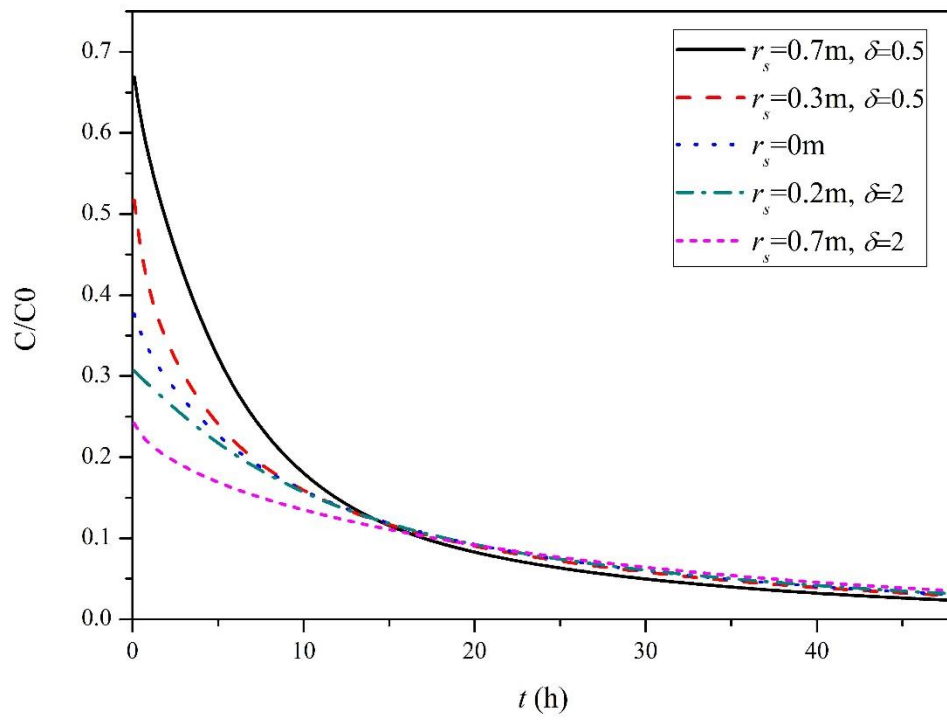
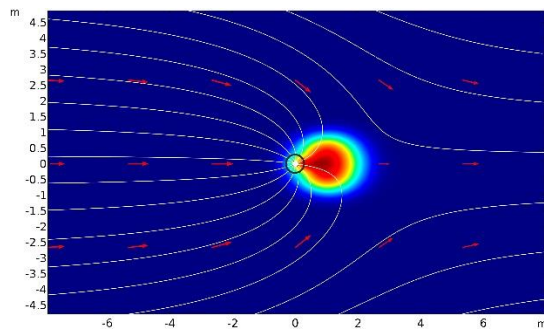
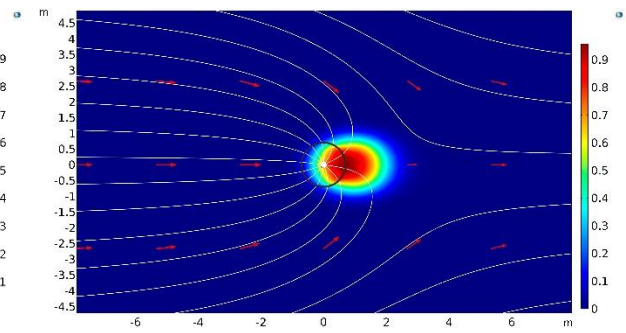


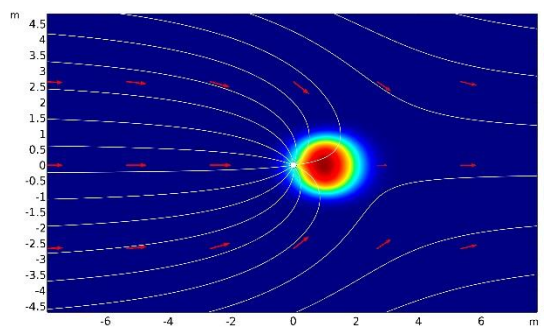
Fig.9



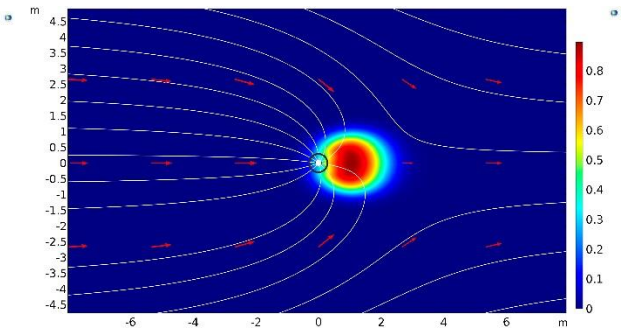
a. $r_s = 0.7$ m, $\delta = 0.5$



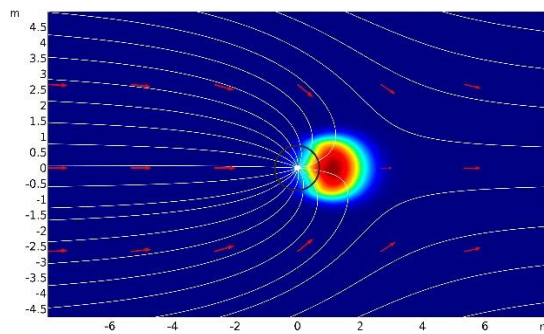
b. $r_s = 0.3$ m, $\delta = 0.5$



c. $r_s = 0$



d. $r_s = 0.3$ m, $\delta = 2$



e. $r_s = 0.7$ m, $\delta = 2$

Fig.10

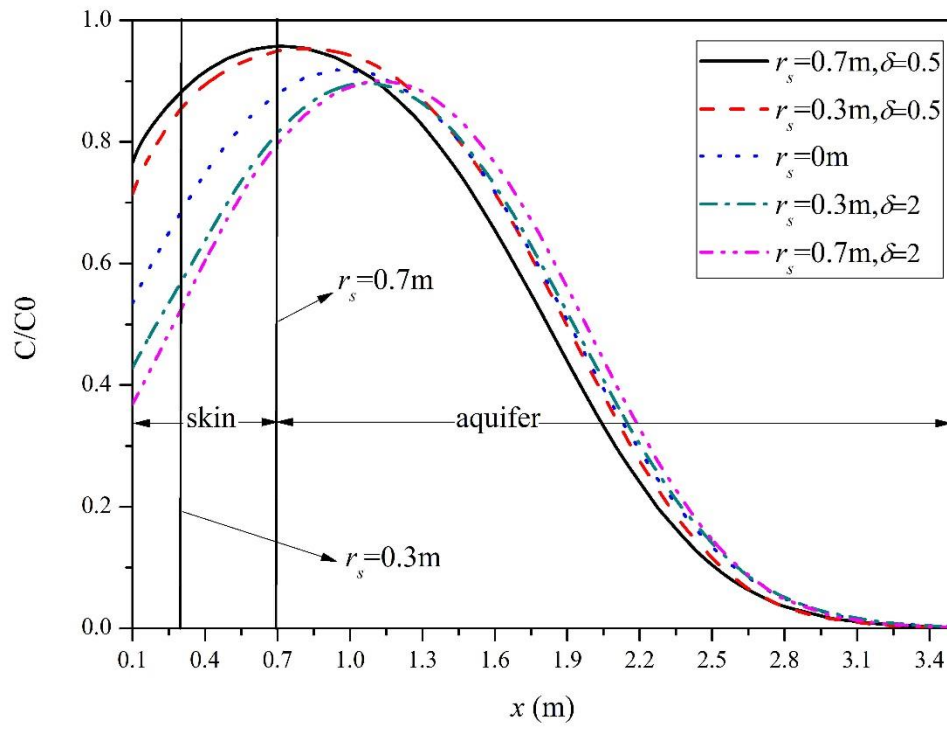


Fig.11

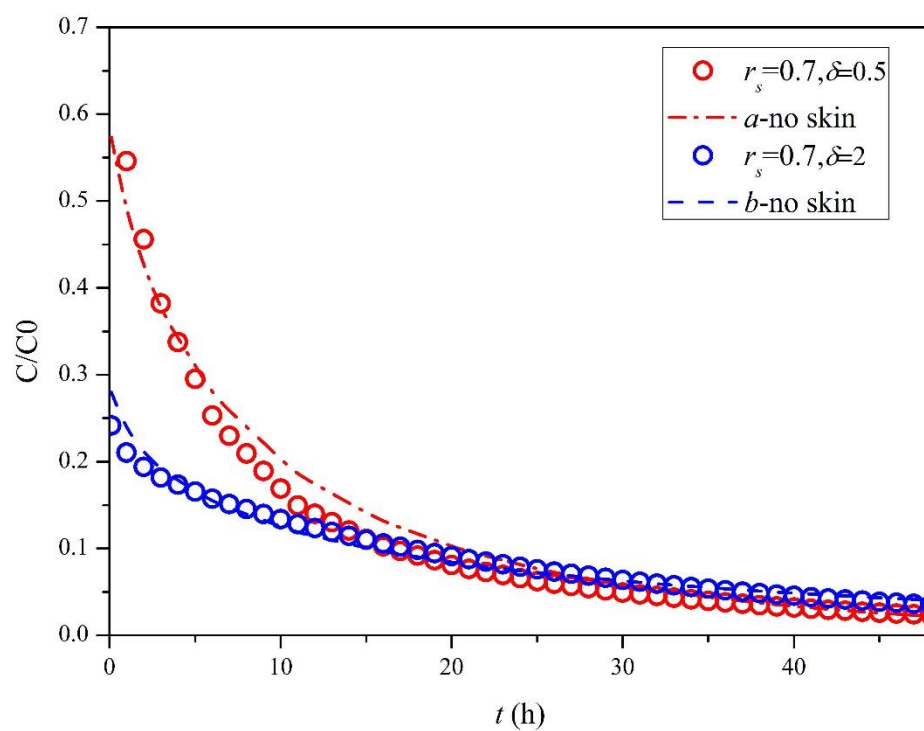


Fig.12