Responses on the comments of Referee 2 on the submitted manuscript "Geostatistical interpolation by Quantile Kriging "hess-2018-276

We thank the reviewer for the valuable comments. Instead of going into detail with the individual points we give a description of the model, which could explain the questionable details.

Description of the Process

Obviously, there is an underlying process assumption behind the model. A sketchy description is as follows:

Let $Y_0(x,t)$ be independent (for each different t) normal stationary spatial fields with $E[Y_0] = 0$ and $D^2(Y_0) = 1$ for each t.

Let:

$$Y_1(x,t) = Y_0(x,t) + M(t)$$
(1)

where M(t) is a process (in time) with zero mean. We may assume that the distribution of M(t) is normal. In this case each $x Y_1(x,t)$ is normally distributed with N(0,d). Further for each t the distribution of $Y_1(x,t)$ is N(M(t),1). Now Y_2 is defined as:

$$Y_2(x,t) = \Phi_{0,d}(Y_1(x,t)) \tag{2}$$

where $\Phi_{0,d}$ is the distribution function of N(0,d). By definition $0 \le Y_2(x,t) \le 1$

The rainfall is then generated as:

$$Z(x,t) = F_r^{-1}(Y_2(x,t))$$
(3)

where F_x is the distribution function of rainfall at location x. The F_x s can be different due to topography and other influencing factors. These F_x -s can be interpolated - example see also in Mosthaf and Bardossy (2017).

We use $Y_2(x,t)$ for each t and assume that it follows a beta distribution. In fact its distribution depends on M(t). If M(t) = 0 for all t-s then monthly rainfall is fully characterised by independent realizations over space. In this case the distribution of Y_2 is uniform for each t.

This however is not the case with observed data. The reason is that wet and dry conditions occur simultaneously over the whole area. This is controlled by M(t). One can take M(t) for example as independent random variables or as an ARMA process. If $M(t) \neq 0$ then the distribution of $Y_2(x,t)$ for this t is not uniform. The reason for assuming it as beta was due to the fact that beta distributions are very flexible and can well describe distributions in [0,1]. The exact form of the corresponding distribution would be something like:

$$G_t(v) = \Phi_{0,1} \left(\Phi_{M(t),1}^{-1}(v) \right)$$

However the use of this would require the exact knowledge of M(t) for each t. We decided to use a simple beta distribution instead.

The introduction of M(t) is reasonable as it explains the difference between the correlation between stations and the spatial correlation calculated using a variogram type approach for a given time. The later correlations are usually lower (smaller ranges) which are increased by the common large scale weather described with M(t).

In our procedure we start with Z(x,t), estimate and interpolate F_x . The calculate Y_2 for the observation locations. We interpolate Y_2 and come back to Z(x,t).

Spatial variograms are calculated for Y_2 for each t, and Y_2 is stationary in space. Non-stationarity and non-Gaussian distributions occur only for Z. That is the reason why we concentrate on Y_2 .

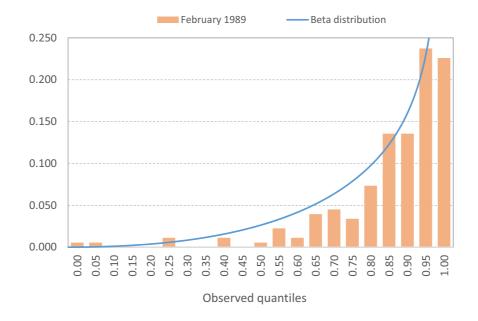


Abbildung 1: Example of the histogram of the observed quantiles in February 1989, along with the fitted Beta-distribution

Reference:

Mosthaf, T. and A. Bárdossy, Regionalizing non-parametric precipitation amount models on different temporal scales, Hydrology and Earth System Sciences, **21**, 2463-2481, 2017