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Discussions

Interactive comment on "Dynamics of wormhole formation in fractured karst aquifers" *by* Wolfgang Dreybrodt and Franci Gabrovšek

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The manuscript "Dynamics of wormhole formation in fractured karst aquifers" looks at the evolution of apertures and permeabilities in a network of fractures. The authors show that such a network will dissolve nonuniformly, with the flow and dissolution being focused in just a few flow paths (wormholes). The authors then further analyze the growth of the wormholes, the interactions between them and the impact of heterogeneity on the pattern.

The paper is interesting, with a large number of deep insights on dissolution-driven pattern formation. Some of the model systems considered by the authors (e.g. the system with non-soluble transverse connectors or the transverse connectors of differ-

ent width than horizontal ones) are really ingenious and allow one to understand the details of the wormhole-wormhole interactions which would be hard to grasp otherwise. I have just a couple of comments (as detailed below) – the two main ones being 1) the necessity of putting these results in the context of previous work, 2) the applicability of this particular model to the real systems. To be more precise:

1. **Relevance of the previous work and references**

   Some of the scientific questions analyzed in the manuscript have been tackled before, which – in my opinion – needs to be properly acknowledged and put into context:

   (a) Upadhyay et al., J. Geophys. Res. Solid Earth, 120, 6102-6121, 2015 looks at the effect of the seeded wormhole on the dissolution pattern, compares the pattern with and without the seeded wormhole (Fig. 12 there). It also identifies the region of influence of a wormhole as a region of the width approximately equal to the wormhole length. Many of the conclusions of this paper are similar to the conclusions of the present manuscript.

   (b) There are several other systems with similar competitive dynamics, where longer fingers screen the shorter ones, thus making them grow more slowly. For example, side-branch growth in two-dimensional dendrites (see e.g Couder et al, Phys. Rev. E 71, 31602 (2005) – look at Fig. 2 there) or anisotropic viscous finger growth (Budek et al, Phys. Fluids., 27, 112109 (2015) – look at Fig. 9 there) – the authors of these papers describe the deterministic nature of growth of such systems – a property shared with the present one. A larger class of such deterministic "hierarchical growth" systems is described by the review article by J. Krug, J. Adv. in Physics, 46(2), 139-282, 1997). Additionally, in the context of wormholes, a similar system (also with deterministic dynamics) was considered by Cabeza, Y et al: Controlling factors of wormhole growth in karst aquifers, in *Hydrogeological and Environmental Investigations in Karst Systems*, pp. 379–385 (2014).

In all of these system, a hierarchical distribution of the fingers orwormholes was observed, with a number of fingers longer than L, scaling as approximately as L^{-1}. I believe this is so also in the case of the patterns analyzed in the present m/s. In fact, such a scaling is not accidental, but closely related to the observation that the region of influence of a wormhole as a region of the width approximately equal to the wormhole length – the link between the two is shown in Upadhyay et al

(c) Upadhay et al (2015) also looks at the impact of heterogeneities on the dissolution patterns in dissolving fractures (something that the authors look at in Sec. 3.6). Interestingly, their conclusions on the influence of noise on the pattern are somewhat different from those of the authors – in particular, the role of lengthscale and timescale is emphasized. As shown e.g. in Fig. 4 of this paper, whereas at the beginning the flow paths are controlled largely by heterogeneities, after a while an instability wavelength appears, on scales much larger than the characteristic correlation length of the noise (on such scales the system can again be considered uniform)

2. Applicability of the model to the real systems: the authors claim that individual fractures in the model will dissolve uniformly – I have serious doubts as to the validity of this statement. To test it, I have carried out the simulations of a single fracture element of Dreybrodt & Gabrovšek network as well as four elements of this network joined in series. I have taken the Péclet and Damköhler numbers as quoted in the m/s (btw, the estimate of the penetration length on p. 25 of the manuscript seems to be erroneous – $l_p = va/2K = a/2\text{Da}$ gives $l_p = 31.25$cm for $a = 0.02$cm and $\text{Da} = 0.00032$ and not 16cm as reported in the paper). The results are presented in the attached Figures

They clearly show that even a single element of the network dissolves nonuni-

formly, contrary to what is stated on p. 25 of the m/s (where it is suggested that the dissolution in the first five elements of the network should be uniform). Nonuniform dissolution of individual elements of the network means also that one cannot impose constant pressure boundary conditions across the fracture-fracture intersections (white lines in Fig. 2), as the pressure will be highly nonuniform there, with the maximum along the developing wormhole. As a result, the pattern which would develop in such a system will consist of circular conduits and not uniformly opening fractures. The flow focusing will not change that much in this picture – our simulations indicate that the widths of the channel scale sublinearly (as $q^{1/3}$) with the flow rate, thus the conduit will still remain localized (only getting thicker with time) and the pressure will not be uniform across the fracture width. In general, I find the assumption of the uniform pressure across the lines joining fractures really dubious – it is true that such an assumption is often made in the case of network models where each link is a capillary/conduit and the intersection is supposed to be small in size compared to the capillary length; here however the width of the intersection (2m) is the same as the length of individual fractures. Imposing a uniform pressure along each such intersections (white lines in Fig. 2) changes the dynamics of the system in a dramatic way.

These remarks are not intended to diminish the value the present manuscript. As I was commenting above (point 1b), many features of flow focusing systems are rather generic and independent of the particular model. The present manuscript offers a lot of interesting insights in the dynamics of such systems and it is qualitatively correct. But I do not think that it has quantitative predictive power (in terms of breakthrough time in years etc) for the system it attempts to model (system of intersecting fractures).

Minor comments:

- What is the origin of factor of 3 in Eq. 7

- p. 17., line 4 – "Outflow remains low because the hydraulic gradients close to the tip of the shorter wormhole stay similar" – the meaning of this sentence is unclear
- What is plotted in Fig. 22? In the text it is stated that "...all horizontal fractures at $y = 150$ have aperture widths $A_0 = 0.02$cm, while the rest of the fractures have generally different initial aperture $0 < a_0 < 0.025$cm. Figure 22 shows the dependence of the breakthrough times on the aperture widths of the net for various A_0". But the caption of this Figure suggests that the results are plotted as a function of a_0 and not A_0.

Technical comments:

- The lettering in many figures (particularly Fig. 3, 5, 6, 7, 12, 15, 17, 20, 23) is illegible. Small bitmapped fonts are used which are almost unreadable. This is a real pity, since a lot of information is lost in this way – I urge the authors to prepare good quality figures, preferably in vector format

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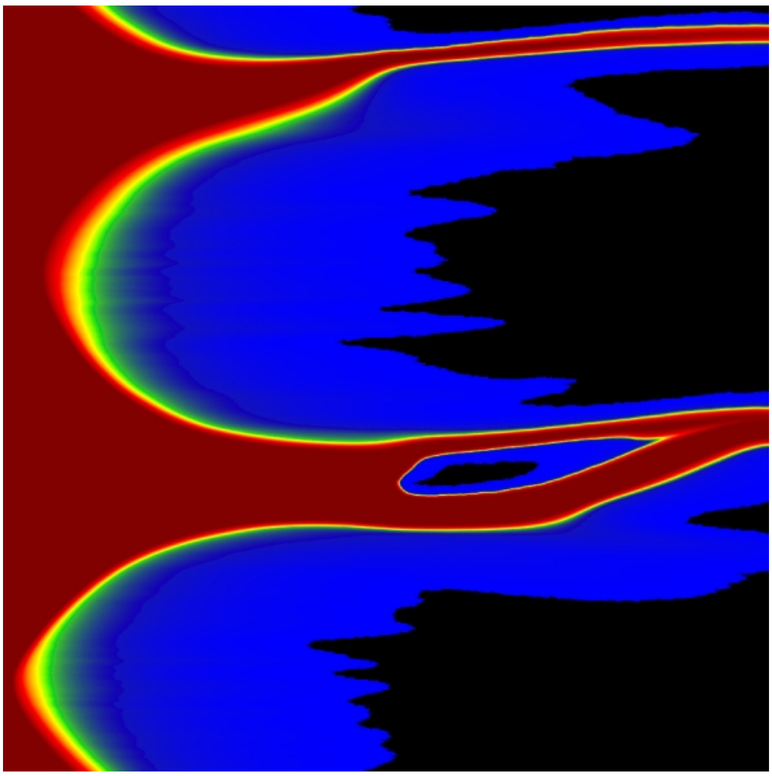

Fig. 1. Dissolution of a single fracture of Dreybrodt and Gabrovsek network. Colors indicate deep erosion (red), intermediate erosion (yellow and green), low erosion (blue), no erosion (black).

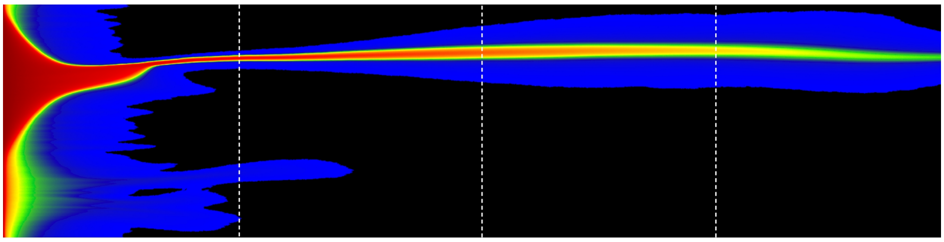

Fig. 2. Simulation of the dissolution of four elements of Dreybrodt and Gabrovsek network.