Response to all RCs and SCs

Dear Professor Carlo De Michele,

First, we deeply appreciate your precious time and efforts for handling our submission. The comments were extremely constructive and insightful, which dramatically helped us to improve the quality of the original submission.

While we have addressed all comments point by point, the major changes done on the manuscript are as follows:

- We further investigated the model's performance to reproduce rainfall extremes. This was done by fitting the annual maxima of both observed and synthetic rainfall time series to the GEV distribution and compare the parameters of the fitted GEV distribution.
- We validated our model to the ungagged temporal period. This was done by comparing the statistics of the synthetic rainfall to those of the rainfall time series observed during the validation period (1951-1980). The calibration period (1981-2010) does not overlap with the validation period.

Thank you again for your time and consideration.

Sincerely,

Dongkyun Kim, PhD.
Associate Professor,
Department of Civil Engineering,
Hongik University, Seoul, Korea

Dear Anonymous Referee #1,

We sincerely appreciate your constructive comments on our manuscript. All your comments tremendously helped us to improve the quality of the article. Our responses are as follow:

Comment 1. The equation for the relationship between the variance of rainfall at one time scale and another together with the covariances (Equation 1) should be clearly derived from the preceding equation.

Authors' Response. We agree with this point. In order to clarify the relationship between preceding equation, we changed Equation 1 like below.

We can then write the variance at time-scale nh as:

$$V_{nh} = \mathrm{Var}\left(Y^{(nh)}\right)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right)$$

$$= \sum_{i=1}^{n}\sum_{j=i}^{n} \mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} \mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right)$$

$$= n\mathrm{Var}\left(Y^{(h)}\right) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} \mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right)$$

In the meantime,

$$\mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right) = \mathrm{Cov}\left(Y_j^{(h)}, Y_i^{(h)}\right)$$

So

$$V_{nh} = n\mathrm{Var}\left(Y^{(h)}\right) + 2\sum_{i=1}^{n}\sum_{j=1, j>i}^{n} \mathrm{Cov}\left(Y_i^{(h)}, Y_j^{(h)}\right) \qquad (1)$$

, where V_h is the variance of rainfall depths at scale h hours and $\mathrm{Cov}(\cdot, \cdot)$ is the covariance operator between the two random variables.

The second term of the right-hand side of Equation 1, which represents the rainfall correlation between individual records separated by $(i - j)$ time-steps of the time series of rainfall depths at scale h hours, is likely to be underestimated by the Poisson cluster rainfall model because …

Comment 2. The comparison of extreme rainfall modeled vs observed was conducted via linear regression. First it is not clear how the "Extreme" observed rainfall points were obtained. Second it is not clear how the modeled values were obtained. Presumably they correspond to similar probability levels obtained from a GEV? In any case the subsequent assessment of bias is reasonable. However it may be more transparent to display the tails of the probability distributions side by side to more clearly visualize how well the model reproduces the observed data. Alternatively, the parameters of the GEV distributions could be compared.

Authors' Response. Yes. We assumed that the monthly maxima follow the Generalized Extreme Value distribution. Here, we separated the analysis for each calendar month, so we have 12 sets of extreme rainfall values corresponding to each gauge station. Therefore, we produced each scatter plot of Figure 13 based on 348 points (12 months/gauge x 29 gauges). We provided the explanation regarding this in the article as follow:

"The scatters in Figure 13 compare the 20-, 50-, 100-, and 200-year rainfall estimated from the observed rainfall (x) and the synthetic rainfall (y) generated by the hybrid model (red) and the MBLRP model (blue) at hourly through daily time scale. The Generalized Extreme Value (GEV) distribution was used to model the distribution of the annual maxima, and the three parameters of the GEV distribution were determined using the method of L-moments. Here, we separated the analysis for each calendar month, so we have 12 sets of extreme rainfall distributions corresponding to each gauge station. Therefore, we produced each scatter plot of Figure 13 based on 348 points (12 months/gauge x 29 gauges)."

We prepared additional figures (Figure 15 and Figure 16) according to your suggestion. The figures and the corresponding explanations are as follow:

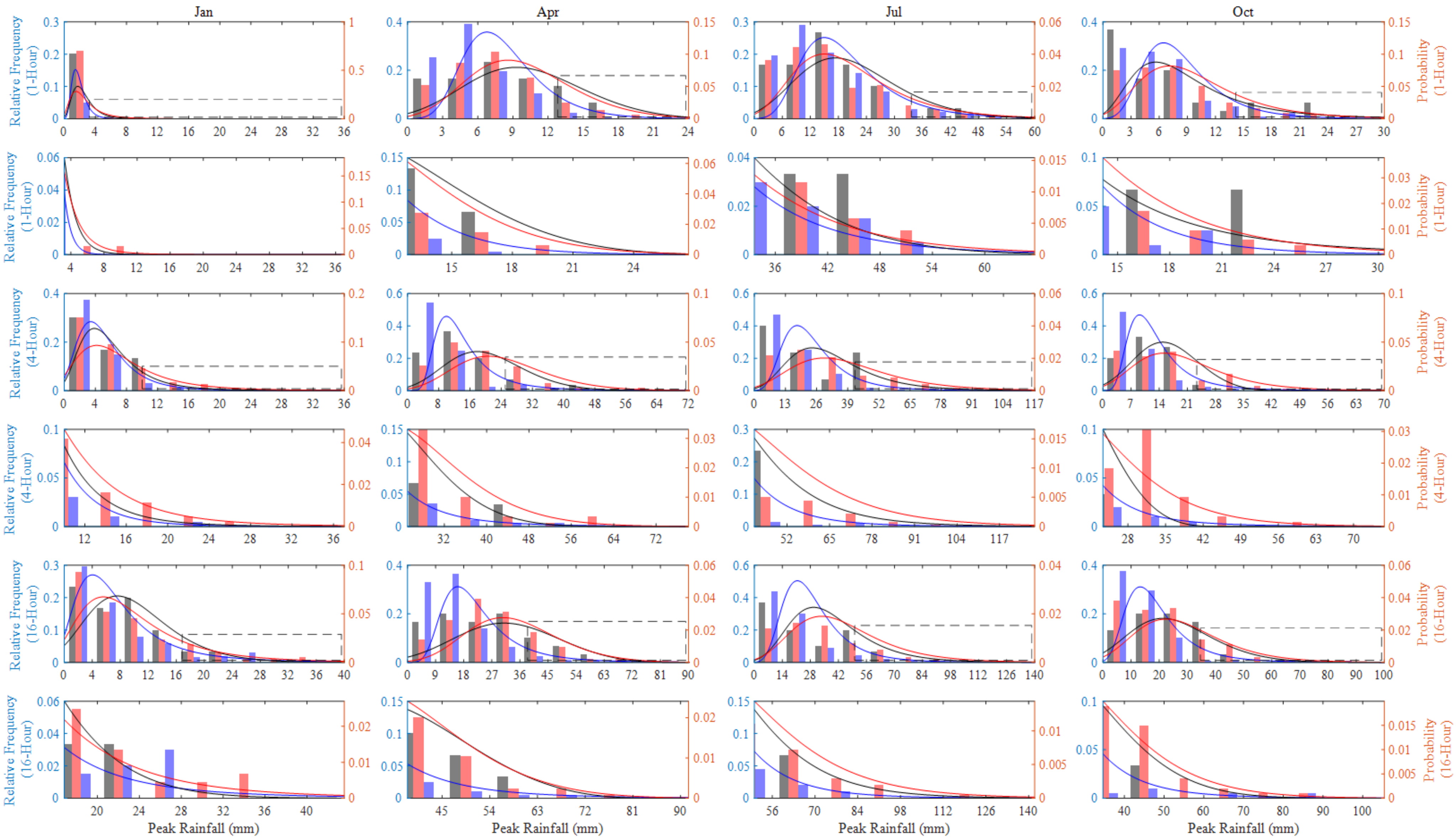


Figure 15. Relative frequency and the fitted GEV distribution of the 1-, 4-, and 16-hour monthly maxima of January, April, July, and October rainfall at NCDC gauge 132203. Results of Observed rainfall (black), our hybrid model (red), and the traditional MBLRP model (blue) are shown. The upper 10 percentile part of the distribution (dashed box in the plots in the first, third, and fifth row) is magnified in the lower rows (plots in the second, fourth, and sixth row).

Figure 15 shows the relative frequency and the fitted GEV distribution of the monthly maxima of January, April, July, and October at NCDC gauge 132203. The black, red, and blue line correspond to the result of observed rainfall, our hybrid model, and the traditional MBLRP model, respectively. The GEV distribution of the 1-hour, 4-hour, and 16-hour duration rainfall are shown in the plots of the first, third, and fifth row, respectively. The plots in the second, fourth, and the sixth row magnifies the upper 10-percentile part of the distribution of the upper figures that is denoted as the dashed box.

For all months and durations, our hybrid model outperforms the traditional MBLRP model in reproducing the head through tail part of the distribution. The distribution of the traditional MBLRP model was skewed toward the lower values. The similar tendency was observed at most gauge locations while at some of the gauges our hybrid model showed the similar or slightly degraded performance compared to the traditional MBLRP model in reproducing distribution of the extreme values.

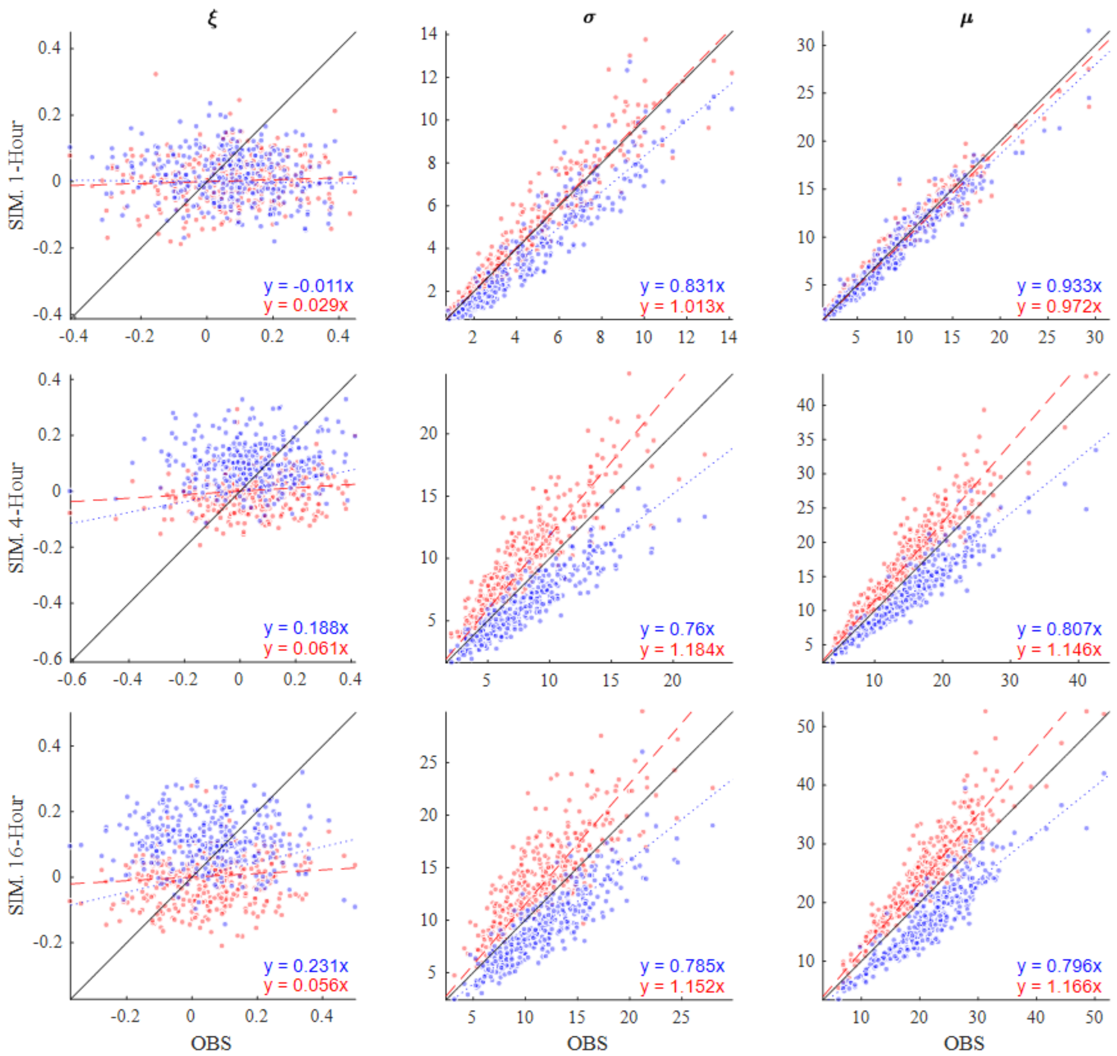


Figure 16. Comparison of the shape (ξ), scale (σ), and location (μ) parameters of the fitted GEV distribution of the monthly maxima. The results based on the observed rainfall (x), our hybrid model (red), and the traditional model (blue) are shown. The results of 1-hour, 4-hour, and 16-hours of rainfall durations are shown.

Figure 16 compares the shape (ξ), the scale (σ), and the location (μ) parameter of the fitted GEV distribution of the monthly maxima of the observed rainfall (x) and of the synthetic rainfall generated by our hybrid model (red scatters) and by the traditional MBLRP model (blue scatters). The results of 1-hour, 4-hours, and 16-hours of rainfall durations are shown. Each scatter point represents the result of one calendar month at one gauge. A total of 348 scatter points (12 months/gauge x 29 gauges) are shown for each of the plot. The traditional MBLRP model underestimates the location parameters at all rainfall durations, and the degree of underestimation increases with the increase of the duration. Our hybrid model showed the opposite trend. The location parameters tend to be overestimated with the increase of the duration, but the degree of overestimation was not as significant as the case of the traditional model. The traditional model compensates the underestimated location of the distribution with the overestimated scale parameters, which were observed for all three durations investigated. Our hybrid model also compensates the overestimated location of the distribution with the underestimated scale parameters, but the degree of compensation was not as significant as the case of the traditional model. However, the shape parameter of the observed monthly maxima was not well reproduced by both models. This result shows the difficulty of precisely reproducing the rainfall extreme values. This is mainly because the extreme values of the rainfall are indeed extreme. For example, 1-hour 100-year rainfall of ninety nine years of rainfall record is theoretically the greatest value of all 867,240 hourly rainfall records (24hours/day x 365days/year x 99year), and precisely reproducing a value with such a low probability of occurrence can be a daunting task using the models with only a limited number of parameters.

Technical notes 1. There are only a few minor editorial suggestions (which have been included in a scanned document)

Authors' Response. Thank you for your thorough review. The attached supplementary material has only the odd pages, so we will incorporate all your correction suggestions once we receive the complete document.

Dear Anonymous Referee #2,

This manuscript reports on a new rainfall model that is an extension of the commonly employed Bartlett-Lewis approach. The model was applied to 29 rain gages, which at first glance seems like perhaps not enough. The authors argue that the approach is very laborious and they justify that this is a good start. Overall the work will make a positive contribution. Some suggestions for improvement are included below.

Response: We deeply appreciate this thoughtful review, which dramatically improved the quality of the article. We also recognize the complexity of implementing our model caused by its non-parsimonious nature. We explicitly addressed this issue in the discussion section of the article. Our responses are as follow:

Major comments

Comment 1. The model performance should be assessed based not just on "effectiveness" (fitting data), but also on "efficiency" (how many parameter). The latter criteria is quantified using AIC or similar metrics. Such criteria are already embedded within ARIMA frameworks so it is clear that the authors should be familiar with the importance of AIC etc. However, the authors have chosen not to describe how many parameters the overall model uses? This is very important information that should be explicitly compared against the MBLRP model, which very clearly uses 6 parameters. The proposed model is acknowledged by the authors (p.26, line 20) as "not easy to implement". Yet it is argued throughout the paper to perform better than the 'standard' MBLRP model, which is easy to implement. So the critical question is whether the improved performance is worth the cost in complexity. AIC or similar should be used to compare the gains in model performance vs the cost of increased complexity. It seems that the current model is perhaps far more complex than the MBLRP, yet this complexity is currently hidden from the reader.

Response: We agree. While the parsimonious nature of the MBLRP model is its greatest strength, there is a need to acknowledge the limitations of modeling the extremely complex rainfall process of reality only with the six parameters, one of which our article addresses and resolves. We explicitly addressed this issue in the discussion section as follow:

4.3. An Issue with Model Parsimoniousness: six versus fifty five

Our hybrid model uses one MBLRP model parameter set per one simulation year while the MBLRP model needs only 6 parameters regardless of the simulation length. However, this does not mean that our model requires 600 MBLRP model parameters (6 per month x 100 months) to generate 100 months of rainfall. This is because parameters are estimated based on the short-term rainfall statistics that are synthetically generated through the process of the SARIMA model and the regression analysis (See Figure 5). Therefore, the parameters of the SARIMA model and the parameters of the regression analyses shown in Figure 5 should be considered as the "true" parameters of this model because once these parameters are given, our model can generate infinite length of rainfall record. The SARIMA model has 6 parameters, and a set of regression analyses shown in Figure 5 has 49 parameters (2 per ten solid arrows in Figure 5 = 20, 3 per 8 bivariate normal distributions relating two subsequent residual terms (ε_i) in Figure 5 = 24, and one per 5 normal distributions perturbing autocorrelation terms (c_i) = 5). Therefore, our model has a total of 55 parameters. This discrepancy of number of parameters (6 of the traditional of MBLRP model versus 55 of our hybrid model) can be considered as a cost taken to reproduce the large-scale rainfall variability that the traditional MBLRP model cannot.

We admit that this large discrepancy of model parsimoniousness is an issue to be resolved for our model to be applied in practice. Regarding this, we are planning to apply our model to additional gauge locations across the world and share the result through the website (http://www.letitrain.info). The work has been already initiated for the rainfall data of Korean Peninsula.

2. Twenty-nine gages were selected. But the spatial distribution is not uniform. For example, most of the gages are in a similar climate region in the Midwestern USA with only two gages in the southeastern USA. What was the reason for this disproportionate representation? It needs to be specified if these are the only gages in the study region where sufficient data are available or whether this is just a sub-section of available data. If it is a sub-selection of available data, then the disproportionate representation needs some explanation.

Response: The gauge locations randomly chosen out of potential 3,000+ gauge locations. The reason why we chose only 29 gauges is because we were concerned about the difficulty of precisely identifying the cause of potential issues of the model if too many gauges were investigated together. In the meantime, we were also curious about the result at the remaining gauges, so we integrated the results at 6 extra gauges in the article. The new analysis result did not significantly alter the original result.

3. I do not understand the necessity of the first model module (SARIMA model). From Figure 4 it seems that the input to the second module is "monthly rainfall mean". If this is the only output from module 1 that is then used in module 2, then why is synthetic data created at all? It seems that the measured monthly means should be used here.

Response: We agree that our model without the first module can still downscale the observed monthly rainfall. However, such model will be only a downscaling model. The first module enables our model to generate the monthly rainfall of which statistical property is similar to the observation including the large-scale (e.g. interannual) temporal variability, which the traditional MBLRP model fails to reproduce. Then, this generated monthly rainfall is downscaled by the remaining modules of the model. To sum up, this unique combination of the modules enables the generation of rainfall time series that is different from the observation but with the similar statistical properties including the large-scale temporal variability, which we believe, is the primary purpose of the rainfall "generators"

4. Also, From Figure 4 it seems that the second module is generating more outputs than are supported by the model inputs. If only monthly means are input, then it seems impossible to determine just from this all of the outputs that are listed. This wide array of outputs would have to be non-unique given only monthly means as an input. So something must be missing here.

Response: We believe that Figure 5, Figure 6 and the corresponding explanation, which addresses the second module in detail, may resolve your concern. Even though the input of the second module is a single monthly rainfall value, it uses the additional information obtained from the regression analysis between various fine time-scale statistics shown in Figure 6 to acquire module output values.

5. Several figures use data from one selected gage as illustration. However, different gages are used between the figures. In such cases it should be specified whether similar trends are observed at all gages or whether this is only seen at this particular gage. The reader should not be led to suspect that the "best" gage is being selected for each figure.

Updated Response: After the suggestion of another reviewer (the third reviewer), we decided to replace the figures in the methodology section (Figure 6, 7, and 8) such that they represent the result of the identical station (NCDC-200164). In the result section, Figure 10 is the only one corresponding to an individual gage, but Figure 11 conveys the information of Figure 10 for all gauge locations. All other figures in the results section (Figure 9, 11, 12, 13, and 14) display the analysis of all gauge locations. The discussion section shows the analysis on the gage NCDC-460582 (Figure 15 and 16), which clearly showed our point of discussion.

Original Response: We completely agree with your concern, which is the reason why the figures that summarize the results of our study were prepared so that they represent the entire study area. The figures showing the analysis of individual gauge were limited in the methodology (Figure 6, 7, and 8 from NCDC gauge ID 200164) and discussion section (Figure 15 and 16 from NCDC gauge ID 406582). The selection of these two gauges were random because using the same gauge for all these five figures conversely give an impression that we are only showing the best results.

In the result section, Figure 10 is the only one that corresponds to individual gauge, but Figure 11 conveys the information of Figure 10 for all gauge locations. All other figures in the result section (Figure 9, 11, 12, 13, and 14) display the analysis of all gauge locations.

Other comments

1) P.1,Line 11, “fine tuned” here is vague and not descriptive. Suggest more specific description of the approach that is used.

The words were replaced into “carefully calibrated.” The detailed description of this careful calibration is given in the next sentence of the article, which were almost impossible to be integrated into one sentence.

2) P.1,Line 11, “fine scale” is similarly vague, so I suggest to be specific.

The words were replaced into “sub-daily statistical.”

3) P.1, Line 14, change to “at 29 gages”

Done as suggested.

4) P.2,lines 1-2, Change to “provide rainfall input data to modeling studies for risk. . .”

Done as suggested.

5) P.2, lines 8-21, This direct quote from another paper was to me much too long to be appropriate. Either summarize briefly in your own words, or just cite the other paper. Do not extract large sections of another paper – even if some authors are shared between the papers.

We agree. The sentence was removed. Our original purpose was to make this article as independent as possible so that readers do not have to find another article to understand the MBLRP model. The sentence was removed and was replaced into the following sentence:

….The model assumes that a series of rain storms (black circles) comprising a sequence of rain cells (red circles), arrives in time according to a Poisson process. The MBLRP model has six parameters of which brief descriptions are provided in the lower text box of Figure 1.

6) P. 3, lines 9-12, This are far too many references cited here. Generally three references are enough to support a point. It is not necessary to cite in one sentence every study that has ever been done.

We agree that three references are enough to support a point, but we believe that the articles cited here describes the gradual progress of the model over the last 30+ years, so we would like to keep them per reviewers' permission.

7) P. 4, line 31, Here it is not specified what model parameters were used nor how those parameters were determined. At a minimum it should say here that these were determined "as specified below/in section XX".

The following sentence was added.

"Here, the MBLRP model used the parameter set that was calibrated to reproduce the observed rainfall mean, variance, lag-1 auto-covariance, and proportion of dry periods at sub-daily aggregation intervals (1-, 2-, 4-, 8-, and 16-hours), which is a typical practice of MBLRP model calibration (Rodriguez-Iturbe et al., 1987; Rodriguez-Iturbe et al., 1988; Kim et al., 2013a)"

8) P. 9, line 9, The justification for 24 is not clear here

The sentence was changed as follow:

In so doing we are now considering the monthly rainfall, when divided by the number of days in the month times 24, as providing us with an estimate of the mean hourly rainfall for that particular month.

9) P. 10, line 11, This seems to be equation 1 re-stated

Removed as suggested.

10) P. 13, line 4, cite figure number, and describe which data these are based on

Done as suggested.

11) P.15, line 29, It is not clear here what data are meant by "generated fine time scale statistics" and "statistics of the generated synthetic hourly rainfall". These sound too similar.

Changed as follow according to your suggestion:

Second, the discrepancy between *the fine time scale statistics generated by the second module of the model (Figure 5)* and the statistics of *the synthetic hourly rainfall time series generated by the MBLRP model* is calculated using the following formula:

12) P. 16, lines 11-12, It is not appropriate to switch between R2 and R when the R2 value is not high enough to support your point.

Removed as suggested.

13) P. 18, section 4.3, It is not clear why variance is described in section 4.2 and then standard deviation

in section 4.3. These are not two independent parameters. Pick only one.

Figure 12 was changed into variance.

14) P.17, line 12, The equation should not be invoked here when it has not been presented yet

The equation was presented previously. It is the following equation:

$$V_{nh} = nV_h + 2\sum_{k=1}^{n-1} C_h(k)$$

15) P. 20, line 2, It is not clear how the 100 or 200 year recurrence intervals are determined from measured data

The sentence was modified as follow:

The scatters in Figure 13 compare the 20-, 50-, 100-, and 200-year rainfall estimated from the observed rainfall (x) and the synthetic rainfall (y) generated by the hybrid model (red) and the MBLRP model (blue) at hourly through daily aggregation levels. The extreme rainfall value were estimated in the following manner: (1) time series of monthly rainfall maxima corresponding to a given duration is obtained; (2) The Generalized Extreme Value (GEV) distribution was assumed to model the distributions, and the three parameters of the distribution were determined using the method of L-moments. So, there are 348 points for each model (29 gauges x 12 months).

16) P. 20, lines 7-9, These sentences should probably be removed. Readers of this journal do not need to be told that R2 approaching 1 means the model is good.

Removed as suggested.

17) P. 22, lines 10-11, Revise this sentence. Currently “rainfall” is repeated four times.

It was changed as follow:

“A good rainfall model should reproduce not only the extreme values but also the distribution of the maxima from which extreme values are derived.”

18) P. 24, all of this seems like it should be in the methods rather than discussion.

While we agree that the results shown here are closely associated with the methodology, we believe that they would better fit in the discussion section because understanding the message of the figure would require understanding on the results.

19) Figure 2 – the meaning of the gray boxes should be described in the caption

20) Figure 3 – the caption should explain the importance of the BIC values

21) Figure 5 – the different shading of the boxes should be described in the caption

22) Figure 6 – the caption should specify which data sets these results correspond to

23) Figure 9 – it is not clear if these are the mean for all months or if there is one data point for each month.

24) Figure 13- make the symbol sizes bigger. I could not see triangles

25) Figure 14 – change the y-axis limits to better show the data range. Since the lower limit is already not equal to zero, just make it 0.7 and the upper limit 1.1 or 1.2

Dear Anonymous Referee #3,

The authors here propose and test a composite method which appears able to generate synthetic rainfall time series for a wide range of aggregation time scales. The issue of producing reliable synthetic rainfall time series is undoubtedly a central problem in hydrology, and the extension to a wide range of scales attempted in this study has the potential to be a relevant contribution. Therefore, the scope of this study makes it suitable for consideration in Hydrology and Earth System Sciences. However, there are a number of issues that needs clarification and/or correction, which I detail below. In particular, my main comment concerns the validation of the method (comment 6 below). I would recommend the paper for publication only after these issues have been addressed by the authors.

Authors' Response. We sincerely appreciate your constructive comments on our manuscript. All your comments tremendously helped us to improve the quality of the article. Our responses are as follows:

Major Comments

Comment 1. The so-called Module 2 uses only one quantity (mean hourly rainfall, as obtained from the 1st module) to generate finer scale rainfall statistics (mean, variance, correlation, and dry fraction). Is not clear to me how the regression analysis for module 2 is carried out (Page 10, line 10). For example, the variance is computed from the mean assuming the two quantities are linearly related. How is the slope $\alpha_{[6]}$ computed? Using the entire dataset available for each station and each month? If this is the case, is not true that the proposed model solely use monthly information (as stated e.g., in Figure 4) to produce fine scale rainfall statistics, and this should be clarified. I find that section 3.2 is overall not very clear, and could be improved. You use 'functional relations' between many quantities, but a relation is only shown for the mean vs standard deviation relationship. Perhaps this linear relation could be introduced more generally (e.g., X = \alpha_i Y + \epsilon_i), and then state that X and Y can be the variables of interest, for example mean, variance, … ?

Authors' Response. We agree that the explanation about the second module is vague. We modified the manuscript as follows:

Revised Contents.

3.2 Generation of fine time scale rainfall statistics

The second module generates the fine time scale statistics corresponding to each monthly rainfall value generated through the SARIMA model. These synthetic fine time scale statistics will be later used for the calibration of the MBLRP model to downscale the monthly rainfall into hourly level. In so doing we first consider the monthly rainfall, when divided by the number of days in the month times 24, as *providing us with an estimate of the mean hourly rainfall for that particular month. Then, this estimated mean hourly rainfall is provided as the input variable of the module that generates the mean, variance, auto-correlation coefficient, and the proportion of dry periods at 1-, 2-, 4-, 8-, and 16-hour aggregation intervals, as described in Figure 5. In this process, the module employs the information obtained from univariate regression analysis between the fine-scale statistics of the observed rainfall (Figure 6) and the mathematical formula relating rainfall variance and auto-covariance at different time scales (Equation 4).*

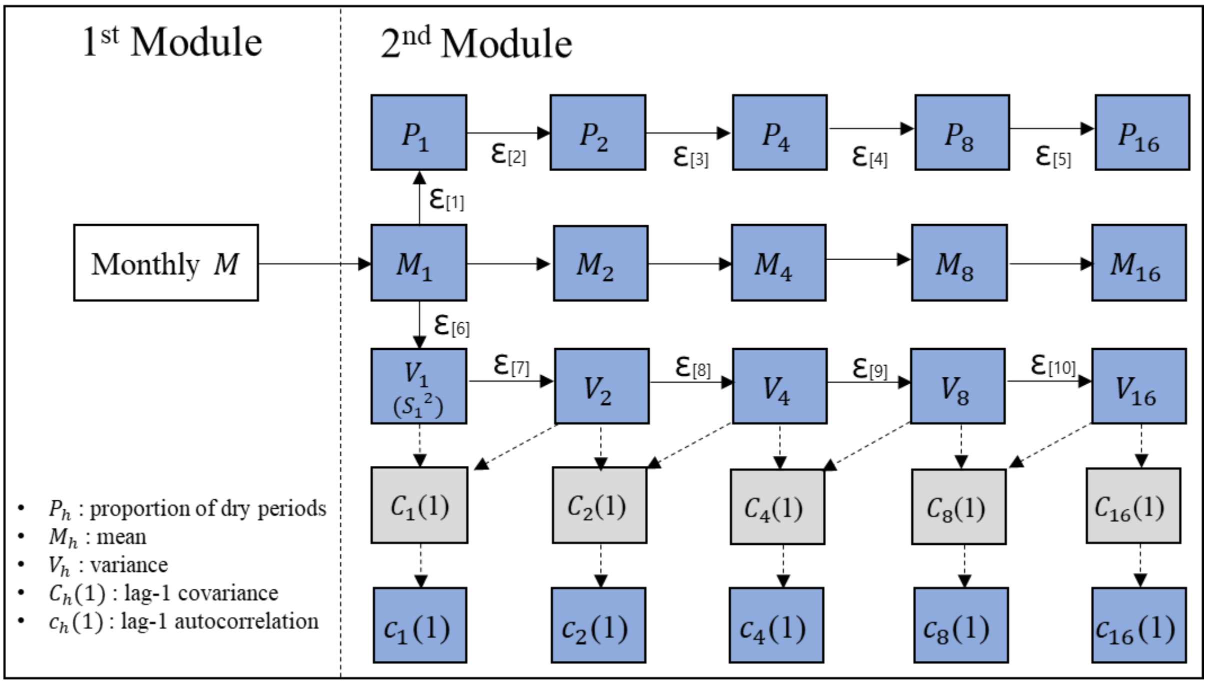


Figure 5: Schematic of the algorithm to generate fine time-scale rainfall statistics. The statistics in the blue boxes are used to calibrate the MBLRP model and the statistics in gray boxes are used to estimate the 1ag-1 autocorrelation.

Figure 5 is the schematic of the second module. In the figure, M_h, S_h, $V_h, c_h(1) = C_h(1)/V_h$ *and* P_h *in each rectangle represent the rainfall mean, standard deviation, variance, lag-1 autocorrelation, and proportion of dry periods at time-scale h hours, respectively. The statistic connected to each solid arrow head is stochastically generated based on its linear relationship to the one connected to the tail of the same arrow. In other words, the following equation is used:*

$$Y = a_{[i]}\, X + b_{[i]} + \varepsilon_{[i]} \qquad (2)$$

where Y is the variable being generated, and the X is the variable being used as the basis of the generation. Here, the variable X and Y can be substituted by any combination of two variables connected by the solid arrow; $a_{[i]}$ and $b_{[i]}$ are the parameters of the regression analysis, and $\varepsilon_{[i]}$ is a random number drawn from the normal distribution $\varepsilon_{[i]} \sim N\left(0, \sigma_{[i]}^2\right)$ fitted to the residuals of the regression analysis. Here, these three parameters are estimated from the univariate regression analysis relating the two variables observed during a given calendar month over multiple years as shown by black scatters in each plot of Figure 6, which shows the linear relationship between the rainfall statistics observed at the gage NCDC-200164 (star mark in Figure 3) during the month of July of different years.

The statistic connected to the dashed arrow head is calculated based on the ones connected to the tail of the same arrow using the mathematical (deterministic) relationship connecting these variables. For instance, in Figure 5, V_1 and V_2 are connected to $C_1(1)$ through a dashed arrow, which means that $C_1(1)$ is derived from V_1 and V_2. The following equations establish the relationship between the variances at time-scales h and 2h from which we shall obtain the relationship between V_1 and V_2: .....

Comment 2. Also, it is not immediate to me how all these relations between rainfall statistics can be linearly related, especially rainfall mean and wet fraction. I think it would be helpful to show how these linear relations hold for all the stations in the study, not just a sample rain gauge. Is it possible they depend on season/rainfall regimes?

Authors' Response. Regarding the linearity, we prepared the plots for all gauge locations and all seasons, which can be accessed through the following website:

http://www.letitrain.info/

Here are some notes about our linearity assumptions:

(i) We assumed that the hourly standard deviation (S_1), but not the hourly variance (V_1), is linearly correlated to the hourly mean (M_1) as suggested by the black scatters in Figure 6(a). After we generated S_1 from M_1 based on this relationship, we took the square of it to obtain the hourly variance (V_1). We believe that the linearity between M_1 and S_1 is not a bold assumption considering numerous previous studies that models the rainfall distribution as exponential (mean = λ^{-1}, standard deviation = λ^{-1}) or gamma (mean = $k\theta$, standard deviation = $k^{0.5}\theta$) distribution;

(ii) The linearity between the variance at different aggregation intervals can be explained by the following equation given in the manuscript.

$$Var\left(Y_i^{(2h)}\right) = Var\left(Y_{2i-1}^{(h)}\right) + Var\left(Y_{2i}^{(h)}\right) + 2Cov\left(Y_{2i-1}^{(h)}, Y_{2i}^{(h)}\right)$$

$$V_{2h} = 2V_h + 2C_h(1)$$

We can consider two extreme cases. First, if $Y_{2i-1}^{(h)}$ and $Y_{2i}^{(h)}$ are independent, then we get a linear regression with the gradient of 2 ($V_{2h} = 2V_h$). Second, if $Y_{2i-1}^{(h)}$ and $Y_{2i}^{(h)}$ were identical, then the covariance is equal to the variance, so we would get $V_{2h} = 4V_h$.

(iii) We could not find the studies that explicitly deals with relationship between hourly mean and hourly proportion of dry periods (M_1 vs P_1). However, our empirical analysis at all 34 stations suggests a strong linear relationship between the two variables. Please see the figures at:

http://www.letitrain.info/

(iv) Regarding the relationship between the proportions of dry periods at different aggregation intervals, Onof et al. (1994) showed that the mean number of events at time scale *h,* is given by the following relation to the proportions dry period:

$$E(N_h) = \frac{P_h}{P_h - P_{2h}}$$

By rearranging the equation, we get:

$$P_{2h} = P_h\left(1 - \frac{1}{E(N_h)}\right).$$

This, therefore, suggests looking at whether the coefficient here is reasonably stable and therefore whether there is a linear relationship between these two proportions dry.

Reference. Onof, C., Wheater, H. S. and Isham, V.: Note on the analytical expression of the inter-event time characteristics for Bartlett-Lewis type rainfall models, Journal of hydrology, 157, 197-210, 1994.

Comment 3. In general, figure captions should be improved and expanded throughout the manuscript, explaining more in depth what is in the figure. For example, the caption of figure 1 should state what the blue areas (cells) and shaded lines (total rainfall intensity I guess) are. Also, even if it is just a schematic figure, Figure 1 should have axes for time and rainfall depth. Figure 6 caption should specify that the results are for a single gauge, etc. The caption of figure 14 is probably swapped (blue and red lines – please check).

Authors' Response. We completely agree. Done as suggested as follow:

Revised Contents.

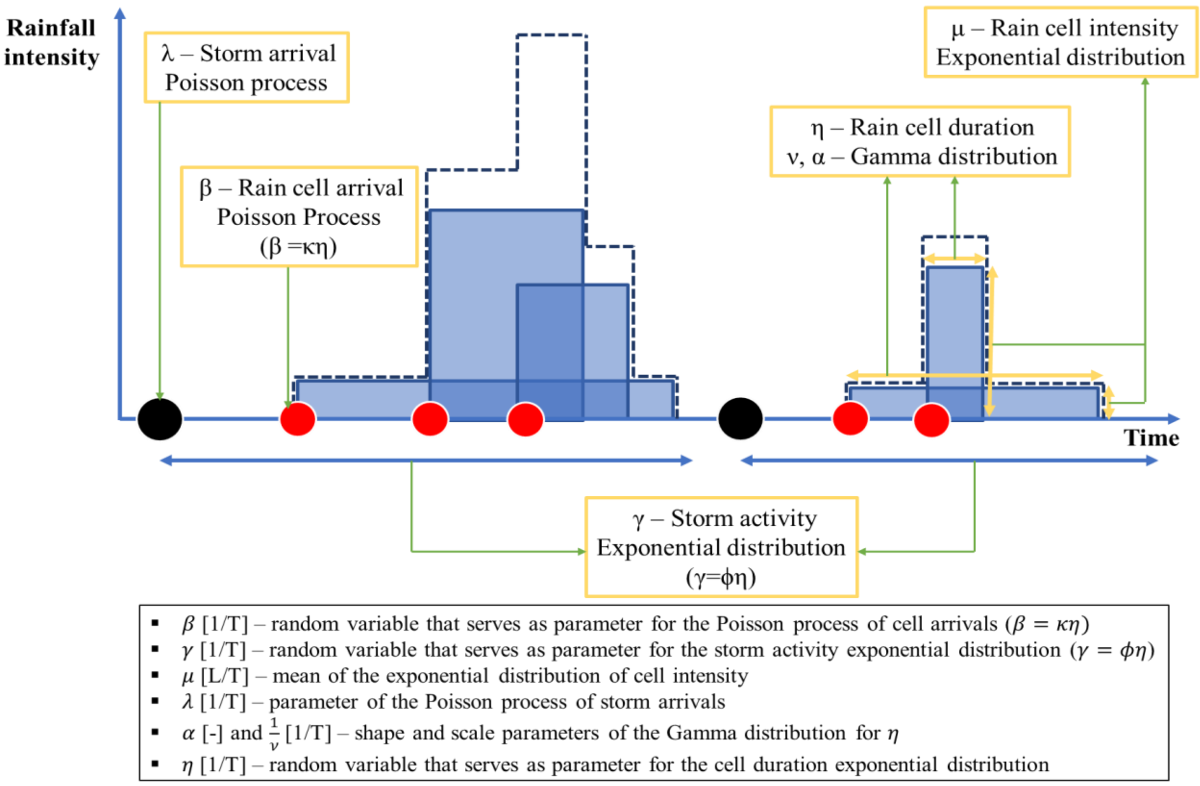


Figure 1: Schematic of the Modified Bartlett-Lewis Rectangular Pulse Model. Blue area represents duration (width) and intensity (height) of each rain cell, respectively. Dashed line represents superposed sum of the rain cell intensities.

Figure 6: Linear relationship between various fine time-scale statistics of the rainfall observed for the month of July of different years ***at the gage NCDC-200164*** *(black dots). The solid black line represents the least squares regression line. Based on this regression relationships, a set of the 20 fine-time scale statistics are generated, which are immediately used as the basis of the MBLRP model parameter calibration. If the objective function of the parameter calibration corresponding to the generated set is greater than a given threshold, the set is rejected (blue squares), and the set with the objective function lower the threshold value is only chosen (red squares).*

Figure 7: (a) Comparison of estimator $\widehat{c(1)}$ *(horizontal axis) with estimator* $\frac{\widehat{V_2}}{2\widehat{V_1}} - 1$ *(vertical axis) of the autocorrelation lag-1 of hourly rainfall, (b) The histogram of the discrepancies between these two estimators* ***at the gage NCDC-200164***.

Figure 8: (a) Relationship between $\varepsilon_{[7]}$ *and* $\varepsilon_{[8]}$ *and the fitted bivariate distribution. (b) Color map of the correlation coefficient between different* $\varepsilon_{[i]}$*s* ***at the gage NCDC-200164 on September***.

*Figure 14: Degree of over/underestimation of extreme values by our model **(red)** and the traditional MBLRP model **(blue)**. $\boldsymbol{ER_{syn}}$ and $\boldsymbol{ER_{obs}}$ are extreme rainfall estimated from synthetic rainfall and observed rainfall, respectively.*

Comment 4. The authors present results for a particular station in some of the figures, and this station is not the same throughout the paper. I think it would be better to be consistent and present the results for the same station.

Authors' Response. We agree with your concern. We replaced the figures in the methodology section (Figure 6, 7, and 8) such that they represent the result of the identical station (NCDC-200164). In the result section, Figure 10 is the only one corresponding to an individual gage, but Figure 11 conveys the information of Figure 10 for all gauge locations. All other figures in the results section (Figure 9, 11, 12, 13, and 14) display the analysis of all gauge locations. The discussion section shows the analysis on the gage NCDC-460582 (Figure 15 and 16), which clearly showed our point of discussion.

Comment 5. Page 15, line 17: it is stated that the Module 2 may fail in generating realistic fine scale rainfall statistics. The Authors should include a bit more explanation for this. How often does it happen? Given that Module 2 is based on linear relations, is it possible for some of this relations not to be linear in some cases, and cause these failures? (This may vary with precipitation types/rainfall regimes). This could be assessed checking if, in a cases of 'failure', any of the relations between rainfall statistics in Module 2 exhibit a divergence from linearity more marked than in other cases.

Authors' Response. We appreciate this thorough review. The cases of failure in which the parameter set satisfying the objective function criteria could not be generated even after the 50 iterations were identified at the 11 months over 8 gage locations. (NCDC-85663(Jun and Dec), 111549(Mar), 138706(Jan and Dec), 360106(May and Dec), 401094(Nov), 431081(Dec), 447201(May), and 460582(Jul)), which comprises 0.01 percent of the entire months simulated (200 years × 12 months per gage × 34 gages = 81,600 months). For these months, the set with the lowest objective function was selected among the 50 generated statistics set. It should be noted that the case of failure was not repeated on the same month at the same gage. If the linearity assumption was not correct, there should be a repetitive (or systematic) behavior of failure. Therefore, we cannot say that the linearity assumption is the primary reason of the failure.

Comment 6. Figure 12 summarizes the performance of the rainfall generation method, comparing observed and simulated rainfall statistics. I think the method should be validated using an independent set of observations, not the same used for calibration (i.e., used for computing the regression coefficients). For example, the authors could divide the gauge time-series in two independent samples, using one of them for calibration and the second for validation. In the end, this is what we want to achieve with a synthetic rainfall generator: match as much as possible statistics of time series that we do not have available. This analysis could also resolve a second issue: Since the proposed model is more complex/has more parameters that the original MBLRP, using independent samples for validation would show to what extent the additional model complexity can improve method performance. Also for figures 9, 10 and 11 it would be helpful to show results obtained using independent samples.

Authors' Response. We completely agree with this view. We used the period of the year 1951 to 1980 as the validation period, which is the previous 30 years to the calibration period. Figure 9 to Figure 12 were redrawn, and the following paragraphs were added in the manuscript.

Revised Contents.

3.5 Validation for Ungaged Periods

One of the primary purposes of the stochastic rainfall model is to provide the synthetic rainfall for the ungaged periods, which can be the period of missing data or future. For this reason, we separated the period of model calibration and validation at some gage locations (square marks in Figure 2) where record length of each period is sufficiently long (60+ years). Then, we tested our model not only based on the statistics of the calibration period (1981-2010) but also based on the validation period (1951-1980).

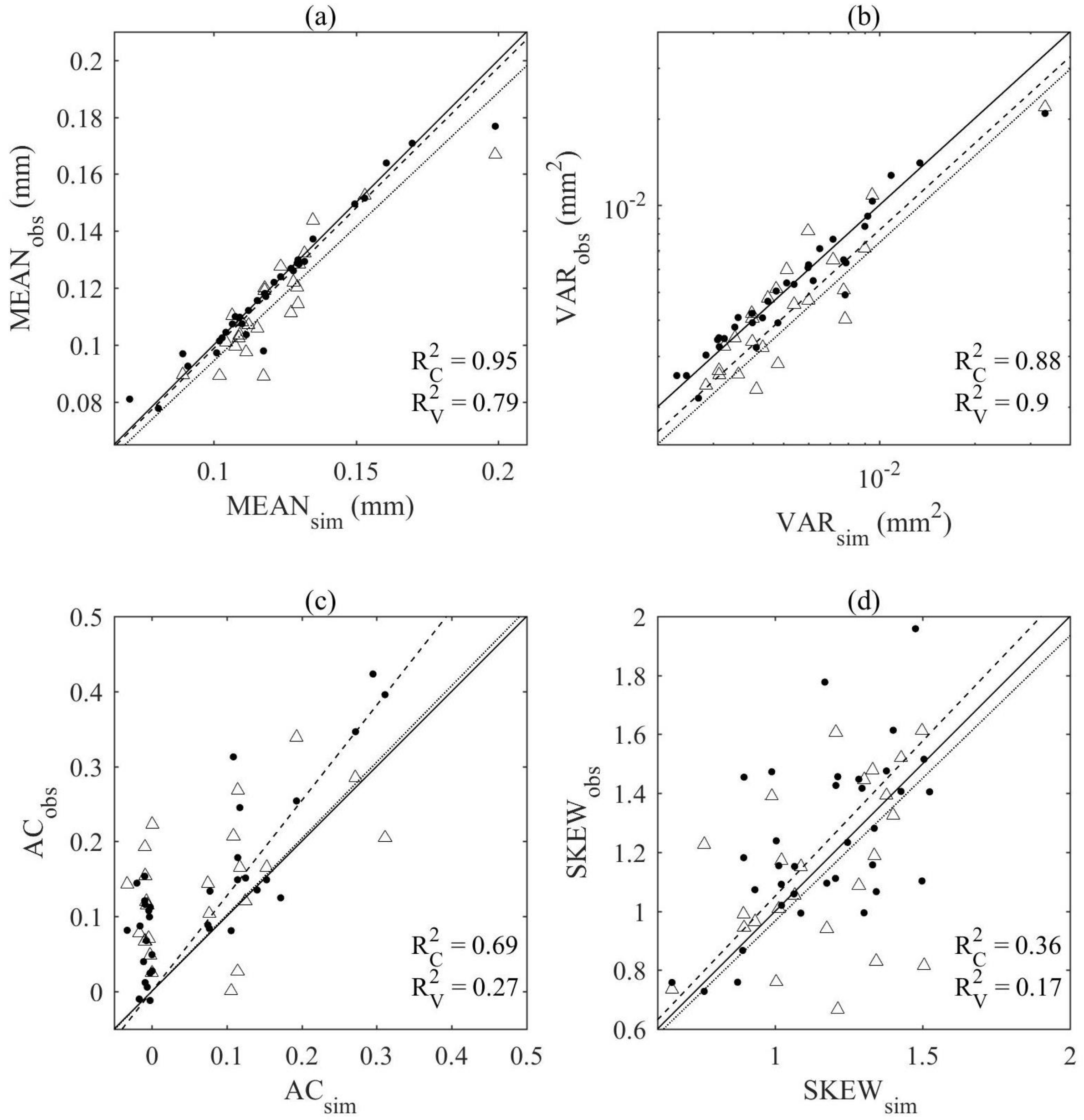


Figure 9: Comparison of (a) mean, (b) variance, (c) lag-1 autocorrelation, and (d) skewness of the synthetic (x) and observed (y) monthly rainfall. Filled circles (dashed line) and hollow triangles (dotted line) correspond to the calibration (1981-2010) and validation period (1951-1980), respectively.

4.1 Monthly Rainfall Statistics Reproduction

Figure 9 compares the mean, variance, lag-1 autocorrelation, and skewness of the monthly rainfall time series generated by the SARIMA model (x) to those of the observed monthly rainfall time series (y). Each scatter represents one rainfall gage. On the calibration period (1981-2010), the first and the second-order moments were reproduced accurately with the coefficient of determination ranging from 0.69 to 0.95. Skewness was reproduced fairly well with the coefficient value of 0.36. For the validation period (1951-1980), mean and variance were reproduced, but not 1ag-1 autocorrelation and skewness. However, this discrepancy cannot be solely attributed to the limitation of the model because the *discrepancy in each plot of Figure 9 is solely caused by the discrepancy of the statistics between calibration and validation period. In other words, if the statistics of the calibration period is similar to those of the validation period, we can expect the similar performance for both periods, and vice versa.*

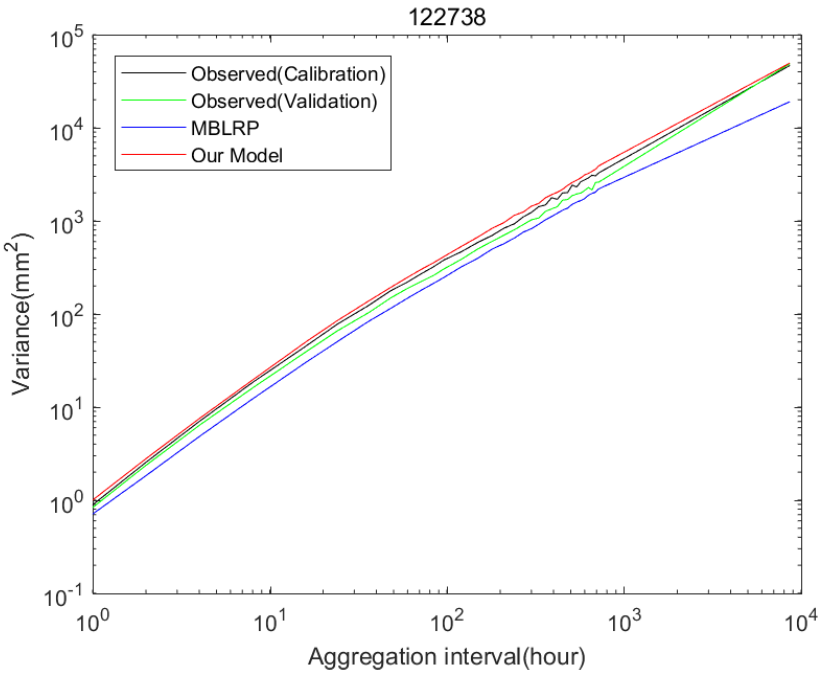


Figure 10: Behaviour of the rainfall variance with regard to the aggregation interval of rainfall time series at the gage NCDC-122738. The behaviour corresponding to the observed-calibration (black, 1981-2010), observed-validation (green, 1951-1980), MBLRP (blue) and our hybrid model (red) are shown together.

4.2 Reproduction of Large Scale Rainfall Variability

Figure 10 shows the behaviour of the rainfall variance varying with temporal aggregation interval between 1 hour and 1 year at the gage NCDC-122738. The behaviour corresponding to the observed-calibration (black, 1981-2010), observed-validation (green, 1951-1980), MBLRP (blue) and our hybrid model (red) are shown together. …

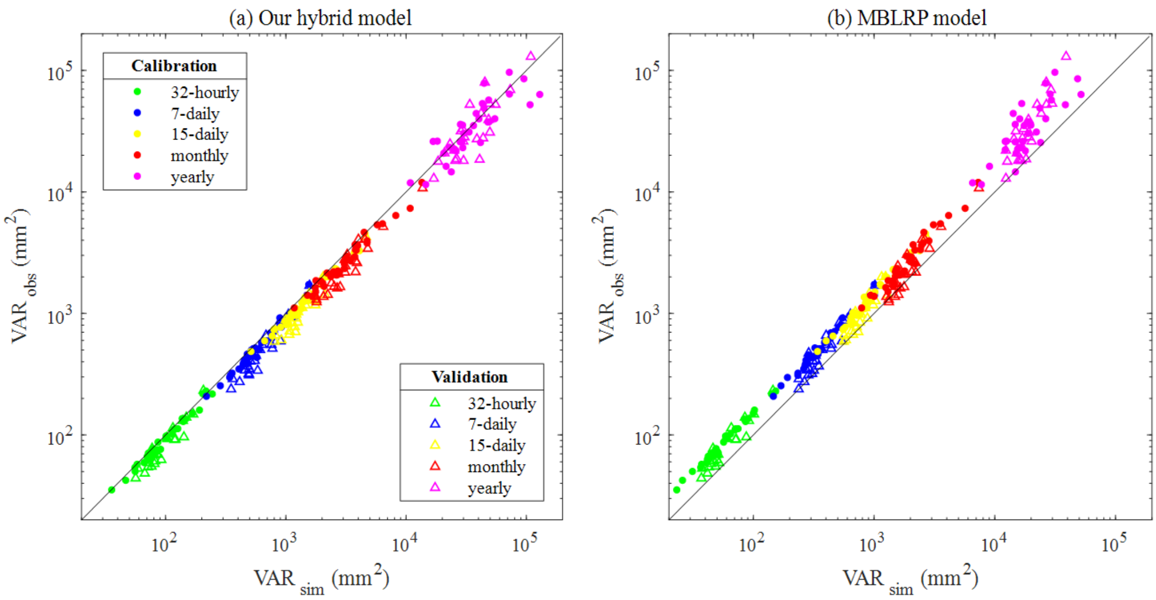
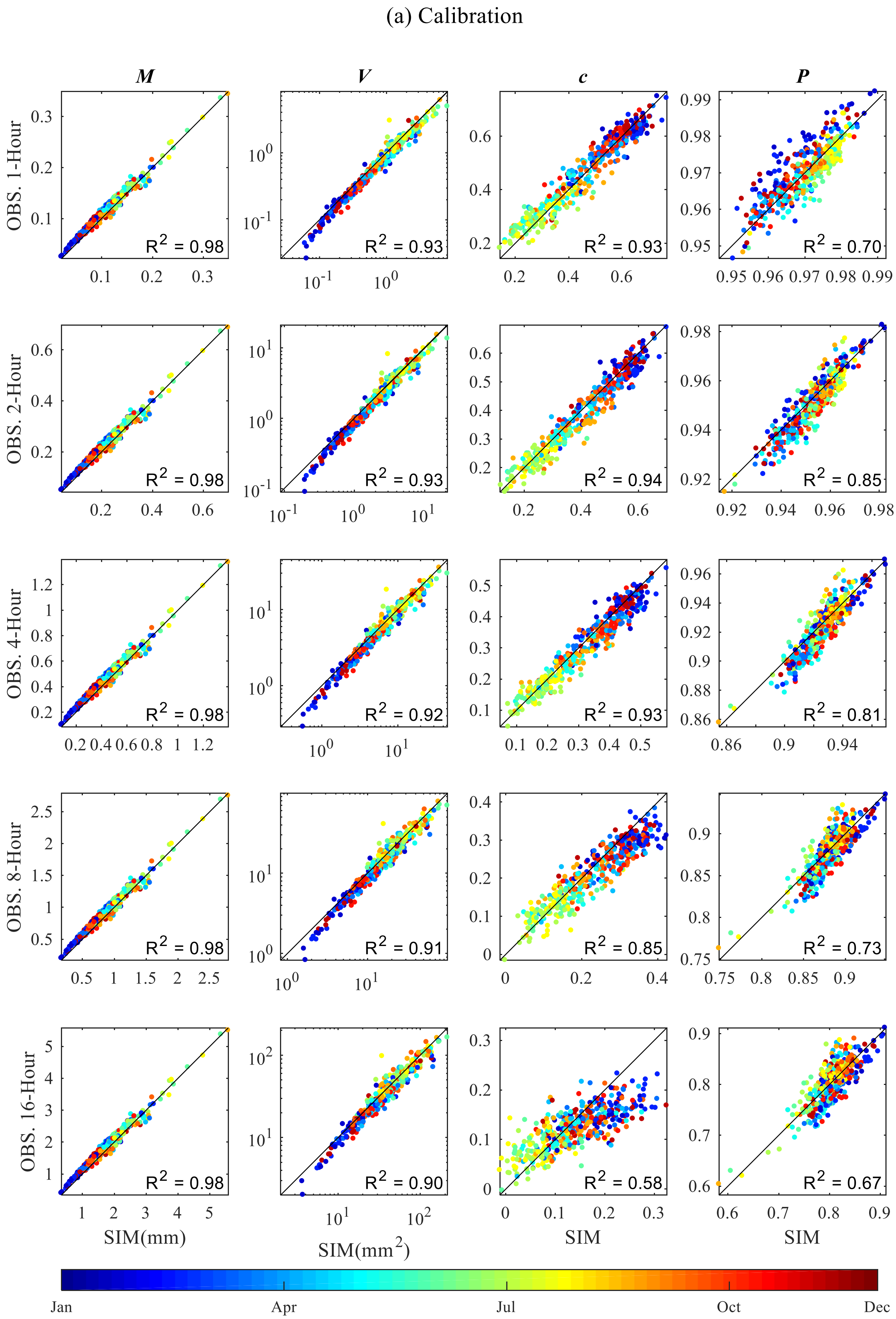


Figure 11: (a) Comparison of the large scale rainfall variance of the rainfall generated by our hybrid model (x) and the observed rainfall (y); (b) Comparison of the large scale rainfall variance of the rainfall generated by the traditional MBLRP model (x) and the observed rainfall (y). The different colours of the scatter correspond to the different aggregation interval of rainfall time series. Filled circles and hollow triangles correspond to the calibration and validation period, respectively.

…Figure 11 compares the variance of the synthetic (x) and observed (y) rainfall time series at yearly (purple), monthly (red), 15-daily (yellow), weekly (blue), and 32-hourly (green) aggregation levels. The comparison of the variance at the finer time scale is carried out in the following section.

As indicated by the concentration of the scatters above the 1:1 line in Figure 11b, the traditional MBLRP model systematically underestimates the variability at time scales greater than 32 hours. Our model did not show any bias in this range of large time-scales as shown in Figure 11a.


(a) Calibration
M
V
c
P
OBS. 1-Hour
OBS. 2-Hour
OBS. 4-Hour
OBS. 8-Hour
OBS. 16-Hour
R2 = 0.98
R2 = 0.93
R2 = 0.93
R2 = 0.70
R2 = 0.98
R2 = 0.93
R2 = 0.94
R2 = 0.85
R2 = 0.98
R2 = 0.92
R2 = 0.93
R2 = 0.81
R2 = 0.98
R2 = 0.91
R2 = 0.85
R2 = 0.73
R2 = 0.98
R2 = 0.90
R2 = 0.58
R2 = 0.67
SIM(mm)
SIM(mm2)
SIM
SIM
Jan
Apr
Jul
Oct
Dec

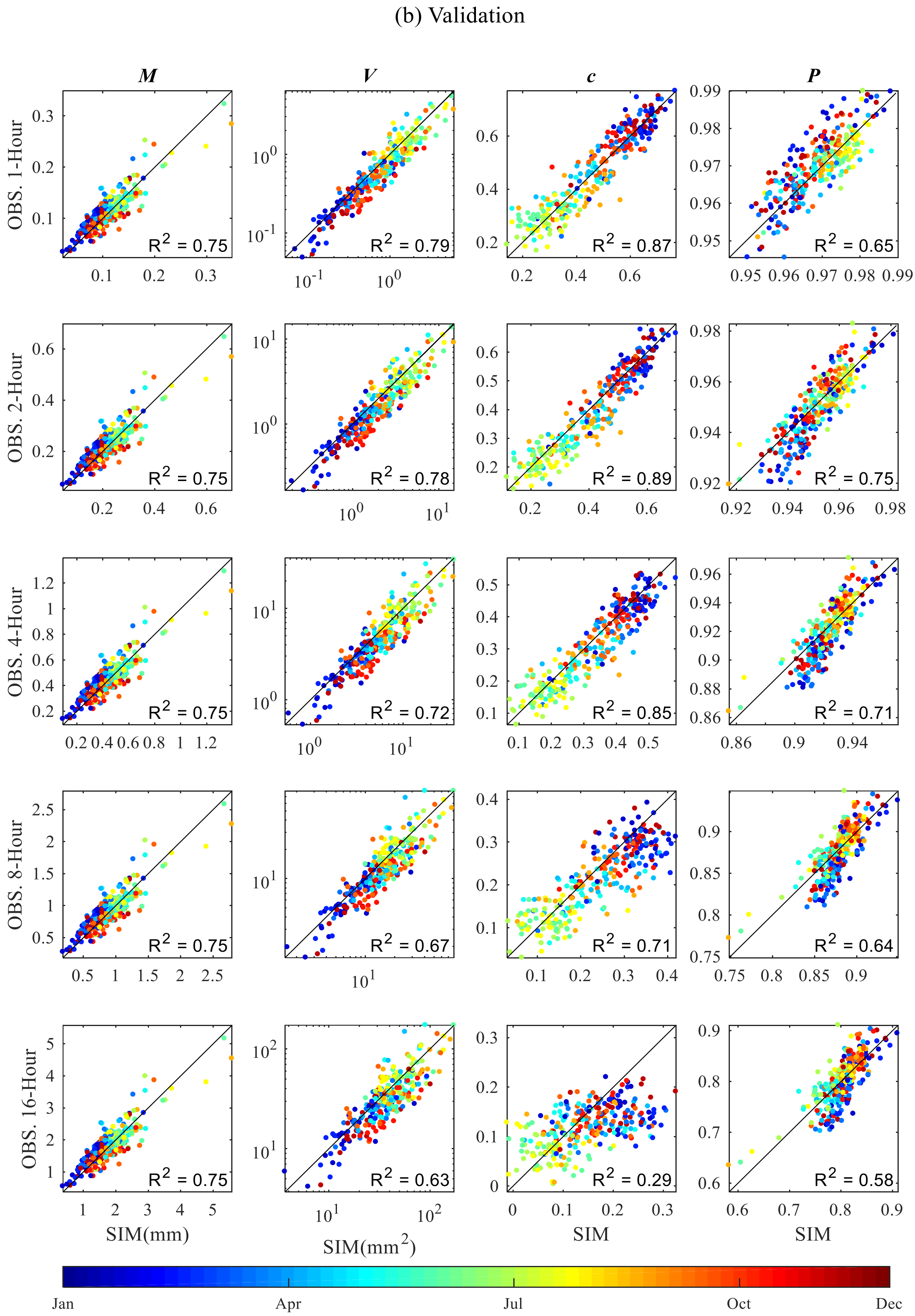


Figure 12: Comparison of the statistics of the synthetic (x) and observed (y) rainfall time series at sub-daily time scale. The colour of the dots represents the statistics of each calendar month. The results of (a) the calibration period (1981-2010) and (b) the validation period (1951-1980) are shown.

4.3 Reproduction of Sub-Daily Rainfall Statistics

Figure 12 compares the mean, variance, lag-1 autocorrelation, and the proportion of dry periods of the synthetic (x) and observed (y) rainfall time-series at hourly through 16 hourly aggregation levels. Each scatter represents the statistics at a given gage of a given calendar month. The colour of the scatters represents the calendar months. In each plot, the coefficient of determination (R^2) of the linear regression between the two variables is shown. All four statistics were accurately reproduced across various sub-daily time scales with R^2 equal to 0.98 (mean), and varying between the following limits for the other statistics: 0.90 and 0.93 (variance), 0.58 and 0.93 (lag-1 autocorrelation), and 0.67 and 0.85 (proportion of dry periods) on the calibration period (Figure 12a). The similar degree of coefficient of determinations were obtained for the validation period (Figure 12b).

5.3 Calibration versus validation

Our approach of separating the period of calibration and validation adopted to some gage locations, at one viewpoint, is paradoxical because most stochastic rainfall generators are calibrated based on the statistics reflecting the temporal stationarity of rainfall process. According to this assumption, the statistics of the period of calibration and the validation should be the same, which obviates the needs of validating the model for separate periods. However, this assumption is often broken, for example, in case that the observation period is too short (e.g. a few extreme events are included in only one part of the time period) or in case that the time series is indeed non-stationary. For this reason, the discrepancy of the model performance between the calibration and the validation period not only attributed to the model's limitation but also to the discrepancy of the statistics between the two periods. In this context, the term "validation" may be substituted into the term "prediction."

Taken these, our primary purpose of separating the period of calibration and validation was to figure out the applicability of our model for rainfall generation of future period: From the view point of calibration period, validation period is ungagged period just as future period, and our model can be easily extended to the future period by adding a term accounting for long-term rainfall non-stationarity to the SARIMA model (first module). Our hybrid model assumes not only the stationarity of the typical rainfall statistics such as mean, variance, covariance and proportion of dry periods but also the relationship between them (See Figure 6). The latter has not been explicitly discussed by previous studies, so it will be very interesting to see whether such relationship between the statistics holds over different temporal periods and how the discrepancy affects the final model performance, if there is any. Figure 19 compares the slope of the regression analysis between the statistics shown in Figure 6 for the calibration (x) and validation (y) period. The plots corresponding to the variances at different scales are not shown because they have a firm analytical background to have solid slope close to 2 (See

Equation 5 and the preceding equations). There is not a significant discrepancy between slopes estimated using statistics on calibration and validation period implying that relationships between the fine time scale statistics are stationary and that our model can be used for future or ungagged periods.

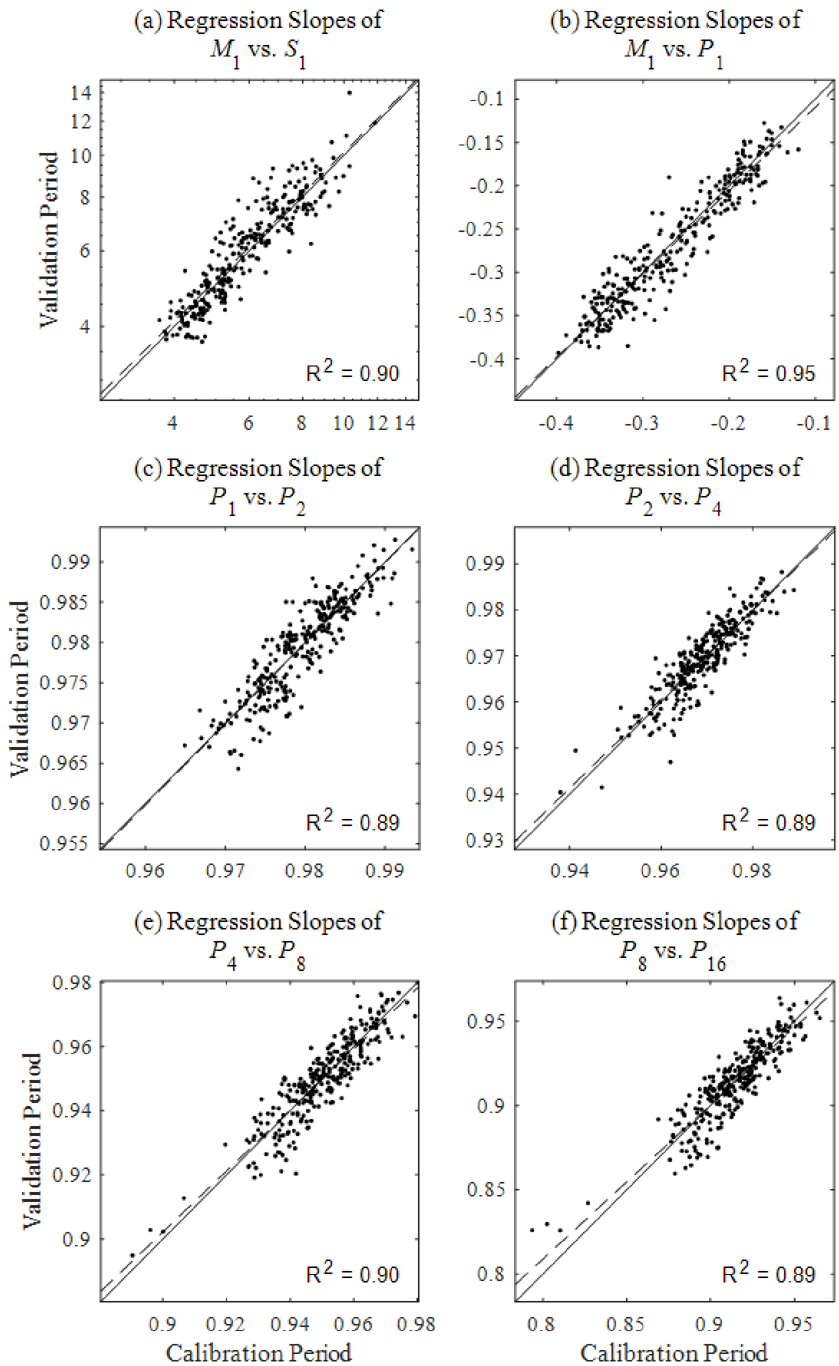


Figure 19: Comparison of the slope of regression analysis between the statistics shown in Figure 6 for the calibration (x) and validation (y) period. The slopes of regression analysis (a) between mean and standard deviation and (b) between mean and proportion of dry periods and (c)-(f) between proportion of dry periods at the different time scale were compared. Solid lines are 1:1 line and dashed lines represent the regression lines.

Comment 7. In the MBLRP model, cell durations are represented by a double stochastic process which produces a long-tail matching observation as stated at line 14. However, rainfall intensities are also known to be of sub-exponential nature (Wilson and Toumi 2005). While MBLRP uses an exponential distribution for rainfall intensities, the method proposed here appear to improve the performance in reproducing rainfall extremes. You state that this results on rainfall extremes (better performance compared to MBLRP) is 'surprising': However, this is likely due to the inter-annual variation of the parameters as you also discuss in section 5.1. On this effect, see Zorzetto et al (2016), where it is shown that the interannual variation of exponential-type rainfall distributions, as is the case with your findings, can explain the emergence of fattailed extreme value distributions. The fact that your model generates time series with improved extreme value properties is very appealing, as this is a traditional shortcoming of Poisson-cells based rainfall models.

Authors' Response. This is the best compliment commented on this article. Indeed, the heavier tail could be reproduced after introducing stochasticity to the parameters of the exponential distribution representing the cell intensity. Please see the following figure that we added to the revised manuscript.

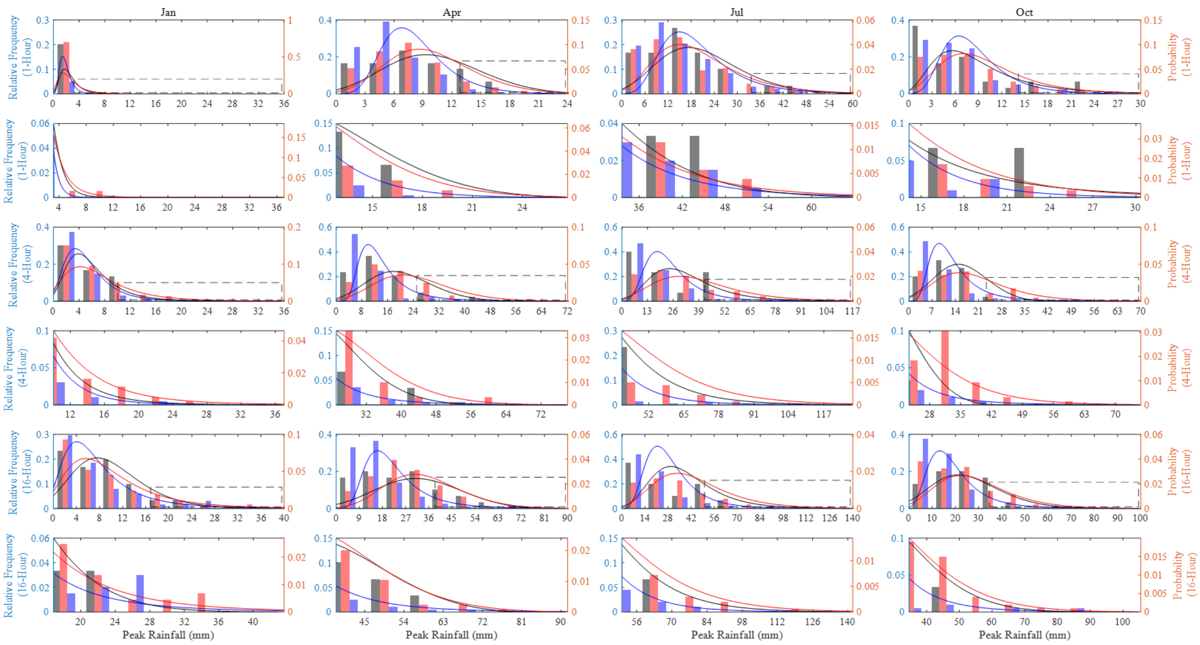


Figure 15. Relative frequency and the fitted GEV distribution of the 1, 4, and 16-hour monthly maxima of January, April, July, and October rainfall at the gage NCDC-132203. Results of Observed rainfall (black), our hybrid model (red), and the traditional MBLRP model (blue) are shown. The upper 10 percentile part of the distribution (dashed box in the plots in the first, third, and fifth row) is magnified in the lower rows (plots in the second, fourth, and sixth row).

In addition, we added the following sentences in the manuscript:

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Zorzetto et al. (2016) also briefly discussed this matter. They introduced a novel framework of meta-

statistical extreme value (MEV) analysis. In this MEV formulation, one can show that interannual-variation of exponential-type rainfall process leads to fat-tail of its extreme value.

Reference. Zorzetto, E., Botter, G. and Marani, M.: On the emergence of rainfall extremes from ordinary events, Geophys. Res. Lett., 43, 8076-8082, 2016.

Minor Comments

1) Page 4, line 29 – Rain gauge with ID 85663, please correct

Authors' Response. Done as suggested.

2) Page 5, line 2 – 'the discrepancy between their quartiles/ their range..'. Please also specify in the caption what the whiskers of the boxplots are (quartiles?)

Authors' Response. Done as suggested.

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Figure 2: Box plots of the observed monthly rainfall at the gage NCDC-85663 in Florida, USA (red). The box plots of the synthetic monthly rainfall generated by the Modified Bartlett-Lewis Rectangular Pulse model at the same gage are shown for reference (blue). Whiskers reach to minimum and maximum values of monthly rainfall during the period between 1961 and 2010 and gray shaded boxes represent the discrepancy of the variability of the two monthly rainfalls.

3) The sentence at lines 27-28 is not clear, please clarify. The variance is not represented by the vertical axis in Figure 2, even though the boxplots do give an idea of the variability of the seasonal monthly rainfall distributions.

Authors' Response. Done as suggested.

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In Figure 2, the red box plots represent the distribution of the monthly rainfall observed at the gage NCDC-85663 located in Florida, USA during the period between 1961 and 2010.

4) Figure 3: please state in the legend what the values in the map are (SARIMA parameters) and what the coloring is, even if it is already explained in the text.

Authors' Response. Done as suggested.

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Figure 3: Study area and 34 NCDC hourly rainfall gages. The label of the markers is presented in the following format: aaaaaa(i,j,k)(x,y,z)$_{12}$, where aaaaaa represents the NCDC gage ID, (i, j,k) represent the orders of the autoregressive, differencing, and moving average terms of SARIMA model, and (x,y,z) represent the orders of the seasonal autoregressive, differencing, and moving average terms of SARIMA model. Colour of the markers represent the Bayesian Information Criterion (BIC) value of the SARIMA model. The lower BIC indicates either parsimonious parameterization, larger likelihood, or both. Model description of SARIMA is detailed in Section 3.1.

5) Page 22, line 10: 'It is important TO NOTE THAT..' or something on this line.

Authors' Response. This line was already pointed from RC2. It was changed as follows:

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A good rainfall model should reproduce not only the extreme values but also the distribution of the maxima from which extreme values are derived.

6) Page 26, line 1: 'and the subsequent EFFECTS ON human..'

Authors' Response. Done as suggested.

7) Page 26, line 3; 'time scaleS'

Authors' Response. Done as suggested.

Dear Dr. Nadav Peleg,

We sincerely appreciate your constructive comments on our manuscript. All your comments tremendously helped us to improve the quality of the article. Our response is as follow:

Comment 1. Is the rainfall generator capable of simulating the diurnal cycle of precipitation? I didn't find a mention to it in the text.

Authors' Response. We appreciate this comment. This rainfall generator was not designed to consider the diurnal cycle of precipitation. Diurnal pattern of rainfall could be integrated by introducing separate model parameter sets to for day and night period, but at the expense of model accuracy and parsimony. Therefore, we would like not to discuss the topic of rainfall diurnal pattern in this article.

Comment 2. You mentioned that ". . . Poisson cluster rainfall models are designed to reflect the original spatial structure of rain storms containing multiple rain cells" [page 3 line 5], which implies that the rainfall generator can be used in a multisite context. Yet, the examples and discussion are given for single sites. Can the model be used to simulate hourly rainfall at multi-sites? And if so, can you please provide some examples to the ability of the model to preserve the spatial structure of the rainfall?

Authors' Response. The model is applicable only for a single site at the current stage, and we believe that the extension of the model to multiple sites may be treated in a separate research. As a matter of fact, there have been efforts to extend the point process models to generate spatially distributed rainfall. The first kind adopted the concept of spatial and temporal correlation (Rodriguez-Itrube et al., 1986), and the second kind adopts adopted the spatial concept in the stage of rain storm and cell generation (Cowpertwait, 1995). However, none of this research can account for the complex dynamics of storm movement. We believe that the recent models assuming movement of structured rain cells (Paschalis et al., 2013; Peleg and Morin, 2014) represent a considerable improvement, and that further research in this direction may ultimately resolve the problem of space-time rainfall generation in the near future.

Reference. Cowpertwait, P. S.: A generalized spatial-temporal model of rainfall based on a clustered point process, Proc.R.Soc.Lond.A, 450, 163-175, 1995.

Paschalis, A., Molnar, P., Fatichi, S. and Burlando, P.: A stochastic model for high-resolution space-time precipitation simulation, Water Resour. Res., 49, 8400-8417, 2013.

Peleg, N. and Morin, E.: Stochastic convective rain-field simulation using a high-resolution synoptically conditioned weather generator (HiReS-WG), Water Resour. Res., 50, 2124-2139, 2014.

Rodriguez-Iturbe, I., Cox, D. R. and Eagleson, P.: Spatial modelling of total storm rainfall, Proc.R.Soc.Lond.A, 403, 27-50, 1986.

Comment 3. In page 4, line 4, it is mentioned that ". . .Poisson cluster rainfall models have also been used to generate future rainfall scenario under climate change". How can the model be re-set to simulate rainfall for a future period following the methodology you are suggesting here?

Authors' Response. Thank you for your suggestion. The concept of Change Factor (CF) may be incorporated to extend our model for future rainfall generation. Here, the change factor represents the ratio of the future to the current rainfall depth, which are typically derived from the information drawn from the General Circulation Model result. Fatichi et al. (2011) also adopted this approach. This change factor may be applied between the first and the second module of our methodology.

Reference. Fatichi, S., Ivanov, V. Y. and Caporali, E.: Simulation of future climate scenarios with a weather generator, Adv. Water Resour., 34, 448-467, 2011.

Comment 4. You state that "Our hybrid model is not easy to implement because it requires extensive analysis of the correlation structure of the fine-scale rainfall statistics to fine-tune the MBLRP model to downscale the generated monthly rainfall" [page 26 line 20]. Can you give some further information in this direction? e.g. what is the minimum number of years that are required for a proper analysis of the statistics? How many gauges are required to cover a given area?

Authors' Response. Regarding the number of gauges, our model requires the records from one gauge to generate the rainfall time series at one gauge. It is one-to-one relationship. Regarding the minimum rainfall record length, we performed additional analyses. However, we would like to keep this information internally because a more extensive analysis targeting every model component is necessary to obtain a comprehensive answer, which may be addressed in a separate article.

Here, we provide a guideline on the length of the rainfall record that is required for successful implementation of the model. The key step of our model that is influenced by the rainfall record length is the algorithm to generate the monthly rainfall statistics to be used for model parameter estimation (Figure 5). This step is composed of a total of 14 linear regressions between various short-term rainfall statistics, and the reliability of these 14 linear regressions influences the subsequent steps, which eventually determines the overall model performance. The relationships between M_1 and V_1 and between M_1 and P_1 are especially important because they impose the greatest uncertainty among all 14 regressions (compare the data spreads of Fig.6a,b and those of the remaining plots of Fig. 6). For this reason, we investigated the sensitivity of the slope of these regression analyses to the rainfall record length. Figure SC1-1 summarizes the result. For both plots, the x-axis represents the rainfall record length, and the y-axis represents the relative error of the regression slope values. Here, the relative error was calculated through the following procedure: (a) Calculate the regression slope at one gauge based on all 30 years of rainfall record (1981-2010). This value was regarded as the true value; (b) Remove the record of the oldest year from the original rainfall data (1982-2010) and calculate the regression slope; (c) Calculate the absolute value of the difference between the regression slope of (a) and (b); (d) Repeat the process (b) and (c) for all 29 gauges and calculate the 5- (lower red line in Figure SC1-1), 50- (middle black line in Figure SC1-1), and 95-percentile (upper red line in Figure SC1-1) value of the 29 absolute values of regression slope difference; (e) Repeat the process (b) through (d) by subsequently removing additional one year of data (1983-2010, 1984-2010, 1985-2010, … , 2000-2010). Fig. SC1-1a and b corresponds to the regression analysis between M_1 and V_1, and M_1 and P_1. The result suggests that systematic bias is introduced in the regression slope values with the decrease in the size of the data set. When the record length was reduced from 30 years to 20 years, 4 to 8 percent, and 3 to 5 percent of systematic bias are introduced into the estimates of the slope of regression relating M_1 and V_1 and the one relating M_1 and P_1 respectively. The plot can be also used for quantifying the systematic bias corresponding to different record lengths.

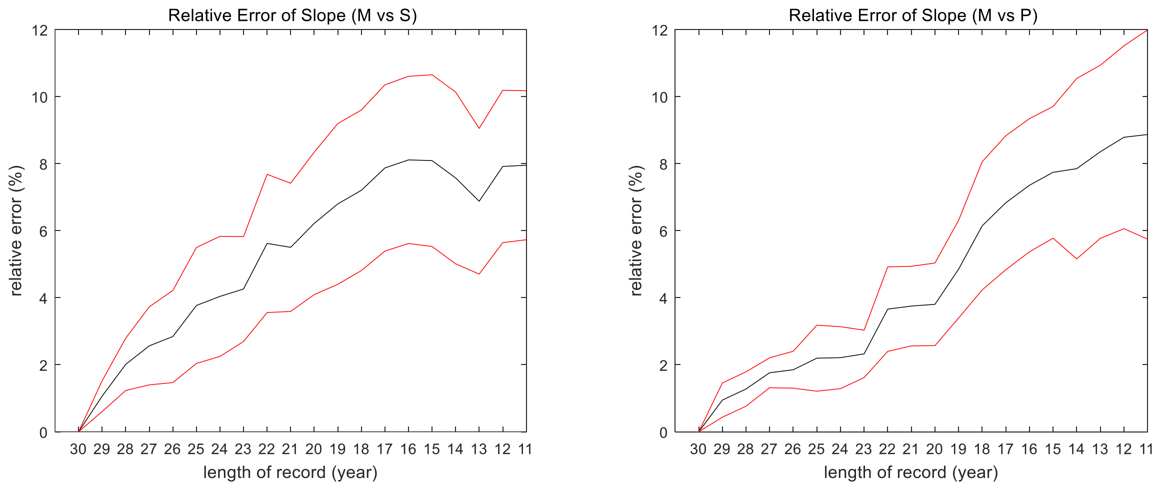

Figure SC1-1. The 5- (lower red line), 50- (middle black line), and 95- (upper red line) percentile of relative error of the regression slope values relating M1 and V1, and M1 and P1, respectively, that vary with the record length. The values can be regarded as the systematic bias introduced in the regression analysis as the record length decreases.

Comment 5. A short comment from my side – you mention in the introduction the importance of rainfall at small scales. The paper here focused on the hourly scale, and you mention in the conclusion that the model can be (with implementing some methods, as random cascades) also used to simulate rainfall at sub-hourly scales. It is maybe worth mentioning in this context that there are already several gridded sub-hourly rainfall generator models that are presented in the literature (e.g. Paschalis et al. 2013 – WRR; Peleg and Morin 2014 – WRR; Peleg et al. 2017 – JAMES; Benoit et al. 2018 - WRR) and to briefly discuss the advantages/limitations of using the tool you are suggesting for simulating rainfall at fine scales in comparison to the others.

Authors' Response. Thank you for your suggestion. We changed the manuscript as below.

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[(old) page 26 line 18 / (new) page 30 line 5] … *Southern Oscillation (ENSO) and North Atlantic Oscillation (NAO). Lastly, the genuine structure of our model that is composed of a large scale rainfall generation module and a downscaling module, may be integrated to existing space-time rainfall generators to enhance their ability to generate large temporal-scale rainfall variability (Burton et al., 2008, Müller and Haberlandt, 2015, Paschalis et al., 2013; Peleg and Morin, 2014; Peleg et al., 2017; Benoit et al., 2018).*

Reference. Benoit, L., Allard, D. and Mariethoz, G.: Stochastic Rainfall Modelling at Sub-Kilometer Scale, Water Resour. Res., 2018.

Burton, A., Kilsby, C., Fowler, H., Cowpertwait, P. and O'Connell, P.: RainSim: A spatial–temporal stochastic rainfall modelling system, Environmental Modelling & Software, 23, 1356-1369, 2008.

Müller, H. and Haberlandt, U.: Temporal rainfall disaggregation with a cascade model: from single-station disaggregation to spatial rainfall, J. Hydrol. Eng., 20, 04015026, 2015.

Paschalis, A., Molnar, P., Fatichi, S. and Burlando, P.: A stochastic model for high-resolution space-time precipitation simulation, Water Resour. Res., 49, 8400-8417, 2013.

Peleg, N. and Morin, E.: Stochastic convective rain-field simulation using a high-resolution synoptically conditioned weather generator (HiReS-WG), Water Resour. Res., 50, 2124-2139, 2014.

Peleg, N., Fatichi, S., Paschalis, A., Molnar, P. and Burlando, P.: An advanced stochastic weather generator for simulating 2-D high-resolution climate variables, Journal of Advances in Modeling Earth Systems, 9, 1595-1627, 2017.

Dear Dr. Hannes Müller,

We sincerely appreciate your constructive comments on our manuscript. All your comments tremendously helped us to improve the quality of the article. We have prepared the following responses to your comments:

Comment 1. The rainfall model generates hourly rainfall time series. Are there any investigations/plans to extend it to e.g. 5 min resolution (instead of applying cascade models as suggested in the outlook)? Do you see any restrictions or arising problems, if the method would be extended to a finer temporal scale?

Authors' Response. Yes, we do have a plan, and we invited you for the co-work in a very near future. I see no arising technical problems.

Comment 2. To enable comparisons with other rainfall generators, maybe the authors want to spend a few words on the total number of parameters required to generate the hourly rainfall time series? E.g. for the MBLRP-module you use 6 parameters per set and one set per month, resulting in 72 parameters for module 3. What is the total number of parameters?

Authors' Response. Thank you for your suggestion. We added the following sentence according to your suggestion:

3.3 MBLRP Model Parameter Estimation

4.3. An Issue with Model Parsimoniousness: six versus fifty five

Our hybrid model uses one MBLRP model parameter set per one simulation year while the MBLRP model needs only 6 parameters regardless of the simulation length. However, this does not mean that our model requires 600 MBLRP model parameters (6 per month x 100 months) to generate 100 months of rainfall. This is because parameters are estimated based on the short-term rainfall statistics that are synthetically generated through the process of the SARIMA model and the regression analysis (See Figure 5). Therefore, the parameters of the SARIMA model and the parameters of the regression analyses shown in Figure 5 should be considered as the "true" parameters of this model because once these parameters are given, our model can generate infinite length of rainfall record. The SARIMA model has 6 parameters, and a set of regression analyses shown in Figure 5 has 49 parameters (2 per ten solid arrows in Figure 5 = 20, 3 per 8 bivariate normal distributions relating two subsequent residual terms (ε_i) in Figure 5 = 24, and one per 5 normal distributions perturbing autocorrelation terms (c_i) =

5). Therefore, our model has a total of 55 parameters. This discrepancy of number of parameters (6 of the traditional of MBLRP model versus 55 of our hybrid model) can be considered as a cost taken to reproduce the large-scale rainfall variability that the traditional MBLRP model cannot.

We admit that this large discrepancy of model parsimoniousness is an issue to be resolved for our model to be applied in practice. Regarding this, we are planning to apply our model to additional gauge locations across the world and share the result through the website (http://www.letitrain.info). The work has been already initiated for the rainfall data of Korean Peninsula.

Comment 3. P15l18-20 ". . .the MLRP model that reflects the original spatial structure of rainfall in reality, . . ." What is the connection between the point statistics mentioned before and the spatial structure? Is it planned to extend the introduced model to be able to generate spatial rainfall? Even if not, maybe the authors want to include in their outlook, how this can be achieved, for example i) during the rainfall generation by e.g. the circle approach with the radius of single pulses or their velocities as additional parameters (Cox and Isham, 1988, Bordoy and Burlando, 2014) or ii) subsequently be a resampling approach (Müller and Haberlandt, 2015)? Where do the authors see opportunities/limitations?

Authors' Response. Thank you for your suggestion. That sentence you mentioned in our article can indeed confuse to many readers because Poisson cluster model mentioned in this study is purely a single-site model. We added a sentence in the conclusion according to your suggestion as follow:

An additional model could be integrated to our hybrid model to incorporate further rainfall variability. For example, an approach based on random cascades (Molnar and Burlando, 2005; Müller and Haberlandt, 2016; Pohle et al., 2018) can be integrated to our model for reproducing the rainfall variability at the time scale as fine as five minutes. In addition, the SARIMA model that was adopted in this study could be further modified to account for the coarser rainfall variability caused by El Niño-Southern Oscillation (ENSO) and North Atlantic Oscillation (NAO). Lastly, the genuine algorithm of our model of sequential generation and disaggregation may be integrated to improve the performance of existing space-time rainfall generators (Burton et al., 2008, Müller and Haberlandt, 2015, Paschalis et al., 2013; Peleg and Morin, 2014; Peleg et al., 2017; Benoit et al., 2018).

Technical notes 1. Fig. 13: For the observations extreme values with return periods of 200 yrs are shown for all stations, taken from a fitted GEV distribution. The observation length is only 30 yrs (1981-2010) and it can be questioned if the comparison of extrapolated values for 200 yrs return periods is reliable from a statistical point of view. I would rather limit the comparison to 50 yrs or 100 yrs.

Authors' Response. While we agree with the reliability issue of the extrapolated values, we still think that presenting the 200-year value may be necessary because it can clearly show the limitation of our model and the traditional MBLRP model.

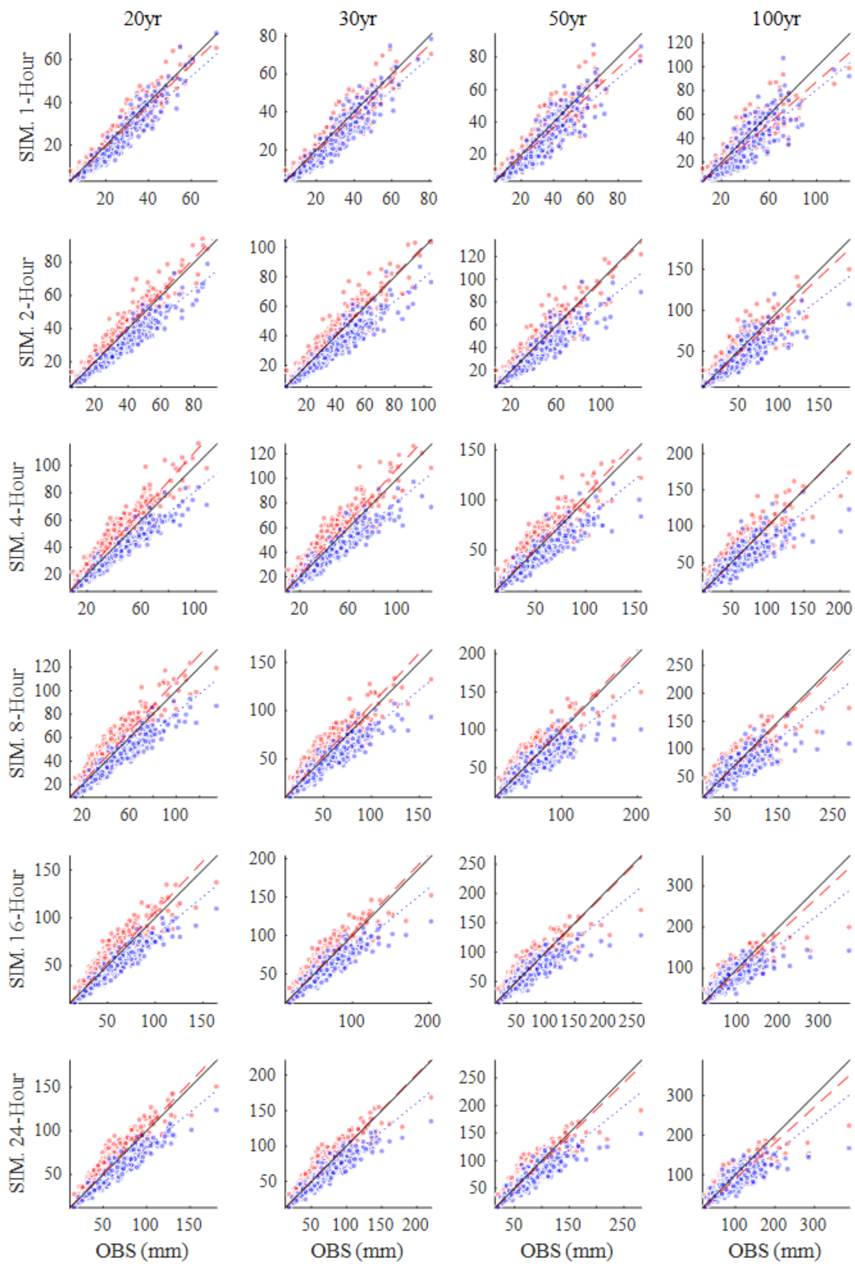


Fig. 13

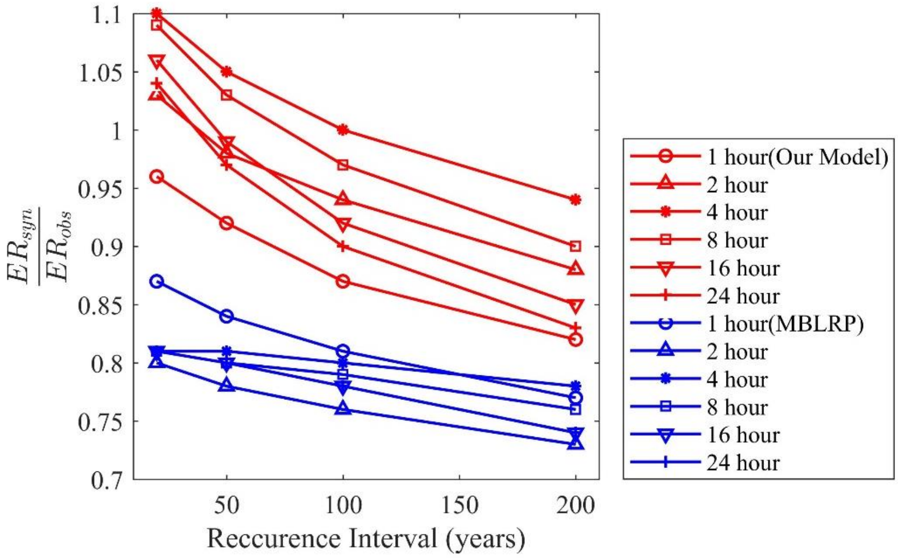


Fig. 14

Technical notes 2. Fig. 14 (caption): I think the model colors have been swapped by mistake: ’our model’ should be red, ’MBLRP’ should be blue.

Authors’ Response. Your comment is correct. We changed the caption of the figures.

Figure 14: Degree of over/underestimation of extreme values by our model (red) and the traditional MBLRP model (blue).

A Hybrid Stochastic Rainfall Model That Reproduces Rainfall Characteristics at Hourly through Yearly Time Scale

Jeongha Park[1], Christian Onof[2], and Dongkyun Kim[1]

[1]Department of Civil Engineering, Hongik University, Seoul, 04066, Republic of Korea
[2]Department of Civil and Environmental Engineering, Imperial College, London, SW7 2AZ, UK

Correspondence to: Dongkyun Kim (kim.dongkyun@hongik.ac.kr)

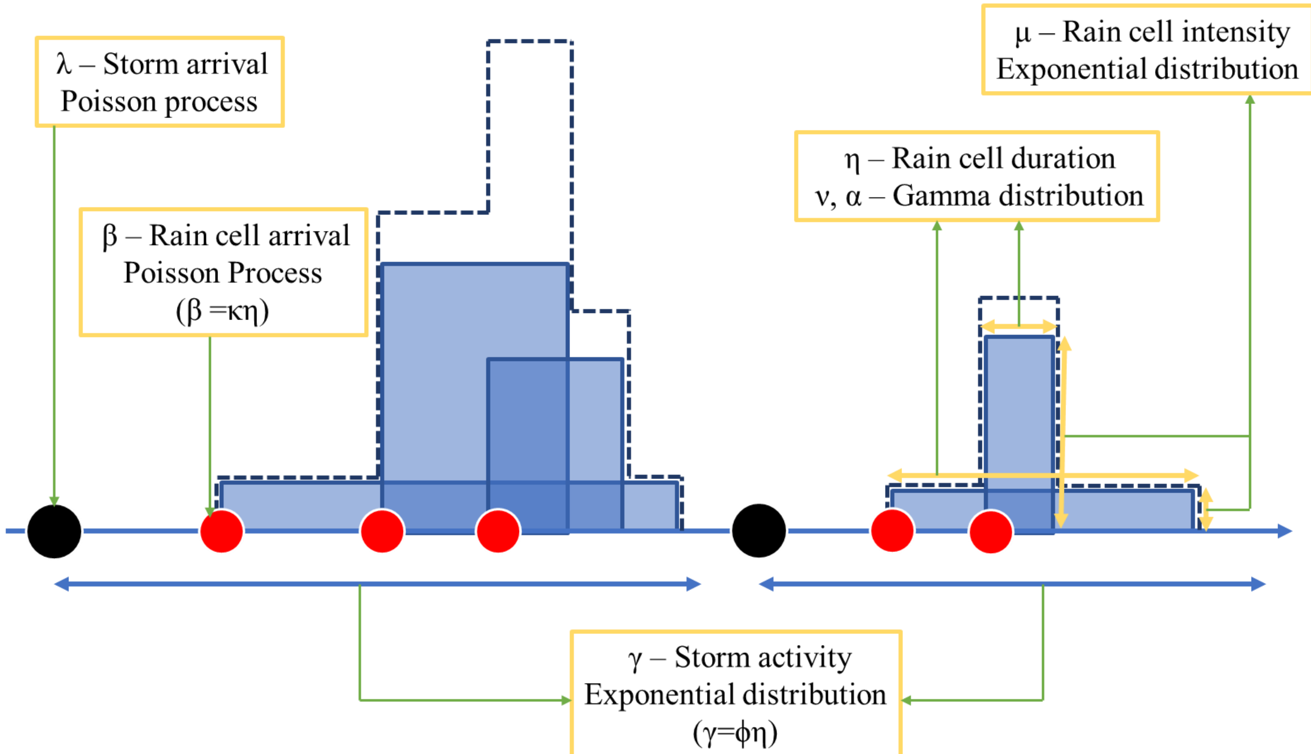
Abstract. A novel approach ~~of~~to stochastic rainfall generation that can reproduce various statistical characteristics of observed rainfall at hourly through yearly time scale is presented. The model uses the Seasonal Auto-Regressive Integrated Moving Average (SARIMA) model to generate monthly rainfall. Then, it downscales the generated monthly rainfall to the hourly aggregation level using the Modified Bartlett-Lewis Rectangular Pulse (MBLRP) model, a type of Poisson cluster rainfall model. Here, the MBLRP model is ~~fine-tuned~~carefully calibrated such that it can reproduce the ~~fine-scale~~sub-daily statistical properties of observed rainfall. This was achieved by first generating a set of fine scale rainfall statistics reflecting the complex correlation structure between rainfall mean, variance, auto-covariance, and proportion of dry periods, and then coupling it to the generated monthly rainfall, which were used as the basis of the MBLRP ~~parameters to downscale monthly rainfall.~~parameterization. The approach was tested ~~at the 29 gauges~~on 34 gages located in the Midwest to the East Coast of the Continental United States with a variety of rainfall characteristics. The results of the test suggest that our hybrid model accurately reproduces the first through the third order statistics as well as the intermittency properties from the hourly to the annual time scale; and the statistical behaviour of monthly maxima and extreme values of the observed rainfall ~~was well~~were reproduced~~ as~~ well.

1 Introduction and Background

Most human and natural systems affected by rainfall react sensitively to temporal variability of rainfall across small (e.g. quarter-hourly) through large (e.g. monthly, yearly) time ~~scale~~scales. Small scale rainfall temporal variability influences short-term watershed responses such as flash flood (Reed et al., 2007) and subsequent transport of sediments (Ogston et al., 2000) and contaminants (Zonta et al., 2005). Large scale rainfall temporal variability influences long-term resilience of human-flood systems (Yu et al., 2017), human health (Patz et al., 2005), food production (Shisanya et al., 2011), and evolution of human society (Warner and Afifi, 2014) and ecosystems (Borgogno et al., 2007; Fernandez-Illescas and Rodriguez-Iturbe, 2004). While the risk exerted by these impacts needs to be precisely assessed for the management of ~~the~~such systems, the observed rainfall record is oftentimes not long enough (Koutsoyiannis and Onof, 2001). Furthermore, the rainfall records do not exist when the risks need to be assessed for the future. For this reason, stochastic rainfall generators, which can ~~generate~~create synthetic rainfall record with infinite length, have been frequently used to provide the rainfall input data to the modelling studies for the risk assessment.

The Poisson cluster rainfall generation model (Rodriguez-Iturbe et al, 1987; 1988) is one of the most widely applied stochastic rainfall generators. Figure 1 shows a schematic of the Modified Bartlett-Lewis Rectangular Pulse (MBLRP) model, which is a typical Poisson cluster rainfall model. The model assumes that a series of rain storms (black circles) comprising a sequence of rain cells (red circles), arrives in time according to a Poisson process. Kim et al. (2013a) summarized the MBLRP model structure as follow: The MBLRP model has six parameters of which brief description is provided in the lower text box of Figure 1.

"In the MBLRP model, X1 [T] is a random variable that represents the storm arrival time, which is governed by a Poisson process with parameter λ [1/T]; X2 [T] is a random variable that represents the duration of storm activity (i.e., the time window after the beginning of the storm within which rain cells can arrive), which varies according to an exponential distribution with parameter γ [1/T]; X3 [T] is a random variable that represents the rain cell arrival time within the duration of storm activity, which is governed by a Poisson process with parameter β [1/T]; X4 [T] is a random variable that represents the duration of the rain cells. The distribution of the rain cell durations is known to have a long-tailed distribution (Rodriguez-Iturbe et. al., 1987), which was assumed to vary according to an exponential distribution with parameter η [1/T] that, in turn, is a random variable represented by a gamma distribution with parameters ν [T] and α [dimensionless]; and X5 [L/T] is a random variable that represents the rain cell intensity, which varies according to an exponential distribution with parameter μ [L/T]. From the physical viewpoint, λ is the expected number of storms that arrive in a given period, 1/γ is the expected duration of storm activity, β is the expected number of rain cells that arrive within the duration of storm activity, 1/η is the expected duration of rain cells and μ is the average rain cell intensity. Parameters ν and α do not have a clear physical meaning, but the expected value and variance of η can be expressed as α/ν and α/ν2. Therefore, the model has six parameters: λ, γ, β, ν, α and μ; however, it is customary to use the dimensionless ratios φ = γ/η and κ = β/η as parameters instead of γ and β."



- β – random variable that serves as parameter for the Poisson process of cell arrivals ($\beta = \kappa\eta$)
- γ- random variable that serves as parameter for the storm activity exponential distribution ($\gamma = \phi\eta$)
- μ – mean of the exponential distribution of cell intensity
- λ – parameter of the Poisson process of storm arrivals
- α and $\frac{1}{\nu}$ – shape and scale parameters of the Gamma distribution for η
- η – random variable that serves as parameter for the cell duration exponential distribution

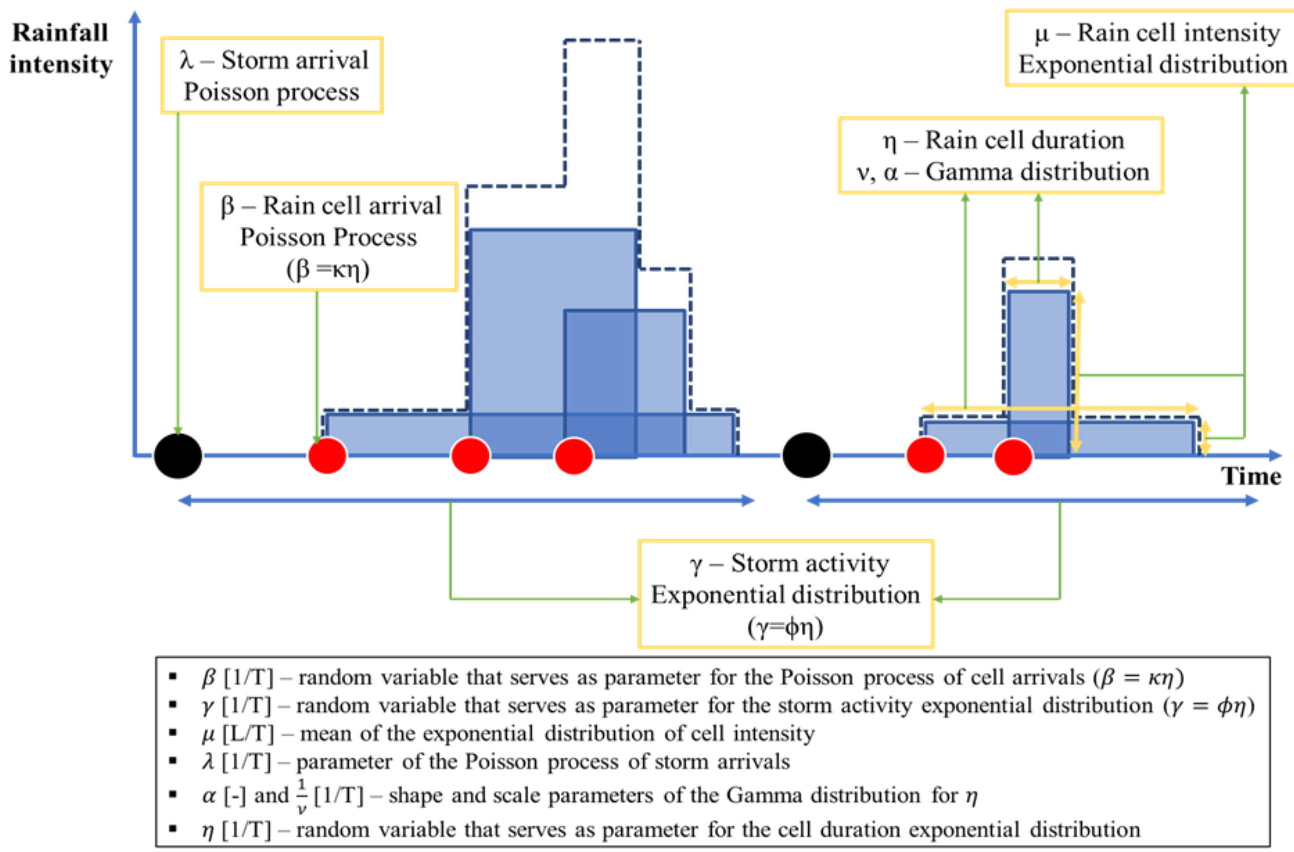


Figure 1: Schematic of the Modified Bartlett-Lewis Rectangular Pulse Model. **The blue area represents duration (width) and intensity (height) of each rain cell, respectively. The dashed line represents superposed sum of the rain cell intensities.**

As suggested by the figure, Poisson cluster rainfall models are designed to reflect the original spatial structure of rain storms containing multiple rain cells (Austin and Houze Jr., 1972; Olsson and Burlando, 2002), so they are good at reproducing the first through the third order statistics of the observed rainfall at quarter-hourly through daily accumulation levels, as well as other hydrologically important statistics such as proportion of non-rainy period (Olsson and Burlando, 2002). The performance of the Poisson cluster rainfall models in reproducing the statistical properties of observed rainfall has been validated for various climates at numerous locations across the globe (Bo et al., 1994; Cameron et al., 2000; Cowpertwait, 1991; Cowpertwait et al., 2007; Derzekos et al., 2005; Entekhabi et al., 1989; Glasbey et al., 1995; Gyasi-Agyei and Willgoose; 1997, Gyasi-Agyei; 1999, Islam et al.; 1990, Kaczmarska et al., 2014; Khaliq and Cunnane, 1996; Kim et al., 2016; Kim et al., 2013b; Kim et al., 2014; Kossieris et al., 2015; Kossieris et al., 2016; Onof and Wheater, 1994a; Onof and Wheater, 1994b; Onof and Wheater, 1993; Rodriguez-Iturbe et al., 1988; Rodriguez-Iturbe et al., 1987; Smithers et al., 2002; Velghe et al., 1994; Verhoest et al., 1997; Wasko et al., 2015~~).~~; Ritschel et al., 2017). For this reason, they have been widely applied to assess the risks exerted on human and natural systems such as floods (Paschalis et al., 2014), water availability (Faramarzi et al., 2009), contaminant transport (Solo-Gabriele, 1998), and landslides (Peres and Cancelliere, 2014; 2016~~).~~, Thomas et al., 2018). Recently, Poisson cluster rainfall models have also been used to generate future rainfall scenario under climate change (Kilsby et al., 2007; Burton et al., 2010; Fatichi et al., 2011).

In the meantime, Poisson cluster rainfall models have an intrinsic limitation derived from a fundamental model assumption. As described by Figure 1, they generate the rainfall time series assuming that the rain storms arrive according to a Poisson process, which assumes that rain storm occurrences are independent. In addition, the rain cells in different storms are independent with each other. These model assumptions deprive the model of the ability to reproduce the long-term memory of rainfall that is often observed in reality (Marani, 2003).

Let us introduce some notation. The aggregated process $Y^{(h)}$ at time-scale h hours is defined in terms of the continuous time process Y by the equation:

$$Y_i^{(h)} = \int_{(i-1)h}^{ih} Y(t)dt$$

We can then write the variance at time-scale nh as:

$$V_{nh} = Var\left(Y^{(nh)}\right) \sout{= nVar\left(Y^{(h)}\right) + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} Cov\left(Y_i^{(h)}, Y_j^{(h)}\right)}$$

~~So~~

$$\sout{V_{nh} = nV_h + 2\sum_{k=1}^{n-1} C_h(k) \qquad (1)}$$

$$= \sum_{i=1}^{n} Cov\left(Y_i^{(h)}, Y_i^{(h)}\right) + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} Cov\left(Y_i^{(h)}, Y_j^{(h)}\right)$$

$$= nVar\left(Y^{(h)}\right) + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} Cov\left(Y_i^{(h)}, Y_j^{(h)}\right)$$

Since $Cov\left(Y_i^{(h)}, Y_j^{(h)}\right) = Cov\left(Y_j^{(h)}, Y_i^{(h)}\right)$

$$V_{nh} = nVar\left(Y^{(h)}\right) + 2\sum_{i=1}^{n}\sum_{j=1,j>i}^{n} Cov\left(Y_i^{(h)}, Y_j^{(h)}\right) \qquad (1)$$

, where V_h is the variance of rainfall depths at scale h hours and ~~$C_h(k)$~~ $Cov(\cdot, \cdot)$ is the covariance ~~of lag k at scale h hours~~operator between the two random variables.

The second term of the right-hand side of Equation 1, which represents the rainfall correlation between individual records separated by ~~k~~$(i-j)$ time-steps of the time series of rainfall depths at scale h hours, is likely to be underestimated by the Poisson cluster rainfall model because it can only reproduce short-term memory in the rainfall signal through its model structure, i.e. through the clustering of rain cells. The degree of underestimation will increase as the correlation between the individual records ($Y_i^{(h)}$) of the observed rainfall time series increases and as the aggregation level n increases. This underestimation was consistently observed in the rainfall data of the United States (Kim et al., 2013a). If $h = 1$ in Equation

1, i.e. hourly rainfall, and $n \cong 720$ (24hours/day x× 30 days = 720 hours ≅ 1 month), the left-hand side of Equation 1 will represent the variance of monthly rainfall, which can be represented on the vertical axis of the box plots in Figure 2.

In Figure 2, the red box plots represent the variabilitydistribution of the monthly rainfall observed at thegage NCDC rain gauge 85663 located in 85663, Florida, USA during the period between 1961 and 2010 The blue box plots represent the variability of the monthly rainfall estimated from the 50 years of hourly synthetic rainfall data generated by the Modified Bartlett-Lewis Rectangular Pulse (MBLRP) model, a type of Poisson cluster rainfall generators. Here, the MBLRP model used the parameter set that was calibrated to reproduce the observed rainfall mean, variance, lag-1 auto-covariance, and proportion of dry periods at sub-daily aggregation intervals (1, 2, 4, 8, and 16-hour), which is a typical practice of MBLRP model calibration (Rodriguez-Iturbe et al., 1987; Rodriguez-Iturbe et al., 1988; Kim et al., 2013a). Note that the vertical length of the red box plots are greater than that of the blue box plots in general, which implies that the variability of the observed rainfall is systematically greater than that of the synthetic rainfall. The discrepancy between the two are shown as the gray shading in the figure. In addition, the monthly extreme values shown as star marks are also underestimated by synthetic rainfall. This is, in particular, caused by the aforementioned limitations of the Poisson cluster rainfall models.

Considering that the management strategies of the water-prone human and natural systems may be governed by the few extreme rainfall values observed in the shaded domain of Figure 2, the risk analysis based on the rainfall data generated by Poisson cluster rainfall models may miss the system behaviour that is crucial for development of the management plans. As a matter of fact, other rainfall models have the similar issueissues: they cannot reproduce the temporal variability of observed rainfall across all time scales (Paschalis et al, 2014). For example, Markov chains, alternating renewal processes, and generalized linear models can reproduce the variability only at time scales coarser than one day. Models based on autoregressive properties of rainfall are typically good at reproducing the observed rainfall variability only for a limited range of scales, for instance from one month to a year or two (Mishra and Desai, 2005; Modarres and Ouarda, 2014; Yoo et al., 2016).

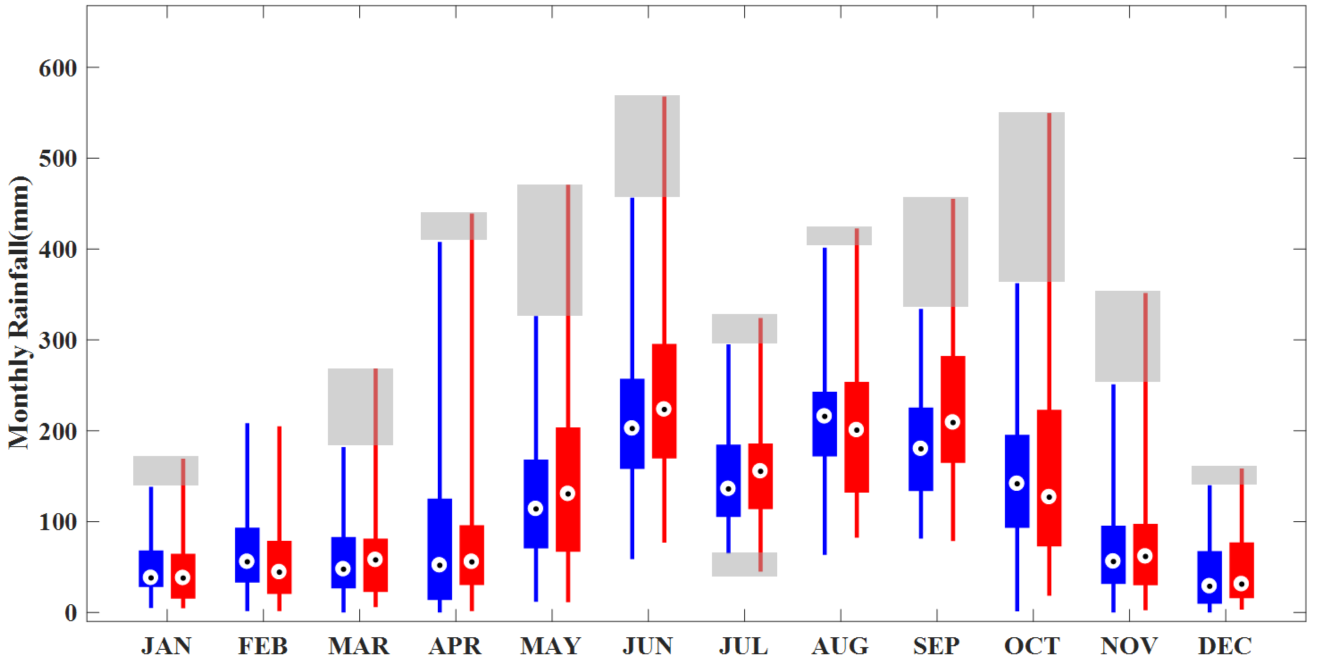


Figure 2: Box plots of the observed monthly rainfall at ~~the~~gage NCDC~~ Gauge -~~85663 in Florida, ~~US.~~USA (red). The box plots of the synthetic monthly rainfall generated by the Modified Bartlett-Lewis Rectangular Pulse model at the same ~~gauge are shown for reference (blue).~~gage are shown for reference (blue). Whiskers reach to minimum and maximum values of monthly rainfall during the period between 1961 and 2010 and gray shaded boxes represent the discrepancy of the variability of the two monthly rainfalls.

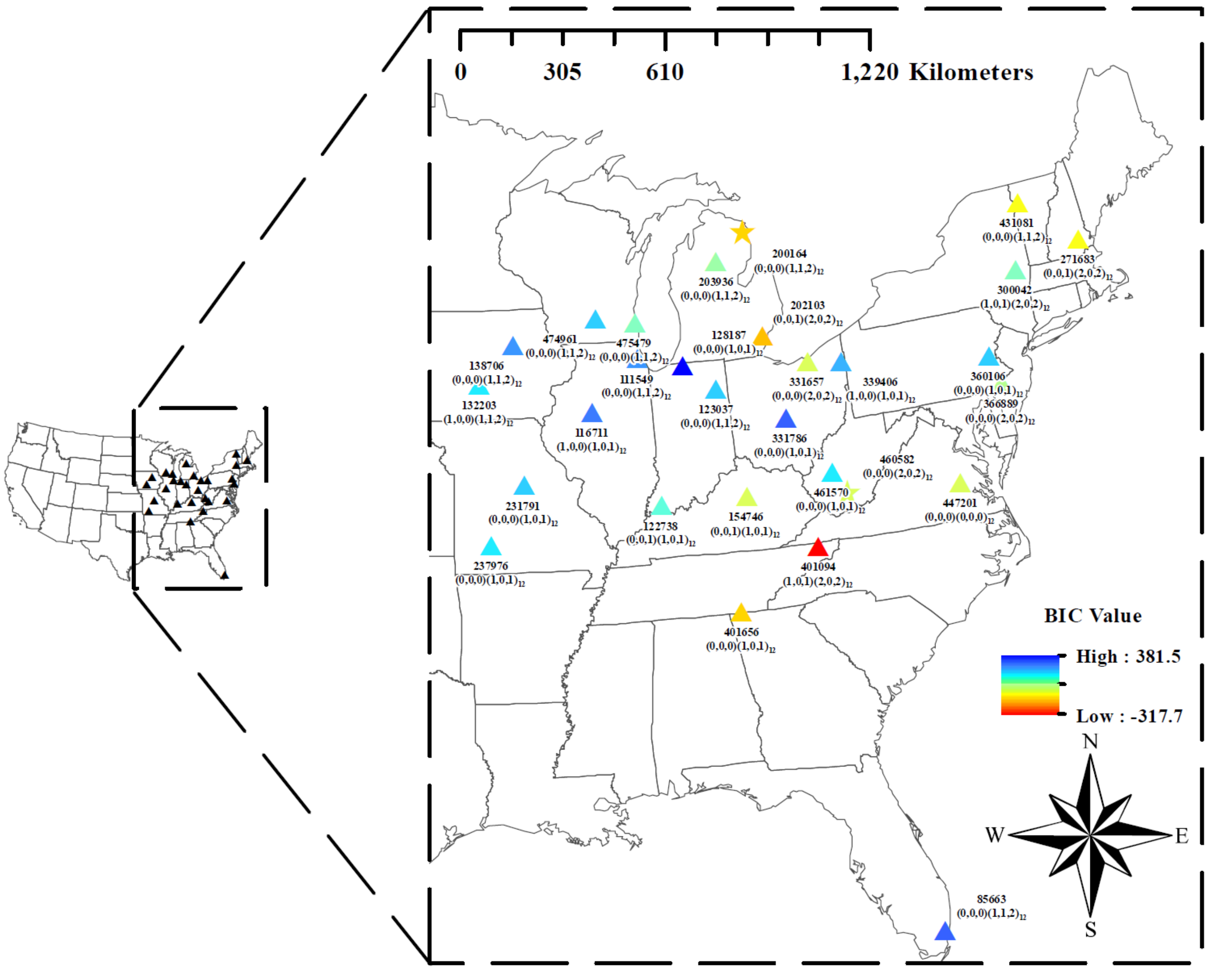
Several studies discussed the need to use composite rainfall models to resolve this scale problem of rainfall models. Koutsoyiannis (2001) used two seasonal autoregressive models with different temporal resolution to generate two different time series referring to the same hydrologic process. Then, they adjusted the fine scale time-series using their novel coupling algorithm so that this series becomes consistent with the coarser scale time series without affecting the second-order statistical properties. Menabde and Sivapalan (2000) combined the alternating renewal process with a multiplicative cascade model in which a multi-year rainfall time series generated by a Poisson process based model is disaggregated using a bounded random cascade model. Their model reproduced the observed scaling behaviour of extreme events very well up to 6 minutes of temporal resolution. Fatichi et al. (2011) developed a model that generates monthly rainfall using an autoregressive model and disaggregating the generated monthly rainfall using a Poisson cluster rainfall model. Their composite model showed improved performance in reproducing the rainfall interannual variability that the latter often fails to reproduce. Kim et al. (2013a) proposed a model where the Poisson cluster rainfall model is used to disaggregate the monthly rainfall that is randomly drawn from a Gamma distribution. They found that incorporating the observed rainfall interannual variability through their composite approach also helps reproduce the statistical behaviour of rainfall annual maxima and extreme values at time scales ranging from 1 to 24 hours. Paschalis et al. (2014) introduced a composite model consisting of a Poisson cluster rainfall model or

Markov chain model for large time scale and a multiplicative random cascade model for small time scale, which performed better than individual models across a wide range of scales at four different sites with distinct climatological characteristics.

This study proposes a composite rainfall generation model that can reproduce various statistical properties of observed rainfall at time scales ranging between one hour and one year. First, the model generates the monthly rainfall time series using the Seasonal Auto-Regressive Integrated Moving Average (SARIMA) model. Then, it downscales the generated monthly rainfall time series to the hourly aggregation level using a Poisson cluster rainfall model. Compared to the previous studies with similar methodology (Fatichi et al., 2011; Paschalis et al., 2014), our model has a novelty in that: (i) it models the monthly rainfalls so as to generate monthly statistics that will serve to calibrate the MLBRP model; (ii) each of the generated individual monthly rainfalls are downscaled using month-specific MLBRP model parameter sets that reflect the complex correlation structure of various rainfall statistics at fine time scale such as mean, variance, covariance, and proportion of dry periods. This distinctive approach of our model enables an accurate reproduction of the first through the third order statistics as well as the proportion of dry periods from the hourly to the annual time scale; and the statistical behaviour of monthly maxima and extreme values of the observed rainfall is well reproduced.

2 Study Area

Figure 3 shows the study area, which encompasses the Midwest to the East Coast of the Continental United States. We used the National Climatic Data Centre (NCDC) hourly rainfall data observed at ~~29 gauge~~34 gage locations (triangles in Figure 3) for the period between 1981 and 2010. The study area has a variety of rainfall characteristics (Kim et al., 2013b). The northern, middle, and the southern part of the study area are classified as Humid Continental (warm summer), Humid Continental (cool summer), and Humid Subtropical climate, respectively, according to the Köppen Climate Classification (Köppen, 1900; Kottek, 2006). The annual rainfall of the study area varies from 750 mm to 1500 mm.



~~**Figure 3: Study area and 29 NCDC hourly rainfall gauges.**~~

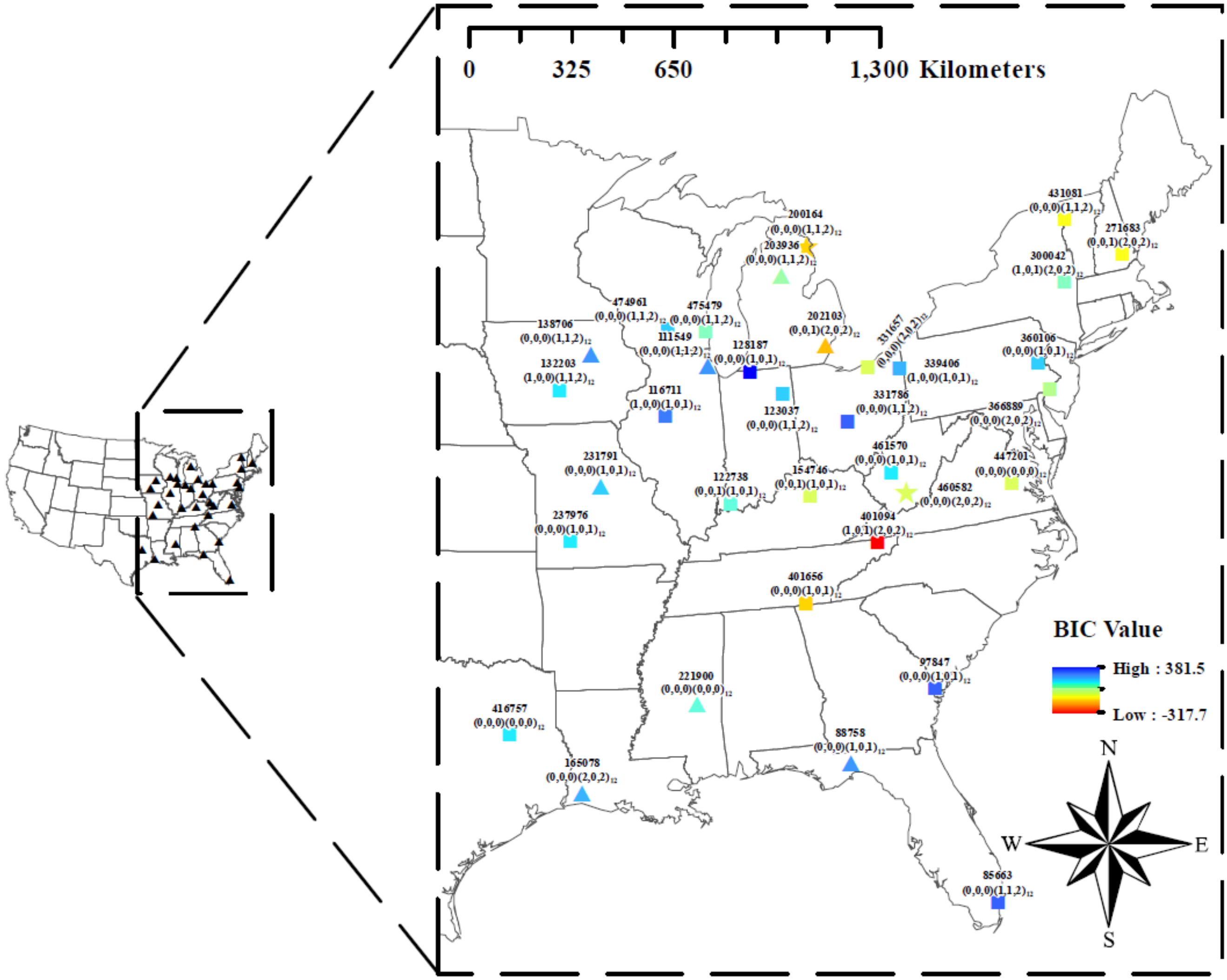
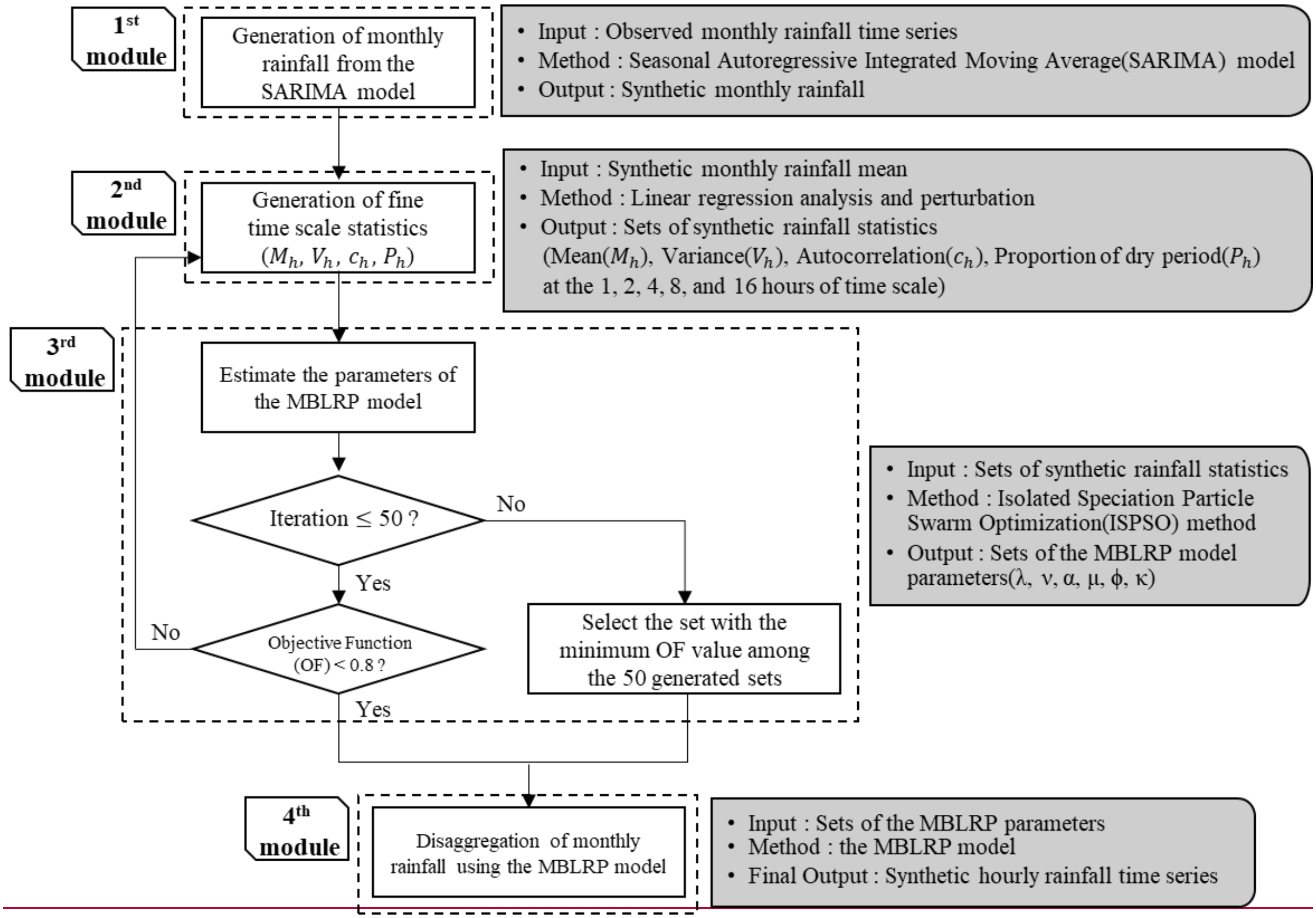


Figure 3: Study area and 34 NCDC hourly rainfall gages. The label of the markers is presented in the following format: aaaaaa(i,j,k)(x,y,z)$_{12}$, where aaaaaa represents the NCDC gage ID, (i, j,k) represent the orders of the autoregressive, differencing, and moving average terms of the SARIMA model, and (x,y,z) represent the orders of the seasonal autoregressive, differencing, and moving average terms of SARIMA model. The colour of the markers represent the Bayesian Information Criterion (BIC) value of the SARIMA model. The lower BIC indicates more parsimonious parameterization, larger likelihood, or both. Model description of SARIMA is detailed in Section 3.1.

3 Methodology

Figure 4 describes the model structure of this study. The model is composed of four distinct modules. The first module generates the monthly rainfall. The second module generates the fine-scale (1 hour through 16 hours) rainfall statistics corresponding to each of the generated monthly rainfall values in the first module. The third module estimates the parameters of the MBLRP model based on the fine-scale rainfall statistics generated by the second module. As a result of this process, each of the generated monthly rainfalls is coupled with the MBLRP parameter set reflecting its fine-scale statistical characteristics. The fourth module downscales each of the monthly rainfalls using the MBLRP model based on the parameters obtained in the third module.



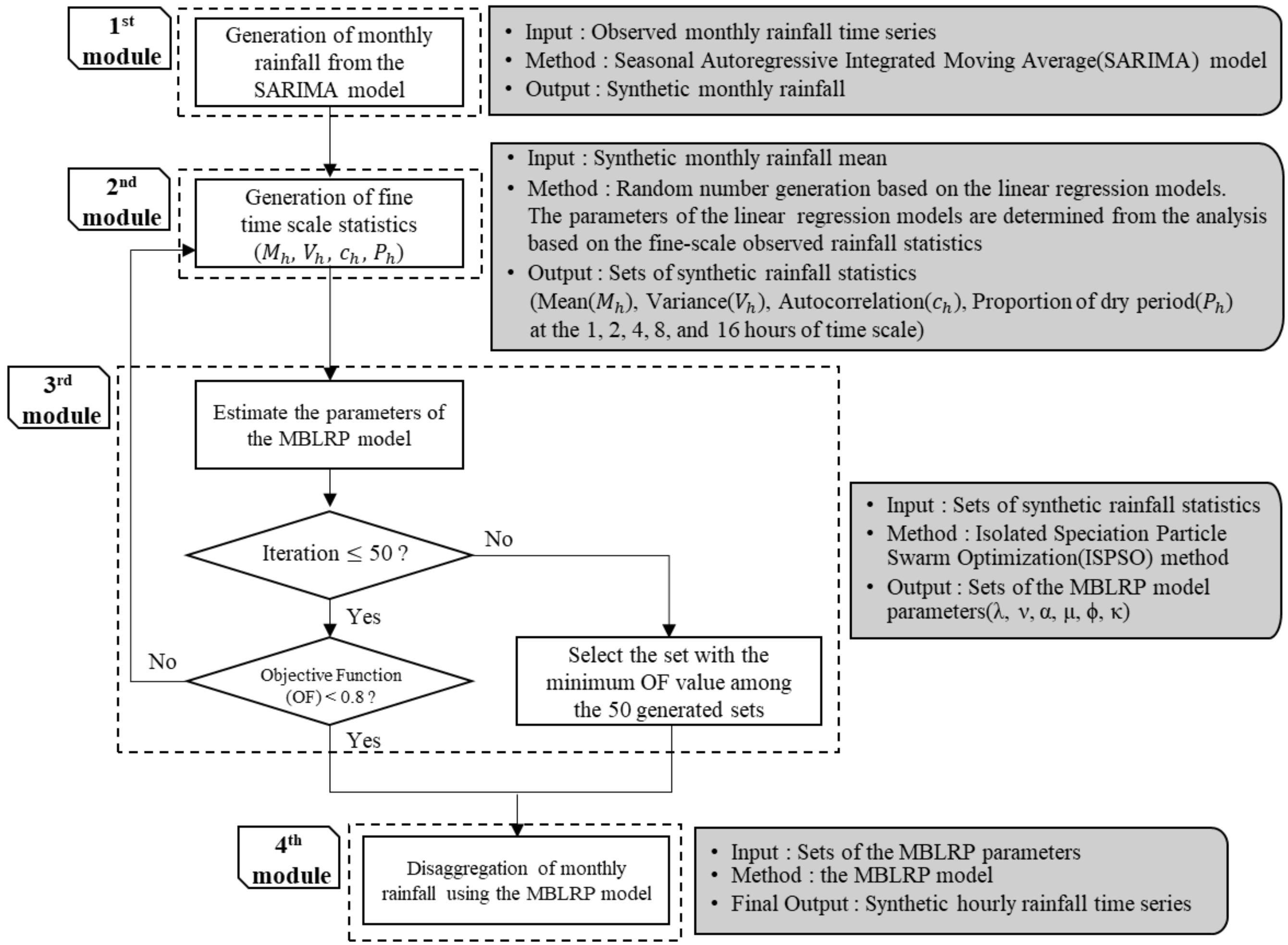


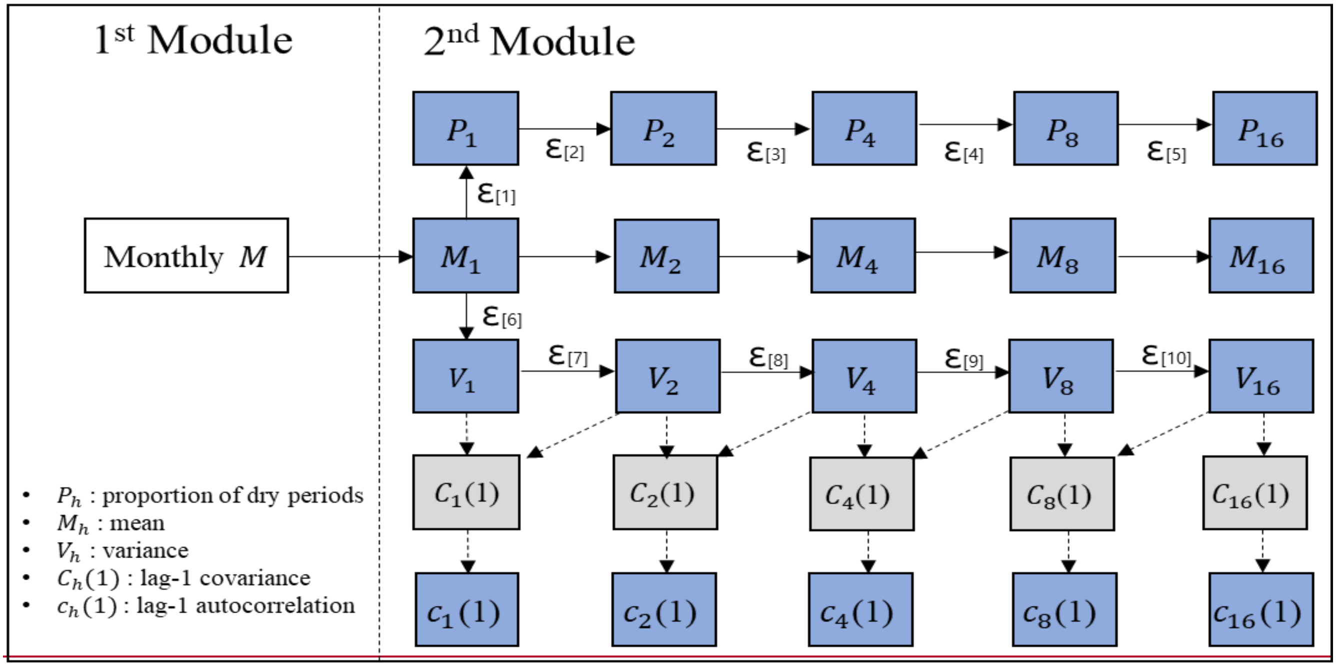
Figure 4: Four different modules of the model of this study

3.1 Monthly Rainfall Generation

We applied the Seasonal Auto-Regressive Integrated Moving Average (SARIMA) model to generate monthly rainfall. Generation of monthly rainfall based on Autoregressive relationship has been widely applied due to its parsimonious nature (Mishra and Desai, 2005) and was proven to successfully reproduce the first through the third-order statistics of the observed rainfall at monthly time scale (Delleur and Kavvas, 1978; Katz and Skaggs, 1981; Ünal et al., 2004; Mishra and Desai, 2005). Rainfall data of different stations have different temporal persistence, so we applied the SARIMA model with different autoregressive(p), differencing(d), and moving average terms(q) to different stations. The choice of the optimal model for each station was determined through the following processes: First, a model structure of SARIMA(p, d, q)(P, D, Q)$_m$ is assumed, where P, D, Q represent the numbers of seasonal autoregressive, differencing, and moving average terms, respectively, and m represents the number of periods (here, months) in each season – here $m = 12$. Second, the parameters of the SARIMA model are determined through the method of maximum likelihood. Third, the the Bayesian Information Criterion (BIC) are calculated for the fitted SARIMA model. Lastly, the first to third steps are repeated for a combination of different values of p (0≤p≤2), d (0≤d≤2), q (0≤q≤2), P (0≤P≤2), D (0≤D≤2), and Q (0≤Q≤2), and the model structure with the lowest BIC is selected for the station. Therefore, a total of 729 (=3^6) SARIMA model structures were tested to obtain the optimal model for a station. The ~~optimal~~selected model structure and the BIC values were shown in Figure 3. Through this process, we generated 200 years of monthly rainfall for the ~~29 gauges~~34 gages.

3.2 Generation of fine time scale rainfall statistics

The second module generates the fine time scale ~~(1hour through 16 hours)~~ statistics corresponding to each monthly rainfall value generated through the SARIMA model. These synthetic fine time scale statistics will later be used for the calibration of the MBLRP model to downscale the monthly rainfall to the hourly level. In so doing we ~~are now considering~~first consider the monthly rainfall, when divided by the number of days in the month times 24, as providing us with an estimate of the mean hourly rainfall for that particular month. ~~The second module consists of univariate regressions and functional relations linking the~~ Then, this estimated mean hourly rainfall ~~to~~ is provided as the input variable of the ~~other~~ module that generates the statistics ~~that are required~~needed to fit the MBLRP model~~. With these statistics MLBRP model parameters are obtained and these will be used to disaggregate the generated monthly rainfall.~~, namely the mean, variance, auto-correlation coefficient, and the proportion of dry periods at 1-, 2-, 4-, 8-, and 16-hour aggregation intervals, as described in Figure 5~~ describes the~~ . In this process~~ of~~ , the module employs the information obtained from univariate regression analyses between the fine~~ time~~ -scale statistics of the observed rainfall (Figure 6) and the mathematical formulae relating rainfall ~~generation.~~variance and auto-covariance at different time scales (Equation 4) as explained below.



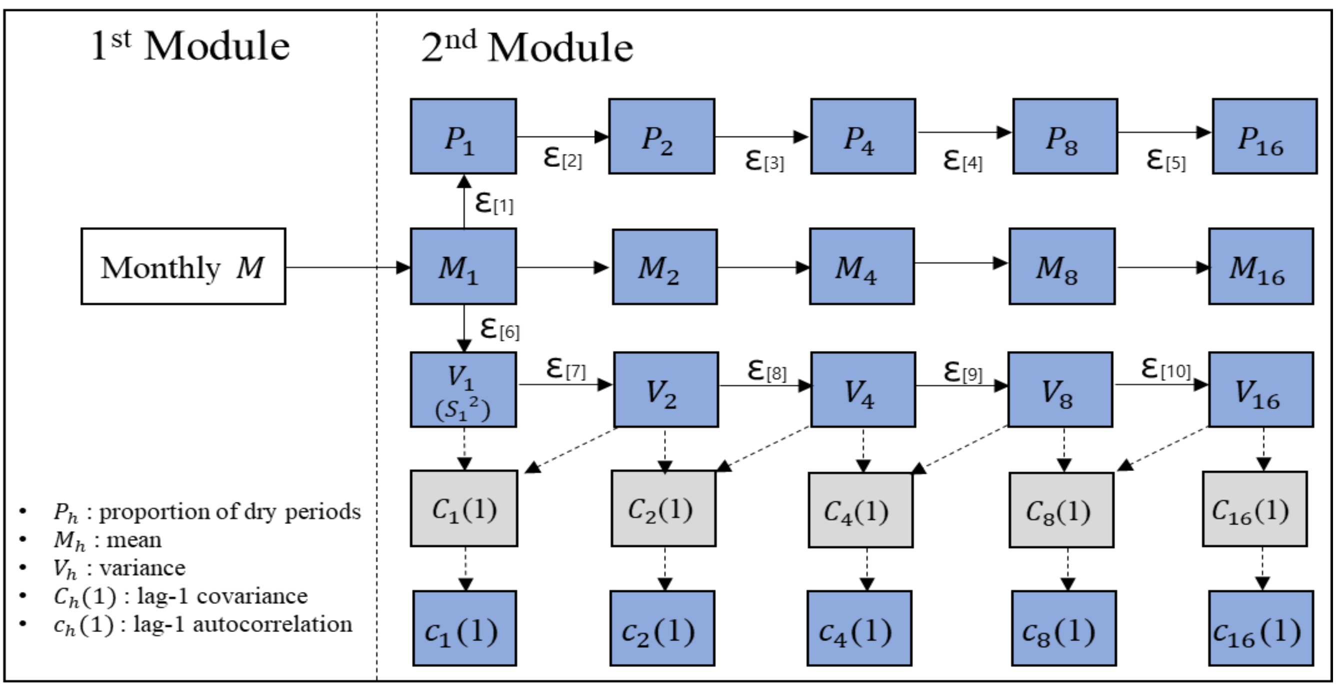


Figure 5: Schematic of the algorithm to generate fine time-scale rainfall statistics. The statistics in the blue boxes are used to calibrate the MBLRP model and the statistics in gray boxes are used to estimate the 1ag-1 autocorrelation.

Figure 5 shows a schematic of the second module. In the figure, M_h, S_h, ~~,~~ V_h, $c_h(1) = C_h(1)/V_h$ and P_h in each rectangle represent the rainfall mean, standard deviation, variance, lag-1 autocorrelation, and proportion of dry periods at time-scale h hours, respectively. The statistic connected to each solid arrow head is stochastically generated based on its linear relationship to the one connected to the tail of the same arrow. ~~The statistic connected to the dashed arrow head is calculated based on the ones connected to the tail of the same arrow using the mathematical (deterministic) relationship connecting these variables as we explain below.~~ In other words, the following equation is used:

~~Let ε represent the residual of the linear regression between the two statistics connected by an arrow. Consider, for example, statistic M_1 which is connected to V_1~~

$$Y = a_{[i]}\, X + b_{[i]} + \varepsilon_{[i]} \qquad (2)$$

where Y is the variable being generated, and the X is the variable being used as the basis of the generation. Here, the variable X and Y can be substituted by any combination of two variables connected by the solid arrow; $a_{[i]}$ and $b_{[i]}$ are the parameters of the regression analysis, and $\varepsilon_{[i]}$ is a random number drawn from the normal distribution $\varepsilon_{[i]} \sim N\left(0, \sigma^2_{[i]}\right)$ fitted to the residuals of the regression analysis. Here, these three parameters are estimated from the univariate regression analysis relating the two variables observed during a given calendar month over multiple years as shown by black scatters in each plot of Figure 6, which shows the linear relationship between the rainfall statistics observed at gage NCDC-200164 (star mark in Figure 3) during the month of July of different years.

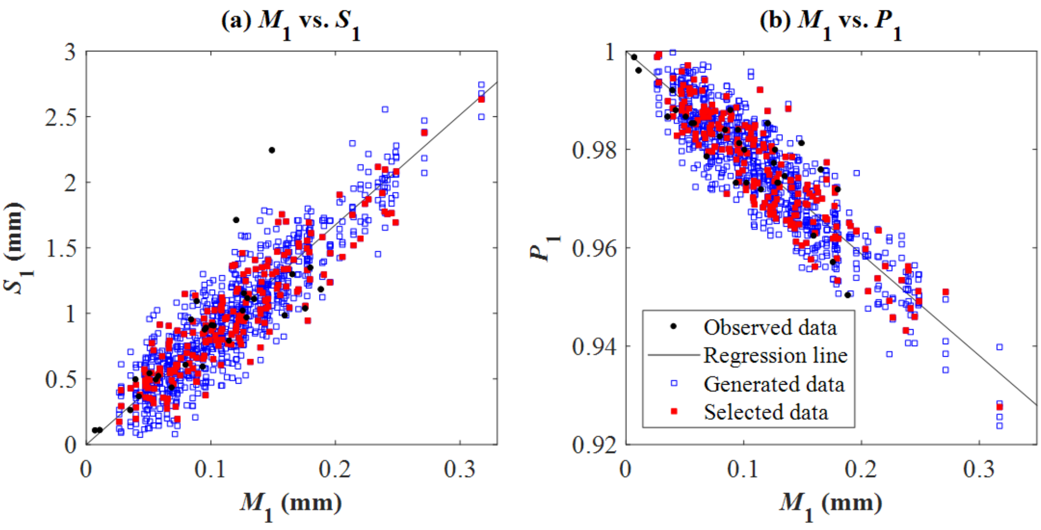
Consider, for example, statistic M_1 which is connected to $V_1 (= {S_1}^2)$ through the solid arrow in the figure, which means that the variance of one-hour rainfall ($V_1 = {S_1}^2$) is stochastically generated using its relationship to one-hour rainfall mean (M_1) (scatter of black dots in Figure 6a) using the following formula:

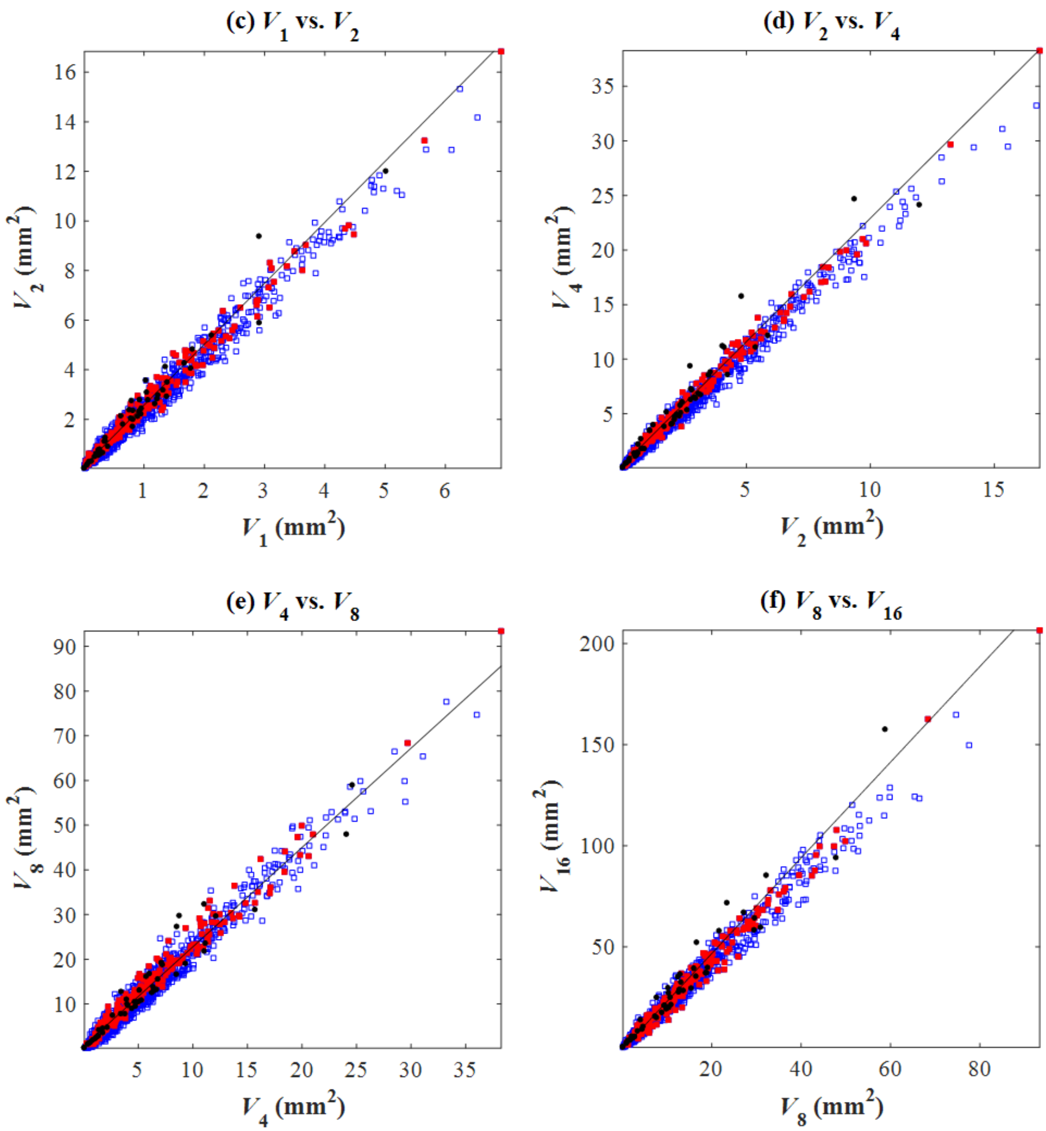
$$S_1 = a_{[6]}\, M_1 + \varepsilon_{[6]} \qquad (~~2~~3)$$

$$V_1 = {S_1}^2 \qquad (~~3~~4)$$

where subscripts with square brackets are used for the residuals so as to avoid confusion with the time-scale, and where $a_{[6]}$ is the coefficient determined from the regression analysis (note that the constant term is zero here since, trivially, $S_1 = 0$ when $M_1 = 0$), and $\varepsilon_{[6]}$ is ~~normally distributed: $\varepsilon_{[6]} \sim N(0, \sigma^2_{[6]})$. Note that M_1 which is the mean hourly rainfall for the month in question is just the monthly total obtained using the SARIMA model, divided by the number of hours in the month.~~ a random number drawn from a normal distribution: $\varepsilon_{[6]} \sim N(0, \sigma^2_{[6]})$.

Similar principles can be applied to the remaining statistics connected through solid arrows in Figure 5. The black scatters in Figure 6 shows the linear relationship between the rainfall statistics observed at ~~the~~gage NCDC~~-gauge ID -~~200164 (star mark in Figure 3) during the month of July of different years.


(a) M1 vs. S1
(b) M1 vs. P1
S1 (mm)
P1
M1 (mm)
M1 (mm)
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.92
0.94
0.96
0.98
1
Observed data
Regression line
Generated data
Selected data


(c) V_1 vs. V_2
V_2 (mm^2)
V_1 (mm^2)
(d) V_2 vs. V_4
V_4 (mm^2)
V_2 (mm^2)
(e) V_4 vs. V_8
V_8 (mm^2)
V_4 (mm^2)
(f) V_8 vs. V_{16}
V_{16} (mm^2)
V_8 (mm^2)

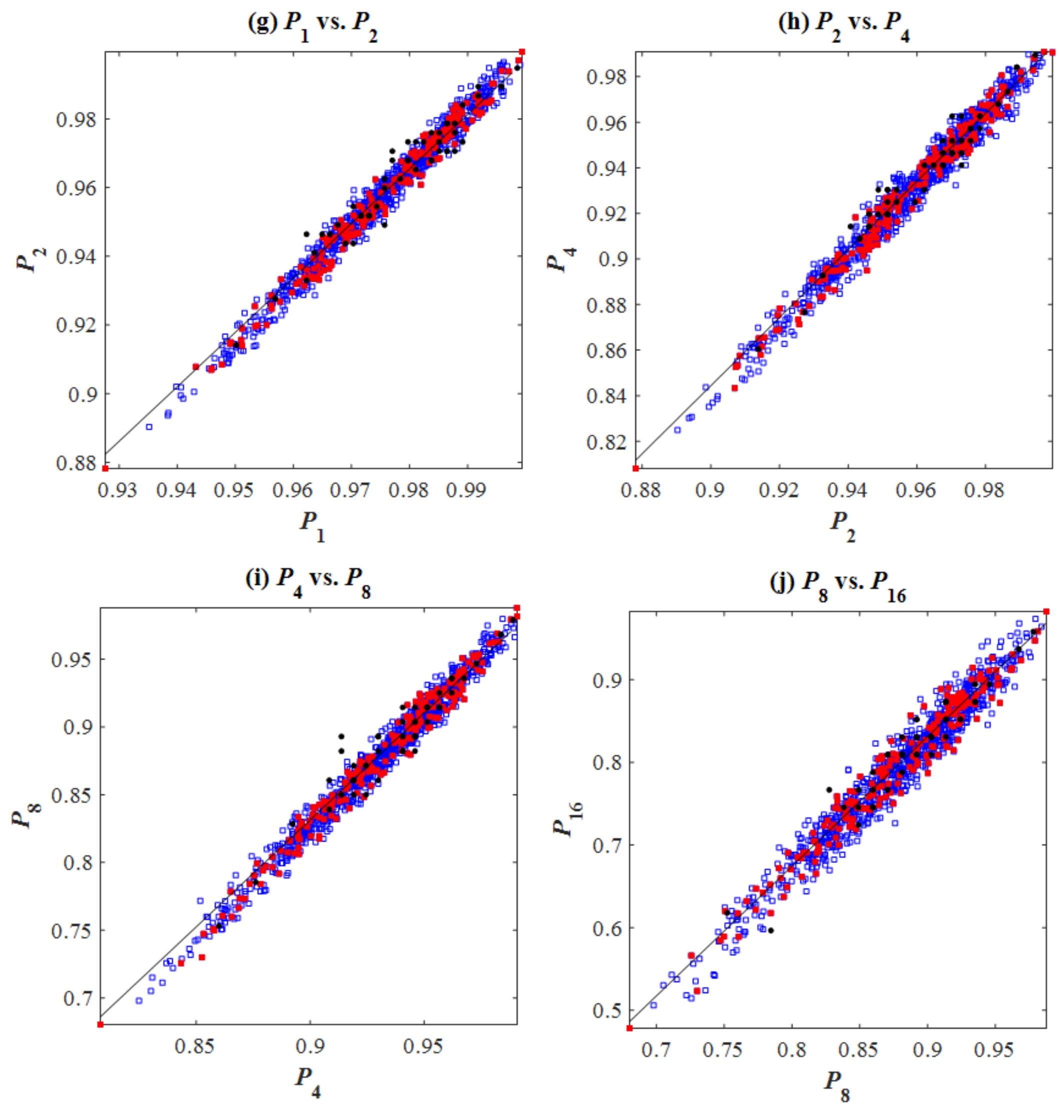


Figure 6: Linear relationship between various fine time-scale ~~rainfall~~ statistics of the rainfall observed ~~rainfall~~for the month of July of different years at gage NCDC-200164 (black dots). The solid black line represents the least squares regression line. Based on this regression relationship, ~~up to 50 sets~~a set of the 20 fine-time scale statistics are generated ~~for each of the months (hollow blue squares) until~~, which are immediately used as the basis of the MBLRP model parameter calibration. If the objective function of the ~~set becomes smaller~~parameter calibration corresponding to the generated set is greater than a given threshold ~~value. Then,~~, the set is rejected (blue squares), and the set with the objective function ~~less than~~lower the threshold ~~in the MBLRP parameter calibration process is finally~~value is only ~~chosen (red squares).~~e

~~Let us look at this process in a little more detail, focusing first upon the dashed arrows:~~

The statistic connected to the dashed arrow head is calculated based on the ones connected to the tail of the same arrow using the mathematical (deterministic) relationship connecting these variables (Equation 4). For instance, in Figure 5, V_1 and V_2 are connected to $C_1(1)$ through a dashed arrow, which means that $C_1(1)$ is derived from V_1 and V_2. The following equations establish the relationship between the variances at time-scales h and $2h$ from which we shall obtain the relationship between V_1 and V_2:

$$Var\left(Y_i^{(2h)}\right) = Var\left(Y_{2i-1}^{(h)}\right) + Var\left(Y_{2i}^{(h)}\right) + 2Cov\left(Y_{2i-1}^{(h)}, Y_{2i}^{(h)}\right)$$

Or, in simplified notation:

$$V_{2h} = 2V_h + 2C_h(1)$$

The autocorrelation lag-k is $c_h(k) = C_h(k)/V_h$, so, for $k = 1$ and $h = 1$ hour, we obtain the relation:

$$\boxed{c(1) = \frac{V_2}{2V_1} - 1} \qquad (~~4~~5)$$

If we estimate the lag-one autocorrelation using standard estimators of the terms in the right-hand side of this relation, i.e. by using $\frac{\widehat{V_2}}{2\widehat{V_1}} - 1$, how good is the estimation likely to be? ~~The figure below~~Figure 7 compares this estimator with the standard estimator $\widehat{c(1)}$ of the autocorrelation.

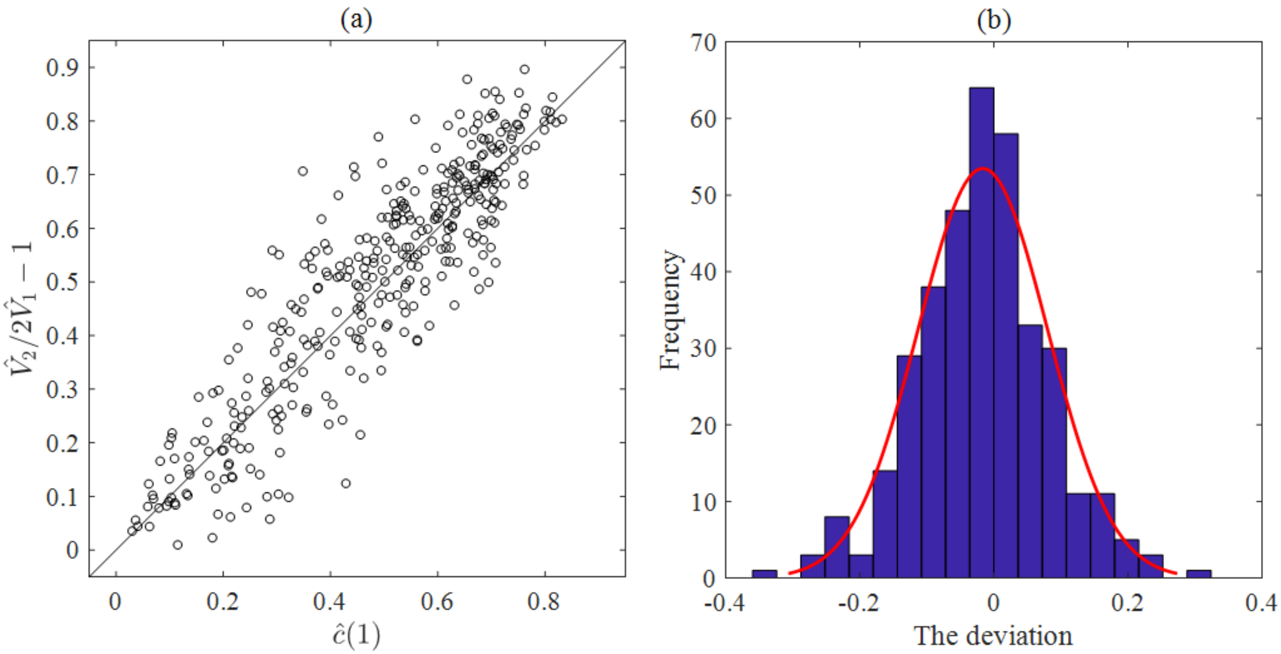


Figure 7: (a) Comparison of estimator $\widehat{c(1)}$ (horizontal axis) with estimator $\frac{\widehat{V_2}}{2\widehat{V_1}} - 1$ (vertical axis) of the autocorrelation lag-1 of hourly rainfall, (b) The histogram of the discrepancies between these two estimators at gage NCDC-200164.

Using the discrepancies ε between these two estimators which are approximately normally distributed as shown in Figure ~~7(b),~~7b, i.e. $\varepsilon \sim N(0, \sigma^2)$ we therefore estimate the autocorrelation lag-1 of hourly rainfalls using $\frac{\widehat{V_2}}{2\widehat{V_1}} - 1 + \varepsilon$.

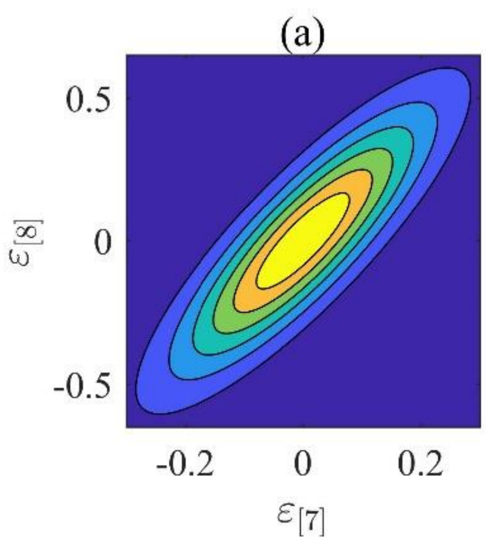
~~Looking now at the solid arrows in Figure 5, we see that the residual terms (denoted $\varepsilon_{[i]}$) are likely to be correlated. For example, consider the following equations relating V_1 to V_2 and V_2 to V_4:~~

$$V_2 = a_{[7]} V_1 + \varepsilon_{[7]} \quad (~~5~~6)$$

$$V_4 = a_{[8]} V_2 + \varepsilon_{[8]} \quad (~~6~~7)$$

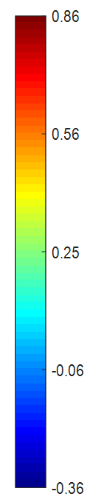
From equation (~~4~~6), it is clear that the term $\varepsilon_{[7]}$ is dependent upon the hourly autocorrelation (lag-1) coefficient, and similarly therefore that $\varepsilon_{[8]}$ in equation (~~6~~7) is dependent upon the two-hourly (lag-1) autocorrelation coefficient.

The autocorrelations at various time scales are known to be correlated with each other (Kim et al., 2013a, Kim et al., 2014), which means that $\varepsilon_{[7]}$ and $\varepsilon_{[8]}$ should be correlated with each other. Figure ~~8(a)~~8a shows the bivariate probability density function of these two variables at ~~the~~gage NCDC~~ gauge 366889~~ 200164 for the month of September. Figure ~~8(b)~~8b shows the colour map of the correlation coefficient between different $\varepsilon_{[i]}$s. This study developed bivariate probability density functions for consecutively numbered random variables ε, i.e. $\varepsilon_{[i]}$ and $\varepsilon_{[i+1]}$ (for *i* ranging from 1 to 4 and 6 to 9 respectively - see Figure 5). These were then used to sample values of $\varepsilon_{[i+1]}$ conditional upon $\varepsilon_{[i]}$, This procedure in effect assumes that a Markov structure governs the sequences $\{\varepsilon_{[i]}\}_{i=1,\dots,5}$ and $\{\varepsilon_{[i]}\}_{i=6,\dots,10}$. ~~.~~The bivariate probability density functions were developed using the Gaussian Copula and its parameters are determined using the maximum likelihood method.



(b)

	ε[1]	ε[2]	ε[3]	ε[4]	ε[5]	ε[6]	ε[7]	ε[8]	ε[9]	ε[10]
ε[1]	-	0.01	-0.21	-0.36	-0.15	0.32	-0.11	-0.25	-0.28	0.19
ε[2]	0.01	-	0.47	0.28	-0.16	-0.10	0.32	0.18	0.07	0.07
ε[3]	-0.21	0.47	-	0.38	0.14	-0.12	0.39	0.46	0.29	-0.04
ε[4]	-0.36	0.28	0.38	-	0.25	-0.28	0.22	0.28	0.34	-0.09
ε[5]	-0.15	-0.16	0.14	0.25	-	0.14	0.21	0.25	0.31	0.20
ε[6]	0.32	-0.10	-0.12	-0.28	0.14	-	-0.15	-0.27	0.08	0.19
ε[7]	-0.11	0.32	0.39	0.22	0.21	-0.15	-	0.86	0.54	0.17
ε[8]	-0.25	0.18	0.46	0.28	0.25	-0.27	0.86	-	0.65	0.18
ε[9]	-0.28	0.07	0.29	0.34	0.31	0.08	0.54	0.65	-	0.16
ε[10]	0.19	0.07	-0.04	-0.09	0.20	0.19	0.17	0.18	0.16	-



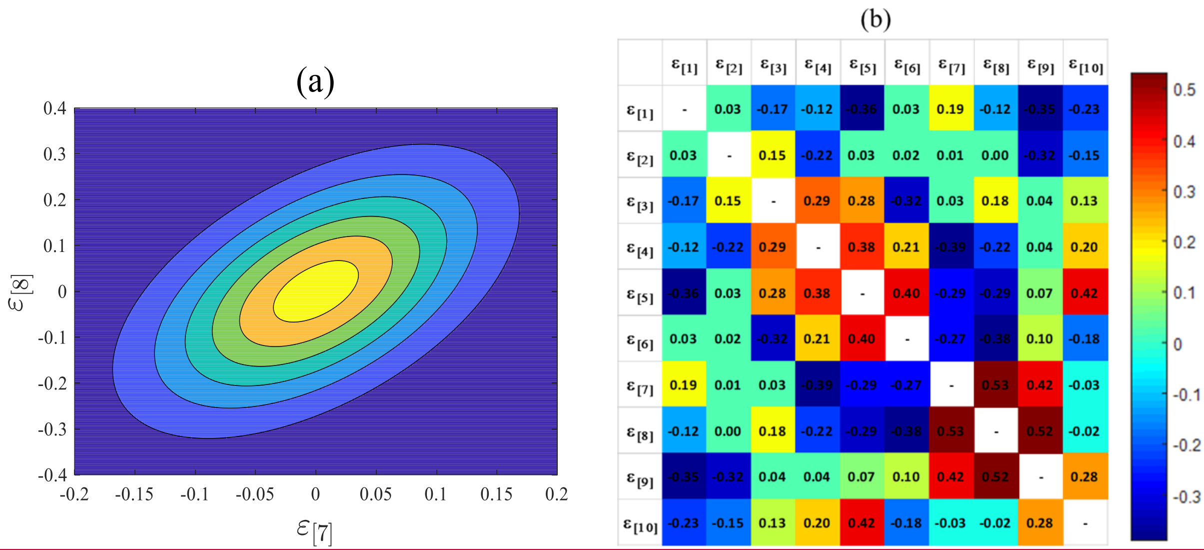


Figure 8: (a) Relationship between $\varepsilon_{[7]}$ and $\varepsilon_{[8]}$ and the fitted bivariate distribution. (b) Color map of the correlation coefficient between different $\varepsilon_{[i]}$s at gage NCDC-200164 on September.

Residual terms ($\varepsilon_{[i+1]}$) are thus generated using the conditional distribution:

$$f_{\varepsilon_{[i+1]}}(y|\varepsilon_{[i]} = x) = \frac{f_{\varepsilon_{[i]},\varepsilon_{[i+1]}}(x,y)}{f_{\varepsilon_{[i]}}(x)} \quad (~~7~~8)$$

, where i = 1, 2, 3, 4, 6, 7, 8, and 9, and $f_{\varepsilon_{[i+1]}}(y|\varepsilon_{[i]} = x)$ is the probability density function of $\varepsilon_{[i+1]}$ conditional upon $\varepsilon_{[i]} = x$, and $f_{\varepsilon_{[i]},\varepsilon_{[i+1]}}$ is the bivariate distribution function of $\varepsilon_{[i]}$ and $\varepsilon_{[i+1]}$.

As a result of this process, a total of 20 rainfall statistics at fine time scale (mean, variance, lag-1 autocorrelation, and proportion of dry period at 1~~-~~, 2~~-~~, 4~~-~~, 8~~-~~, and 16-hourly aggregation interval) are sampled using these conditional distributions and the individual monthly rainfall that is generated by the SARIMA model.

3.3 MBLRP Model Parameter Estimation

In this process, each of the monthly rainfall values generated by the SARIMA model is coupled with one set of six MBLRP model parameters that define the random nature of rain storm and rain cell arrival frequency, and the intensity and duration of rain cells (Figure 1).

In this study, the parameters of the MBLRP model were determined such that the rainfall statistics of the generated rainfall resemble the 20 fine-scale rainfall statistics that were coupled with the monthly rainfall generated by the SARIMA model. The Isolated-Speciation Particle Swarm Optimization (ISPSO, Cho et al., 2011) algorithm was employed to identify a set of parameters that minimizes the following objective function:

~~$OF = \sum_{i=1}^{20} w_i \cdot \left[1 - \frac{F_i(\lambda,\nu,\alpha,\mu,\phi,\kappa)}{f_i}\right]^2$~~ $OF = \sum_{i=1}^{20} w_i \cdot \left[1 - \frac{F_i(\lambda,\nu,\alpha,\mu,\phi,\kappa)}{f_i}\right]^2$ (~~8~~9)

F_i is the i^th^ statistic of the synthetic rainfall time series (e.g. mean of hourly rainfall, standard deviation of 4-hourly rainfall, etc.). The mathematical formulae for the F_is were derived by Rodriguez-Iturbe et al. (1988) as a function of the six parameters $(\lambda, \nu, \alpha, \mu, \phi, \kappa)$; f_i is the i^{th} generated statistic, and w_i the weighting factor given to the i^{th} rainfall statistic depending on the use of the synthetic rainfall time series (Kim and Olivera, 2011). Here, it should be noted that a time step with rainfall less than 0.~~5mm~~5 mm was considered dry when the proportion of non-rainy period was calculated because small rainfall values are known to distort the "true" proportion of non-rainy period exerting an adverse effect on calibration process (Kim et al, 2016, Cross et al., 2018).

It is noteworthy that Module 2 may fail to generate a realistic set of fine scale rainfall statistics due to the complex interdependencies between them. The unrealistic fine scale rainfall statistics cannot be represented by the MBLRP model that reflects the original spatial structure of rainfall in reality, which entails poorly calibrated model parameters with high objective function value of Equation 8. To exclude the poorly calibrated parameter sets caused by the unrealistic fine scale rainfall statistics generated by Module 2, we repeated the process of Module 2 and Module 3 until the objective function value of Equation 8 becomes lower than a given threshold value (0.8 in this study). If the algorithm fails to find the parameter set after 50 repetitions, the parameter set with the lowest objective function value is chosen. Figure 4 describes this filtering process, and the red squares in Figure 6 shows the chosen parameter sets.

3.4 Downscaling of Monthly Rainfall Using the MBLRP Model

The MBLRP model was used to downscale the monthly rainfall to the hourly aggregation level. First, the MBLRP model generates the hourly rainfall time series using the parameter set for the monthly rainfall being downscaled. Second, the discrepancy between the ~~generated~~ fine time scale statistics generated by the second module of the model (Figure 5) and the statistics of the ~~generated~~ synthetic hourly rainfall time series generated by the MBLRP model is calculated using the following formula:

$$D^j = \sum_{i=1}^{20} \left[\frac{S_i^j - f_i}{R_i}\right]^2 \qquad (\sout{9}10)$$

,where D^j is the discrepancy between the generated statistics and statistics of j^{th} synthetic hourly rainfall time series. S_i^j is the i^{th} statistic of j^{th} time series and R_i is the difference between maximum and minimum values of S_i^j about i^{th} statistic.

Third, the first and the second process are repeated 300 times. Then the synthetic hourly rainfall time series with the lowest discrepancy value is chosen. Finally, we repeated the entire process for 200 times to obtain 200 synthetic hourly rainfall time series for each of the generated monthly rainfall.

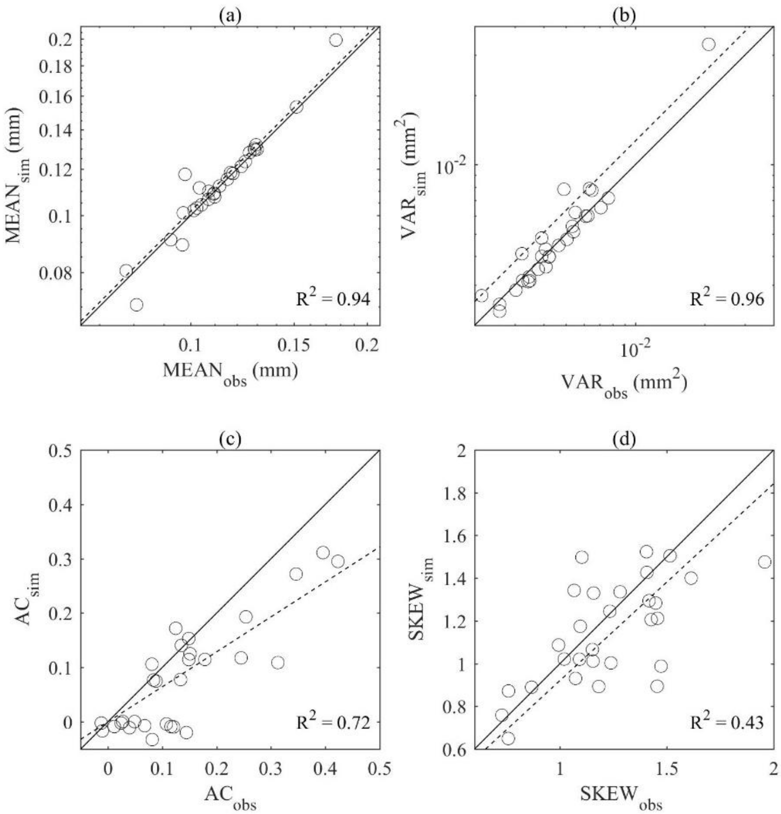
3.5 Validation for Ungaged Periods

One of the primary purposes of the stochastic rainfall model is to provide synthetic rainfall for the ungaged periods, which can be the periods of missing data or future periods. For this reason, we separated the period of model calibration and validation at some gage locations (square marks in Figure 2) where record length of each period is sufficiently long (60+ years). Then, we tested our model not only based on the statistics of the calibration period (1981-2010) but also based on the validation period (1951-1980).

4 Result

4.1 Monthly Rainfall Statistics Reproduction

Figure 9 compares the mean, variance, lag-1 autocorrelation, and skewness of the ~~observed (x) and~~monthly rainfall time series generated ~~rainfall~~ by the SARIMA model (x axis) to those of the observed monthly rainfall time series (y axis). Each scatter represents one rainfall ~~gauge. The~~gage. For the calibration period (1981-2010), the first and the second-order moments were reproduced accurately with the coefficient of determination ranging ~~between 0.72 and 0.96. Skewness was reproduced fairly well with the correlation coefficient of 0.43~~from 0.69 to 0.95. Skewness was reproduced fairly well with the coefficient value of 0.36. For the validation period (1951-1980), mean and variance were reproduced, but not 1ag-1 autocorrelation and skewness. However, this discrepancy cannot be attributed solely to the limitations in the model because the discrepancy in each plot of Figure 9 directly results from the differences between the statistics of the calibration and validation periods. In other words, had the statistics of the calibration period been similar to those of the validation period, we would have expected similar performance for both periods, and vice versa.



(a)
MEAN$_{sim}$ (mm)
MEAN$_{obs}$ (mm)
$R^2 = 0.94$
(b)
VAR$_{sim}$ (mm^2)
VAR$_{obs}$ (mm^2)
$R^2 = 0.96$
(c)
AC$_{sim}$
AC$_{obs}$
$R^2 = 0.72$
(d)
SKEW$_{sim}$
SKEW$_{obs}$
$R^2 = 0.43$

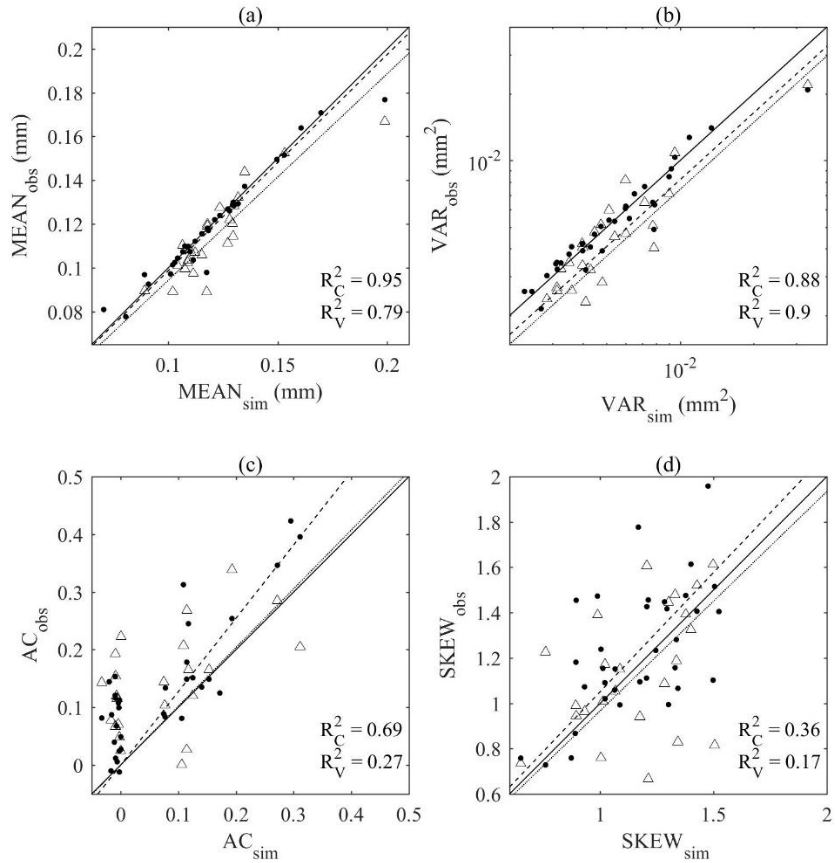
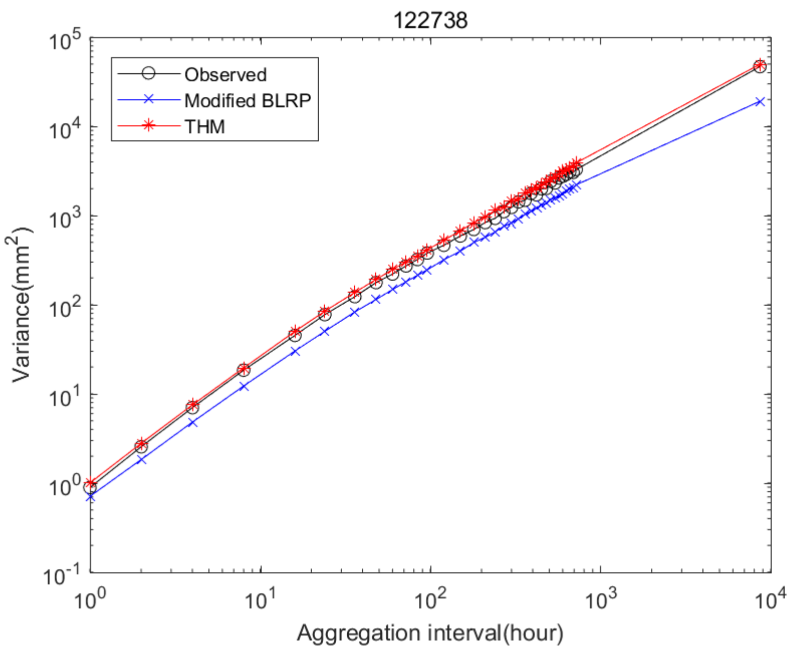


Figure 9: Comparison of (a) mean, (b) variance, (c) lag-1 autocorrelation, and (d) skewness of the ~~observed (x) and~~ synthetic (x) and observed (y) monthly rainfall. **Filled circles (dashed line) and hollow triangles (dotted line) correspond to the calibration (1981-2010) and validation period (1951-1980) respectively.**

4.2 Reproduction of Large Scale Rainfall Variability

Figure 10 shows the behaviour of the rainfall variance varying with temporal aggregation interval between 1 hour and 1 year at ~~the~~gage NCDC~~ gauge -~~122738. The behaviour corresponding to the observed~~ rainfall~~ calibration (black~~) and the 200 years of synthetic rainfall generated by the~~, 1981-2010), observed-validation (green, 1951-1980), MBLRP ~~model~~ (blue) and ~~by~~ our hybrid model (red) are shown together. While our model successfully reproduces the rainfall variance across the time scale, the MBLRP model is successful in reproducing the rainfall variance only at the hourly accumulation level. This reflects the fact that Poisson cluster rainfall models are not designed to preserve the rainfall persistence at the aggregation interval that is greater than the typical model storm duration, i.e. a few hours. See Figure 1 for example. Within the duration of one storm, rainfall at different time steps may be similar insofar as a portion of it is from the same rain cell. However, the rainfall within one storm is independent of the rainfall within another storm. Therefore, it is natural that Poisson cluster rainfall models tend to underestimate the observed rainfall variance (which reflects the covariance structure - see Equation 1) at time scales exceeding the rain storm duration. Kim et al. (2013b), when mapping the average model storm duration across the continental United States using Equation 11, showed that the model storm duration of the MBLRP model approximately ranges from 2 to 100 hours, so it is not only at the annual scale, but already at the scale of several hours (depending upon the location) that the variability may be underestimated by the MBLRP model.



$$\textit{Average storm duration (hr)} \cong \frac{1}{\phi\frac{\alpha}{\nu}\left[1+\phi(\kappa+\phi)-\frac{1}{4}\phi(\kappa+\phi)(\kappa+4\phi)+\frac{1}{72}\phi(\kappa+\phi)(4\kappa^2+27\kappa\phi+72\phi^2\right]} \qquad (11)$$

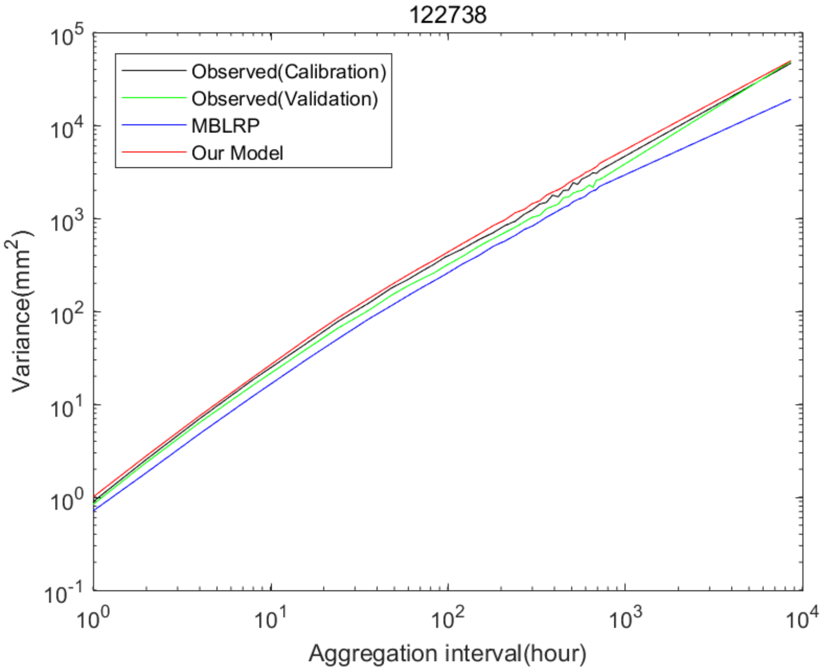
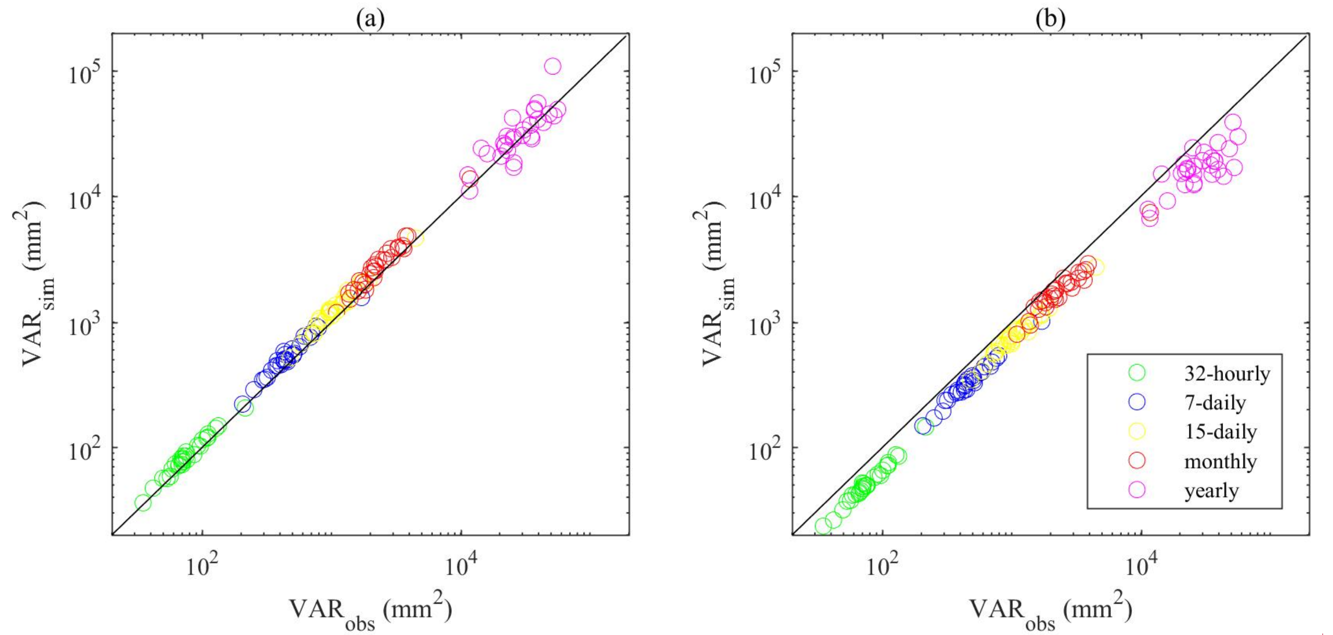


Figure 10: Behaviour of the rainfall variance with regard to the aggregation interval of the observed rainfall time series at gage NCDC-122738. The behaviour corresponding to the observed-calibration (black) and the 200 years of synthetic rainfall generated by the, 1981-2010), observed-validation (green, 1951-1980), MBLRP model (blue) and our hybrid model (red).) are shown together.

A similar trend as exhibited in Figure 1011 was observed at all of the 29 gauges34 gages. Figure 11(a) compares the variance of the observedsynthetic (x) and syntheticobserved (y) rainfall time series at yearly (purple), monthly (red), 15-daily (yellow), weekly (blue), and 32-hourly (green) aggregation levels. The comparison of the variance at the finer time scale is carried out in the following section. Figure 10(b) compares the observed (x) and synthetic rainfall time series generated by the traditional MBLRP model (y).



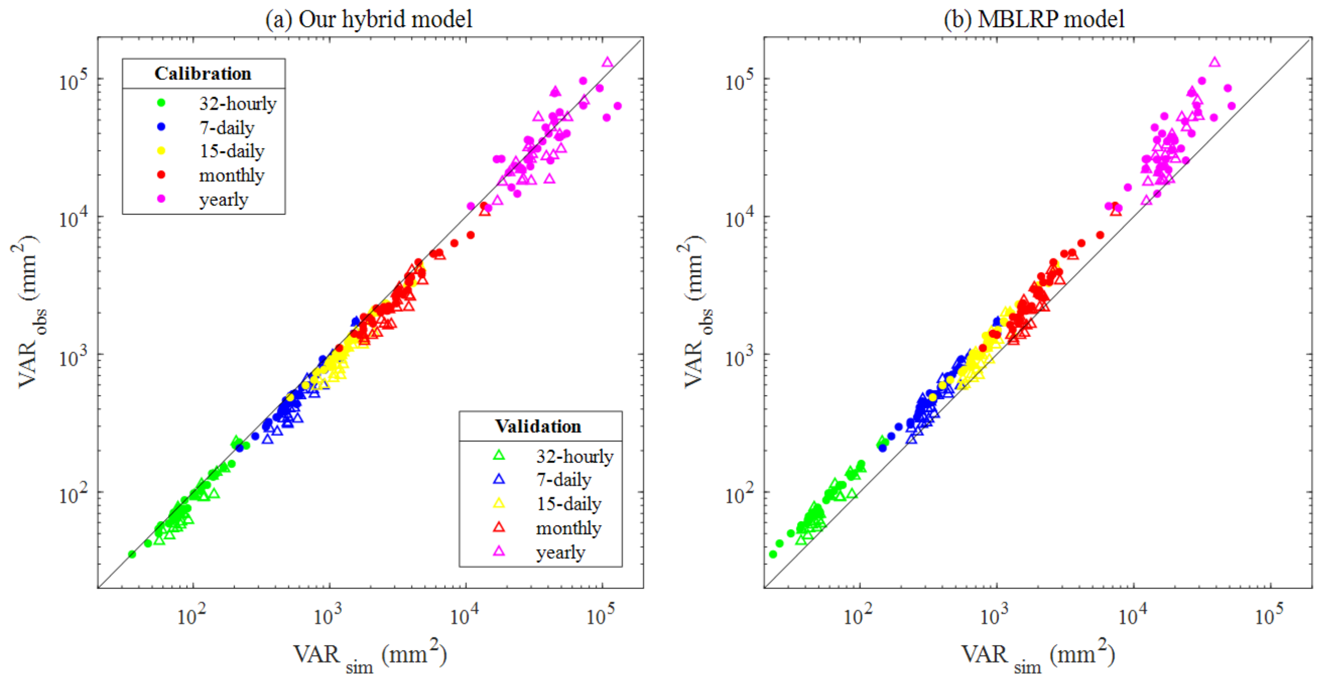
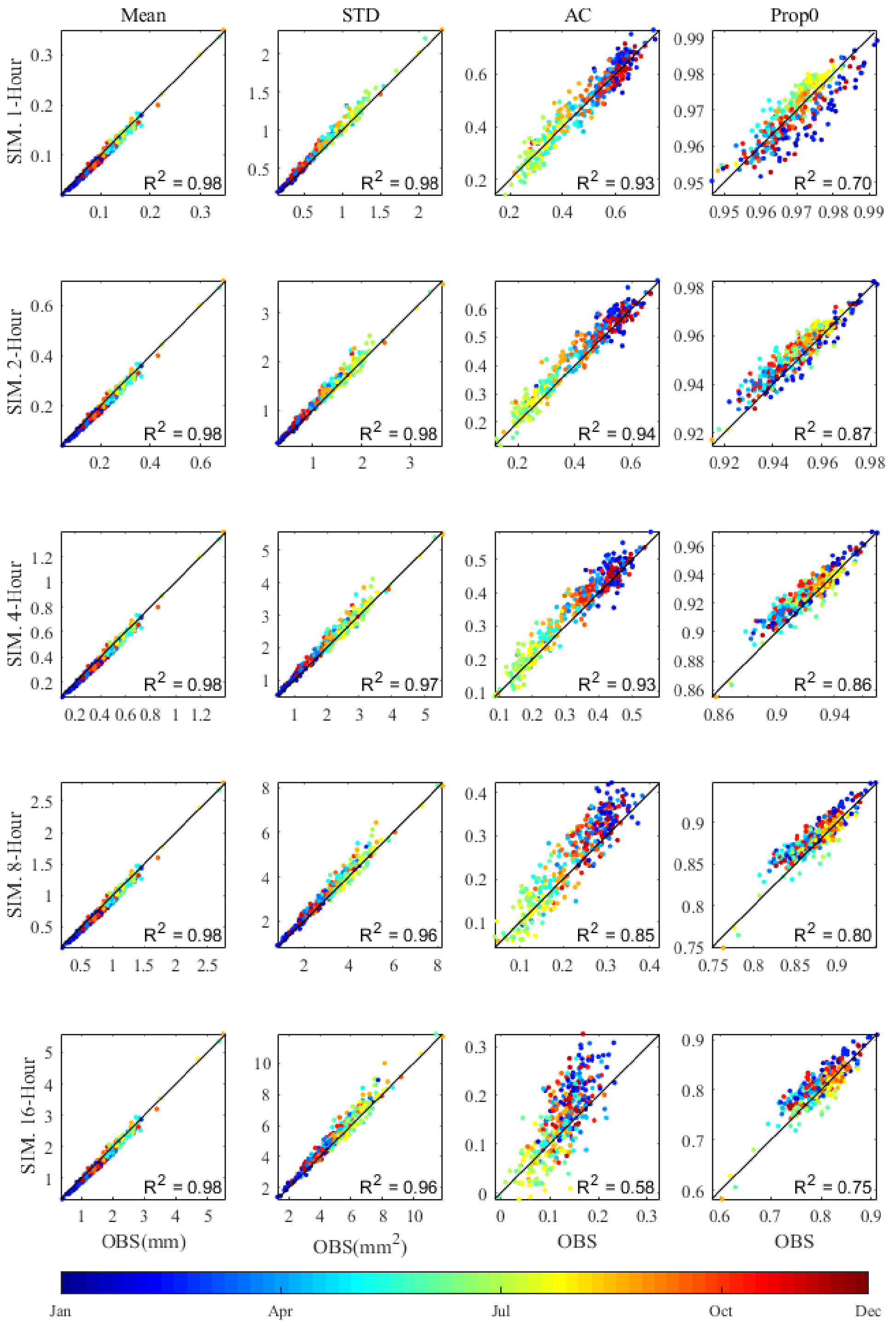


Figure 11: (a) Comparison of the large scale rainfall variance of the ~~observed rainfall (x) and the~~ rainfall generated by our hybrid model (x) and the observed rainfall (y); (b) Comparison of the large scale rainfall variance of the ~~observed rainfall (x) and the~~ rainfall generated by the traditional MBLRP model (x) and the observed rainfall (y). The different colours of the scatter correspond to the different aggregation interval of rainfall time series. Filled circles and hollow triangles correspond to the calibration and validation periods respectively.

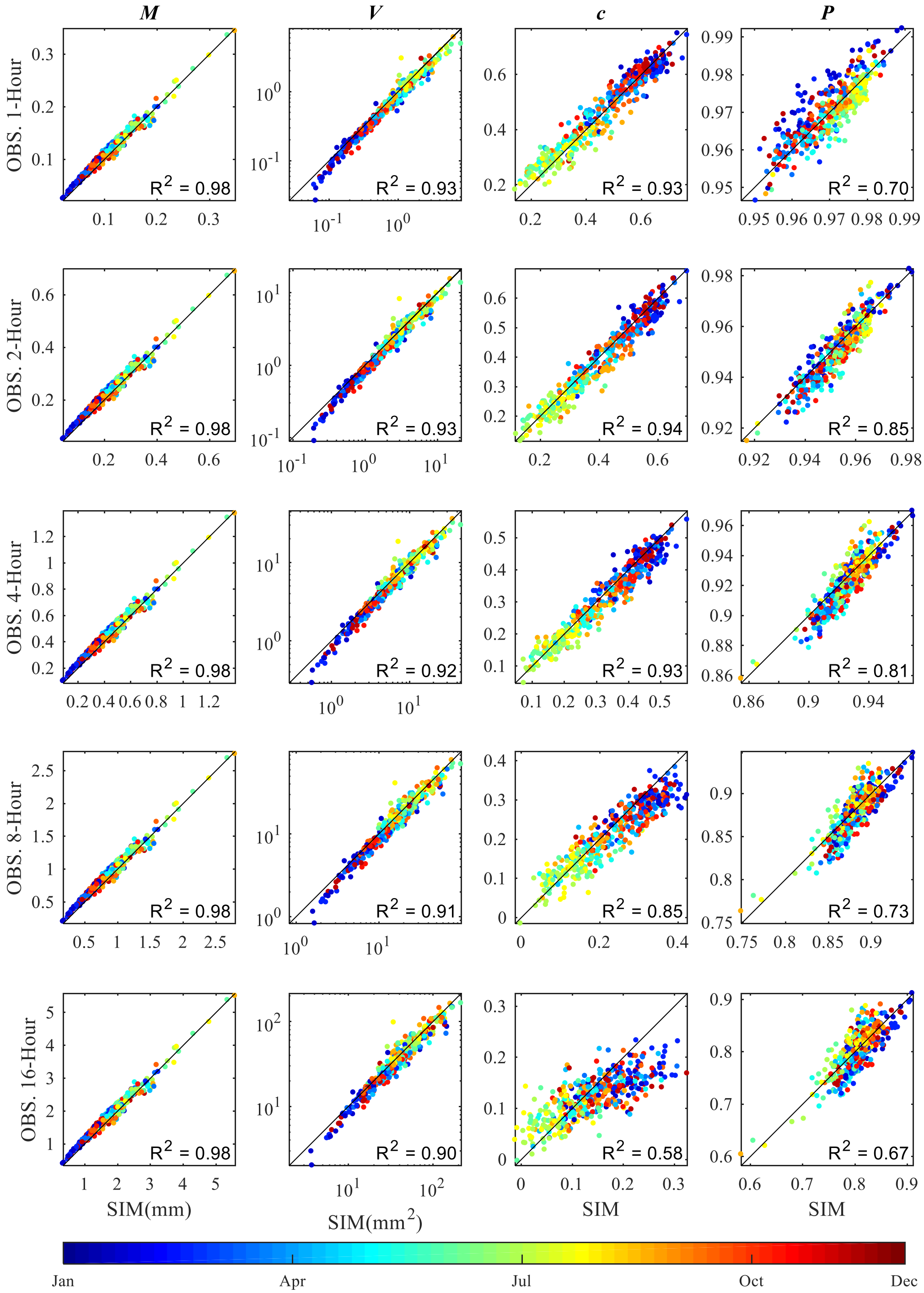
As indicated by the concentration of the scatters ~~below~~above the 1:1 line in Figure ~~11(b),~~11b, the traditional MBLRP model systematically underestimates the variability at time scales greater than 32 hours. Our model did not show any bias in this range of large time-scales as shown in Figure 11a.

4.3 Reproduction of Sub-Daily Rainfall Statistics

Figure 12 compares the mean, ~~standard deviation~~variance, lag-1 autocorrelation, and the proportion of dry periods of the ~~observed (x) and~~ synthetic (x) and observed (y) rainfall time-series at hourly through 16 hourly aggregation levels. ~~The colour~~Each scatter represents the statistics at a given gage for a given calendar month. The colours of the scatters ~~represents~~represent the calendar months. In each plot, the coefficient of determination (R^2) of the linear regression between the two variables is shown. All four statistics were accurately reproduced across various sub-daily time scales with R^2 equal to 0.98 (mean), and varying between the following limits for the other statistics: 0.~~96~~90 and 0.~~98 (standard deviation~~93 (variance), 0.58 and 0.~~94~~93 (lag-1 autocorrelation), and 0.~~70~~67 and 0.~~87~~85 (proportion of dry periods) on the calibration period (Figure 12a). Similar ranges of coefficient of determinations were obtained for the validation period (Figure 12b).


Mean
STD
AC
Prop0
SIM. 1-Hour
SIM. 2-Hour
SIM. 4-Hour
SIM. 8-Hour
SIM. 16-Hour
R2 = 0.98
R2 = 0.98
R2 = 0.93
R2 = 0.70
R2 = 0.98
R2 = 0.98
R2 = 0.94
R2 = 0.87
R2 = 0.98
R2 = 0.97
R2 = 0.93
R2 = 0.86
R2 = 0.98
R2 = 0.96
R2 = 0.85
R2 = 0.80
R2 = 0.98
R2 = 0.96
R2 = 0.58
R2 = 0.75
OBS(mm)
OBS(mm2)
OBS
OBS
Jan
Apr
Jul
Oct
Dec

(a) Calibration



(b) Validation

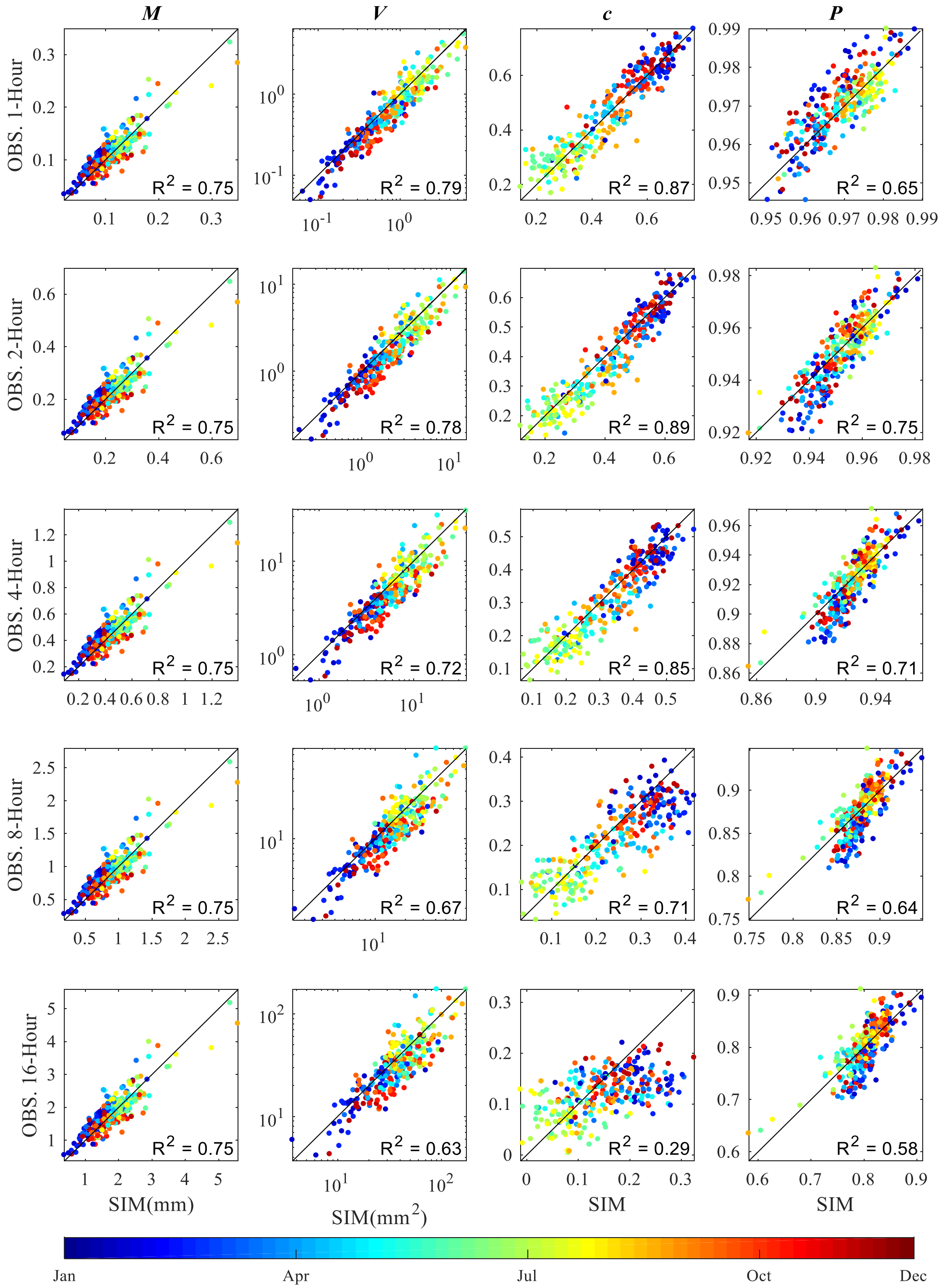
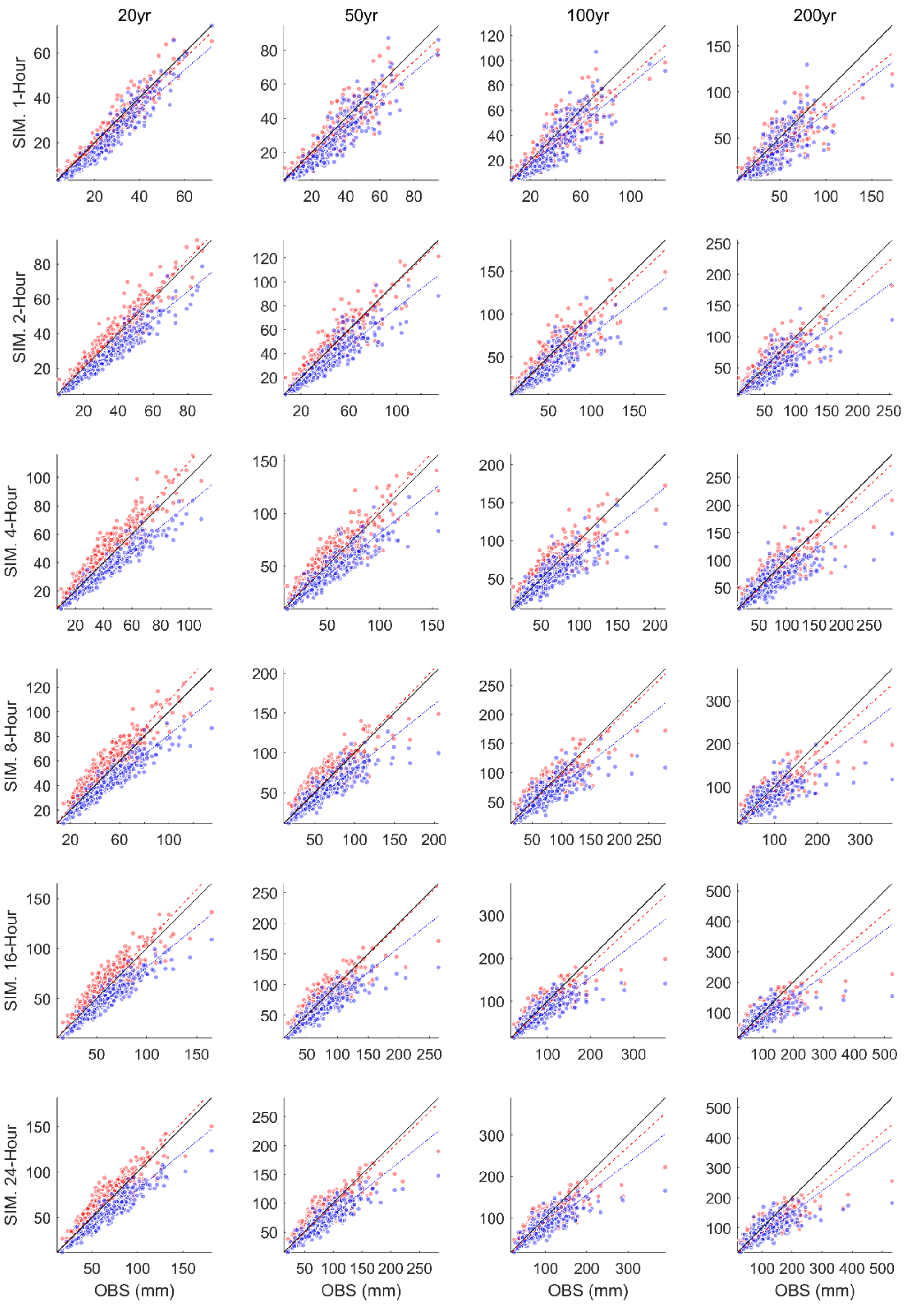


Figure 12: Comparison of the statistics of the ~~observed~~synthetic (x) and ~~synthetic~~observed (y) rainfall time series at sub-daily time scale. The colour of the dots represents the statistics of each calendar month. The results of (a) the calibration period (1981-2010) and (b) the validation period (1951-1980) are shown.

4.4 Reproduction of Extreme Values and Distribution of Annual Maxima

The ~~scatter of circles~~ scatters in Figure 13 ~~compares~~compare the ~~(a)~~ 20~~, (b)~~, 50~~, (c)~~, 100~~, (d)~~, and 200-year rainfall estimated from the observed rainfall (x) and the synthetic rainfall (y) generated by the ~~composite~~hybrid model ~~of this study (y). The colour of~~(red) and the ~~scatters represents the duration of the extreme rainfall.~~MBLRP model (blue) at hourly through daily time scale. The Generalized Extreme Value (GEV) distribution was ~~assumed~~used to model the distribution of the annual maxima ~~of both rainfall time series~~, and the three parameters of the GEV distribution were determined using the method of L-moments. Here, we separated the analysis for each calendar month, so we have 12 sets of extreme rainfall distributions corresponding to each gage station. Therefore, we produced each scatter plot of Figure 13 based on 408 points (12 months/gage × 34 gages).

A linear regression line passing through the origin is shown in each plot. ~~As the slope of the regression line approaches the value of one, the less biased the extreme values reproduced by the model. As the R^2 of the regression line approaches the value of one, the more consistent the extreme values reproduced by the model.~~ In all cases, our hybrid model did not show the tendency of underestimating extreme values, which is one of the most widely discussed issues in Poisson cluster rainfall modelling (Cowpertwait, 1998; Cross et al., 2018; Furrer and Katz, 2008; Verhoest et al., 2010; Kim et al., 2013a; Onof et al., 2013; Kim et al., 2016). This is a somewhat surprising result: our algorithm to incorporate large scale variability of the observed rainfall not only served its original purpose but also enhanced the capability of the model to reproduce extreme rainfall values.


20yr
50yr
100yr
200yr
SIM. 1-Hour
SIM. 2-Hour
SIM. 4-Hour
SIM. 8-Hour
SIM. 16-Hour
SIM. 24-Hour
OBS (mm)
OBS (mm)
OBS (mm)
OBS (mm)

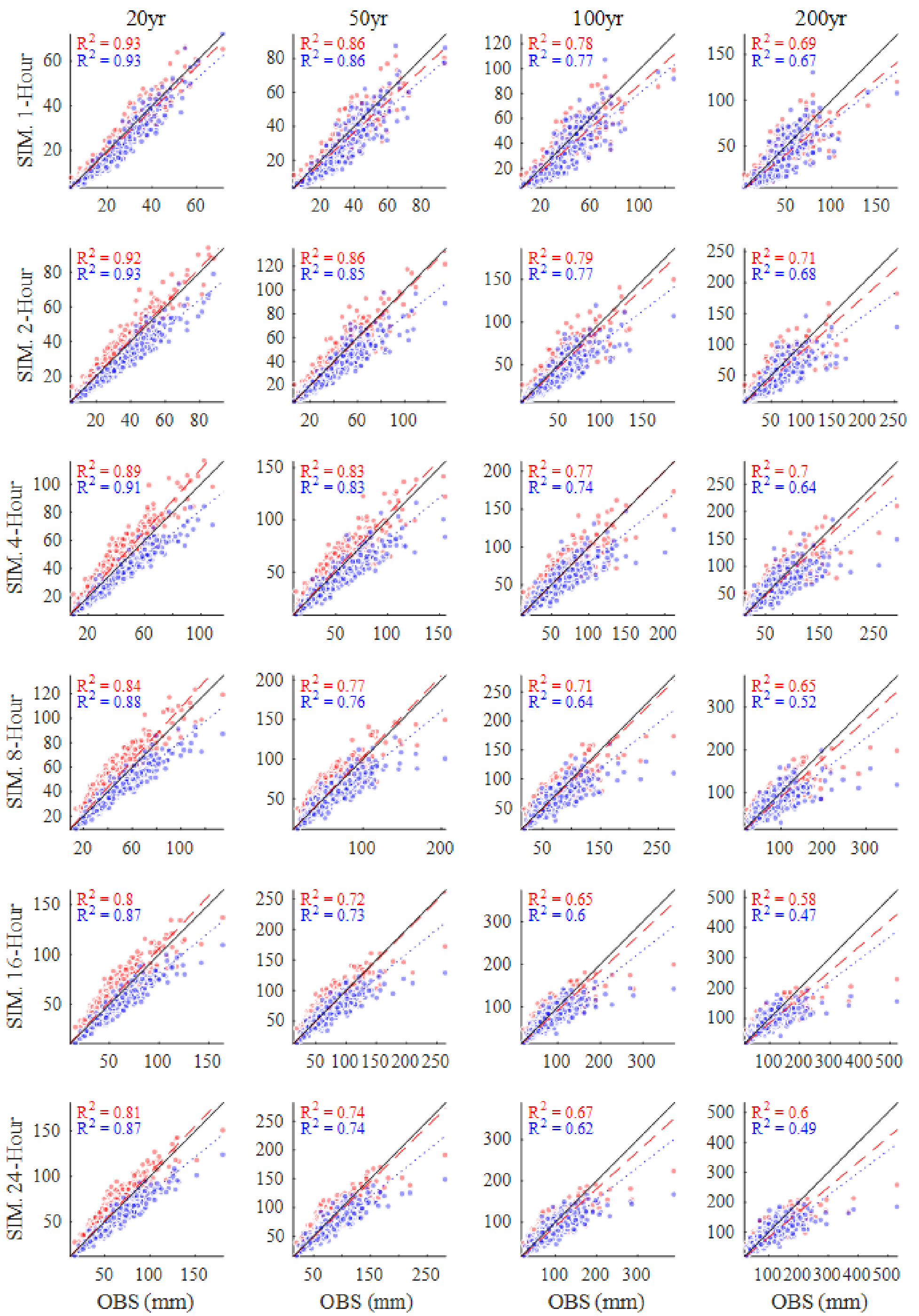
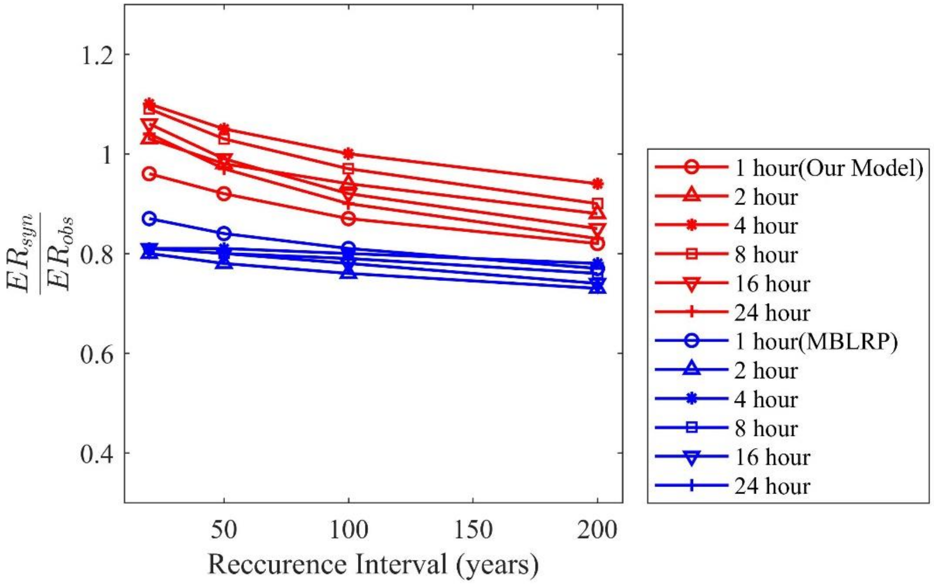


Figure 13: Comparison of the extreme rainfall values estimated from the observed rainfall (x) and synthetic rainfall (y) generated by the model of this study (~~y).~~red) and the MBLRP model (blue). The plots ~~comparing (a)~~compare 20~~, (b)~~, 50~~, (c)~~, 100~~,~~, and ~~(d)~~ 200-year rainfall ~~are shown. Different colours of the scatter represent the rainfall duration. A circle represents the result according~~

~~the hybrid model, and a triangle represents the result according to the traditional MBLRP model~~at hourly through daily aggregation levels.

Figure 14 shows the degree of bias of extreme value reproduction (slope of the regression line in Figure 13) varying with recurrence interval. The values corresponding to the traditional MBLRP model is also shown. The degree of underestimation of the traditional methods varies between 73% and 87%, and it tends to increase as the recurrence interval increases. A similar tendency was observed for our model, but the degree of underestimation was significantly reduced. For our model, the degree of underestimation is the greatest for the 1-hour extreme rainfall and tends to decrease as the duration of the rainfall increases. This tendency was not observed with the traditional MBLRP model.



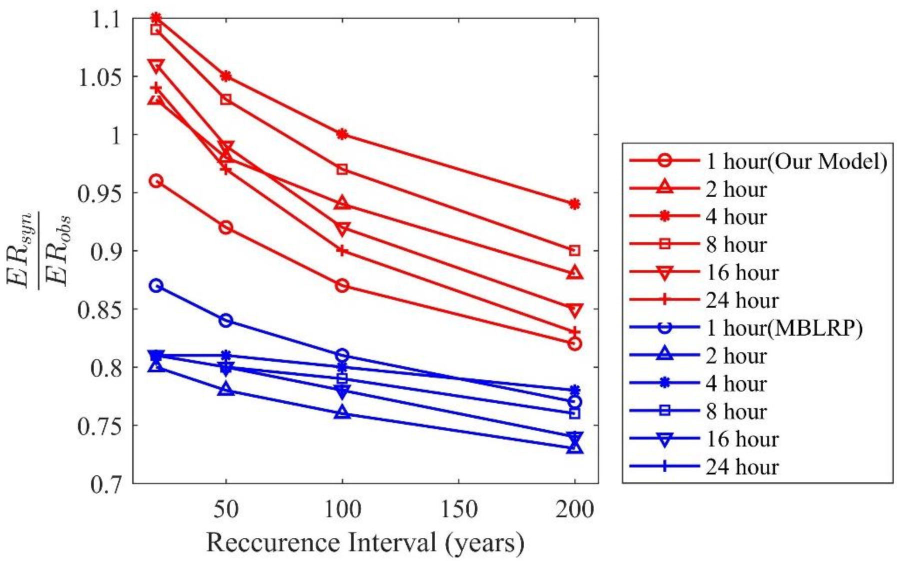


Figure 14: Degree of over/underestimation of extreme values by our model (~~blue~~red) and the traditional MBLRP model (~~red~~blue). ER_{syn} and ER_{obs} are extreme ~~rainfall~~rainfalls estimated from synthetic rainfall and observed rainfall, respectively.

~~It is important that the~~A good rainfall model should reproduce not only the extreme ~~rainfall~~ values but also the distribution of the ~~rainfall monthly~~ maxima from which extreme ~~rainfall~~ values are derived. We performed the two-sample Kolmogorov-Smirnov (K-S) test between the monthly maxima of the synthetic rainfall and the observed rainfall. A significance level of 5% was used. Among all ~~348~~408 calendar months (~~29 gauges x~~34 gages × 12 months), the null hypothesis of assuming that two distributions are the same could not be rejected at ~~334, 318, 267, 248, 272~~384, 368, 317, 301, 323, and ~~282~~333 months for the 1~~-~~, 2~~-~~, 4~~-~~, 8~~-~~, 16~~-~~, and 24-hour rainfall, respectively (~~82~~83 percent of all ~~gauges~~gages). On the contrary, the traditional approach successfully reproduced the observed monthly maxima distribution only at 292, 243, ~~202, 189, 165, 168~~219, 200, 220, and ~~172~~219 months (~~47~~57 percent of all ~~gauges).~~gages).

Figure 15 shows the relative frequency and the fitted GEV distribution of the monthly maxima of January, April, July, and October at NCDC gage 132203. The black, red, and blue line correspond to the result of observed rainfall, our hybrid model, and the traditional MBLRP model, respectively. The GEV distribution of the 1, 4, and 16-hour rainfall durations are shown in the plots of the first, third, and fifth row, respectively. The plots in the second, fourth, and the sixth row magnify the upper 10th percentile part of the distribution of the upper figures that is denoted as the dashed box. For all months and durations, our hybrid model outperforms the traditional MBLRP model in reproducing the head through tail part of the distribution. The distribution of the traditional MBLRP model was skewed toward the lower values. A similar tendency was observed at most gage locations while at some of the gages our hybrid model showed similar or slightly degraded performance compared to the traditional MBLRP model in reproducing the distribution of extreme values. We discuss about this finding further in Discussion 5.1.

Figure 16 compares the shape (ξ), the scale (σ), and the location (μ) parameter of the fitted GEV distribution of the monthly maxima of the observed rainfall (x) and of the synthetic rainfall generated from our hybrid model (red scatters) and from the traditional MBLRP model (blue scatters). The results for 1, 4, and 16-hour rainfall durations are shown. Each scatter point represents the result of one calendar month at one gage. A total of 408 scatter points (12 months/gage × 34 gages) are shown for each of the plot. The traditional MBLRP model underestimates the location parameters at all rainfall durations, and the degree of underestimation increases with increased duration. Our hybrid model showed the opposite trend. The location parameters tend to be overestimated with an increase in the duration, but the degree of overestimation was not as significant as in the case of the traditional model. The traditional model compensates the underestimated location of the distribution with the overestimated scale parameters, which were observed for all three durations investigated. Our hybrid model also compensates the overestimated location of the distribution with the underestimated scale parameters, but the degree of compensation was not as significant as in the case of the traditional model. However, the shape parameter of the observed monthly maxima was not well reproduced by both models. This result shows the difficulty of precisely reproducing the rainfall extreme values. This is mainly because the rainfall extreme values are indeed extreme. For example, 1-hour 100-year rainfall of 100 years of rainfall record is theoretically the greatest value of all 72,000 hourly rainfall records (24 hours/day × 30 days/month × 100 years), and precisely reproducing a value with such a low probability of occurrence can be a daunting task using the models with only a limited number of parameters.

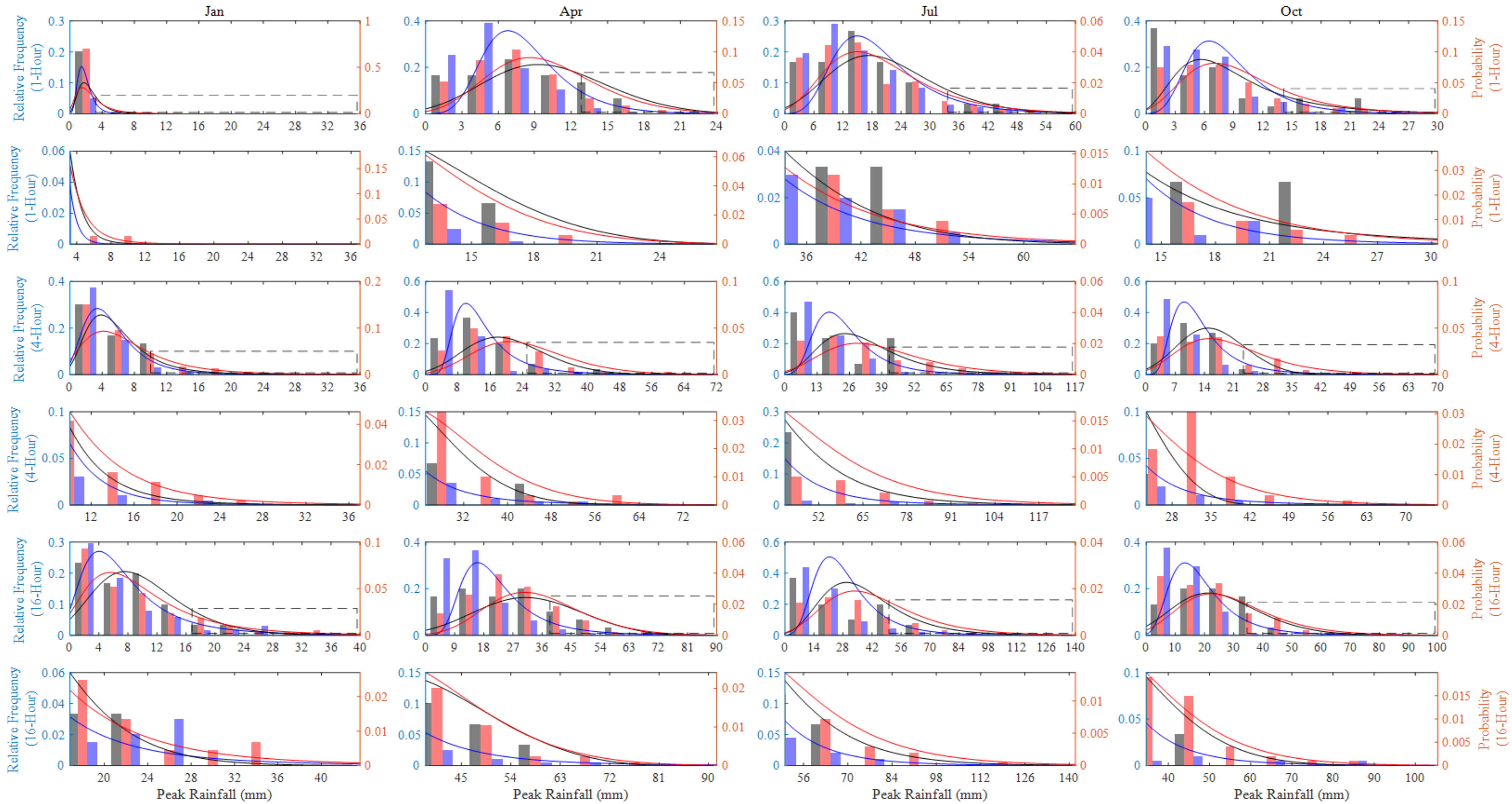


Figure 15. Relative frequency and the fitted GEV distribution of the 1, 4, and 16-hour monthly maxima of January, April, July, and October rainfall at NCDC gage 132203. Results of Observed rainfall (black), our hybrid model (red), and the traditional MBLRP model (blue) are shown. The upper 10 percentile part of the distribution (dashed box in the plots in the first, third, and fifth row) is magnified in the lower rows (plots in the second, fourth, and sixth row).

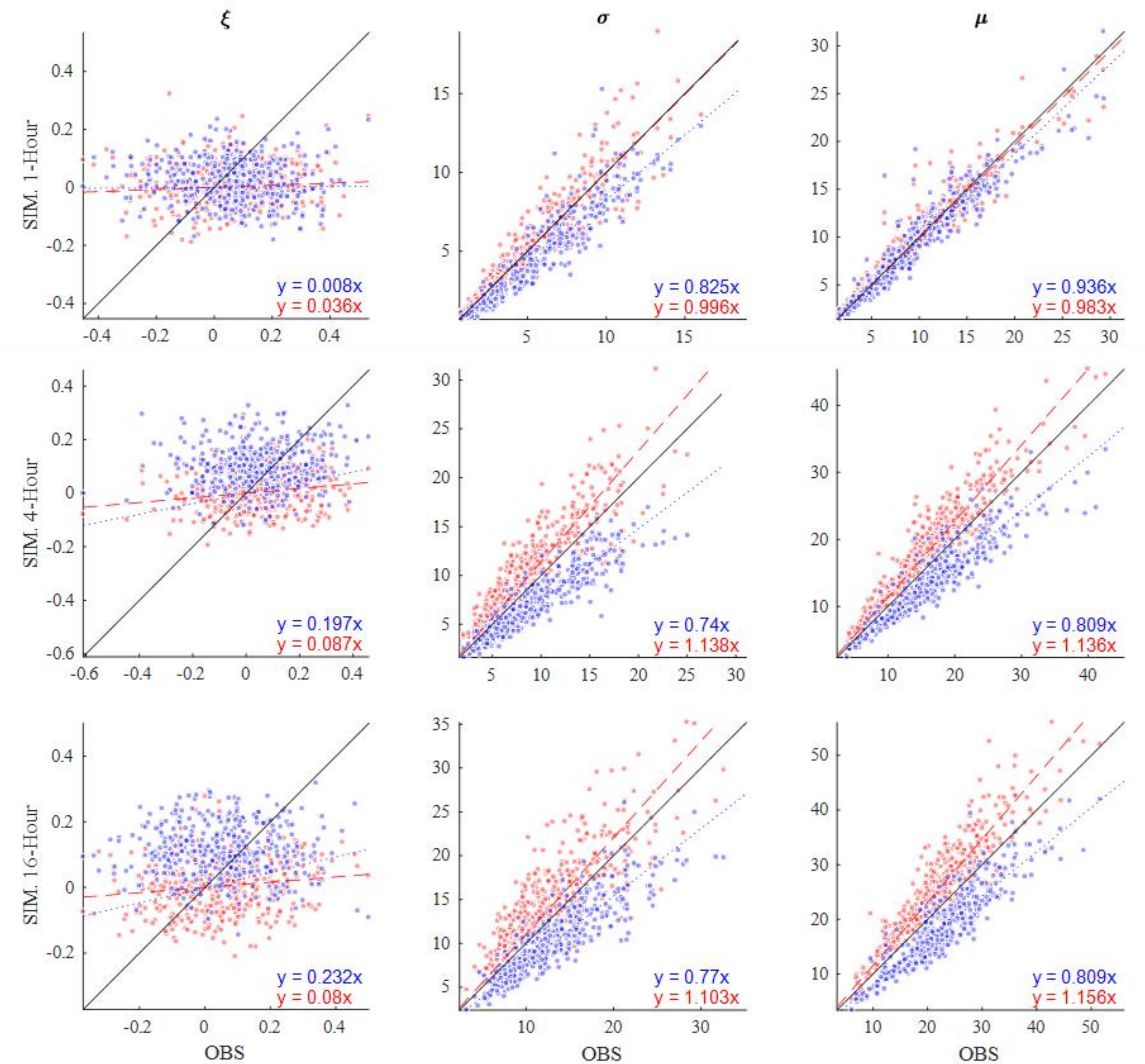


Figure 16. Comparison of the shape (ξ), scale (σ), and location (μ) parameters of the fitted GEV distribution of the monthly maxima. The results based on the observed rainfall (x), our hybrid model (red), and the traditional model (blue) are shown. The results of 1, 4, and 16-hour rainfall durations are shown.

5 Discussion

5.1 Variability of the Parameters of the MBLRP model and Extreme Values

Our model uses different parameter sets of the MBLRP model to disaggregate different monthly rainfalls. This means that one given calendar month can have many different parameter sets. By contrast, the traditional MBLRP model uses one parameter set for each calendar month. Therefore, if we look at the variability of each month's parameters, we can see how the model of this study explains the variability of rainfall unlike the MBLRP model. Figure ~~15~~17 shows a box plot of the parameters for each month at ~~the~~gage NCDC~~ Gauge ~~ 460582. The parameters of the traditional MBLRP model are shown together for reference (triangles).

While significant variability is observed for all six parameters, the parameter μ, which represents the average rain cell intensity, showed the greatest variability, ranging over two orders of magnitudes. This explains why our model is good at both reproducing large scale rainfall variability and small scale extreme values: the variability of the rain cell intensity parameter has the effect of stretching out the distribution of rainfall depths at a range of levels of aggregation, thereby increasing the probability of very large values. And~~ it is course~~ the variability of this cell intensity parameter~~ that~~ is also the most important factor responsible for the increase in the large scale rainfall variance. Zorzetto et al. (2016) also briefly discussed this matter. They introduced a novel framework of meta-statistical extreme value (MEV) analysis. In this MEV formulation, one can show that interannual-variation of exponential-type rainfall process leads to a fat-tail for its extreme values.

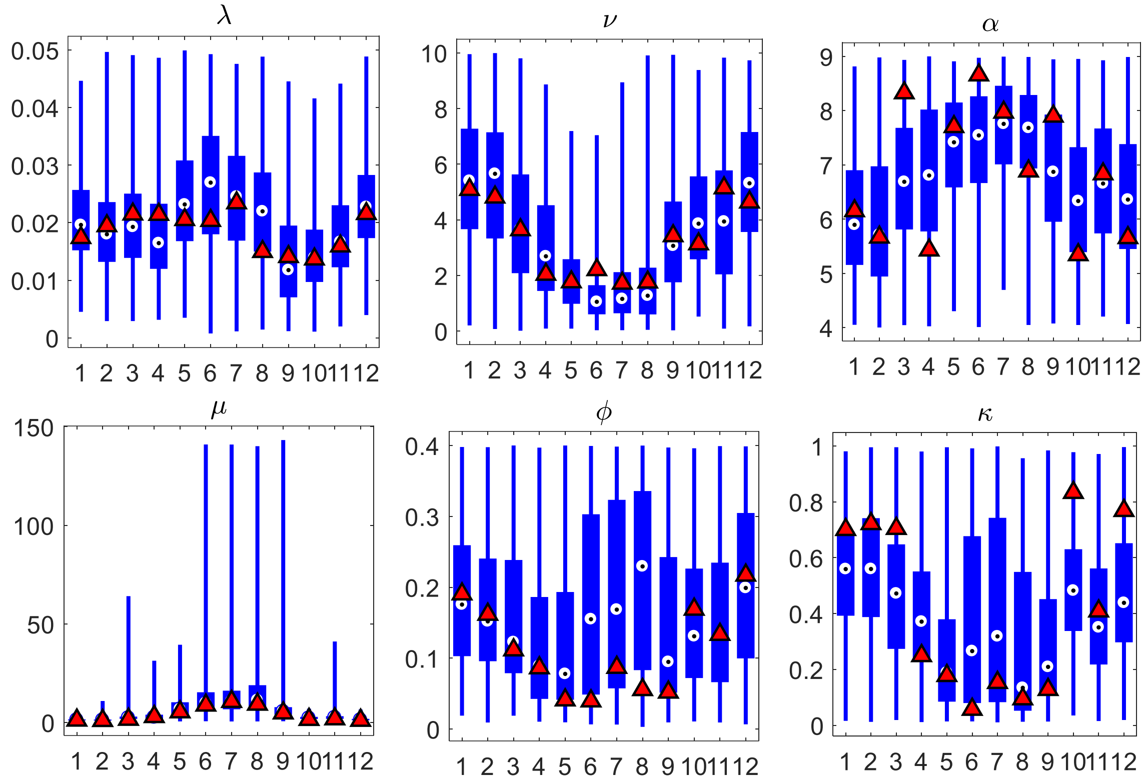


Figure ~~15~~17: Variability of the six parameters of the MBLRP model of this study (box plot) at ~~the~~gage NCDC~~-Gauge-~~-460582 (star mark in Figure 3). The parameters of the traditional MBLRP model are shown together for reference (triangle).

The physical characteristics of rainfall can be estimated using Equation ~~10~~11 and Equation 12 through ~~Equation 14~~15. We repeated the same analysis on these variables to compare the variability of the rainfall characteristics of our hybrid mode and that of the MBLRP model.

~~Average rainfall depth per storm (mm~~ $Average\ rainfall\ depth\ per\ storm\ (mm) = (1 + \frac{\kappa}{\phi})(\frac{\nu}{\alpha})$ ~~μ~~ ~~(10)~~

~~Average storm duration (hr) $\cong \frac{1}{\phi\frac{\alpha}{\nu}\left[1+\phi(\kappa+\phi)-\frac{1}{4}\phi(\kappa+\phi)(\kappa+4\phi)+\frac{1}{72}\phi(\kappa+\phi)(4\kappa^2+27\kappa\phi+72\phi^2)\right]}$ (11)~~

~~Average number of rain cells per storm $= 1 + \frac{\kappa}{\phi}$~~ μ (12)

~~Average rain cell arrival rate (hr^{-1}~~ $Average\ number\ of\ rain\ cells\ per\ storm = 1 + \frac{\kappa}{\phi}$ (13)

$Average\ rain\ cell\ arrival\ rate\ (hr^{-1}) = \kappa\frac{\alpha}{\upsilon}$ ~~(13)~~

~~$Average\ rain\ cell\ duration(hr) = \frac{\nu}{\alpha}$~~ (14)

$$Average\ rain\ cell\ duration(hr) = \frac{\nu}{\alpha} \quad (15)$$

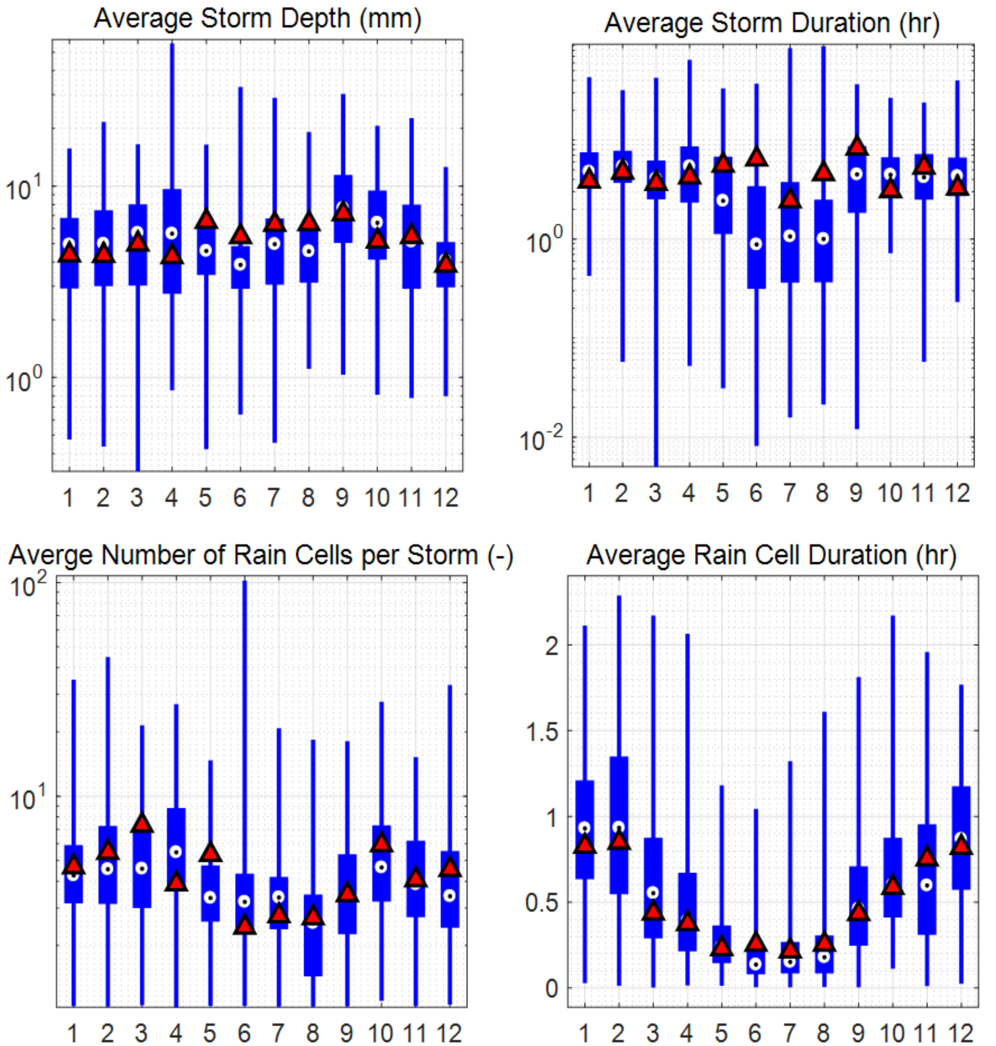


Figure ~~16~~18: Variability of the rainfall characteristics of the MBLRP model of this study (box plot) at ~~the~~gage NCDC ~~Gauge~~ -460582 (star mark in Figure 3). The rainfall characteristics of the traditional MBLRP model are shown together for reference (triangle).

Figure ~~16~~18 shows box plots of the various rainfall characteristics for each month at ~~the~~gage NCDC ~~Gauge~~ -460582. The values were calculated using Equations ~~10~~11 through ~~14~~15. The rainfall characteristics of the traditional MBLRP model are shown together for reference (triangles). The variability of the average storm depth, the average storm duration, and the average number of rain cells per storm was significant, so the y-axes of the box plots were drawn in log-scale. This result suggests that the parameter variability that is incorporated in our model's distinct algorithm contributes to the highly variable external (average storm depth, average storm duration) and internal (average number of rain cells per storm, average rain cell duration) properties of the generated rainfall.

5.2 An Issue with Model Parsimoniousness: six versus fifty five

Our hybrid model uses one MBLRP model parameter set per one simulation month of one year while the MBLRP model needs only 6 parameters regardless of the simulation length. However, this does not mean that our model requires 600 MBLRP model parameters (6 per month × 100 months) to generate 100 months of rainfall. This is because parameters are estimated based on the sub-daily scale rainfall statistics that are synthetically generated through the process of the SARIMA model and the regression analysis (See Figure 5). Therefore, the parameters of the SARIMA model and the parameters of the regression analyses shown in Figure 5 should be considered as the “true” parameters of this model because once these parameters are given, our model can generate infinite length of rainfall record. The SARIMA model has 6 parameters, and a set of regression analysis shown in Figure 5 has 49 parameters (2 for each of ten solid arrows in Figure 5 = 20, 3 per 8 bivariate normal distributions relating two subsequent residual terms (ε_i) in Figure 5 = 24, and one for each of 5 normal distributions perturbing autocorrelation terms (c_i) = 5). Therefore, our model has a total of 55 parameters. This discrepancy of number of parameters (6 for the traditional of MBLRP model versus 55 of our hybrid model) can be considered as a cost taken to reproduce the large-scale rainfall variability that the traditional MBLRP model cannot.

We admit that this large discrepancy of model parsimoniousness is an issue to be resolved for our model to be applied in practice. Regarding this, we are planning to apply our model to additional gage locations across the world and share the result through the website (http://www.letitrain.info). The work has been already initiated for the rainfall data of Korean Peninsula.

5.3 Calibration versus validation

Our approach of separating the period of calibration and validation adopted to some gage locations, may seem surprising because most stochastic rainfall generators are calibrated based upon the statistics under an assumption of temporal stationarity of the rainfall process. According to this assumption, the statistics of the periods of calibration and the validation should be the same, which obviates the needs for validating the model for separate periods. However, this assumption often does not obtain, for example, in case that the observation period is too short (e.g. a few extreme events are included in only one part of the time period) or in when the time series is indeed non-stationary. For this reason, the discrepancy of the model performance between the calibration and the validation period should not only attributed to the model's limitations but also to the difference between statistics from the two periods. In view of these considerations, our primary purpose of separating the period of calibration and validation should be understood as an assessment of the model’s applicability to rainfall generation for a future period. From the point of view of the calibration period, the validation period is an ungagged period just as any future period, and our model can be easily extended to the future period by adding a term accounting for long-term rainfall non-stationarity to the SARIMA model (first module). Our hybrid model assumes not only the stationarity of the typical rainfall statistics such as mean, variance, covariance and proportion of dry periods but also the relationship between them (See Figure 6). The latter has not been explicitly discussed by previous studies, so it was also interesting to see whether such relationships between the statistics hold over different temporal periods and how the discrepancy affects the final model performance, if there is any. Figure 19 compares the slope of the regression analysis between the statistics shown in Figure 6 for the calibration (x axis) and validation (y axis) periods. The plots corresponding to the variances at different scales are not shown because there are theoretical reasons for having a solid slope close to 2 (See Equation 5 and the preceding equations). There is no a significant discrepancy between slopes estimated using statistics on calibration and validation period implying that relationships between the fine time scale statistics are stationary and that our model can be used for future or ungagged periods.

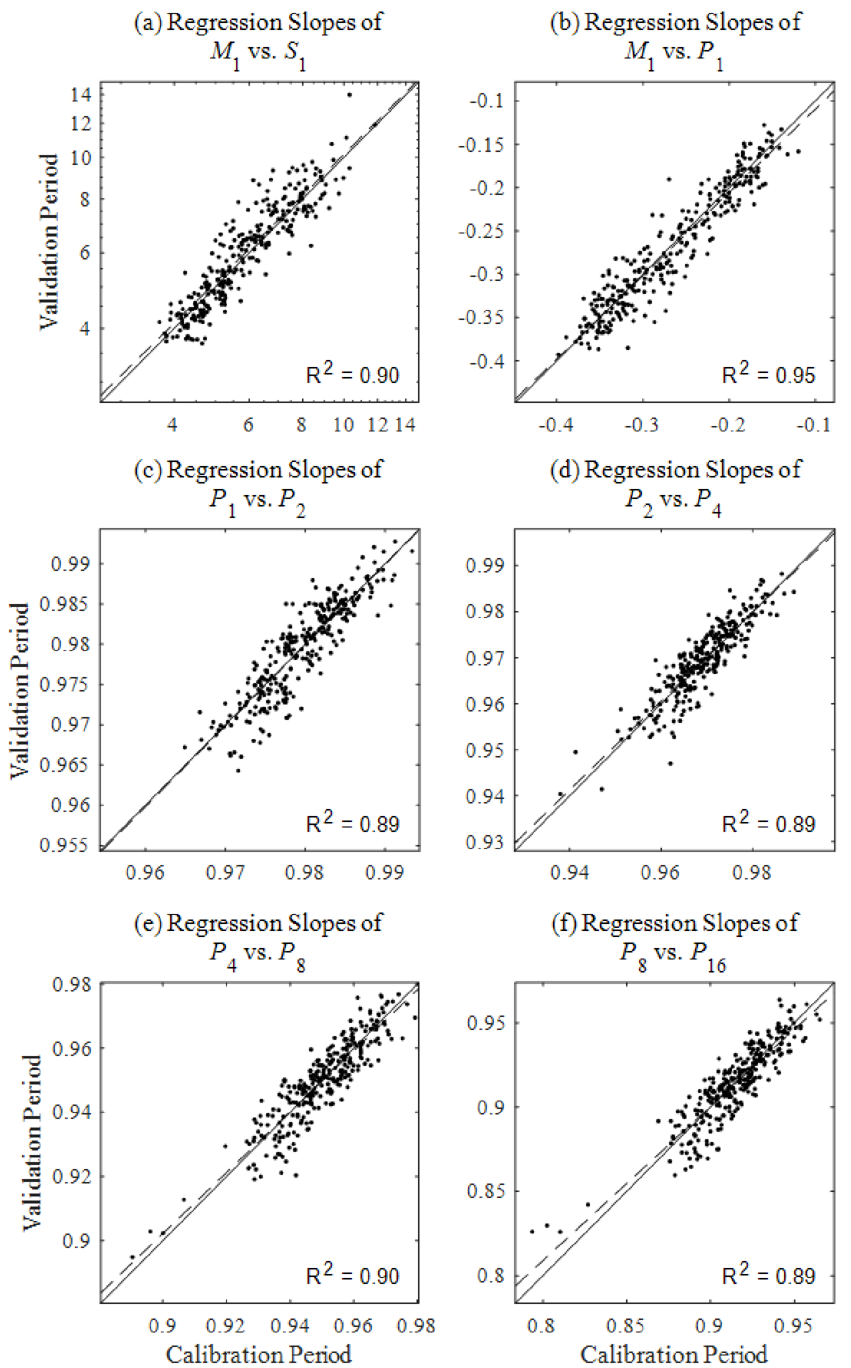


Figure 19: Comparison of the slope of regression analysis between the statistics shown in Figure 6 for the calibration (x) and validation (y) period. The slopes of regression analysis (a) between mean and standard deviation and (b) between mean and proportion of dry periods and (c)-(f) between proportion of dry periods at the different time scale were compared. Solid lines are 1:1 line and dashed lines represent the regression lines.

6 Conclusion

The phenomena observed in hydrologic systems and the subsequent effects on human and environmental systems are the consequences of the complex interactions between the components that are influenced by rainfall variability at various ranges of time ~~scale~~scales. Therefore, a good or realistic rainfall model must properly reflect the rainfall variability at all hydrologically relevant time scales. Its importance will gather more attentions because of the recent trend of the hydrologic societies that started to recognize the hydrologic, human, and environmental systems from a holistic view point and interpret them based on continuous and dynamic simulation as opposed to the event-based ones (Wagener et al., 2010).

This study is one of many recent efforts in this regard (Fatichi et al., 2011; Kim et al., 2013a; Paschalis et al., 2014). First, we showed that the Poisson cluster rainfall model, which is probably the most widely applied stochastic rainfall models, cannot reproduce large-scale rainfall variability due to in-built limitations that lie in the model assumptions. Then, we showed that a combination of an autoregressive model for monthly time scale and the "well-tuned" Poisson cluster rainfall model for the finer ranges of time scale is capable of reproducing not only the first through the third order statistics of the rainfall depths, but also the intermittency properties of the observed rainfall.

An additional model could be integrated to our hybrid model to incorporate further rainfall variability. For example, an approach based on random cascades (Molnar and Burlando, 2005; Müller and Haberlandt, 2016; Pohle et al., 2018) can be integrated to our model for reproducing the rainfall variability at the time scale as fine as five minutes. In addition, the SARIMA model that was adopted in this study could be further modified to account for the coarser rainfall variability caused by El Niño-Southern Oscillation (ENSO) and North Atlantic Oscillation (NAO). Lastly, the genuine structure of our model that is composed of a large scale rainfall generation module and a downscaling module, may be integrated to existing space-time rainfall generators to enhance their ability to generate large temporal-scale rainfall variability (Burton et al., 2008, Müller and Haberlandt, 2015, Paschalis et al., 2013; Peleg and Morin, 2014; Peleg et al., 2017; Benoit et al., 2018).

7 Data Availability

Our hybrid model is not easy to implement because it requires extensive analysis of the correlation structure of the fine-scale rainfall statistics to fine-tune the MBLRP model to downscale the generated monthly rainfall. For this reason, we shall continue our work on all possible rain ~~gauge~~gage locations across the world and share the results (several hundred years of synthetic rainfall data in text format) through the following website: http://letitrain.info. We ask for cooperation from the international community to share their rainfall data.

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