



1 **Exploring the relationship between warm season precipitation, potential evaporation, and**  
2 **“apparent” potential evaporation at site scale**

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21 **Abstract**

22 Bouchet’s complementary relationship and the Budyko hypothesis are two classic frameworks  
23 that are inter-connected. To systematically investigate the connections between the two  
24 frameworks, we analyze precipitation, pan evaporation and potential evaporation data at 259  
25 weather stations across the United States. The precipitation and pan evaporation data are from  
26 field measurement and the potential evaporation data are computed from a remote-sensing  
27 dataset. We use pan evaporation to represent “apparent” potential evaporation, which is different  
28 from potential evaporation. With these data, we study the correlations between precipitation and  
29 potential evaporation, and between precipitation and “apparent” potential evaporation. The  
30 results show that 93% the study weather stations exhibit a negative correlation between  
31 precipitation and “apparent” potential evaporation. Also, the aggregated data cloud of  
32 precipitation versus “apparent” potential evaporation with 5312 warm season data points from  
33 259 weather stations shows a negative trend in which “apparent” potential evaporation decreases  
34 with increasing precipitation. On the other hand, no significant correlation is found in the data  
35 cloud of precipitation versus potential evaporation, indicating that precipitation and potential  
36 evaporation are independent. We combine a Budyko-type expression, the Turc-Pike equation,  
37 with the Bouchet’s complementary relationship to derive upper and lower Bouchet-Budyko  
38 curves, which display a complementary relationship between “apparent” potential evaporation  
39 and actual evaporation. The observed warm season data follow the Bouchet-Budyko curves  
40 well. Our study shows the consistency between Budyko’s framework and Bouchet’s  
41 complementary relationship, with the distinction between potential evaporation and “apparent”  
42 potential evaporation. The formulated complementary relationship can be used in quantitative  
43 modeling practices.



## 44 1. Introduction

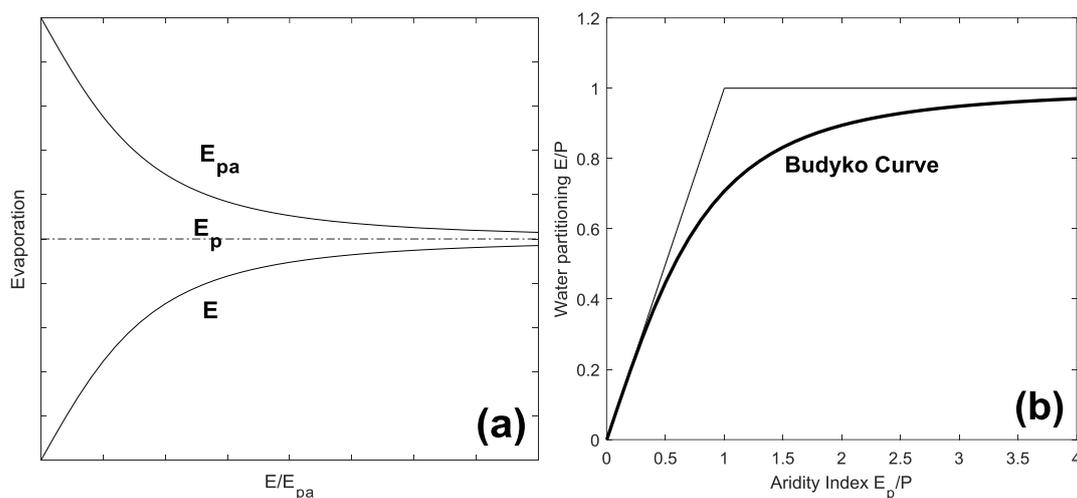
45 Potential evaporation ( $E_p$ ) is a widely used physical variable in hydrologic frameworks. It is the  
46 evaporation rate under two conditions: firstly, the land surface water supply is unlimited  
47 (Thornthwaite, 1948); secondly, the surface vapor pressure is saturated (Van Bavel, 1966;  
48 Brutsaert, 2015). Pan evaporation ( $E_{pan}$ ) measurement is often used as a surrogate of potential  
49 evaporation. However, these two variables are not the same (Brutsaert and Parlange, 1998;  
50 Roderick, et al., 2009). The main difference between potential evaporation and pan evaporation  
51 is that pan evaporation is not measured under saturated surface vapor pressure. As a result,  
52 potential evaporation can be considered to depend only on the energy supply of climate while  
53 pan evaporation is driven by both energy supply and humidity deficit in the atmosphere  
54 (Rotstayn, et al., 2006). In Brutsaert and Parlange (1998), the term “apparent” potential  
55 evaporation ( $E_{pa}$ ) is introduced to distinguish pan evaporation from potential evaporation.  
56 “Apparent” potential evaporation can be measured by evaporation pan, while potential  
57 evaporation cannot.

58 Because potential evaporation is energy-driven, it can be used as a physical variable to  
59 describe energy supply in a hydrologic system. For instance, the well-established Budyko  
60 framework (Budyko, 1958; 1974) uses the relationship between precipitation ( $P$ ) and potential  
61 evaporation to represent the relationship between water supply and energy supply. The Budyko  
62 framework has been extensively used to analyze interactions between hydrology, climate,  
63 vegetation and other elements in watersheds (Milly, 1994; Zhang et al., 2001; Yang et al., 2007;  
64 Donohue et al., 2007; Yang, et al., 2011; Xu, et al., 2014; Zhou et al., 2015; Zhou et al., 2016).  
65 Several studies have made connections between the Budyko framework and the Bouchet’s  
66 complementary relationship (CR) (Bouchet, 1963). Yang et al. (2006) used the Fu’s equation



67 (Fu, 1981), which is one of the commonly used equations to represent the Budyko's curve, to  
68 describe the relationship between actual evaporation and potential evaporation in the CR.  
69 Roderick et al. (2009) presented a complementary relationship normalized by net irradiance and  
70 compared it with the Budyko framework. Lhomme and Moussa (2016) combined Turc-Pike  
71 equation (Turc, 1954; Pike, 1964), which is another commonly used Budyko-type equation, with  
72 the CR to show the dependence of Budyko curve on the drying power of the air.

73 When linking Budyko framework with the CR, it is crucial to have a clear definition of  
74 different types of evaporation used in these two frameworks. Brutsaert and Parlange (1998) and  
75 Brutsaert (2015) generalized the CR and provided definitions of the evaporation terms in the CR,  
76 namely actual evaporation ( $E$ ), potential evaporation ( $E_p$ ), and “apparent” potential evaporation  
77 ( $E_{pa}$ , see Fig. 1a). As described previously, potential evaporation, following the original  
78 potential evaporation definition (Thornthwaite, 1948; Van Bavel, 1966), is the evaporation rate  
79 under saturated surface vapor pressure and unlimited land surface water supply; while “apparent”  
80 potential evaporation, which can be measured by evaporation pan, is the evaporation rate under  
81 unlimited land surface water supply, but not under saturated vapor pressure. It is clarified in  
82 Brutsaert and Parlange (1998) that the complementary relationship is between actual evaporation  
83 and “apparent” potential evaporation, not between actual evaporation and potential evaporation.  
84 In the Budyko framework (Fig. 1b), the definition of potential evaporation is also following the  
85 original potential evaporation definition that it is under unlimited land surface water supply and  
86 saturated vapor pressure (Budyko, 1974).



87

88 Fig. 1. Conceptual representations of (a) the complementary relationship and (b) Budyko

89 framework.

90 Our study investigates the relationship between precipitation and potential evaporation as  
91 well as between precipitation and “apparent” potential evaporation. We collect warm season  
92 precipitation, potential evaporation and pan evaporation data from 259 weather stations across  
93 the contiguous US. Studying the relationship between  $P$ ,  $E_p$  and  $E_{pa}$ , advances our understanding  
94 of the well-established classic Budyko framework and the CR. Furthermore, based on insights  
95 provided by previous studies (Yang et al., 2006; Roderick et al., 2009; Lhomme and Moussa,  
96 2016), we use a Budyko-type expression to develop a new formulation for the CR.

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## 98 2. Methodology

### 99 2.1 Theoretical development

#### 100 2.1.1 Budyko framework



101 The Budyko curve (Fig. 1b) describes the relationship between long-term water partitioning,  
102 represented by the ratio of actual evaporation over precipitation, and long-term climate,  
103 represented by the ratio of potential evaporation over precipitation, namely aridity index  
104 (Budyko, 1958; 1974). In recent decades, the Budyko framework has been examined with  
105 annual data (e.g. Yang et al., 2007; Potter and Zhang, 2009; Cheng et al., 2011). A number of  
106 Budyko-type functions have been developed to mathematically describe the Budyko curve (Turc,  
107 1954; Fu, 1981; Zhang, et al., 2001; Yang et al., 2008; Wang and Tang, 2014). Within these  
108 functions, the Turc-Pike equation is a parsimonious single parameter equation (Turc, 1954; Pike,  
109 1964):

$$110 \quad \frac{E}{P} = \left[ 1 + \left( \frac{E_p}{P} \right)^{-v} \right]^{-\frac{1}{v}} \quad (1)$$

111 where  $E$  is actual evaporation,  $E_p$  is potential evaporation,  $P$  is precipitation, and  $v$  is a parameter  
112 to represent landscape properties such as vegetation coverage and soil properties (Zhang, et al.,  
113 2001; Yang, et al., 2008). The parameter  $v$  needs to be a positive number, and its typical value is  
114 2.0.

### 115 2.1.2 Generalized complementary relationship

116 Bouchet's complementary relationship (Bouchet, 1963) is to describe the relationship between  
117 actual evaporation  $E$  and potential evaporation  $E_p$ . Brutsaert and Parlange (1998) introduced the  
118 term "apparent" potential evaporation  $E_{pa}$  and clarified that the CR is between  $E$  and  $E_{pa}$ , not  $E$   
119 and  $E_p$  (Fig. 1a). They also proposed a generalized complementary relationship:

$$120 \quad bE + E_{pa} = (1 + b)E_p \quad 0 \leq E \leq E_p \leq E_{pa} \quad (2)$$



121 where  $b$  is a proportionality parameter not less than one. When  $b$  is equal to one, Eq. (2)  
122 represents the original complementary relationship (Kahler and Brutsaert, 2006). “Apparent”  
123 potential evaporation will be higher than potential evaporation under dry condition; while it  
124 gradually approaches potential evaporation as ratio of  $E$  over  $E_{pa}$  increases (Fig. 1a). As  
125 suggested by Morton (1976) and Brutsaert and Stricker (1979), potential evaporation can be  
126 estimated using the Priestley-Taylor equation (Priestley and Taylor, 1972). “Apparent” potential  
127 evaporation can be estimated using the Penman equation (Penman, 1948; Linacre, 1994;  
128 Rotstayn et al., 2006) or using data measured at evaporation pans (Brutsaert, 1982; Brutsaert and  
129 Parlange, 1998):

$$130 \quad E_{pa} = aE_{pan} \quad (3)$$

131 where  $E_{pan}$  is the pan evaporation and  $a$  is the pan coefficient. The pan coefficient varies from  
132 location to location (Stanhill, 1976; Linacre, 1994). In Kahler and Brutsaert (2006), a pan  
133 coefficient of  $a = 1.0$  is recommended for mixed natural vegetation, which will be used in this  
134 study. It should be noted that the linear relationship between  $E_{pa}$  and  $E_{pan}$  given in Eq. (3) and  
135 the choice of “ $a$ ” value will not affect the correlations between  $P$ ,  $E_p$  and  $E_{pa}$ .

### 136 2.1.3 Relationships between $P$ , $E_p$ and $E_{pa}$

137 The x-axis of the complementary relationship is a ratio between  $E$  and  $E_{pa}$  (Bouchet, 1963).  
138 Ramírez et al. (2005) used the water-energy framework to link the CR with Budyko approach  
139 and changed the x-axis in the CR to moisture availability. Following this idea, several studies  
140 have used precipitation or wetness index ( $P/E_p$ ) to represent moisture availability in the CR  
141 (Yang et al., 2006; Roderick et al., 2009). In this study, we also use  $P$  to represent moisture  
142 availability in the CR. The  $E_p$  is a horizontal line in the CR that is in parallel with the x-axis



143 (Fig. 1a), which is now represented by  $P$ . Therefore, the modified CR is indicating that  $P$  and  $E_p$   
 144 are independent. On the other hand, the upper curve of the CR, which is representing “apparent”  
 145 potential evaporation  $E_{pa}$ , is declining along the x-axis, indicating that  $E_{pa}$  and  $P$  are not  
 146 independent. After changing the x-axis in the CR to  $P$ , to have a dimensionless CR, we  
 147 normalize the x and y axes in the CR. The normalized CR describes the relationship between  
 148  $\frac{E_{pa}}{E_p}$ ,  $\frac{E}{E_p}$ , and  $\frac{P}{E_p}$  (Fig. 2).

149 To connect Budyko framework with the normalized CR, and therefore to formulate the  
 150 Bouchet-Budyko curves, we first transform Eq. (1) into a relationship between  $\frac{E}{E_p}$  and  $\frac{P}{E_p}$ :

$$151 \quad \frac{E}{E_p} = \left[ \left( \frac{P}{E_p} \right)^{-v} + 1 \right]^{-\frac{1}{v}} \quad (4)$$

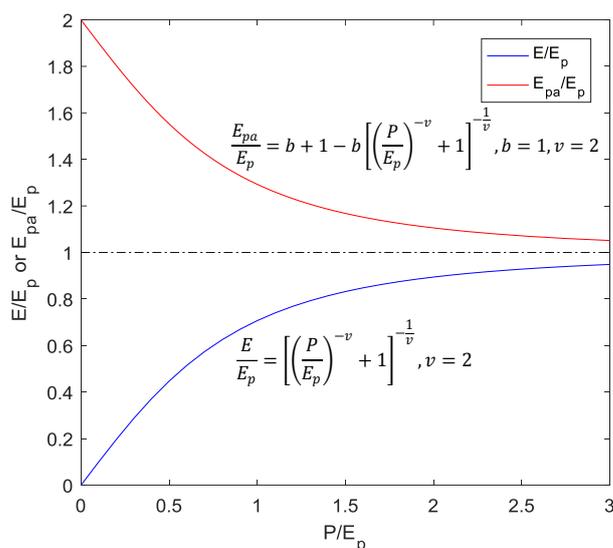
152 Yang et al. (2006) did similar transformation using Fu’s equation (Fu, 1981). Dividing both  
 153 sides of Eq. (2) by  $E_p$  yields:

$$154 \quad b \frac{E}{E_p} + \frac{E_{pa}}{E_p} = 1 + b \quad (5)$$

155 Combining Eqs. (4) and (5), gives a relation between  $\frac{P}{E_p}$  and  $\frac{E_{pa}}{E_p}$ :

$$156 \quad \frac{E_{pa}}{E_p} = b + 1 - b \left[ \left( \frac{P}{E_p} \right)^{-v} + 1 \right]^{-1/v} \quad E_{pa} \geq E_p \quad (6)$$

157 Equations (4) and (6) represent the lower and upper curves of the normalized CR  
 158 respectively (Fig. 2). Roderick et al. (2009) presented a similar framework, without the  
 159 formulation of the curves. To verify the relationships between precipitation, potential  
 160 evaporation and “apparent” potential evaporation and to examine the Bouchet-Budyko curves in  
 161 Eqs. (4) and (6), we analyze climate data from 259 weather stations across the contiguous US.



162

163 Fig. 2. Dimensionless Bouchet-Budyko curves in the normalized complementary relationship.

164 2.2 Data sources

165 Monthly precipitation and pan evaporation are collected from the National Oceanic and  
 166 Atmospheric Administration (NOAA) at the National Climatic Data Center (NCDC). The data  
 167 can be downloaded at: <https://www.ncdc.noaa.gov/IPS/cd/cd.html>. The precipitation data are  
 168 measured using standard rain gauge and the pan evaporation data using Class A evaporation  
 169 pans. We collect data for the period 1984-2015 from a total of 259 weather stations (Fig. 3a).  
 170 Since pan evaporation is collected only during warm months (when temperatures remain above  
 171 freezing), the weather stations at cold regions have less than 12 months of pan readings in a year.  
 172 We calculate the monthly average of pan evaporation and precipitation using only the warm  
 173 months with available pan evaporation data for each year each weather station. The calculated  
 174 warm month averages are used to represent warm season pan evaporation and precipitation in  
 175 each year. For short, it is called warm season data (i.e., warm season pan evaporation, warm  
 176 season precipitation). We also calculate the annually averaged warm season data to represent the



177 long term average level of pan evaporation and precipitation at each station. For short, it is  
178 called long term average data. Over the 259 selected stations, there is an average of seven  
179 months per year with warm season pan evaporation. As Fig. 3 shows, the number of available  
180 months decreases from Southern regions to Northern regions. On the other hand, not all 259  
181 weather stations have the full record from 1984 to 2015, the average number of years with  
182 available data for each location is 18. A complete summary of the information available at all  
183 259 weather station is provided in Table S1. In order to minimize the uncertainty from various  
184 warm periods in a year from station to station, we repeat the analysis using an alternative source  
185 of pan evaporation in the NCDC dataset containing homogenized warm month data from May to  
186 October (Hobbins, et al., 2017). A total of 93 weather stations overlap both sets of pan  
187 evaporation data for the period 1984 to 2001 (Fig. 3b). After data collection, we convert pan  
188 evaporation to “apparent” potential evaporation using Eq. (3). The potential evaporation  $E_p$  data  
189 are generated using the Priestley-Taylor equation with remotely sensed net radiation (Zhang et  
190 al., 2010):

$$191 \quad E_p = \alpha \frac{\Delta}{\Delta + \gamma} (R_n - G) \quad (7)$$

192 where  $\alpha$  is a coefficient to account for the effect of surface characteristics and vegetation, and is  
193 set to 1.26;  $\Delta$  is the slope of the saturated vapor pressure curve;  $\gamma$  is the psychometric constant;  
194  $R_n$  is the net radiation; and  $G$  is the heat flux into the ground. The  $E_p$  data cover the period 1983-  
195 2006. Similar with  $P$  and  $E_{pan}$ , we calculate the warm season  $E_p$  and long term annually  
196 averaged warm season  $E_p$  based on the monthly  $E_p$  data.

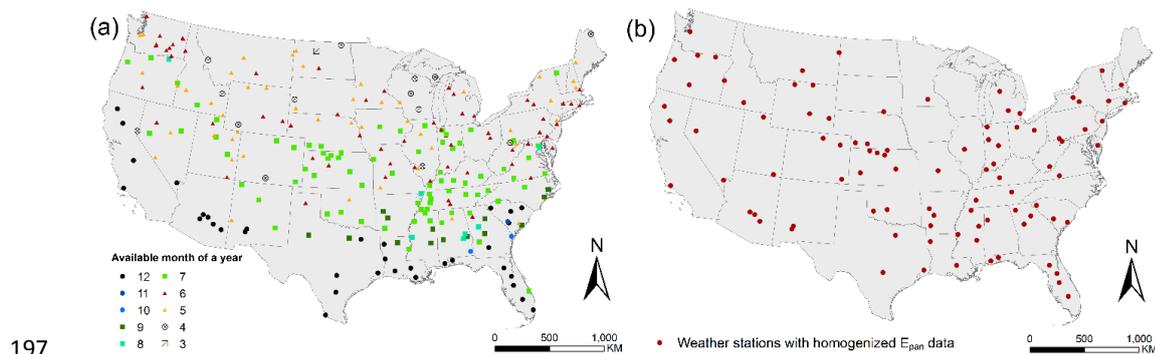


Fig. 3. (a) Map of 259 weather stations. The available month of a year of pan evaporation data for each weather station is presented using legends with different colors and shapes. (b) Map of 93 weather stations with homogenized pan evaporation data.

### 2.3 $P$ , $E_p$ and $E_{pa}$ correlation analysis

Using the weather station data of precipitation and pan evaporation for the period 1984 to 2015, we first calculate the Pearson correlation coefficient between warm season  $P$  and warm season  $E_{pa}$  for each location (Fig. 3a). We then perform the same correlation analysis of  $P$  and  $E_{pa}$  using the homogenized pan evaporation dataset (Hobbins et al., 2017) (Fig. 3b). Secondly, we use data of warm season  $P$  and warm season  $E_p$  for the period of 1984 to 2006, which is the period both  $P$  and  $E_p$  data are available, to investigate the correlation between  $P$  and  $E_p$ . Finally, to validate the newly derived Bouchet-Budyko curves, the relationship between  $\frac{P}{E_p}$  and  $\frac{E_{pa}}{E_p}$  is plotted using the collected data at both seasonal and long term average time scales.

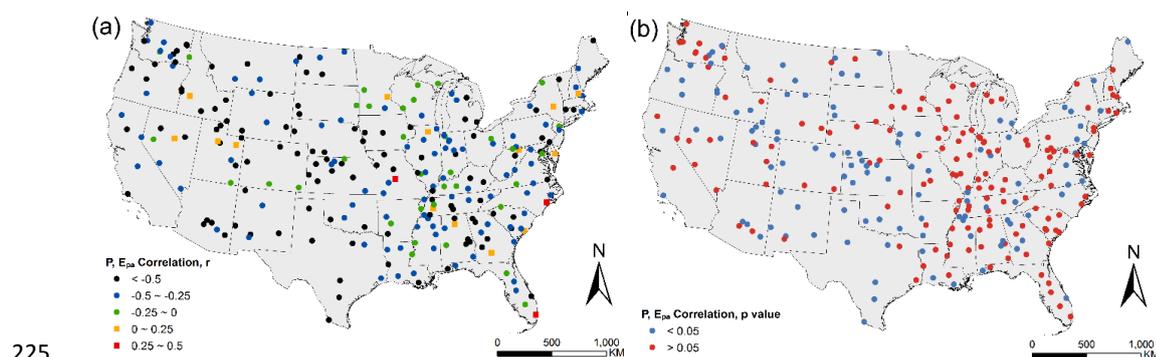
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## 211 3. Results

### 212 3.1 Correlations among $P$ , $E_p$ , and $E_{pa}$



213 In the 259 weather stations, 93% of the stations have a negative correlation between  $P$  and  $E_{pa}$   
214 (Fig. 4a). However, only 43% of the stations have statistically significant correlation ( $p < 0.05$ )  
215 between  $P$  and  $E_{pa}$  (Fig. 4b). All the weather stations with significant  $P$ ,  $E_{pa}$  correlation have  
216 negative correlation. The weather stations located in the western region are more likely to have a  
217 significant  $P$ ,  $E_{pa}$  negative correlation than in the east. This spatial difference may be related to  
218 climate characteristics that eastern region has higher precipitation and lower aridity index, while  
219 the western region has lower precipitation and higher aridity index. The Bouchet's  
220 complementary relationship is more significant in arid regions (Ramírez et al., 2005),  
221 corresponding to the left side of the CR curves; while it is less significant in humid regions,  
222 corresponding to the right side of the CR curves (Fig. 1a). As a result, the negative correlation  
223 between precipitation and “apparent” potential evaporation is more significant in the west than in  
224 the east.



226 Fig. 4. Map of point scale annual  $P$ ,  $E_{pa}$  correlation at 259 weather stations, (a)  $r$  value and (b)  $p$   
227 value.

228 We then plot all the warm season data of each year each station, totally 5312 data points,  
229 on a  $P$  vs.  $E_{pa}$  figure (Fig. 5a). The data cloud shows a negative trend in general. We also plot  
230 the long term annually averaged values of warm season  $P$  and  $E_{pa}$  of the 259 weather stations



231 (Fig. 5b), which shows a similar negative trend. Hobbins et al. (2004) showed a similar negative  
232 trend between  $P$  and  $E_{pan}$  with watershed scale data. To represent the spatial distribution of the  
233 weather stations, we color code the data points based on their spatial coordinates of latitude and  
234 longitude. The climate in the eastern US is much wetter than the western US, and therefore the  
235 data cloud of  $E_{pa}$  vs.  $P$  is separated into two parts horizontally. The right side cloud is mostly  
236 green and brown, representing the northeastern and southeastern areas of the US, respectively;  
237 while the left side cloud is mostly yellow and red, representing the northwestern and  
238 southwestern areas, respectively. The left side cloud is more vertically oriented, indicating that  
239 the western region has higher  $E_{pa}$  variability than  $P$  variability. Southwestern region has the  
240 highest  $E_{pa}$  in the US, represented by the red and orange points. Northwestern region has much  
241 lower  $E_{pa}$ , represented by the yellow points. On the other hand, the right side cloud is more  
242 horizontally oriented, indicating that the eastern region has higher  $P$  variability than  $E_{pa}$ . Unlike  
243 the western region, the difference between the northeastern and southeastern regions are not very  
244 distinguishable. Southeastern region of the US has a wide range of precipitation; while points of  
245 the northeastern region are more concentrated.

246 As explained before, we also use an alternative pan evaporation dataset (Hobbins et al.,  
247 2017) to further validate our analysis result. This dataset is homogenized to have the same  
248 period of pan evaporation data record in each year from May to October and therefore to  
249 minimize the uncertainty from the various length of warm period from station to station. It is  
250 derived from the same dataset as our data, namely the NCDC dataset. Based on the  
251 homogenized pan evaporation data, 85 stations out of 93 (91%) have a negative correlation  
252 between  $P$  and  $E_{pa}$ . Only 41% of the stations have a statistically significant correlation ( $p < 0.05$ ).  
253 All the significant correlations are negative. This result is consistent with the analysis result

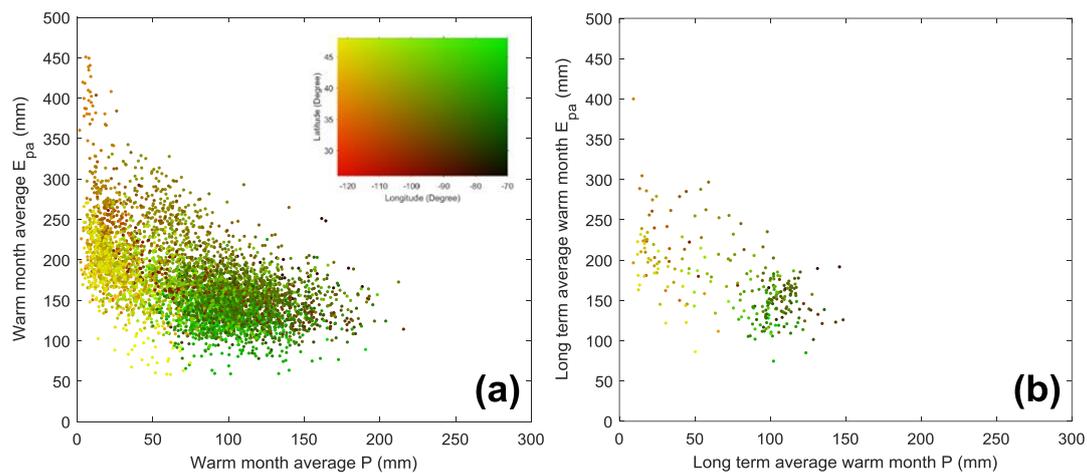


254 based on our collected data from 259 weather stations. We also use the data cloud to show the  
255 relationship between  $P$  and  $E_{pa}$  in the warm period of May to October in each year at each of the  
256 93 stations (Fig. 5c), as well as the relationship of long term annually averaged warm period  $P$   
257 and  $E_{pa}$  (Fig. 5d). The trend of data cloud is similar with the data cloud trend using our collected  
258 data at both seasonal and long term average time scales. In other words, both datasets show a  
259 negative relationship between  $P$  and  $E_{pa}$ .

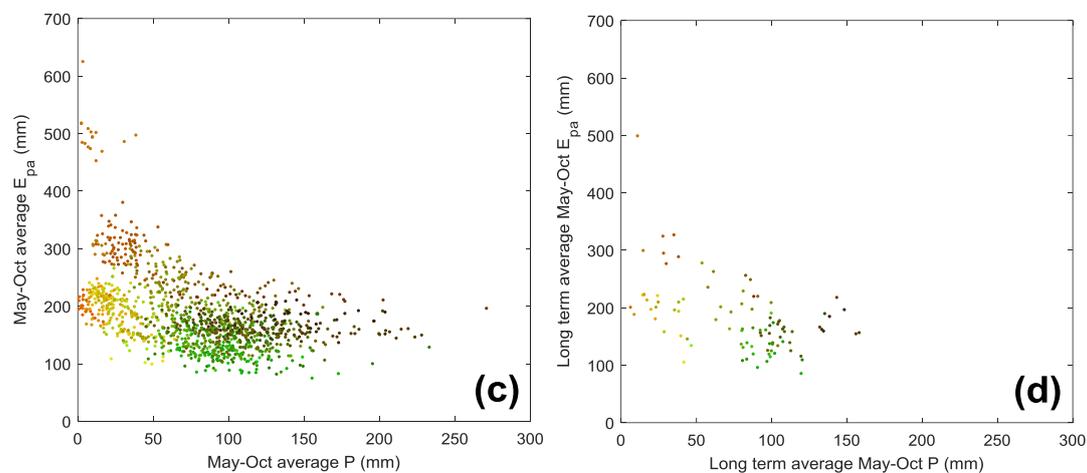
260 We then plot the relationship between  $P$  and  $E_p$  (Fig. 5e and 5f), using the  $E_p$  data  
261 generated by a remote-sensing algorithm based on the Priestley-Taylor equation as explained  
262 previously (Zhang et al., 2010). At both seasonal and long term average time scales, there is no  
263 clear relationship shown between  $P$  and  $E_p$ , confirming the independence between  $P$  and  $E_p$   
264 discussed in Section 2.1.3. This result shows the difference between  $E_p$  and  $E_{pa}$ , that  $E_p$  is  
265 independent from  $P$  but  $E_{pa}$  is not. Therefore, it is important to distinguish  $E_{pa}$  from  $E_p$  and to  
266 understand the different physical mechanisms of the two processes (Brutsaert, 2015).



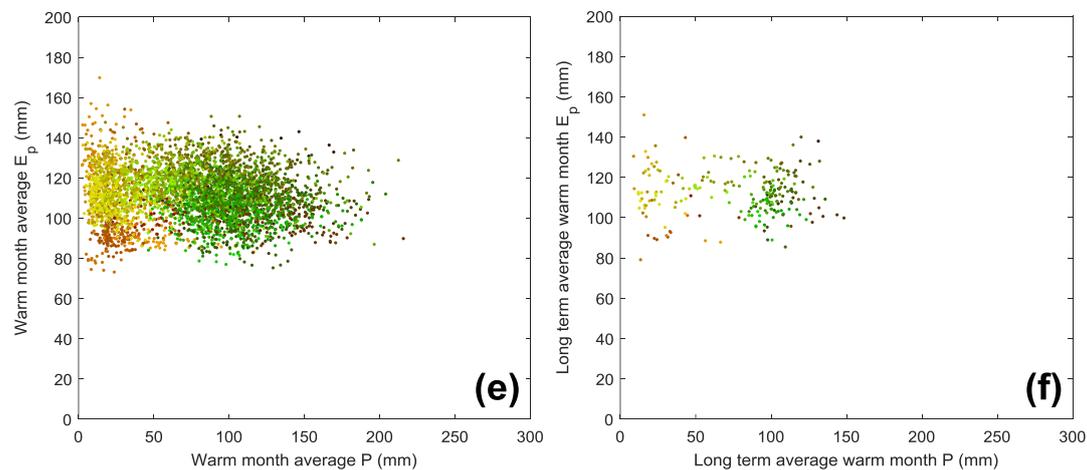
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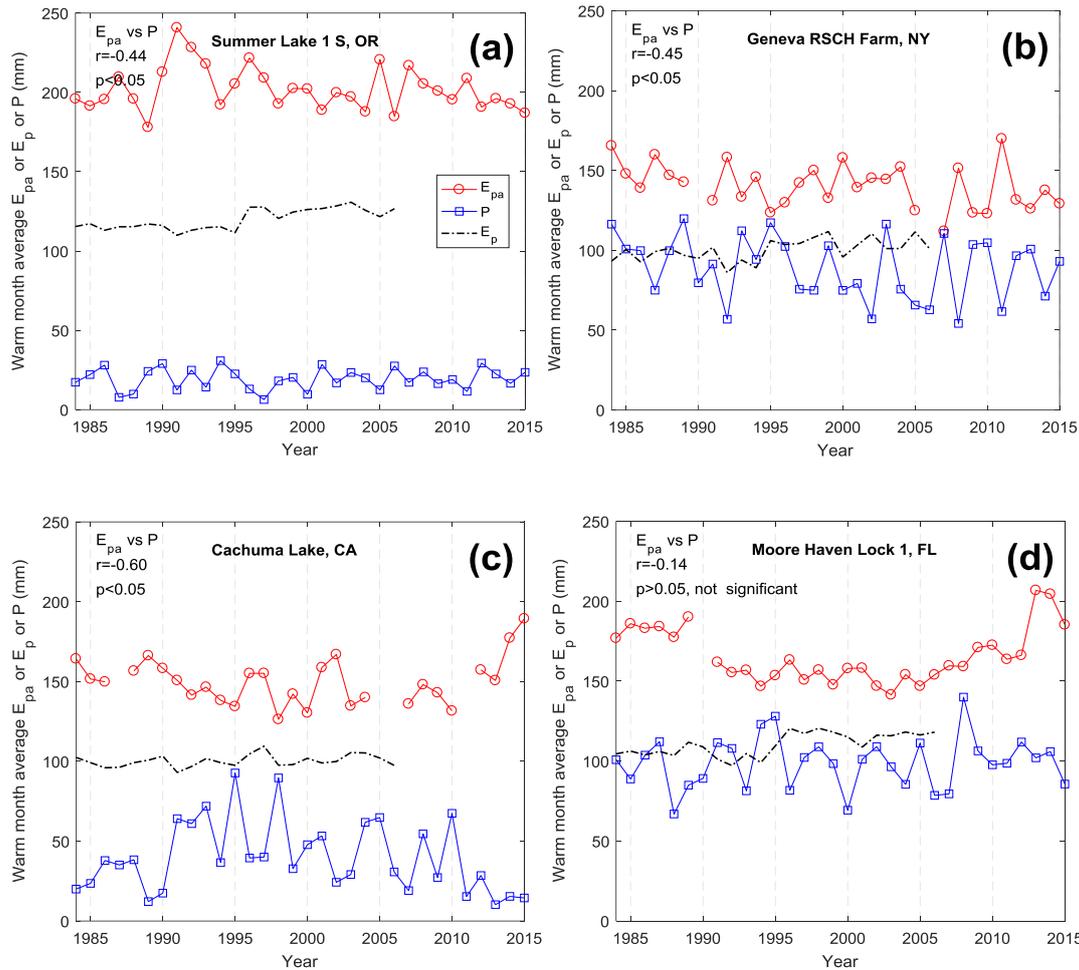
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270 Fig. 5.  $P$  vs.  $E_{pa}$  at 259 weather stations in the US for the period 1984 to 2015 for **(a)** warm  
271 season data (N=5312), and **(b)** long term annually averaged warm season data (N=259). The  
272 data points are color coded based on their latitudes and longitudes.  $P$  vs.  $E_{pa}$  at 93 weather  
273 stations in the US for the period 1984 to 2001 using the homogenized pan evaporation dataset for  
274 **(c)** warm period May-Oct in each year (N=1214), and **(d)** long term annual average warm period  
275 May-Oct data (N=93).  $P$  vs.  $E_p$  at the 259 weather stations for the period of 1984 to 2006 for **(e)**  
276 warm season data (N=5312) and **(f)** long term annual average warm season data (N=259).

277 To present the  $P$ ,  $E_p$  and  $E_{pa}$  relationship at individual locations and therefore to further  
278 investigate the dependence between the three variables, we select four weather stations from the  
279 northwest, northeast, southwest, and southeast regions respectively to show the warm season  $P$ ,  
280  $E_p$  and  $E_{pa}$  in time series (Fig. 6). The two stations in the southern regions have data in all 12  
281 months of a year; while the two stations in the northern regions only have pan evaporation data  
282 in warm months of 6 or 7 months of a year. All four stations show negative correlations between  
283  $P$  and  $E_{pa}$ . This negative correlation at the selected weather station in Florida is not statistically  
284 significant (Fig. 6d). As mentioned before, the  $P$  and  $E_{pa}$  correlation is less significant in the  
285 eastern region than in the west, because of the higher humidity in the east. On the other hand, at  
286 the other three locations, the warm season  $P$  and  $E_{pa}$  are relatively symmetric to each other (Fig.  
287 6a, 6b, and 6c). During years when one series is above average, the other tends to be below  
288 average and vice versa. In terms of the relationship between  $P$  and  $E_p$ , all four locations show no  
289 significant correlations between the two variables ( $p>0.05$ ). This is consistent with the  
290 independence of  $P$  and  $E_p$  shown in Fig. 5e and 5f.



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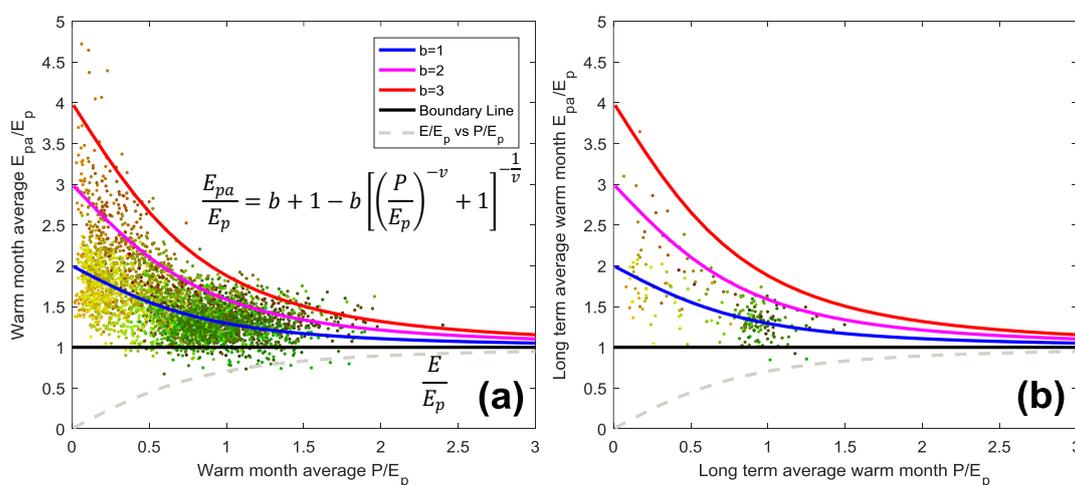
293 Fig. 6. Warm season  $P$ ,  $E_p$  and  $E_{pa}$  time series of four example weather stations in the study  
 294 period of 1984-2015: **(a)** Summer Lake 1 S, OR (N 42°58', W 120°47'); **(b)** Geneva RSCH  
 295 Farm, NY (N 42°53', W 77°20'); **(c)** Cachuma Lake, CA (N 34°35', W 119°59'); and **(d)** Moore  
 296 Haven Lock 1, FL (N 26°50', W 81°50').

297 3.2 Bouchet-Budyko curves

298 There are two Bouchet-Budyko curves (Fig. 2). The upper curve describes the relationship  
 299 between  $E_{pa}$ ,  $E_p$  and  $P$  (Eq. 6) and the lower curve describes the relationship between  $E$ ,  $E_p$  and  $P$



300 (Eq. 4). The lower curve is derived from the Budyko curve based on Turc-Pike equation. This  
 301 relationship between  $E$ ,  $P$  and  $E_p$  has been studied extensively following the Budyko framework,  
 302 and it is therefore not the focus of this study. This study investigates the relationship between  
 303  $E_{pa}$ ,  $E_p$  and  $P$ , which is represented by the upper Bouchet-Budyko curve. Since the collected  
 304 weather station data of  $P$  and  $E_{pan}$  are available 1984 to 2015 and the  $E_p$  data generated from  
 305 remote-sensing algorithm are available 1983 to 2006, we study the relationship between  $P/E_p$  and  
 306  $E_{pa}/E_p$  in the overlapping period of 1984 to 2006. Based on warm season data of  $P$ ,  $E_p$  and  $E_{pan}$ ,  
 307 we plot the relationship between  $P/E_p$  and  $E_{pa}/E_p$  (Fig. 7). We draw three curves using Eq. (6)  
 308 with different  $b$  values of 1, 2, and 3. The  $\nu$  value is set at 2, which is a commonly used value in  
 309 the Budyko framework. When  $b$  exceeds one, the two CR curves are asymmetric. This  
 310 asymmetry is discussed in previous studies (Kahler and Brutsaert, 2006; Brutsaert, 2015).  
 311 Brutsaert (2015) reports  $b$  values around 4.5. The horizontal solid black line in Fig. 7 is the  
 312 boundary of the upper Bouchet-Budyko curve that  $E_{pa} \geq E_p$ .



313  
 314 Fig. 7.  $P/E_p$  vs.  $E_{pa}/E_p$  at 259 weather stations in the US for the period 1984 to 2015 for (a) warm  
 315 season data (N=5312), and (b) long term average data (N=259). The data points are color coded



316 based on their latitudes and longitudes. The three upper Bouchet-Budyko curves are plotted with  
317 different  $b$  values of  $b=1$ ,  $b=2$ , and  $b=3$ , and with the same  $\nu$  value of  $\nu=2$ .

318

#### 319 **4. Discussion**

##### 320 4.1 Relationship between $P$ and $E_{pa}$ , and between $P$ and $E_p$

321 With the weather station data, a negative correlation between warm season precipitation and  
322 “apparent” potential evaporation is shown in 242 out of 259 weather stations (93%). The  
323 negative correlation between  $P$  and  $E_{pa}$  is linked by the humidity deficit. The formation of  
324 precipitation is positively related to the local level of humidity (Pal et al., 2000; Sheffield et al.,  
325 2006; An et al., 2017) while “apparent” potential evaporation is inversely related to humidity or  
326 positively related to the humidity deficit (Penman, 1948; Allen et al., 1998). As a result,  
327 precipitation and “apparent” potential evaporation will tend to exhibit a negative correlation.  
328 Similar with the Bouchet’s complementary relationship, this negative correlation between  $P$  and  
329  $E_{pa}$  is more significant in arid regions than in humid regions.

330 On the other hand,  $P$  and  $E_p$  shows no significant correlation at both seasonal and long  
331 term average time scales. Potential evaporation is driven by energy supply, which is quantified  
332 by the Priestley-Taylor equation using the remote-sensing data (Zhang, et al., 2010). As a result,  
333 our study indicates that energy supply and precipitation, the representation of water supply, are  
334 likely to be independent. This independence is currently under investigation with field data. It  
335 should be noted that the relationship between  $P$  and  $E_p$  and between  $P$  and  $E_{pa}$  we find in this  
336 study are not direct causal relationship, but rather the result of interactions between a number of



337 physical variables. We will collect more data and further investigate the physical mechanisms of  
338 these relationships in future studies.

#### 339 4.2 The Bouchet-Budyko curve and its applications

340 Combining the Bouchet's complementary relationship and the Budyko framework leads to two  
341 dimensionless CR curves, normalized by  $E_p$  (Fig. 2). The upper Bouchet-Budyko curve is  
342 derived from the connection between Budyko framework and the CR, and the lower Bouchet-  
343 Budyko curve is directly derived from Budyko framework, based on the Turc-Pike equation.  
344 The two curves show that when the wetness index  $P/E_p$  is lower than one, the complementary  
345 relationship between  $E$  and  $E_{pa}$  is more significant. In other words, the CR is more significant  
346 under water limited condition. As discussed in Ramírez et al. (2005), the CR can be considered  
347 as an extension of the Budyko framework. With the Bouchet-Budyko curves shown in Fig. 2,  
348 this connection can be quantitatively analyzed, which will be our future study direction.

349 The collected data of  $P$ ,  $E_p$  and  $E_{pan}$  fits with the general trend of the upper Bouchet-  
350 Budyko curve (Fig. 7). The remote-sensing data of  $E_p$  may not have the same level of accuracy  
351 as the field measured  $P$  and  $E_{pan}$  and the value of  $\alpha$  in the Eq. (7) may vary from location to  
352 location (Chen and Brutsaert, 1995; Brutsaert and Chen, 1995). This may explain the deviation  
353 of some data points from the curve in Fig. 7. This upper Bouchet-Budyko curve can be used to  
354 estimate the "apparent" potential evaporation based on the data of precipitation and potential  
355 evaporation. The "apparent" potential evaporation can be measured by evaporation pan, but this  
356 measurement has its limitations. For example, it is only functional in warm period. The  
357 collected data with time averaged pan evaporation levels over weeks, months, and years may  
358 lead to systematic error in surface flux calculations (Brutsaert, 1982; Kahler and Brutsaert,  
359 2006). The Bouchet-Budyko curve can help us to estimate  $E_{pa}$  without the limitation of



360 evaporation pans. Comparing with more physically-based  $E_{pa}$  quantification approaches, such as  
361 Penman equation (Penman, 1948) and “PenPan” model (Rotstayn et al., 2006), our model is  
362 derived from conceptual frameworks and therefore may provide top-down insights about the  $E_{pa}$   
363 level in hydrologic systems.

364 In addition, the lower Bouchet-Budyko curve is based on an alternative form of Budyko-  
365 type equation (Eq. 4), derived from the Turc-Pike equation. This curve can be used to show the  
366 relationship between  $E$  and  $E_p$  under varying climate characteristic. We will collect field  
367 evaporation data to investigate this curve in future studies.

368 Similar to the Budyko framework, the Bouchet-Budyko curves can be used in hydrologic  
369 models and climate models. Furthermore, the upper and lower curves can be used to estimate the  
370 trend of “apparent” potential evaporation and actual evaporation respectively, based on the level  
371 of precipitation and potential evaporation. These Bouchet-Budyko curves can also be used to  
372 examine the fidelity of simulated precipitation and evaporation sequences routinely produced by  
373 general circulation models to drive climate change investigations.

374

## 375 5. Conclusions

376 We collect warm season precipitation, potential evaporation, and pan evaporation data at 259  
377 weather stations to investigate the correlation among these three physical variables. The results  
378 show a negative correlation between  $P$  and  $E_{pa}$  in 93% of the study locations. The physical  
379 reason of the  $P$ ,  $E_{pa}$  negative correlation could be related to the humidity variability. When  
380 humidity increases, the likelihood for precipitation increases while the rate of pan evaporation  
381 decreases. On the other hand, our study results on the relationship between warm season  $P$  and



382  $E_p$  support the assumption that  $P$  and  $E_p$  are independent. By combining the CR with a Budyko-  
383 type equation, we formulate the CR curves, showing the connection between the two  
384 frameworks. As a result, this research may encourage hydrologists to generate new ideas on the  
385 interpretation of the Budyko framework and the CR, promoting new ways of hydrologic  
386 modeling. Future work will investigate the physical mechanism behind the Bouchet-Budyko  
387 curves and explore the application of Bouchet-Budyko curves.

388

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390 The data of precipitation and pan evaporation measurements can be downloaded from the  
391 National Climatic Data Center website: <https://www.ncdc.noaa.gov/IPS/cd/cd.html>. The  
392 homogenized pan evaporation data can be downloaded from the USGS ScienceBase:  
393 <https://www.sciencebase.gov/catalog/>. The data of remote-sensing based potential evaporation is  
394 provided by the Numerical Terradynamic Simulation Group at University of Montana, based on  
395 the study of Zhang et al. (2010). The data can be downloaded from their website:  
396 <http://www.ntsg.umt.edu/about/default.php>.

397

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