

Summary: *The paper introduces a new resampling method for particle filters, well suited to estimate both state variables and model parameters in a sequential DA approach. The well-known Universal Resampling Approach is modified by assigning new weights to the particles that should be duplicated, without actually duplicating the particles. These weights are proportional to the number of times the particles are selected in the universal resampling. To keep constant the ensemble size, new particles (states and parameters) are then generated by sampling from a multivariate Gaussian distribution having the same mean and covariance of the weighted particles. To avoid the degeneracy of the filter, the covariance is inflated using a multiplicative factor.*

The proposed method is applied to a synthetic 1-d infiltration problem in a porous media constituted of two layers. Initial conditions and soil parameters (saturated hydraulic conductivity and two parameters of the Van-Genuchten equations, for both layers) are considered uncertain. The authors demonstrate that, in the considered example, the proposed pf well retrieves the state variables of the system and the soil parameters.

The paper is well written and the method is clear. At my knowledge, the proposed resampling technique is new and I really like it, since it gives the possibility to propagate realizations consistent with the model equations (a limitation of EnKF – see e.g., Pasetto et al. 2012) and, at the same time, the possibility of sampling new particles, which is fundamental to explore the parameter space.

Reply: We thank the reviewer for the detailed comments and suggestions, which will help to improve our manuscript. In the following we provide the answers to the comments.

Specific comments

Comment: *The paper does not present any result on the convergence of the filter with respect to the ensemble size: it would be important to show that at least the first and second moments of state and parameters converge toward the correct solution when N increases, and that the results are insensitive to the particular seed used. This analysis would also help justifying the choice of $N=100$.*

Reply: Thank you for pointing this out. A continuous convergence to the truth with an increasing ensemble size does not occur. For varying seeds combined with a small fixed ensemble size (e.g. 20) the filter converges to

the truth or degenerates for different seeds. Increasing the ensemble size leads to less cases that degenerate and ultimately the filter converges to the true value for every chosen seed. The resulting parameters are in a narrow range around the truth except for the insensitive parameter α_1 . A paragraph about the seed and ensemble size dependence of the resulting parameters will be added to clarify the behaviour of the filter.

Comment: *In a similar way, the sensitivity of the filter to the multiplicative factor gamma for the parameters (selected to be 1.2) should be presented. Could the authors give an advice to the readers on how to choose gamma for a different problem?*

Reply: The multiplicative factor is a tuning factor that depends on the specific problem and can be used to increase the efficiency of the filter. The covariance resampling also works with a neutral factor of 1.0 but needs (in this case) approximately an order of magnitude more ensemble members for convergence. The factor will be discussed in more detail in the course of the convergence analysis.

Comment: *A comparison of the results against the SIR using universal resampling (or the methodology proposed in Moradkhani et al. 2005) would help to understand if the proposed PF is retrieving the correct solution (in terms of both mean and covariance) and which are the practical advantages of the proposed resampling step.*

Reply: Using the SIR with universal resampling leads to filter degeneration after a few assimilation cycles because the model equation in our case study does not have a stochastic model error. Therefore, after resampling, the duplicated particles will stay identical after forward propagation which leads to filter degeneration. For the resampling techniques of Moradkhani et al. (2005), we tested different tuning factors in the interval [0.2, 0.8] for the parameter resampling. The tuning factor modifies the variance of the perturbation. We also tried different ensemble sizes in the interval [100, 1000]. Using the same initial conditions as in the presented case, we were not able to achieve converging results. Changing the state's initial condition from the interpolation to the truth, the resampling of Moradkhani et al. (2005) was able to follow the truth in state space for 400 ensemble members and a factor of 0.6, but without convergence in parameter space and the ensemble of parameter α_1 diverges.

Minor comments

Comment: *Please revise the numbers and labels on the x and y axis for all figures. Probably there was an error with the software used to produce the figures.*

Reply: Thank you very much, we will revise the figures. This is an issue with the used font during the plotting, which is not correctly displayed after uploading the .pdf file. We are in contact with Copernicus to solve it in the revised version.

Comment: *L8, p1: ‘With just 100 particles’. In large scale applications 100 particles are frequently adopted. However, in this 1-d scenario it is difficult to assess if 100 particles are ‘small’, especially without presenting a comparison with other approaches and/or the sensitivity of the results to the number of particles.*

Reply: You are right. The comparison to other approaches is difficult because the case studies have a different setup, therefore we will delete the ‘just’.

Comment: *L8-10, p1: ‘The estimated states and parameters are tested with a free run after the assimilation, which is found to be in good agreement with the synthetic truth’. PFs (and DA in general) are meant to assess not only the mean value of states and parameters, but also their covariance (if not all the pdf). To assess if the covariance computed is correct, a comparison with respect to other DA schemes would be required.*

Reply: The entire ensemble is propagated forward in time and the ensemble mean is calculated using the weights at the last assimilated observation. This way we consider the mean of the propagated distribution. We will clarify this point.

Comment: *L20, p1: ‘The EnKF based on Richards equation..’. EnKF is applied to Richards equation, not ‘based on’. Please rephrase.*

Reply: Thank you for pointing this out. We will rephrase the sentence.

Comment: *L23, p2: what does it mean ‘without additional model evaluation’? This statement would be more relevant if the results of the proposed method are compared against more traditional PFs (e.g. SIR with a standard resampling).*

Reply: In this case we considered the MCMC resampling, which needs additional model runs for the resampling process. These runs are usually expensive and therefore, we think the absence of additional model runs as an advantage. We will clarify the statement.

Comment: L27,p2: missing point.

Reply: Thank you for noticing the missing point. We will correct it.

Comment: Section 3 and part of section 2: particle filters refer to a broad class of methods (see, e.g., Arulampalam 2002). The authors are mainly describing the Sequential Importance Resampling technique. Please clarify this point in the paper, so that readers familiar with PFs can easily understand which technique has been modified.

Reply: We will clarify this point.

Comment: Fig1: Write evapotranspiration instead of evaporation.

Reply: In our simulations we do not have plants and therefore, no transpiration.

Comment: Eq. 12, Page 5. I was expecting to see the weight w_i in the summation. Please, provide a reference for the Bessel correction.

Reply: You are right with expecting the weight w_i in the summation. The missing weight in the equation is a typing error. The correct equation is:

$$\mathbf{P}^f = \frac{1}{1 - \sum_{i=1}^N w_i^2} \sum_{i=1}^N w_i [\mathbf{u}_i - \bar{\mathbf{u}}] [\mathbf{u}_i - \bar{\mathbf{u}}]^\top. \quad (1)$$

It is possible to represent the weights using a larger number of equal weighted particles. For example, a particle with $w_i = 1/2$, $u_i = 1$ and two particles with $w_i = 1/4$, $u_i = 2$ is equal to having two particles with $w_i = 1/4$, $u_i = 1$ and two particles with $w_i = 1/4$, $u_i = 2$.

Inserting equal weighted particles in this equation results in

$$\mathbf{P}^f = \frac{1}{1 - \frac{1}{N}} \sum_{i=1}^N \frac{1}{N} [\mathbf{u}_i - \bar{\mathbf{u}}] [\mathbf{u}_i - \bar{\mathbf{u}}]^\top \quad (2)$$

$$= \frac{1}{N-1} \sum_{i=1}^N [\mathbf{u}_i - \bar{\mathbf{u}}] [\mathbf{u}_i - \bar{\mathbf{u}}]^\top. \quad (3)$$

This is equal to the correction of an unbiased estimate of the covariance.

Comment: Lines 5-10, p9: it is not clear how the covariance matrix for the initial ensemble has been generated. Which matrices are multiplied in step 2? Is step one performed after step 2, to ensure zero correlation of the states across the two layers?

Reply: Thank you for pointing this out. Every entry of the initial covariance matrix is set to 0.003^2 . In the first step, covariances across the layer boundary are set to zero. The Gaspari and Cohn function is multiplied component-wise (not written in the manuscript) with the resulting matrix. Therefore,

both steps are exchangeable. The component-wise multiplication with the Gaspari and Cohn function results in a covariance that decreases with the distance. We will rephrase and extend this paragraph to clarify the steps.

Comment: *Page 13, L1: add 'of' after estimate*

Reply: We will add the 'of'.

References

Hamid Moradkhani, Kuo-Lin Hsu, Hoshin Gupta, and Soroosh Sorooshian. Uncertainty assessment of hydrologic model states and parameters: Sequential data assimilation using the particle filter. *Water Resources Research*, 41(5), 2005. doi:10.1029/2004WR003604. URL <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2004WR003604>.