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Saturated Hydraulic Conductivity and Textural Heterogeneity of Soils

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Abstract. Saturated hydraulic conductivity K_{sat} is an important soil parameter that highly depends on soil's particle size distribution (PSD). The nature of this dependency is explored in this work in two ways, (1) by using the Information Entropy as a heterogeneity parameter of the PSD and (2) using descriptions of PSD in forms of textural triplets, different than the usual description in terms of the triplet of sand, silt and clay contents. The power of this parameter, as a descriptor of K_{sat} and $\log K_{sat}$, was tested on a database of >19K soils. We found coefficients of determination of up to 0.977 for $\log K_{sat}$ using a triplet that combines very coarse, coarse, medium and fine sand as coarse particles, very fine sand as intermediate particles, and silt and clay as fines. The power of the correlation is analysed for different textural classes and different triplets. Overall, the use of textural triplets different than traditional, combined with IE, may provide a useful tool for predicting K_{sat} values.

1 Introduction

Saturated hydraulic conductivity (K_{sat}) is the measure of soil's ability to conduct water under saturation conditions (Klute and Dirksen, 1986). It is an essential parameter of soil hydrology. Soil K_{sat} affects many aspects of soil functioning and soil ecological services, like infiltration, runoff, groundwater recharge, nutrients transport. Knowing values of soil K_{sat} appears to be essential in designing management actions and practices, such as irrigation scheduling, drainage, flood protection, and erosion control.

The dependence of K_{sat} on soil texture has been well documented (Hillel, 1980). Different parameterizations of particle size distributions (PSDs) were suggested to relate K_{sat} and soil texture. It was proposed to use d10, d20, and d50 particle diameters (Chapuis, 2004; Odong, 2007) or slope and intercept of the particle size distribution curve (Alyamani and Sen, 1993). Also various functions were fitted to PSDs, and the fitting parameters were related to K_{sat}. For example, Chapuis et al. (2015) proposed to use two lognormal distributions to fit the detailed particle size distribution and to use the lognormal distribution parameters to predict the K_{sat}.

A common way to parameterize the PSD for K_{sat} estimation purposes is using the textural triplet that provides the percentage of coarse particles (sand), intermediate particles (silt), and fine particles (clay). K_{sat} values are estimated using the contents of one or two triplet fractions or just the textural class (Rawls et al., 1998). Representing PSD by textural triplets is the common way to estimate a large number of soil parameters (Pachepsky and Rawls, 2004). The coarse, intermediate, and fine fractions need not to be sand, silt and clay. Martín et al. (2017) showed that different definitions of the triplet, e.g. coarse sand, sand, and medium sand as coarse, fine sand, very fine sand as intermediate, and silt and clay as fine triplet fractions, provide much better inputs for bulk density estimation compared with the standard textural triplet.

The heterogeneity of particle size distributions appears to be an important factor affecting hydraulic parameters of soils, including the saturated hydraulic conductivity. Values of K_{sat} depend on both distribution of sizes of soil particles, i.e. soil texture, and the spatial arrangement of these particles, i.e. soil structure. Soil structure can be to some extent controlled by soil texture, since packing of particles is affected by the particle size distributions (e.g Gupta and Larson, 1979; Assouline and Rouault, 1997; Jorda et al., 2015). It was recently proposed to use the information entropy as the parameter of the PSD heterogeneity for predicting soil water retention (Martín et al., 2005) and soil bulk density (Martín et al., 2017). Previously, information entropy was used, together with other predictor vaiables to estimate K_{sat}, using multivariate analysis (Boadu, 2000).

The objective of this work was to test the hypothesis that combining two recent developments -the description of the PSD by different textural triplets, that may focus on different soil properties, and the information entropy as a PSD heterogeneity parameter that depends on the triplet used for its description- may improve estimation of K_{sat} and may be seen as a step forward to study the effect of heterogeneity widely recognized in the majority of works that studied the particle size - hydraulic conductivity relationships. We used the large USKSAT database on laboratory measured K_{sat}. containing more than 19000 samples.

2 Materials and Methods

2.1 Database description and textural triplet selection

For this study we used USKSAT database. Detailed information can be found in (Pachepsky and Park, 2015). This database consists on soils from different locations of the USA and contains soils from 45 different sources. We selected only those sources which (a) had data on both K_{sat} and on the seven textural fractions, and (b) presented measurements made in laboratory; K_{sat} was deteremined using the constant head method in 99.6% of cases and falling head metod in 0.4% of cases.From those, we subset those soils whose sum of mass in the seven textural fractions, i.e. (1) very coarse sand, (2) coarse sand, (3) medium sand, (4) fine sand, (5) very fine sand, (6) silt and (7) clay differed from 100% not more than by 2%. The final number of soils considered was 19193. By USDA textural classes the total number of soils are: 12068 sands, 1779 loamy sands, 2123 sandy loams, 106 loams, 178 silt loams, 36 silts, 1982 sandy clay loams, 80 clay loams, 48 silt clay loams, 334 sandy clays, 7 silty clays, 414 clays and 38 soils that were classified into more than one class.

We used all possible triplets formed from seven textural fractions. Triplets consisted of coarse, intermediate, and fine fractions. The symbols for triplet showed how the fractions were grouped. For example the "coarse" fraction for the triplet '3-2-2' included very coarse sand, coarse sand and medium sand, the "intermediate" fraction included fine sand and very fine sand , and "fine" included silt and clay; triplet '5-1-1' was the standard one where "coarse" included all five sand fractions, "intermediate" included silt, and "fine" included clay. The amount of possible triplets with 7 textural fractions was 15.

2.1.1 Heterogeneity metric calculation

The Entropy based parametrization of textures introduced in (Martín et al., 2001) has as central concept in the Information Entropy (Shannon, 1948). Assuming the texture interval divided into k textural size ranges and that the respective textural fraction contents are $p_1, p_2, \ldots, p_k$, $1 \leq i \leq k$, with $\sum_{i=1}^{k} p_i = 1$, the Shannon Information Entropy (IE) (Shannon, 1948) is defined by

$$IE = -\sum_{i=1}^{k} p_i \log_2 p_i \tag{1}$$

where $p_i \log p_i = 0$ if $p_i = 0$. The IE is a widely accepted measure of the heterogeneity of distributions (Khinchin, 1957). In case of three fractions, the minimum value of IE is zero when only one fraction os present, and the maximum value is 1.57 when three fractions are present in equal amounts. (see Fig. 1.)

For each soil in this study, we grouped the 7 available textural fractions in the 15 possible triplet combinations and calculated the respective triplet's IE using formula (1). Fig 2 shows heatmaps of IE calculated for all the soils available in this study but using two different triplets as input. It is clear that, by changing the triplet, the calculated IE values vary differently along the same textural triangle. IE is a measure of heterogeneity, but the triplet used is the substrate for this measure. We will notate this combination together (IE,*triplet*), i.e., (IE,'5-1-1').

We followed the binning method of Martín et al. (2017) to research the relationship between K_{sat} and soil heterogeneity. Specifically, the range of values of IE was divided into ten bins, the average value of K_{sat} was plotted against the average IE for the bin, i.e. the bin midpoint. Linear regressions 'bin midpoint vs. average bin K_{sat}' and 'bin midpoint IE value vs. average bin $\log K_{sat}$' were computed. These regressions were obtained for each of 15 triplets and for those of USDA textural classes that were represented by more than 50 samples, i.e. sands, loamy sands, sandy loams, loams, silt loams, sandy clay loams, clay loams, sandy clays, and clays.

The regression relationships were characterized by the coefficient of determination, R^2, and the Root-mean-square error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_t - y_y)^2}{n}}$$

where $\hat{y}_t$ are the predicted and y_t are the real values of K_{sat}, and n is the number of soils.

3 Results and Discussion

3.1 The Dataset Overview

Fig. 3 presents the 19193 soils used in this study in the USDA textural triangle and in the modified '3-2-2' triangle. The density of points reflects the dominance of coarse textural soils in the database. When the triplet is changed, the distribution of points across the triangle changes. By setting the textural fractions to be the '3-2-2' triplet, the distribution of points/soils in the new textural triangle spreads. While there is still a high concentration of soils in the stripe of bigger than 85% of the Coarsefraction, where Coarse 3 includes very Coarse Sand, Coarse sand a Medium Sand, now those soils spread fully from 10 to 100% of the Intermediate-2 fraction, where Intermediate-2 contains Fine and Very Fine Sand. On the USDA textural triangle, most of the soils are clustered in the subtriangle limited by the lines "more than 70% sand" and "less than 20% silt". This new textural triangle allows for a finer look into the sand fraction, revealing the distribution of soils within the USDA sandy textural classes. This finer look might prove itself useful to study physical properties of these soils that are mainly related to the type and amount of sand in them.

Table 1 shows the K_{sat} statistics for the soils in the study. A total of 19420 soils were used in this study, from which 39 (0.2%) could not be classified as they belonged to more than one textural class. The textural class sand comprises the 62.3% of all the soils, followed by sandy loam (11.12%) and sandy clay loam (10.77%). Six textural classes were poorly represented with percentages less than 1% of the total soils. The K_{sat} values varied between 0.0005 and 841 cm/h being 22.57 the mean value.

3.2 Regression in binned data: IE as a predictor of K_{sat} / log K_{sat})

Linear regressions for K_{sat} and $\log K_{sat}$ were done to find out the predictive power of the proposed parameter, (IE,triplet), with the 15 possible different triplets that could be archived by grouping the available textural data. Table 2 shows the computed R^2 and $RMSE$ values for the linear regressions using 10 interval bins.

The best triplet for the K_{sat} regression was '2-3-2', i.e. 'coarse sand + sand + medium sand' as coarse, 'fine sand + very fine sand' as intermediate and 'silt + clay' sa fime factions, with the highest R^2 value (R^2=0.885) and this triplet also had the lowest RMSE value (RMSE=3.609). Figure 4 shows a heatmap representation of the K_{sat} values of the soils of the study on the textural triangle compared to a heatmap representation of the IE values of the same soils computed using the '2-3-2' triplet. The sandy soils had high K_{sat} values, and the IE values on that part of the triangle were low. The triangle presents high K_{sat} values in a stripe between 0 and 20% sand. This stripe has also low (IE,'2-3-2') values, so there is a reasonable visual relationship between these two values.

For the $\log K_{sat}$ regression, the best triplet in terms of highest R^2 value was '4-2-1', with a R^2 value of 0.977, but the lowest RMSE value (RMSE=0.194) was attained with the '1-2-4' triplet. Figure 5 shows the same comparison as figure 4, but using the $\log K_{sat}$ heatmap and the IE computed with the '4-2-1' triplet. There is a higher visual similarity between these two images, with high $\log K_{sat}$ value zones, near the lower corners (sandy and silty soils) that correspond to low (IE,'4-2-1') values. The $\log K_{sat}$ values tend to decrease towards the centre of the triangle. On the other hand, the (IE,'4-2-1') values tend to increase around this point.

The standard triplet ('5-1-1') yielded, for the K_{sat} regression, a R^2 value of 0.776 and RMSE value of 7.528; for the $\log K_{sat}$ regression, the R^2 value with this triplet was 0.960 and the RMSE value was 0.340. Regressions against $\log K_{sat}$ were in average better: the average R^2 value of the regressions with all possible triplets for $\log K_{sat}$ was 0.734, whilst the average R^2 value for the K_{sat} regression was 0.673.

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3.3 Predictive power of IE among the USDA textural classes

In this section we show how IE works differently among textural classes: using different triplets we can find that the textural classes are predicted differently; what works for some, for others is counterproductive.

Table 3 shows the best triplet, chosen in terms of highest R^2 value of all the possible regressions, for each textural class that had $N > 50$, and the R^2 value for both, K_{sat} and $\log K_{sat}$, linear regressions. The best R^2 values were obtained for the regression of the sand textural class against the (IE,‘5-1-1’), i.e., the IE computed with the standard ‘5-1-1’ clay-silt-sand USDA triplet (R^2=0.907 for the K_{sat} regression and R^2=0.989 for the $\log K_{sat}$). A possible explanation for this triplet being the best among all the other possible triplets, is that sandy soils are the ones that contain percentages of the sand fraction higher than 70%, so their distribution is highly heterogeneous. Minor fractions are now silt and clay, and the information about this two fractions could be very important for the hydraulic properties of the soil, thus the (IE,‘5-1-1’) triplet yielded the best regression result. One might think that, having such a high concentration of sand particles, is now silt and clay the fractions that made the difference in the packing properties, thus in the saturated hydraulic conductivity values. The high value of R^2 indicates that the relation is very strong in this case.

The sandy textural classes had the highest regression coefficients (R^2>0.655 for all classes), in constrast to the non-sandy classes (R^2<0.56). This motivated the classification of all the soils in the study in two superclasses: sandy (`SC`) and not-sandy class (`NSC`). `SC` comprised the textural classes sandy clay, sandy clay loam, sandy loam, loamy sand and sand. `NSC` comprised the other seven textural classes. Total number of soils in `SC` was 18286 (95.27% of total soils in the database). `NSC` contained 869 soils (4.53% of total). Tables 4 and 5 show the R^2 and RMSE values for all regressions (K_{sat} and $\log K_{sat}$) for the soils in `SC` and `NSC`.

For the `SC` we observed that the best regression (R^2=0.888) against K_{sat} was reached with (IE,‘2-2-3’), and being (IE,‘2-3-2’) a close runner-up (R^2=0.880) and a lower RMSE value. Both these triplets make a distinction among the sand fractions, putting very coarse and coarse sand in the coarse fraction in the first case, and adding medium sand in the second case. Also, the fines fraction contains either very fine sand or not. Comparing this to the sandy textural class results, where the best triplet was ‘5-1-1’, we observed that now more information from the sandy fraction was required to infer hydraulic properties. The area that the `SC` soils cover in the textural triangle and the hydraulic property variation of these soils can be related with a heterogeneity metric associated to triplets that distinguish well among the predominant fraction in that area of the triangle, i.e., sand.

For the `NSC`, best triplet in both regressions (K_{sat} and $\log K_{sat}$) was ‘4-1-2’, with R^2=0.232 for K_{sat} and R^2=0.769 for $\log K_{sat}$. Regression results were worse than for `SC`, but this might be just provoked by the nature of `NSC` itself: these are soils with less sand, thus higher content in clays and aggregating particles. The packing -and consequently the K_{sat}- of these soils is not just mainly affected by the PSD, but also by aggregation, which cannot be accounted for in the IE value, regardless of the triplet used.

Furthermore, the best triplet, ‘4-1-2’, also pointed in this direction: the fines fraction contains silt and sand particles, while the intermediate fraction contains only the very fine sand, leaving the coarse fraction with most of the sand, thus giving more importance to the possibly aggregating particles than a triplet like, '1-3-3' which had R^2 values equal to 0.051 (K_{sat}) and 0.019 ($\log K_{sat}$).

3.4 Triplets and Scaling Break

In the regressions made with all the soils, it was noteworthy the behaviour of the (IE,'3-1-3'). The average R^2 for all the K_{sat} regressions was 0.673, but the R^2 using (IE,'3-1-3') gave a R^2 equal to 0.0002, far below the next lowest one, which was (IE,'3-2-2') with a R^2 value of 0.433. The same happened in the $\log K_{sat}$ regressions, where the average value of all triplets was 0.734, but (IE,'3-1-3') gave a exceptionally low R^2 value of 0.111, being the next lowest (IE,'2-2-3') with a R^2 value of 0.229.

The '3-2-3' triplet groups fine sand with silt and clay, and coarse and very coarse sand with medium sand. Kravchenko and Zhang (1998); Wu et al. (1993); Tyler and Wheatcraft (1992) reported the break in scaling where the powerlaw scaling of soil texture occurred in the size range of fine sand The Particle size distribution scales in a different way in two different regions of the size intervals, and that the change of scaling is produced around the fine sands. The triplet '3-1-3' separates these two regions, maybe bringing forth this scaling break effect. Fig 6. shows how the relationship between K_{sat} (and $\log K_{sat}$) and (IE,'3-1-3') could be nonlinear, maybe due to the absence of global selfsimilarity showed in the scaling break.

On the other hand, it is also noteworthy that regressions against (IE,'3-1-3') were actually quite good (R^2=0.855 for K_{sat} and R^2=0.955 for $\log K_{sat}$) in the `SC`, while in the `NSC` they were moderate (R^2=0.204 for K_{sat}, and R^2=0.267 $\log K_{sat}$). Furthermore, even though (IE,'3-1-3') presented the lowest R^2 values for all soils, this triplet yielded the best R^2 results for some soils belonging to silty loam ($N = 178$) and sandy clay loam ($N = 1982$) textures, for K_{sat} regressions. Nevertheless, for other textures, the '3-1-3' triplet had, generally, a very low value.

When all the soils are considered together, then (IE,'3-1-3') might fail, due to the scaling break, but when we restrict the study to a certain part of the textural triangle, that effect might diminish to a point where this triplet is even useful to predict some textural derived properties, or maybe the scaling break effect is also restricted to some textural classes and should be further investigated.

As results show, IE is not powerful K_{sat} predictor by itself, but combined with an input triplet. By changing the triplet, we may focus on certain physical aspects of the soils, but it is also important to keep in mind that this might not work statistically for random groupings of soils that belong to different textures.

3.5 IE variation as a spatial function in the textural triangle

Heatmaps were used to visually correlate the IE values calculated with the K_{sat} (or $\log K_{sat}$) values of the soils in the study. Also, a less visual, but more quantifyable approach, to find out how much of K_{sat} could be explained through IE variation was to find out what ranges of IE are available for soils in different textural classes and compare them to the range of K_{sat} values of soils inside those same textural classes. Also, in order to compare the new tool (IE triplet), we compared these ranges to the ranges computed for (IE,'5-1-1') , i.e. to the values of the IE computed with the usual description of soil texture. We want to find out if, by changing the triplet, we obtain a wider range of variation in IE for a given range of K_{sat}. This way we compare if the new descriptions of texture, in form of different triplets, might be suitable for explaining soil physical properties, in particular K_{sat}.

For each textural class, we calculated the ratio of the range of K_{sat} values inside the textural class versus the range of K_{sat} values of all the soils in the study. The same was done for IE for each triplet. Table 6 shows, for each textural class, the ratio of the percentage of (IE,'5-1-1') against the percentage of K_{sat} range. The same ratio was also calculated using IE for the triplet that gave the best R^2 value in the linear regression against K_{sat}. These values can be thought of as how much range of (IE,*triplet*) can be used to explain a certain variation of K_{sat} inside each textural class, i.e. as how much parametrizing power is available by the IE. In all the textural classes where the regressions were done, the parametrizing power of the alternative triplet was higher than the one by using the usual clay-silt-sand triplet. For the sand textural class, the triplet which gave the best R^2 regression was '5-1-1' thus the results are the same; the average value of the parametrizing power for the usual triplet was 2.46, while when we change the triplet we obtained 4.80. This shows how, by considering different triplets, combined with IE, a better description/parametrization of K_{sat} can be reached.

3.6 Final Comments

Textural heterogeneity is a crucial factor affecting soil K_{sat}, but it acts along many other ecological factors, as animal activity, root exudates, soil aggregation, etc. In this work we showed that a proper representation of textural heterogeneity, by IE, allows one to (1) demonstrate its effect on K_{sat} by binning samples based on the textural heterogeneity and (2) to statistically parametrize this effect for some textures.

This work has limitations, in particular, the limited available texture data of only seven fractions in the database. The boundaries between coarse, intermediate, and fine fractions can be moved with data from continuous measurements of texture in the fine sand-silt-clay range of particle sized. This may bring the improvements in mean bin K_{sat} estimates for non-sandy soils that could not be achieved in this work.

Although globally the IE computed from different triplets show a potential to reflect the effect of soil texture on the K_{sat} values, the different relationship between the IE and the K_{sat} depending on the triplet used might have different possible explanations. While the IE/K_{sat} relationship is found satisfactory in some textural classes, results seem to indicate that the IE parameter cannot reflect with the same efficiency the K_{sat} values in other classes predominating fine particles, in which other processes as aggregation or weathering can not been elucidated by the single textural data input.

4 Conclusions

The PSD coarse, intermediate,and fine fractions in soil textural triplets can be redefined from standard 'sand-silt-clay' to other fraction size ranges. The textural heterogeneity parameters obtained for some of the new triplets correlate with soil saturated hydraulic conductivity averaged by ranges of the heterogeneity parameters. This approach allows one to quantify the effect of the textural heterogeneity of saturated hydraulic conductivity of soils. Given that size boundaries of sand, silt, and clay fractions have not originally been established for the purposes of prediction of soil hydraulic conductivity, it may be beneficial to look for other size based subdivisions of particle size distributions which, when used along with other soil properties such as bulk density and organic matter content, may provide better predictions of the saturated hydraulic conductivity.

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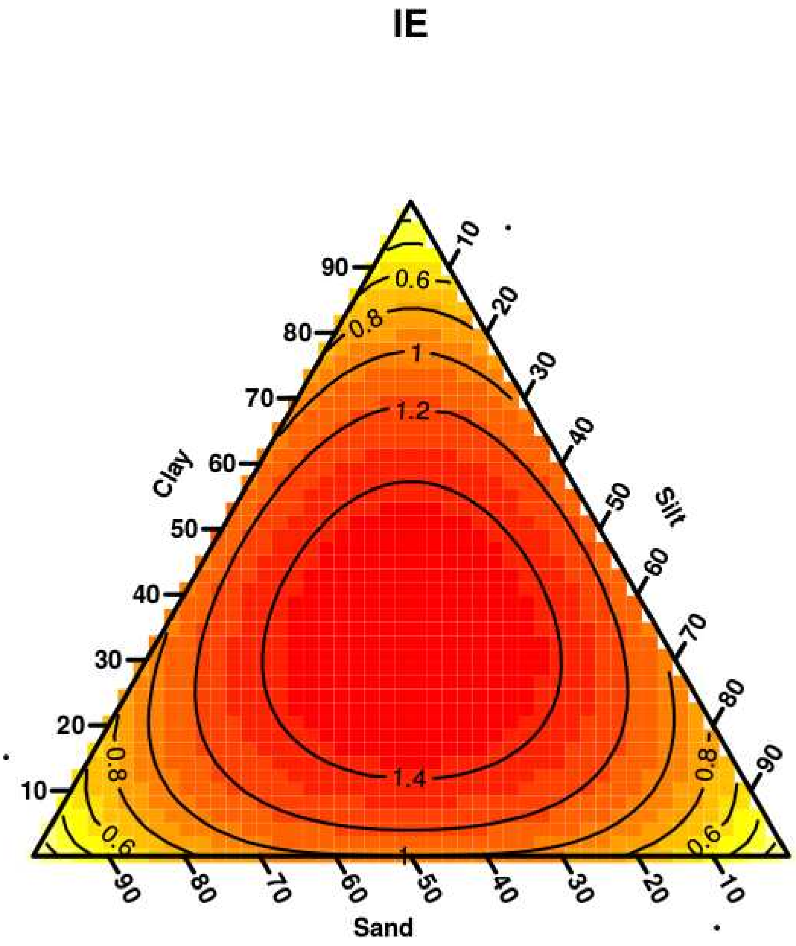

Figure 1. IE numerical approximation heatmap: IE is computed for a sample of 5051 evenly distributed soils in the USDA textural triangle using the clay, silt and sand fractions as input triplet. This distribution of IE is repeated for any textural triangle, when the fractions used for its calculations are the ones at the axes of the triangle. The lowest values for the IE are near the vertex of the triangle, i.e. where one fraction dominates above the others. Biggest values are located towards the centre of the triangle, where the distribution fractions are more balanced.

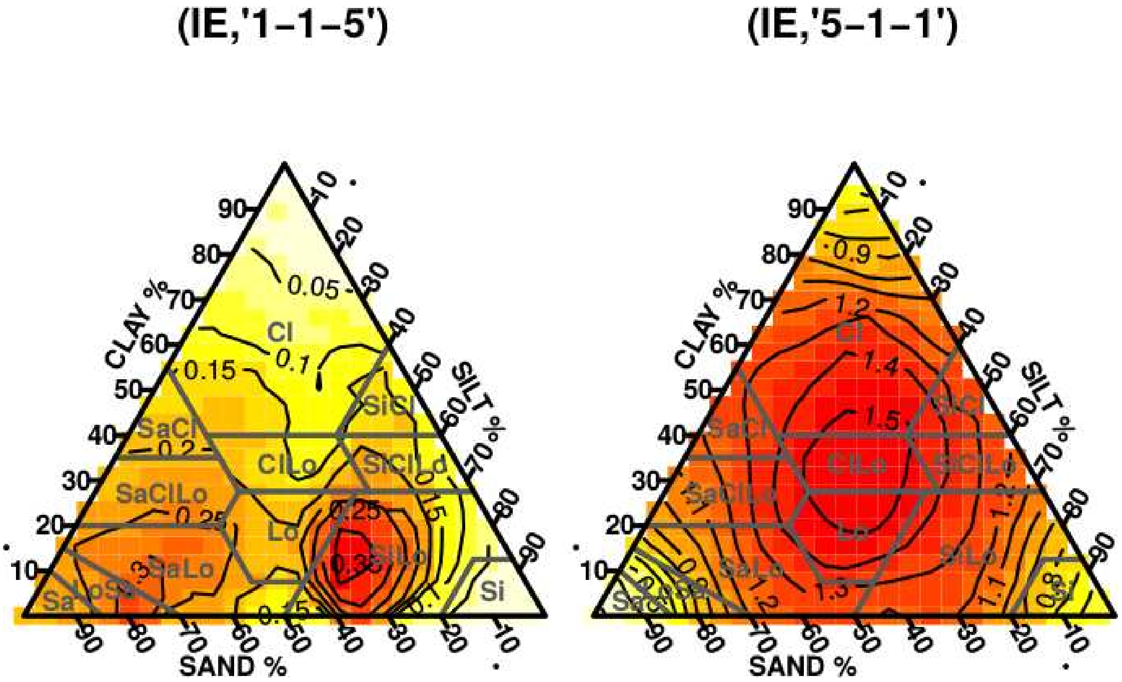

Figure 2. Heatmaps for IE calculated for the soils of the study but using different triplets. The usual clay, silt and sand triplet (‘5-1-1’) was used at the left and the grouping seven textural fractions into ‘1-1-5’ was used as input for the right.

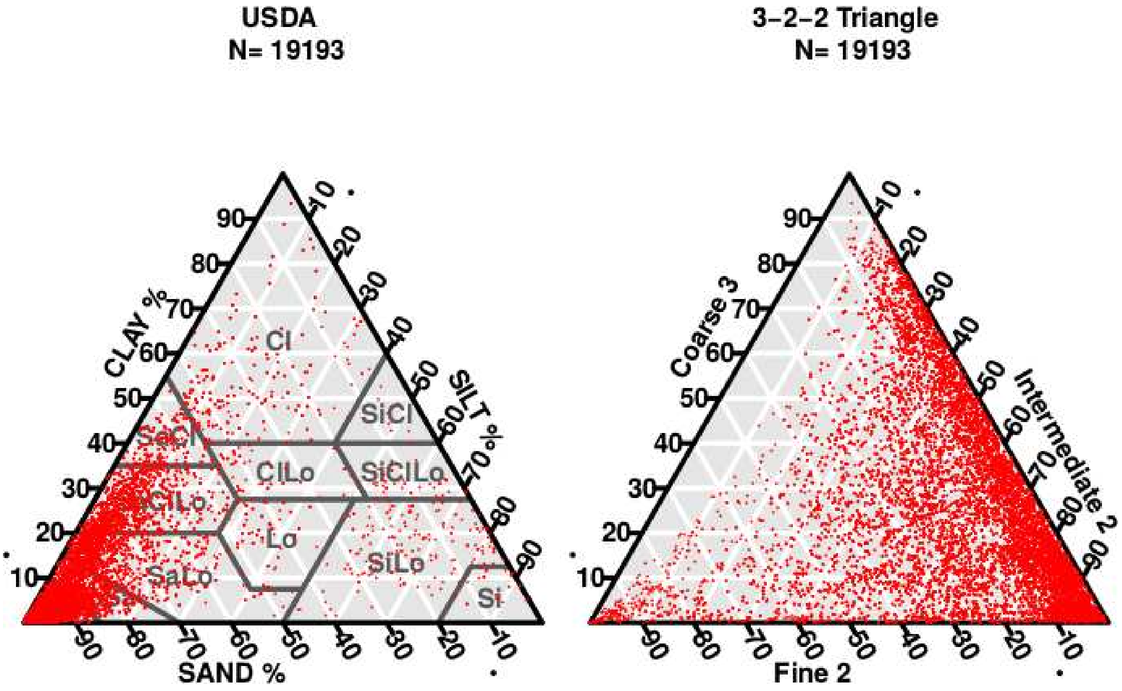

Figure 3. Representation in the USDA textural triangle of the 19193 soils used in this study. (a) standard sand-silt-clay, i.e. '5-1-1' triplet. (b) the '3-2-2' triplet.

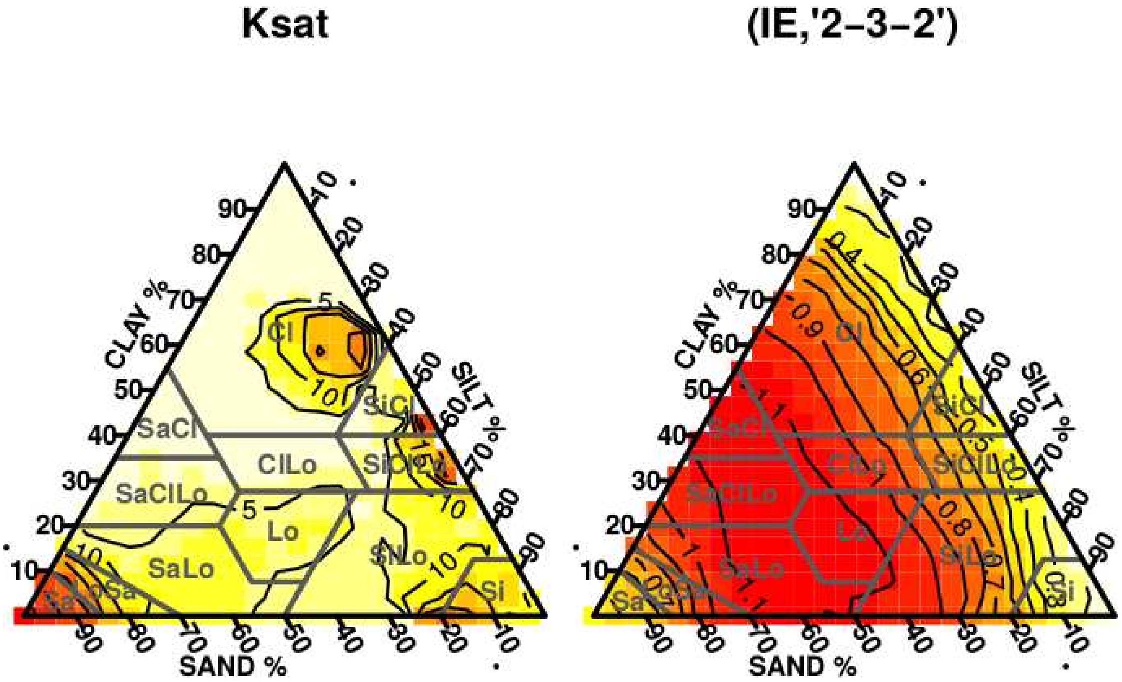

Figure 4. Heatmaps for the K_{sat} and the (IE,‘2-3-2’), represented in the USDA textural triangle.

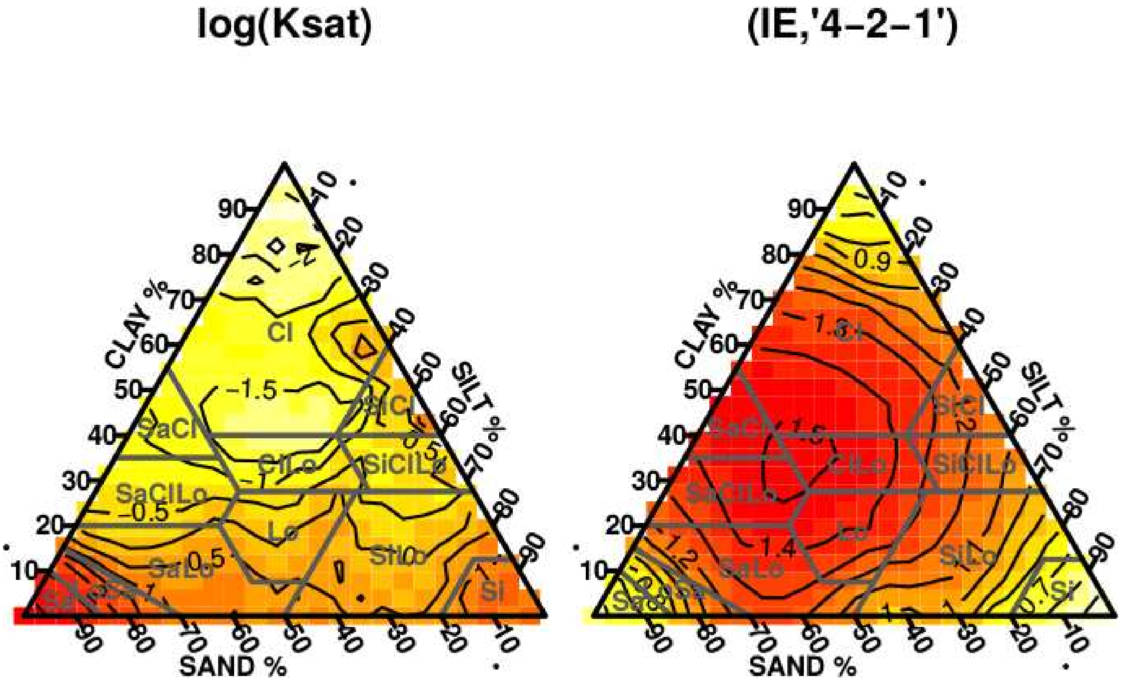

Figure 5. Heatmaps for $\log K_{sat}$ and $(IE$,‘4-2-1’) represented in the USDA textural triangle.

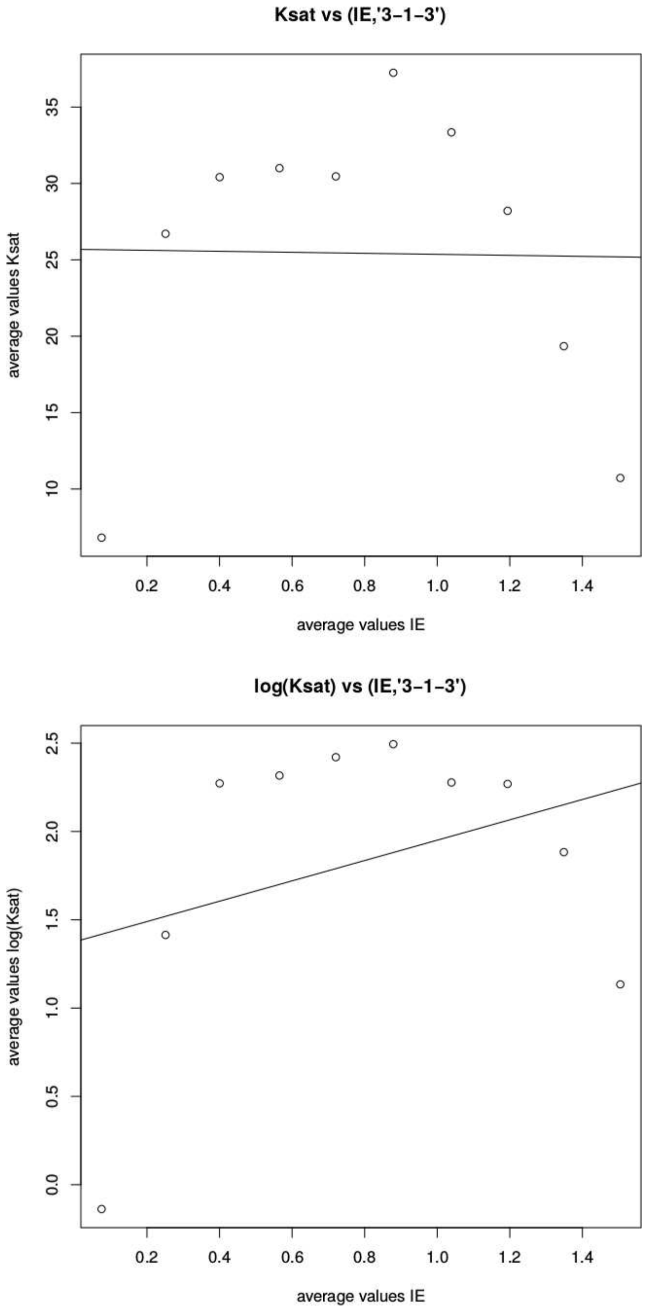

Figure 6. K_{sat} (top) and $\log K_{sat}$ (bottom) values against IE calculated with the '3-1-3' triplet in 10 interval binnings. The lines are the regression lines.

Table 1. Statistical description of K_{sat} values by classes. Soils have also been grouped into two major classes, the sandy class (`SC`) and not sandy class (`NSC`). The multiple class soils are the soils that were classified into more than one textural class. Legend: N, the number of soils in each class; sd, the standard deviation; skew, the skewness number and se, the standard error.

	N	min	1st Quart	Median	Mean	3rd Quart	max	sd	skew	kurtosis	se
Clay	414	0.00	0.04	0.16	4.07	0.92	421.00	25.29	13.12	195.74	1.24
Clay loam	80	0.01	0.05	0.22	1.58	0.73	38.20	5.38	5.44	30.85	0.60
Loam	106	0.01	0.17	0.71	5.67	2.74	52.60	11.17	2.40	5.13	1.09
Loamy sand	1779	0.01	1.37	5.00	9.84	13.80	189.00	13.35	3.85	29.40	0.32
Sand	12068	0.01	11.80	23.95	32.97	43.40	841.00	62.83	4.01	51.09	0.30
Sandy clay	334	0.00	0.10	0.42	2.80	1.32	60.60	8.41	4.65	22.72	0.46
Sandy clay loam	1982	0.00	0.12	0.50	3.26	1.69	405.00	17.20	13.94	240.56	0.39
Sandy loam	2123	0.00	0.28	1.10	4.92	3.67	504.00	18.26	15.57	347.07	0.40
Silt	36	0.27	1.27	5.21	19.16	22.54	213.00	40.62	3.57	13.03	6.77
Silty clay	7	0.00	0.41	3.92	5.85	6.89	22.39	7.90	1.16	-0.14	2.99
Silty clay loam	48	0.00	0.07	0.36	15.90	2.03	159.00	40.40	2.69	6.06	5.83
Silty loam	178	0.00	0.23	1.12	5.14	4.49	53.90	9.61	2.97	9.58	0.72
Multiple class	38	0.01	0.06	0.21	0.78	0.60	8.08	1.60	3.28	10.81	0.26
All	19193	0.00	1.88	12.90	22.82	31.60	841.00	31.03	4.26	51.53	0.22
`SC`	18286	0.00	2.53	13.90	23.69	32.90	841.00	31.15	4.18	51.13	0.23
`NSC`	869	0.00	0.08	0.35	5.55	1.86	421.00	22.64	10.87	158.51	0.77

Table 2. Computed R^2 and RMSE values for the lineal regression of K_{sat} and log K_{sat} against all possible (IE,*triplet*). In green are marked the highest R^2 value, in blue the lowest RMSE value. Summary line of max/min shows the maximum R^2 value and the minimum RMSE value for each column.

	K_{sat}		log K_{sat}	
Triplet	R^2	RMSE	R^2	RMSE
‘1-1-5’	0.798	5.895	0.899	0.208
‘1-2-4’	0.615	5.267	0.901	0.194
‘1-3-3’	0.839	9.366	0.909	0.395
‘1-4-2’	0.726	7.609	0.617	0.853
‘1-5-1’	0.723	6.076	0.748	0.726
‘2-1-4’	0.750	4.883	0.875	0.223
‘2-2-3’	0.500	9.115	0.229	0.866
‘2-3-2’	0.885	3.609	0.745	0.593
‘2-4-1’	0.856	3.796	0.766	0.604
‘3-1-3’	0.000	9.448	0.111	0.739
‘3-2-2’	0.433	6.051	0.519	0.574
‘3-3-1’	0.469	6.829	0.769	0.554
‘4-1-2’	0.880	8.604	0.976	0.239
‘4-2-1’	0.849	10.109	0.977	0.260
‘5-1-1’	0.776	7.528	0.960	0.340
max/min	0.885	3.609	0.977	0.194
average	0.673	6.946	0.734	0.491

Table 3. Summary of best triplets for K_{sat} and log(K_{sat}) regressions using 10 interval binnings. Both the triplet and the R^2 are shown.

		K_{sat}		$\log K_{sat}$	
Textural class	N	Best triplet	R^2	Best triplet	R^2
Silty loam	178	‘3-1-3’	0.346	‘5-1-1’	0.382
Sandy loam	2123	‘2-3-2’	0.689	‘1-1-5’	0.954
Sandy clay loam	1982	‘3-1-3’	0.801	‘2-2-3’	0.948
Sandy clay	334	‘2-4-1’	0.655	‘1-3-3’	0.432
Sand	12068	‘5-1-1’	0.907	‘5-1-1’	0.989
Loamy sand	1779	‘2-2-3’	0.702	‘2-2-3’	0.763
Loam	106	‘2-4-1’	0.446	‘1-5-1’	0.406
Clay loam	80	‘2-3-2’	0.219	‘4-1-2’	0.426
Clay	414	‘1-5-1’	0.560	‘1-2-4’	0.727

Table 4. R^2 and RMSE values for linear regressions of IE vs K_{sat} and log(K_{sat}) using the 15 different triplets for the sandy class selection.

	K_{sat}		$\log K_{sat}$	
	R^2	RMSE	R^2	RMSE
'1-1-5'	0.777	6.134	0.880	0.210
'1-2-4'	0.579	5.270	0.830	0.206
'1-3-3'	0.807	11.139	0.917	0.411
'1-4-2'	0.734	7.582	0.636	0.812
'1-5-1'	0.731	6.024	0.740	0.709
'2-1-4'	0.715	5.028	0.757	0.261
'2-2-3'	0.888	5.234	0.796	0.510
'2-3-2'	0.880	3.807	0.772	0.559
'2-4-1'	0.842	3.889	0.769	0.560
'3-1-3'	0.855	4.253	0.955	0.169
'3-2-2'	0.601	6.206	0.805	0.433
'3-3-1'	0.431	7.220	0.724	0.601
'4-1-2'	0.853	9.997	0.986	0.189
'4-2-1'	0.852	10.016	0.977	0.259
'5-1-1'	0.787	7.399	0.938	0.402
max/min	0.888	3.807	0.986	0.169
average	0.755	6.613	0.832	0.419

Table 5. R^2 and RMSE values for linear regressions of IE vs K_{sat} and log(K_{sat}) using the 15 different triplets for the sandy class selection.

	K_{sat}		$\log K_{sat}$	
	R^2	RMSE	R^2	RMSE
‘1-1-5’	0.075	2.783	0.634	0.535
‘1-2-4’	0.002	3.540	0.468	0.576
‘1-3-3’	0.051	5.220	0.019	0.789
‘1-4-2’	0.095	7.595	0.015	0.818
‘1-5-1’	0.187	5.595	0.633	0.478
‘2-1-4’	0.010	3.262	0.374	0.573
‘2-2-3’	0.099	4.913	0.002	0.830
‘2-3-2’	0.118	6.300	0.041	0.777
‘2-4-1’	0.149	6.132	0.345	0.717
‘3-1-3’	0.204	4.962	0.267	0.439
‘3-2-2’	0.178	6.749	0.374	0.513
‘3-3-1’	0.215	7.046	0.474	0.724
‘4-1-2’	0.232	6.331	0.769	0.294
‘4-2-1’	0.169	11.304	0.565	0.754
‘5-1-1’	0.065	5.336	0.375	0.798
max/min	0.232	2.783	0.769	0.294
average	0.123	5.805	0.357	0.641

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Table 6. Comparison of parametrizing power of (IE,'5-1-1') against IE calculated with other triplets. The ranges of variation of IE calculated with the different triplets are compared to the ranges of variation of K_{sat} for the textural classes. The triplets are chosen to be the ones that gav the highest R^2 values at the linear regressions for K_{sat}.

Textural class	%range '5-1-1' / % range K_{sat}	% range best triplet / % range K_{sat}
Silty loam	8.579	12.923
Sandy loam	0.840	1.002
Sandy clay loam	1.026	1.263
Sandy clay	4.226	4.992
Sand	0.420	0.420
Loamy sand	1.320	2.708
Loam	2.643	9.157
Clay loam	1.064	15.084
Clay	1.477	1.444