Editor response:

Comments to the Author:
Both the Referees recognize that the manuscript has been substantially improved and some of the raised issues have been addressed.
However, I agree with Referee #2 that significant further revisons are needed, and I invite the authors to seriously consider all the comments received by both the Referees.
Best regards,
Roberto Greco

Dear Editor:

Please find in this document the reply to all the comments by the Referees. Also find the revised manuscript with the changes introduced.

Best regards,
Carlos

Reviewer # 1 (Armindo, Robson André rarmindo@gmail.com)

Reviewer #1:
The manuscript was improved in several aspects, mainly in written part giving more details to the readers. I am glad that some of my previous points were helpful to the authors as I may see over this new version. I see also that new graph (Fig 6) and analysis are in this version. By the authors explaining, I understand now that this work does not intend to develop PTFs to estimate Ksat but to present a physical explaining of the relationship between Ksat and PSD. If so, this information should be clearer in the text.

Dear Reviewer:
we are very thankful for the previous helpful comments and how they affected our manuscript. It is true we do not want to develop a PTF, as we are investigating only the linear relationship between the new parameter, combined with the different possible textural descriptions, in terms of the triplets, and the saturated hydraulic conductivity. We have explained this in the introduction and in the results section. We have changed the text in order to explain this more.

Reviewer #1:
Plus, I still have two main comments in order to help authors to get a better text:

- It is pointed out as objective that this work yielded a better description of Ksat. However, I cannot find in conclusion item which was this description of Ksat. I may see that PSD, by means of sand, silt and clay sizes, correlated with Ksat values. In my point of view this is a statistical correlation and not a physical characterization. So, if this work has no provided a physical characterization of Ksat for explaining it, the objective could be rearranged to be presented in another way. In conclusion item is written that PSD correlate with Ksat. Was the aim of this work to verify a possible correlation between both variables or to physically describe Ksat in function of PSD?

Answer:
It is not PSD, by means of sand, silt and clay, that correlates with Ksat values. It is the heterogeneity parameter, obtained from different textural descriptions, that correlates with Ksat. In particular, there is a strong linear relationship between Ksat and this new parameter. This should be interpreted as Ksat being linearly explained, described, by a heterogeneity parameter which depends on the particular data, i.e. triplet, that is chosen from the whole PSD. The point is that PSD is not anymore described by the three usual values, but now we can focalize in different aspects, intervals, from it, thus putting focus on different triplets, i.e. physical features of the soil. For some particular triplets, their heterogeneity, measured in terms of the information entropy correlates extremely well, thus explains, the Ksat. So, when looking at different physical aspects of the soil, in terms of the heterogeneity of different triplets -this is the key- one can actually linearly explain the Ksat.

Reviewer #1:
- Fig. 6 shows in title ln Ksat and in graph log Ksat. Please, be careful with that. Authors should fix it over all text.

Answer:
we have corrected this throught the text. Also, as suggested by the second reviewer, we have eliminated the Ksat relationships and kept only the logarithmic ones.

Reviewer #1:

Minor comments
- Authors are still using the symbol "." over the text to represent values and I think in ESL "," would better describe it. For example, at line 21 where is "19.000" I think it should be "19,000".

Answer:
This has been corrected. Thank you very much.

Reviewer #1:
- Again, I wonder to know if the name "heatmap" is the better name to those maps that do not show values of heat or temperature. Is it because of entropy information? I had no any reply about this previous mentioned query.

Answer:
We have changed from "heatmap" to "ternary graph" throughout the text.

Reviewer #1:
- The quality of Figures 1, 2, 4, 5 and 6 can be improved.

Answer:
we have improved these figures as much as the R software and our computational force allows us.

Reviewer #1:
- Variables Ksat and RMSE presented in Table 4 and 5 are missing units.

Answer:
the variables present in Tables 4 and 5 are R^2 (no units) and RMSE, which has got the same units as Ksat (cm/h).

Reviewer #1:
- Table 5. log10Ksat or lnKsat?

Answer: we are using the natural logarithm. For mathematical purposes it is indistinct as they are the same except a multiplicative factor.

Reviewer # 2.
You therefore must not mention the R2 value in the abstract as it is artificially increased. Likewise, the abstract must better reflect the fact that the regression results are very good for the sandy soils but clearly worse for the finer textured soils (as it is very much expected from previous studies

Answer:
We have removed R2 values from the abstract as the reviewer suggested. We also included a phrase on relatively worse correlations for fine texture soils.

As the reviewer states, our database, of more than 19K soils, contains, sadly, a vast majority of sandy soils. We remind that this database is available for anyone that asks for it, on the reference in our bibliography.

The lack of clayey soils is pointed out in the text and present in the statistical summary table. Nevertheless, our study does not pretend to explain all the possible soils, but just the ones available to us in our database. We wished we had a finer representation of the whole textural triangle.

Reviewer # 2.
I still do not think that including Ks is necessary. It only complicates understanding what you have done. Log Ks is enough. If you do not agree please explain in the manuscript why both are needed. The authors moreover need to make clear whether they are using log_{10} or ln [i.e. log_{e}). the log-notations are furthermore inconsistent throughtout the manuscript.

Answer:
we agree with the reviewer and we have eliminated the Ksat values, and left only the log Ksat values. The logarithm used is the natural one, in all cases. As a reminder: mathematically any two logarithms are just a constant factor away.

Reviewer # 2.
The authors state that it is the aim of this study to test whether the combination of different texturial triplets and the information entropy yields "a better description of K_sat" (p2L16). It is not clear to me what is meant with "description". I am interpreting it as "prediction" from texturial data.

Answer:
Our purpose was not to predict, but to find out if *different* physical representations of the texture -in form or triplets-, which is a key factor in the pore space arrangement, and thus in its hydraulic behavior, would *linearly* correlate to the Ksat values, thus giving insight on what physical aspects of the texture (which types or sizes of particles) can explain better the hydraulic conductivity. We also wanted to do this through the parameter IE, which summarizes the heterogeneity of a given triplet, which we understand as a physical representation of the building blocks of the soil.

The purpose of this work is to focus on how different triplets, i.e. different possible physical descriptions of the soil, and the heterogeneity associated to them, is, or not, linearly correlated to Ksat. The most correlated ones will be understood as the physical sizes that influence the most in the packing of particles yielding that particular hydraulic behaviour.

We do not intend to predict Ksat values from soil texture, but to present other textural representations that would correlate better with this hydraulic parameter

To address the comment, we removed the term “description” which we agree is ambiguous. Instead we included a part of above explanation

Reviewer # 2.
As also stated by the AE, the authors are statistically predicting Ksat from texturial data. They are providing very little physical reasoning/theory for why their approach should work better. In this, they are basically creating pedotransfer functions. In fact, the authors are apparently failing to take soil structure into account, i.e. stable superstructures built from individual texturial particles that are held together by clay sized particles. Such structures are often biopores e.g. dug by earthworms or old root channels. In the "final comments" At P8L2 the authors are writing that "soil aggregation, root exudates, soil aggregation, etc." are other important factors that determine Ks. I completely agree. However, this must already be discussed in the introduction. It will help the authors to explain why the texturial data is performing so much worse for the finer textured soils.

Answer: Please note that we do not predict Ksat. “Prediction” means estimating something which has not been observed. We do not do that.

We are comparing the statistics of the regressions against different textural descriptions, i.e. different physical sizes of particles, in terms of triplets, and the trying to reason why the performance changes in terms of the triplets. We see that different textures correlate with different physical descriptions, thus finding out that for different soils, there are different parts of the texture that matter more in the hydraulic behaviour.

Reviewer # 2.
As also stated by the AE, the authors are statistically predicting Ksat from texturial data. They are providing very little physical reasoning/theory for why their approach should work better. In this, they are basically creating pedotransfer functions. In fact, the authors are apparently failing to take soil structure into account, i.e. stable superstructures built from individual texturial particles that are held together by clay sized particles. Such structures are often biopores e.g. dug by earthworms or old root channels. In the "final comments" At P8L2 the authors are writing that "soil aggregation, root exudates, soil aggregation, etc." are other important factors that determine Ks. I completely agree. However, this must already be discussed in the introduction. It will help the authors to explain why the texturial data is performing so much worse for the finer textured soils.

Answer:
We note that research in this work is a descriptive one. It does not include explanation of what we have observed. However, any explanatory research with mechanisms, models, etc. was historically preceded with the descriptive research. We have included the above passage to the manuscript.

We do not agree with the reviewer’s conclusion that we create the pedotransfer functions. We do not do that. Pedotransfer functions are meant to be used outside the database thay have been created from. WE have not intend to do that, and no component of the manuscript indicaes that we have created PTFs.

Reviewer # 2.
I have for now assumed that the database used by the authors contains undisturbed samples. It is not stated in the manuscript, though. It needs to be stated there because it is a very important information for understanding the results (see comment above).

Answer:
All samples in the database are undisturbed. This is now included in the manuscript.

Reviewer # 2.
Another major concern is the motivation of using the information entropy. It is simply not provided in the manuscript. It is unclear to me why it should be better then using a texturial triplet directly for a regression. An explanation should be given and a respective reference included. The authors aim at testing whether it would yield "a better description of Ksat". They are then however only testing the effect of different texturial triplets. If also the information entropy should be tested for usefulness for predicting Ksat, a comparison to linear regressions without using the information entropy needs to be included. I am urging the authors to do this. It would then be interesting to discuss why the IE is working better. Maybe because it tends to linarize the relationship to log Ksat?

Answer: The information entropy is the heterogeneity parameter. Using simple regression with triplet elements as inputs does not explicitly use the heterogeneity into account.

Reviewer # 2.
I propose another round of major revisions.

More specific comments:

P1L6-13: I agree that the particle size distribution has a large impact on Ksat. Please explain better that it is the soil pore network architecture that determines Ksat (not the texture itself) and that stable structural pores (i.e. biopores, cracks, etc.) are only existing if the clay content is large enough to support them. Please quote some respective references, e.g. Horn, R., H. Taubner, M. Wuttke and T. Baumgartl. 1994. Soil physical properties related to soil structure. Soil Tillage Res. 30: 187-216. Explain that the structural pores are often more important for Ksat because they are larger than the inter-particle pores (q ~ r^2) and mostly well connected. Furthermore explain what advantages the use of the IE brings and what the findings of Boadu (2000) were. Was IE found to be an improvement?

Note: it is P2L6-13
Answer:
it is clear that the Ksat is controlled by the soil pore network. Nevertheless, data relevant to the soil pore network are scarce and not available for our database. Some of the soil pore network is controlled by soil texture. We state: "Soil structure can be to some extent controlled by soil texture, since packing of particles is affected by the particle size distributions". Here the reference suggested by the Reviewer is very useful and we included it. However the suggestion of the reviewer to “Explain that the structural pores are often more important for Ksat because they are larger than the inter-particle pores (q ~ r^2) and mostly well connected.” Means that we need to include a textbook material in the body of the manuscript. We do not see a reason for that.

Reviewer # 2.
P3L17-19: justify the use of the binning approach. The reader should not get the impression that it was only used to make the results look better.

Answer:
the particular value of R2 is not important. So it is not about "results looking better". We want to compare the R2 values from one regression to another, and to able to do this, we use the 10 interval binnings because it allows to use the same number of points in all the regressions. This methodology is described in our paper [M.A. Martin, Ya. A. Pachepsky, C.

García-Gutiérrez and M. Reyes, On soil textural classifications and soil-texture-based estimations, Solid Earth, 9, 159-165, 2018] that we reference.

Reviewer # 2.
P6L21: not just "possibly aggregating". In undisturbed soil the fines are as good as always aggragated. Exception are highly sodic soils.

Answer:
we are sorry, but we are not sure what the reviewer wants us to do.

Reviewer # 2.
P6L23-34: the scale breaking discussion is interesting but needs to be expanded. I still do not find it convincing that what you observe is caused by the scale breaking.

Answer:
we do not insist that the scale breaking is the answer here. We propose it a plausible explanation in the discussion section.

Reviewer # 2.
Figure 6 shows that the relationship between IE-3-1-3 and log Ksat is non-linear (not only "could be" linear). If you would use a quadratic regression, you would again get very nice fits.

Answer:
There are many equations that can be used for nonlinear fit. Our objective was not to "fit" the data.

Reviewer # 2.
P7L14-23: move to the materials and methods section

Answer: Thank you. This has been moved.

Reviewer # 2.
P8L17-20: I do not understand what the authors are trying to state here. First of all, such a passage belongs to the end of the introduction, where the aim of the study is defined.

Answer:
the passage in question belongs to the end of the introduction and we have moved it. We stick to linear relationships, which are not intended for prediction. Nevertheless, we calculate the RMSE value, which can be used to compare the performance of the regression.

Reviewer # 2.
Secondly, PTFs may be linear or non-linear, but they are not created for interpretation. Moreover, this manuscript does does only minimally involve physical relationships.

Answer: We responded to this concluding comment before. To summarize:

- **(a) We do not develop PTFs**
- **(b) We agree that we present the descriptive research, that preds future physical modeling.**

Saturated Hydraulic Conductivity and Textural Heterogeneity of Soils

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Abstract. Saturated hydraulic conductivity (K_{sat}) is an important soil parameter that highly depends on soil's particle size distribution (PSD). The nature of this dependency is explored in this work in two ways, (1) by using the Information Entropy as a heterogeneity parameter of the PSD and (2) using descriptions of PSD in forms of textural triplets, different than the usual description in terms of the triplet of sand, silt and clay contents. The power of this parameter, as a descriptor of ~~K_{sat} and~~ $\ln K_{sat}$, was tested on a database larger than 19,000 soils. Bootstrap analysis yielded coefficients of determination of up to 0.977 for $\ln K_{sat}$ using a triplet that combines very coarse, coarse, medium and fine sand as coarse particles, very fine sand and silt as intermediate particles, and clay as fines. The power of the correlation was analysed for different textural classes and different triplets using a bootstrap approach. Also, it is noteworthy that soils with finer textures had worse correlations, as their hydraulic properties are not solely dependant on soil PSD.

This heterogeneity parameter can lead to new descriptions of soil PSD, other than usual clay, silt and sand, that can describe better different soil physical properties, that are texture dependant.

1 Introduction

Saturated hydraulic conductivity (K_{sat}) is the measure of soil's ability to conduct water under saturation conditions (Klute and Dirksen, 1986). It is an essential parameter of soil hydrology. Soil K_{sat} affects many aspects of soil functioning and soil ecological services, like infiltration, runoff, groundwater recharge and nutrients transport. Knowing values of soil K_{sat} appears to be essential in designing management actions and practices, such as irrigation scheduling, drainage, flood protection, and erosion control.

The dependence of K_{sat} on soil texture has been well documented (Hillel, 1980). Different parameterizations of particle size distributions (PSDs) were suggested to relate K_{sat} and soil texture. It was proposed to use d10, d20, and d50 particle diameters (Chapuis, 2004; Odong, 2007) or slope and intercept of the particle size distribution curve (Arya and Paris, 1980; Alyamani and Sen, 1993). Also various functions were fitted to PSDs, and the fitting parameters were related to K_{sat}. For example, Chapuis et al. (2015) proposed to use two lognormal distributions to fit the detailed particle size distribution and to use the lognormal distribution parameters to predict the K_{sat}.

A common way to parameterize the PSD for K_{sat} estimation purposes is using the textural triplet that provides the percentage of coarse particles (sand), intermediate particles (silt), and fine particles (clay). K_{sat} values are estimated using the contents of one or two triplet fractions or just the textural class (Rawls et al., 1998). Representing PSD by textural triplets is the common way to estimate a large number of soil parameters (Pachepsky and Rawls, 2004). The coarse, intermediate, and fine fractions need not to be sand, silt and clay. Martín et al. (2017) showed that different definitions of the triplet, e.g. coarse sand, sand, and medium sand as coarse, fine sand, very fine sand as intermediate, and silt and clay as fine triplet fractions, provide much better inputs for bulk density estimation compared with the standard textural triplet. These different parametrizations of soil texture might put the focus on different soil physical properties, depending on the different particle sizes represented in the triplet.

The heterogeneity of particle size distributions appears to be an important factor affecting hydraulic parameters of soils, including the saturated hydraulic conductivity. Values of K_{sat} depend on both distribution of sizes of soil particles, i.e. soil texture, and the spatial arrangement of these particles, i.e. soil structure. Soil structure can be to some extent controlled by soil texture, since packing of particles is affected by the particle size distributions ~~(e.g., Gupta and Larson, 1979; Assouline and Rouault, 1997; Jor~~ It was recently proposed to use the information entropy as the parameter of the PSD heterogeneity for predicting soil water retention (Martín et al., 2005) and soil bulk density (Martín et al., 2017). Previously, information entropy was used, together with other predictor variables to estimate K_{sat}, using multivariate analysis (Boadu, 2000).

The objective of this work was to test the hypothesis that combining two recent developments -the description of the PSD by different textural triplets, that may ~~focus on different soil properties~~represent different soil physical properties dependent on the particle sizes present in the triplet, and the information entropy, as a PSD heterogeneity parameter, that depends on the triplet ~~used for its description- may yield a better description of K_{sat}~~ used- may linearly correlate with $\ln K_{sat}$ and may be seen as a step forward to study the effect of heterogeneity widely recognized in the majority of works that studied the particle size - hydraulic conductivity relationships. ~~We~~ By describing the PSD in terms of different triplets, the input information would possibly have different physical interpretations. We wanted to link the heterogeneity of this physical information to the hydraulic behaviour of the soil. Therefore, we explored the possible relationships between ~~K_{sat}~~ $\ln K_{sat}$ values and an entropy metric of soil texture heterogeneity using different size limits of coarse intermediate and fine fractions, using the large USKSAT database on laboratory measured K_{sat} containing more than ~~19.000 samples.~~ 19,000 samples. The triplets with highest correlations will be understood as the physical sizes that influence the most in the packing of particles yielding the particular hydraulic behaviour. While PTFs are a useful tool to predict difficult-to-measure soil properties, they sometimes exhibit highly non-linear relationships which are difficult to interpret. While the objective of this paper was the exploration of the physical relation of the new tools and the saturated hydraulic conductivity, the future development of PTFs for prediction purposes is promising avenue for expanding this research. We note that research in this work is a descriptive one. It does not include explanation of what we have observed. However, any explanatory research with mechanisms, models, etc. was historically preceded with the descriptive research.

2 Materials and Methods

2.1 Database description and textural triplet selection

For this study we used USKSAT database in which detailed information can be found in Pachepsky and Park (2015). This database consists on soils from different locations of the USA and contains soils from 45 different sources. We selected only those sources which (a) had data on both K_{sat} and on the seven textural fractions, and (b) presented measurements of K_{sat} made in laboratory with the constant head method. From those, we subset those soils whose sum of mass in the seven textural fractions, i.e. (1) very coarse sand, (2) coarse sand, (3) medium sand, (4) fine sand, (5) very fine sand, (6) silt and (7) clay ranged from 98 to 102%. The final number of soils considered was 19.121. 19,121. By USDA textural classes the total number of soils are: 12.068 12,068 sands, 1.780 loamy sands, 2.123 2,123 sandy loams, 104 loams, 135 silt loams, 36 silts, 2004 2,004 sandy clay loams, 78 clay loams, 41 silt clay loams, 345 sandy clays, 0 silty clays and 407 clays. All the samples in the database used are undisturbed soil samples.

We used all possible triplets formed from seven textural fractions. Triplets consisted of coarse, intermediate, and fine fractions. The symbols for triplet showed how the fractions were grouped. For example the "coarse" fraction for the triplet '3-2-2' included very coarse sand, coarse sand and medium sand, the "intermediate" fraction included fine sand and very fine sand , and "fine" included silt and clay; triplet '5-1-1' was the standard one where "coarse" included all five sand fractions, "intermediate" included silt, and "fine" included clay. The amount of possible triplets with 7 textural fractions was 15.

2.1.1 Heterogeneity metric calculation

The Entropy based parametrization of textures introduced in Martín et al. (2001) has as central concept in the Information Entropy (Shannon, 1948). Assuming the texture interval divided into k textural size ranges and that the respective textural fraction contents are $p_1, p_2, \ldots, p_k$, $1 \leq i \leq k$, with $\sum_{i=1}^{k} p_i = 1$, the Shannon Information Entropy (IE) (Shannon, 1948) is defined by

$$IE = -\sum_{i=1}^{k} p_i \log_2 p_i \quad (1)$$

where $p_i \log_2 p_i = 0$ if $p_i = 0$. The IE is a widely accepted measure of the heterogeneity of distributions (Khinchin, 1957). In case of three fractions, the minimum value of IE is zero when only one fraction is present, and the maximum value is 1.57 when three fractions are present in equal amounts (see Fig. 1).

For each soil in this study, we grouped the 7 available textural fractions in the 15 possible triplet combinations and calculated the respective triplet's IE using formula (1). Fig 2 shows heatmaps ternary graphs of IE calculated for all the soils available in this study but using two different triplets as input. It is clear that, by changing the triplet, the calculated IE values vary differently along the same textural triangle. IE is a measure of heterogeneity, but the triplet used is the substrate for this measure: (IE,*triplet*), i.e., (IE,'5-1-1').

We As we want to compare the linearity, i.e. the proportionality between the heterogeneity of the particular physical sizes chosen and the hydraulic behaviour, we used coefficient of determination, R^2, as a comparison statistic. As this statistic is highly sensitive to the number of points in the regression, we followed the binning method of Martín et al. (2017) to research the relationship between K_{sat} $\ln K_{sat}$ and soil heterogeneity. Specifically, the range of values of IE was divided into ten bins, the average value of K_{sat} $\ln K_{sat}$ was plotted against the average IE for the bin, i.e. the bin midpoint. This way, the number of points in each relationship was always the same. We want to state that this way, the particular value of R^2 is irrelevant, but it is to be only used as a comparison tool among these regressions.

Linear regressions 'bin midpoint vs. average bin K_{sat}' and 'bin midpoint IE value vs. average bin $\ln K_{sat}$' were computed. The Besides the coefficient of determination value for comparison purposes, the goodness-of-fit of these regressions was tested using the coefficient of determination R^2 and the Root-mean-square error, RMSE

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_t - y_y)^2}{n}}$$

where $\hat{y}_t$ are the predicted and y_t are the measured values of K_{sat} $\ln K_{sat}$, and n is the number of soils.

In order to make some inference on these parameters we employed the bootstrap method, which has been used in a very similar context by Schaap and Leij (2000). The bootstraph method is a tool for assessing statistical accuracy. It assumes that one can obtain multiple samples from a single data set, by randomly drawing data with replacement from the original sample. Thus, one can perform the same statistical analysis multiple times in different data sets, obtaining slightly diferent regression models, thus resulting in an uncertainty in each of the parameters of the model. All of the samples used have the same size as the original sample they were drawn from, so the are generated by random sampling with replacement. We used 1.000 bootstrap data sets, resulting in 1.000 1,000 linear regression models. In particular we obtained not just one R^2 and one $RMSE$ value for each IE vs. triplet regression, but one thousand of them, that were summarized into a mean and a standard deviation values. More information on this method can be found in (Efron and Tibshirani, 1993; T. Hastie, 2003).

We took 1.000 1,000 samples with size equal the total amount of soils, with repetition, and calculated, for each sample, the coefficient of determination, R^2 and the Root-mean-square error, RMSE. Finally, the mean and standard deviation from these two values, for the 1.000 1,000 samples were calculated.

These regressions were obtained for each of 15 triplets and for those of USDA textural classes that were represented in the selected database by more than 50 samples, i.e. all of them except silty clay loams and silts.

2.2 IE variation in the textural triangle

Ternary graphs were used to visually correlate the IE values calculated with the $\ln K_{sat}$ values of the soils in the study. Also, a less visual, but more quantifiable approach, to find out how much of $\ln K_{sat}$ could be explained through IE variation was to find out what ranges of IE are available for soils in different textural classes and compare them to the range of $\ln K_{sat}$ values of soils inside those same textural classes. Also, in order to compare the new tool (IE triplet), we compared these ranges to the ranges computed for (IE,'5-1-1') , i.e. to the values of the IE computed with the usual description of soil texture. We wanted to find out if, by changing the triplet, we would obtain a wider range of variation in IE for a given range of $\ln K_{sat}$. This way we compared if the new descriptions of texture, in form of different triplets, might be suitable for explaining soil physical properties, in particular $\ln K_{sat}$.

For each textural class, we did a sensitivity analisis by calculating the ratio of the range of $\ln K_{sat}$ values inside the textural class versus the range of $\ln K_{sat}$ values of all the soils in the study. The same was done for IE for each triplet.

3 Results and Discussion

3.1 The Dataset Overview

Fig. 3 presents the 19.121 soils used in this study in the USDA textural triangle and in the modified '3-2-2' triangle. The density of points reflects the dominance of coarse textural soils in the database. When the triplet is changed, the distribution of points across the triangle changes. By setting the textural fractions to be the '3-2-2' triplet, the distribution of points/soils in the new textural triangle spreads. While there is still a high concentration of soils in the stripe of bigger than 85% of the Coarse fraction, where Coarse 3 includes very Coarse Sand, Coarse sand a Medium Sand, now those soils spread fully from 10 to 100% of the Intermediate-2 fraction, where Intermediate-2 contains Fine and Very Fine Sand. On the USDA textural triangle, most of the soils are clustered in the subtriangle limited by the lines "more than 70% sand" and "less than 20% silt". This new textural triangle allows for a finer look into the sand fraction, revealing the distribution of soils within the USDA sandy textural classes. This finer look might prove itself useful to study physical properties of these soils that are mainly related to the type and amount of sand in them.

Table 1 shows the K_{sat} statistics for the soils in the study. A total of ~~19.420~~ 19,420 soils were used in this study, from which 299 (1.53%) had to be rejected due to missing values. The textural class sand comprises the 63.1% of all the soils, followed by sandy loam (11.1%) and sandy clay loam (10.48%). Five textural classes were poorly represented with percentages less than 1% of the total soils. The K_{sat} values varied between 0.0005 and 841 cm/h being 22.57 the mean value.

3.2 Regression in binned data: IE as a predictor of K_{sat} and $\ln K_{sat}$

Linear regressions for ~~K_{sat} and~~ $\ln K_{sat}$ were done to find out the predictive power of the proposed parameter, (IE,triplet), with the 15 possible different triplets that could be archived by grouping the available textural data. Table 2 shows the computed R^2 and $RMSE$ values for the linear regressions using 10 interval bins.

~~The best triplet for the K_{sat} regression was '2-3-2', i.e. 'very coarse sand + coarse sand' as coarse, 'medium sand + fine sand + very fine sand' as intermediate and 'silt + clay' as fine factions, with the highest mean R^2 value (mean = 0.885, std = 0.020) and this triplet also had the lowest RMSE value (RMSE=3.649 cm/h). Figure **??** shows a heatmap representation of the K_{sat} values of the soils of the study on the textural triangle compared to a heatmap representation of the IE values of the same soils computed using the '2-3-2' triplet. The sandy soils had high K_{sat} values, and the IE values on that part of the triangle were low. The triangle presents high K_{sat} values in a stripe between 0 and 20% sand. This stripe has also low (IE,'2-3-2')~~

~~values, so there is a reasonable visual relationship between these two values. It is also worth noting that the '4-1-2' triplet, i.e. 'very coarse + coarse + medium + fine sands' as coarse fraction, 'very fine sand' as intermediate and 'silt + clay' as fines had a very similar mean R^2 value (0.880) but a lower standard deviation (0.007).~~

~~For the $\ln K_{sat}$ regression, the~~ The best triplet in terms of highest mean R^2 value was '4-2-1', with a mean of 0.977 and a standard deviation of 0.002, but the lowest mean RMSE $(\ln(cm/h))$ value (mean=0.207, std = 0.030) was attained with the '1-2-4' triplet. Figure 4 shows ~~the same comparison as figure **??**, but using~~ a ternary represenation representation of the $\ln K_{sat}$ ~~heatmap and~~ values of the soils of the study on the textural triangle compared to a ternary representation of the IE ~~computed with~~ values of the same soils computed using the '4-2-1' triplet. There is a ~~higher~~ high visual similarity between these two images, with high $\ln K_{sat}$ value zones, near the lower corners (sandy and silty soils) that correspond to low (IE,'4-2-1') values. The $\ln K_{sat}$ values tend to decrease towards the centre of the triangle. On the other hand, the (IE,'4-2-1') values tend to increase around this point.

The standard triplet ('5-1-1') yielded, for the ~~K_{sat} regression, a mean R^2 value of 0.775 with standard deviation equal to 0.010 and a mean RMSE value of 7.555 with standard deviation of 0.190; for the~~ $\ln K_{sat}$ regression, the R^2 value with this triplet had a mean of 0.960 and a standard deviation of 0.005; the RMSE mean value was 0.339 with a standard deviation of 0.021. ~~Regressions against $\ln K_{sat}$ were in average better: the~~ The average of the R^2 mean values of the regressions with all possible triplets for $\ln K_{sat}$ was 0.727~~, whilst the average R^2 value for the K_{sat} regression was 0.667.~~.

3.3 Predictive power of IE among the USDA textural classes

In this section we show how IE works differently among textural classes: using different triplets we can find that the textural classes are predicted differently; what works for some, for others is counterproductive.

Table 3 shows the best triplet, chosen in terms of highest mean R^2 value of all the possible regressions, for each textural class that had $N > 50$. In the table are shown the mean and standard deviation for R^2, of the ~~1.000~~ 1,000 bootstrap samples for ~~both, K_{sat} and~~ $\ln K_{sat}$ ~~,~~ linear regressions. The best R^2 values were obtained for the regression of the sand textural class against the (IE,'5-1-1'), i.e., the IE computed with the standard '5-1-1' clay-silt-sand USDA triplet~~(means~~. The mean value was equal to R^2=~~0.905 for the K_{sat} regression and R^2=~~0.987 for ~~the $\ln K_{sat}$)~~ all the regressions. A possible explanation for this triplet being the best among all the other possible triplets, is that sandy soils are the ones that contain percentages of the sand fraction higher than 70%, so their distribution is highly heterogeneous. Minor fractions are now silt and clay, and the information about this two fractions could be very important for the hydraulic properties of the soil, thus the (IE,'5-1-1') triplet yielded the best regression result. One might think that, having such a high concentration of sand particles, is now silt and clay the fractions that made the difference in the packing properties, thus in the saturated hydraulic conductivity values. The high value of R^2 indicates that the relation is very strong in this case.

Almost all sandy textural classes had the highest regression coefficients. Table 3 suggested grouping the textural classes into two superclasses: `SC1`, comprising the textures sandy, sandy clay loam, sandy loam and loamy sand; and `SC2`, with sandy clay, clay, clay loam, loam and silty loam. Soils in `SC1` are mostly sandy soils, with the exception of the sandy clay textural class which is within the `SC2` soils which are mostly clayley and loamy soils. The ~~first superclass, `SC1`, had K_{sat} mean R^2 values~~

above 0.608 and for the second superclass, the biggest mean R^2 value was of 0.416. Even more, the lowest mean R^2 value for the $\log K_{sat}$ regressions in the SC1 superclass was 0.742 and the highest one for the SC2 class was 0.604. Total number of soils in SC1 was 17975 17,975 (94.06% of total soils in the database). SC2 contained 1069 1,069 soils (5.59% of total). Tables 4 and **??** show the R^2 and RMSE values for all regressions (K_{sat} and $\ln K_{sat}$) for the soils in SC1 and SC2.

For the SC1 we observed that the best regression (R^2=0.885) against K_{sat} 0.986, $RMSE$=0.184) against $\ln K_{sat}$ was reached with (IE,'2-2-3'), and being (IE,'2-3-2')a close runner-up (R^2=0.868) and a lower RMSE value. Both these triplets make a distinction 4-1-2'). This triplet creates a division among the sand fractions, putting very coarseand coarse sand in the coarsefraction in the first case, and adding medium sand in the second case. Alsogrouping together very coarse, coarse, medium and fines, and leaving alone the very fines sand. Finally, the fines fraction contains either very fine sand or notfractions contains only the silt and clays. Comparing this to the sandy textural class results, where the best triplet was '5-1-1', we observed that now more information from the sandy fraction was required to infer hydraulic properties. The area that the SC1 soils cover in the textural triangle and the hydraulic property variation of these soils can be related with a heterogeneity metric associated to triplets that distinguish well among the predominant fraction in that area of the triangle, i.e., sand.

For the SC2, best triplet in both regressions (K_{sat} and $\ln K_{sat}$) was '1-1-5', with R^2=0.202 for K_{sat} and R^2=0.623for $\ln K_{sat}$. Regression results were worse than for SC1, but this might be just provoked by the nature of SC2 itself: these are soils with less sand, thus higher content in clays and aggregating particles. The packing -and consequently the K_{sat}- of these soils is not just mainly affected by the PSD, but also by aggregation, which cannot be accounted for in the IE value, regardless of the triplet used.

Furthermore, the best triplet, '1-1-5', also pointed in this direction: the fines fraction contains medium sand, fine sand, very fine sand, silt and sand particles, while the intermediate fraction contains only the coarse sand, leaving the coarse fraction with the very coarse sand, thus giving more importance to the possibly aggregating particles than a triplet like, '1-4-2' which had R^2 values equal to 0.014 (K_{sat}) and 0.033($\ln K_{sat}$).

3.4 Triplets and Scaling Break

In the regressions made with all the soils, it was noteworthy the behaviour of the (IE,'3-1-3'). The average mean value R^2 for all the K_{sat} regressions was 0.667, but the R^2 using (IE,'3-1-3') gave a R^2 equal to 0.005, far below the next lowest one, which was (IE,'3-2-2') with a mean R^2 value of 0.428. The same happened in the $\ln K_{sat}$ regressions, where the average value of all triplets was 0.727, but (IE,'3-1-3') gave a exceptionally low R^2 value of 0.087, being the next lowest (IE,'2-2-3') with a mean R^2 value of 0.235.

The '3-2-3' triplet groups fine sand with silt and clay, and coarse and very coarse sand with medium sand. Kravchenko and Zhang (1998); Wu et al. (1993); Tyler and Wheatcraft (1992) reported the break in scaling where the powerlaw scaling of soil texture occurred in the size range of fine sand The Particle size distribution scales in a different way in two different regions of the size intervals, and that the change of scaling is produced around the fine sands. The triplet '3-1-3' separates these two regions, maybe bringing forth this scaling break effect. Fig 5. shows how the relationship between K_{sat} (and $\ln K_{sat}$) and (IE,'3-1-3') could be nonlinear, maybe due to the absence of global selfsimilarity showed in the scaling break.

On the other hand, it is also noteworthy that regressions against (IE,'3-1-3') were actually quite good (R^2=0.837 for K_{sat} and R^2=0.939for $\ln K_{sat}$) in the SC1, while in the SC2 they were moderate (R^2=0.047 for K_{sat}, and R^2=0.045$\ln K_{sat}$). Furthermore, even though (IE,'3-1-3') presented the lowest R^2 values for all soils, this triplet yielded the best R^2 results for soils belonging sandy clay loam ($N = 2.004$)texture, for K_{sat} regressions. Nevertheless, for other textures, the '3-1-3' triplet had, generally, a very low value.).

When all the soils are considered together, then (IE,'3-1-3') might fail, due to the scaling break, but when we restrict the study to a certain part of the textural triangle, that effect might diminish to a point where this triplet is even useful to predict some textural derived properties, or maybe the scaling break effect is also restricted to some textural classes and should be further investigated.

As results show, IE is not powerful K_{sat} $\ln K_{sat}$ predictor by itself, but combined with an input triplet. By changing the triplet, we may focus on certain physical aspects of the soils, but it is also important to keep in mind that this might not work statistically for random groupings of soils that belong to different textures.

3.5 IE variation as a spatial function in the textural triangle

Heatmaps were used to visually correlate the IE values calculated with the K_{sat} (or $\ln K_{sat}$) values of the soils in the study. Also, a less visual, but more quantifiable approach, to find out how much of K_{sat} could be explained through IE variation was to find out what ranges of IE are available for soils in different textural classes and compare them to the range of K_{sat} values of soils inside those same textural classes. Also, in order to compare the new tool (IE triplet), we compared these ranges to the ranges computed for (IE,'5-1-1') , i.e. to the values of the IE computed with the usual description of soil texture. We want to find out if, by changing the triplet, we obtain a wider range of variation in IE for a given range of K_{sat}. This way we compare if the new descriptions of texture, in form of different triplets, might be suitable for explaining soil physical properties, in particular K_{sat}.

For each textural class, we did a sensitivity analisis by calculating the ratio of the range of K_{sat} values inside the textural class versus the range of K_{sat} values of all the soils in the study. The same was done for IE for each triplet. Table 6 shows, for each textural class, the ratio of the percentage of (IE,'5-1-1') against the percentage of K_{sat} $\ln K_{sat}$ range. The same ratio was also calculated using IE for the triplet that gave the best R^2 value in the linear regression against $K_{sat}$$\ln K_{sat}$. These values can be thought of as how much range of (IE,*triplet*) can be used to explain a certain variation of K_{sat} $\ln K_{sat}$ inside each textural class, i.e. as how much parametrizing power is available by the IE. In all the textural classes , except clay, where the regressions were done, the parametrizing power of the alternative triplet was higher than the one by using the usual clay-silt-sand triplet. For the clay textural class, the relative difference was of 2.2%. For the sand textural class, the triplet which gave the best R^2 regression was '5-1-1' thus the results are the same; the average value of the parametrizing power for the usual triplet was 2.460.50, while when we change the triplet we obtained 5.100.79. This shows how, by considering different triplets, combined with IE, a better description/parametrization of K_{sat} $\ln K_{sat}$ can be reached.

3.6 Final Comments

Textural heterogeneity is a crucial factor affecting soil K_{sat}, but it acts along many other ecological factors, as animal activity, root exudates, soil aggregation, etc. In this work we showed that a proper representation of textural heterogeneity, by IE, allows one to (1) demonstrate its effect on ~~K_{sat}~~ $\ln K_{sat}$ by binning samples based on the textural heterogeneity and (2) to statistically parametrize this effect for some textures.

This work has limitations, in particular, the limited available texture data of only seven fractions in the database. The boundaries between coarse, intermediate, and fine fractions can be moved with data from continuous measurements of texture in the fine sand-silt-clay range of particle sized. This may bring the improvements in mean bin ~~K_{sat}~~ $\ln K_{sat}$ estimates for non-sandy soils that could not be achieved in this work.

Although globally the IE computed from different triplets show a potential to reflect the effect of soil texture on the ~~K_{sat}~~ $\ln K_{sat}$ values, the different relationship between the IE and the ~~K_{sat}~~ $\ln K_{sat}$ depending on the triplet used might have different possible explanations. While the IE/~~K_{sat}~~ $\ln K_{sat}$ relationship is found satisfactory in some textural classes, results seem to indicate that the IE parameter cannot reflect with the same efficiency the ~~K_{sat}~~ $\ln K_{sat}$ values in other classes predominating fine particles, in which other processes as aggregation or weathering can not been elucidated by the single textural data input.

Overall, the heterogeneity parameter, IE, combined with the different triplet information, appears to be a strong candidate as an input for the development of new pedotransfer functions (PTFs) to predict ~~K_{sat}~~ $\ln K_{sat}$ and probably other soil physical parameters which are strongly dependant on soil particle size distribution. ~~While PTFs are a useful tool to predict difficult-to-measure soil properties, they exhibit highly non-linear relationships which are difficult to interpret. While the objective of this paper was the exploration of the physical relation of the new tools and the saturated hydraulic conductivity, the future development of PTFs is promising avenue for expanding this research.~~

4 Conclusions

The PSD coarse, intermediate,and fine fractions in soil textural triplets can be redefined from standard ‘sand-silt-clay’ to other fraction size ranges. The textural heterogeneity parameters obtained for some of the new triplets correlate with soil saturated hydraulic conductivity averaged by ranges of the heterogeneity parameters. This approach allows one to quantify the effect of the textural heterogeneity of saturated hydraulic conductivity of soils. Given that size boundaries of sand, silt, and clay fractions have not originally been established for the purposes of prediction of soil hydraulic conductivity, it may be beneficial to look for other size based subdivisions of particle size distributions which, when used along with other soil properties such as bulk density and organic matter content, may provide better predictions of the saturated hydraulic conductivity.

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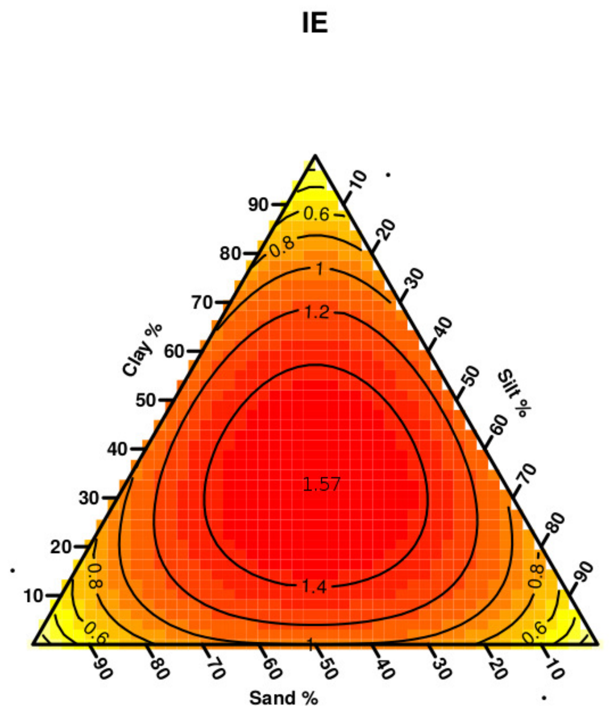

Figure 1. IE numerical approximation ~~heatmap~~ternary representation: IE is computed for a sample of 5051 evenly distributed soils in the USDA textural triangle using the clay, silt and sand fractions as input triplet. This distribution of IE is repeated for any textural triangle, when the fractions used for its calculations are the ones at the axes of the triangle. The lowest values for the IE are near the vertex of the triangle, i.e. where one fraction dominates above the others. Biggest values are located towards the centre of the triangle, where the distribution fractions are more balanced.

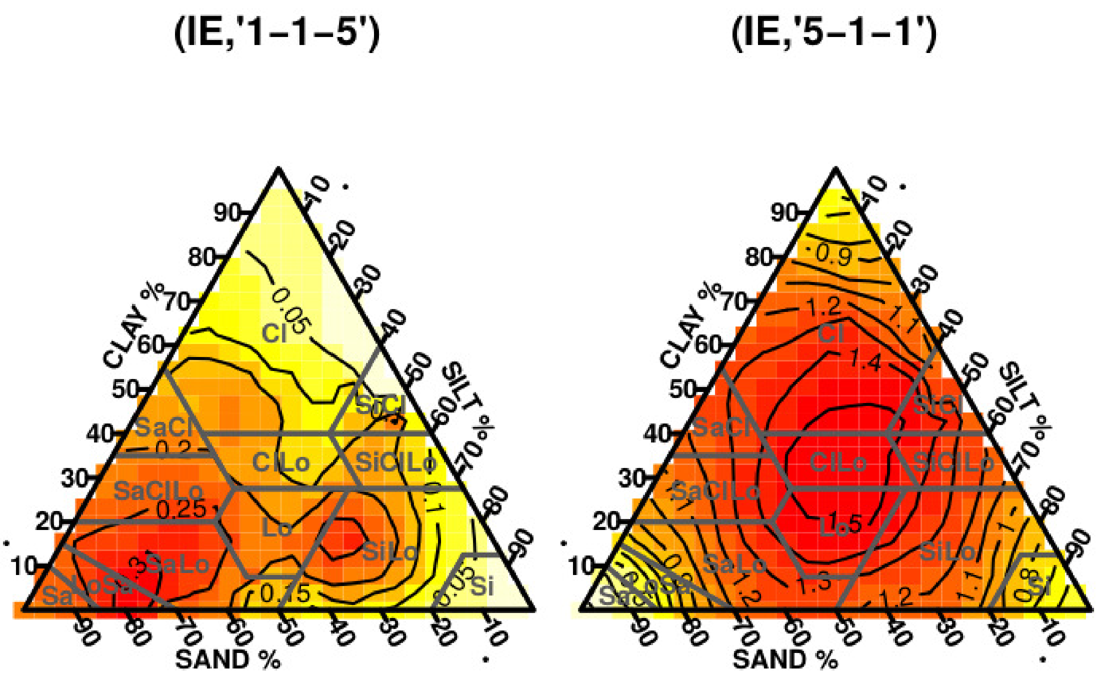

Figure 2. ~~Heatmaps~~ Ternary representations for IE calculated for the soils of the study but using different triplets. The usual clay, silt and sand triplet ('5-1-1') was used at the left and the grouping seven textural fractions into '1-1-5' was used as input for the right.

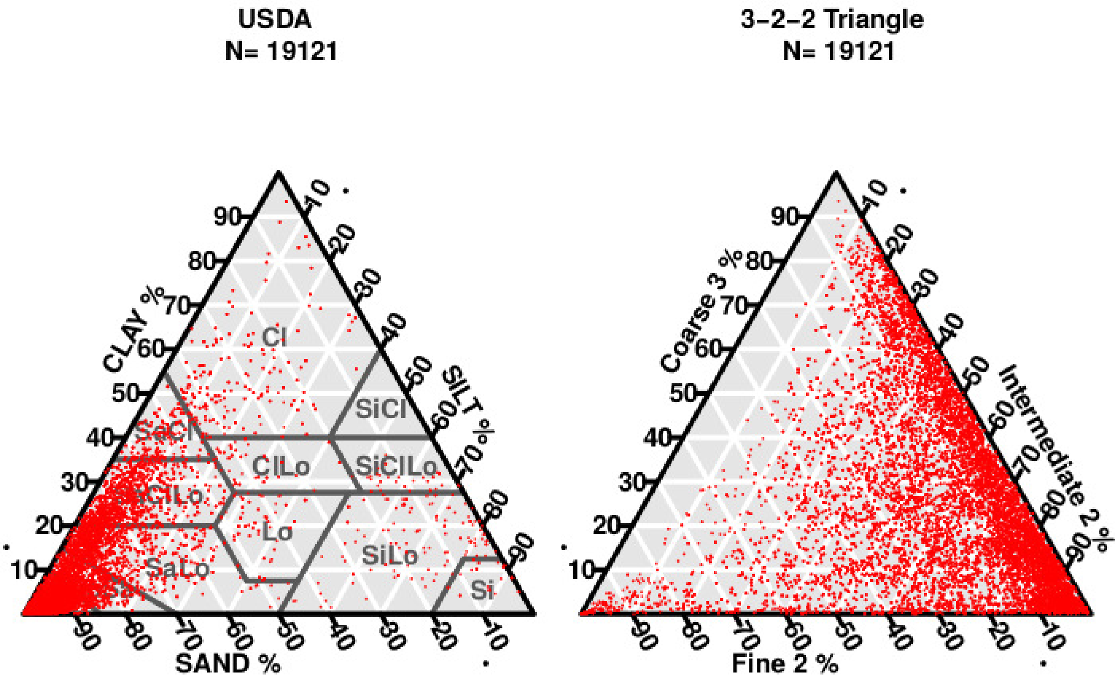

Figure 3. Representation in the USDA textural triangle of the ~~19193~~ 19,193 soils used in this study. (a) standard sand-silt-clay, i.e. '5-1-1' triplet. (b) the '3-2-2' triplet.

Heatmaps for the K_{sat} and the $(IE,$'2-3-2'$)$, represented in the USDA textural triangle.

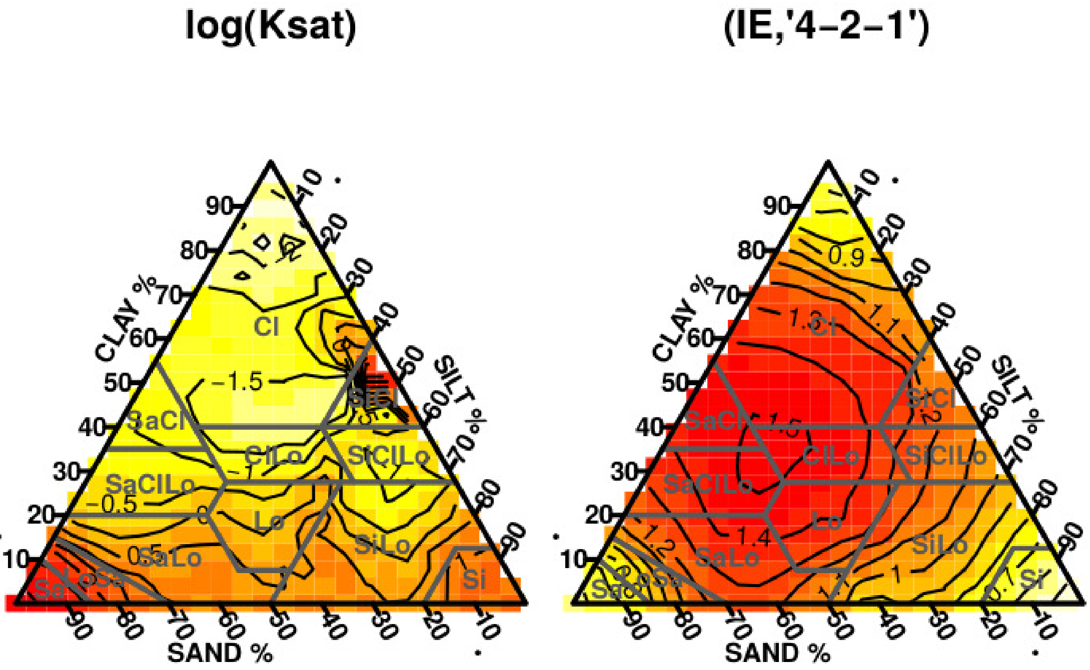

Figure 4. ~~Heatmaps~~ Ternary representations for $\ln K_{sat}$ and $(IE,$'4-2-1'$)$ represented in the USDA textural triangle.

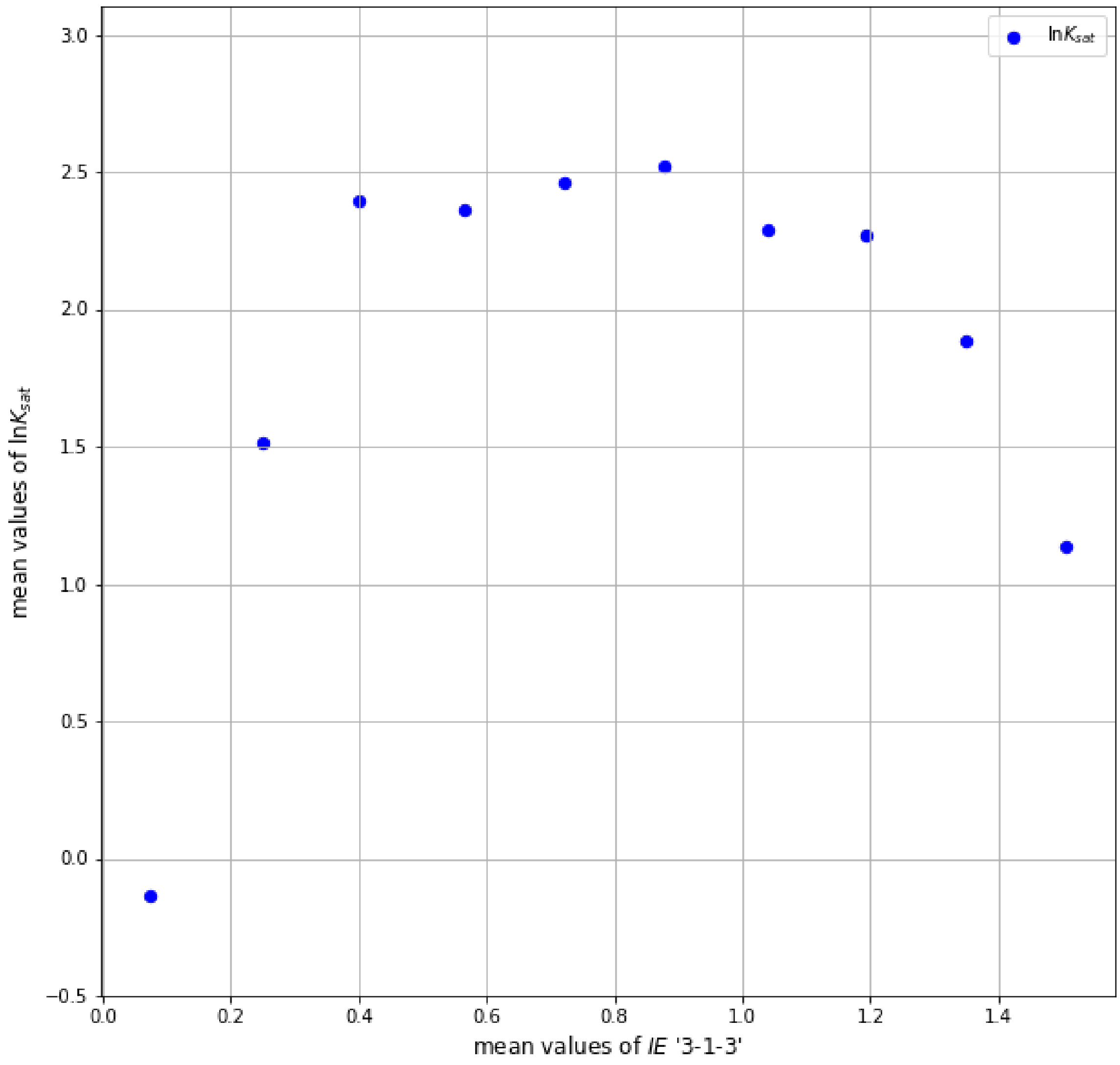

Figure 5. ~~K_{sat} (cm/h) (circles) and~~ $\ln K_{sat}$ ~~(crosses)~~ values against IE calculated with the '3-1-3' triplet in 10 interval binnings.

Table 1. Statistical description of K_{sat} (cm/h) values by classes. Soils have also been grouped into two super classes, `SC1` and `SC2`, which can be interpreted as the *sandy class* and not sandy class, respectively. Legend: N, the number of soils in each class; sd, the standard deviation; skew, the skewness number and se, the standard error.

	N	min	1stQ	Median	Mean	3rdQ	max	sd	skew	kurtosis	se
Sandy clay	345	0.00	0.09	0.41	2.72	1.29	60.60	8.29	4.78	24.12	0.45
Sandy clay loam	2.004	0.00	0.12	0.50	3.23	1.67	405.00	17.11	14.04	244.07	0.38
Sandy loam	2.123	0.00	0.28	1.10	4.92	3.67	504.00	18.26	15.60	348.22	0.40
Loamy sand	1.780	0.01	1.37	5.00	9.84	13.80	189.00	13.35	3.86	29.54	0.32
Sand	12.068	0.01	11.80	23.95	32.97	43.40	841.00	32.83	4.01	51.12	0.30
Clay	407	0.00	0.04	0.16	4.07	0.73	421.00	25.49	13.12	196.18	1.26
Clay loam	78	0.01	0.04	0.22	1.26	0.71	38.20	4.56	7.27	57.93	0.52
Silty clay loam	41	0.00	0.08	0.34	18.02	1.67	159.00	43.36	2.60	5.69	6.77
Loam	104	0.01	0.17	0.72	5.77	2.89	52.60	11.26	2.43	5.42	1.10
Silty loam	135	0.00	0.17	0.69	5.20	4.42	53.90	9.65	2.90	9.40	0.83
Silt	36	0.27	1.27	5.21	19.16	22.54	213.00	40.62	3.88	16.30	6.77
`SC1`	17975	0.00	2.85	14.50	24.05	32.90	841.00	31.28	4.18	51.00	0.23
`SC2`	1069	0.00	0.07	0.31	3.74	1.36	421.00	17.21	16.54	360.54	0.53
All	19.121	0.00	1.92	13.10	22.89	31.60	841.00	31.06	4.25	51.47	0.22

Table 2. Computed mean and standard deviation (std) for R^2 and RMSE ($\ln cm/h$) values using the bootstrap method for the binned lineal regression of ~~K_{sat} and~~ $\ln K_{sat}$ against all possible (IE,*triplet*).

	R^2		$RMSE$	
Triplet	mean	std	mean	std ~~mean std mean std~~
'1-1-5'	~~0.740 0.076 6.782 1.337~~ 0.872	0.048	0.230	0.040
'1-2-4'	~~0.614 0.055 5.138 1.135~~ 0.884	0.029	0.207	0.030
'1-3-3'	~~0.838 0.016 9.365 0.531~~ 0.885	0.042	0.434	0.061
'1-4-2'	~~0.726 0.042 7.620 0.466~~ 0.637	0.084	0.837	0.083
'1-5-1'	~~0.720 0.025 6.108 0.228~~ 0.735	0.051	0.745	0.063
'2-1-4'	~~0.746 0.017 4.911 0.201~~ 0.870	0.017	0.227	0.015
'2-2-3'	~~0.494 0.056 9.300 0.627~~ 0.235	0.064	0.879	0.068
'2-3-2'	~~0.882 0.020 3.649 0.300~~ 0.744	0.019	0.595	0.020
'2-4-1'	~~0.850 0.019 3.860 0.232~~ 0.760	0.023	0.611	0.026
'3-1-3'	~~0.005 0.006 9.745 0.353~~ 0.087	0.031	0.766	0.072
'3-2-2'	~~0.428 0.051 6.146 0.269~~ 0.519	0.075	0.582	0.050
'3-3-1'	~~0.464 0.025 6.882 0.190~~ 0.765	0.009	0.558	0.014
'4-1-2'	~~0.880 0.007 8.615 0.397~~ 0.975	0.004	0.245	0.018
'4-2-1'	~~0.849 0.007 10.099 0.370~~ 0.977	0.002	0.263	0.011
'5-1-1'	~~0.775 0.010 7.555 0.190~~ 0.960	0.005	0.339	0.021
average	~~0.667 7.052~~ 0.727		0.501	

Table 3. Summary of triplets for ~~K_{sat} and log(K_{sat})~~ $\ln K_{sat}$ with highest R^2 mean values for regressions using 10 interval binnings. Both the mean value and the standard deviation of R^2 are shown.

Text class	n soils	triplet	mean R2	std R2 ~~triplet mean R2 std R2~~
Sandy clay	345	~~241 0.309 0.237 124~~ '1-2-4'	0.386	0.194
Sandy clay loam	2004	~~313 0.711 0.105 232~~ '2-3-2'	0.879	0.054
Sandy loam	2123	~~232 0.608 0.140 115~~ '1-1-5'	0.917	0.046
Loamy sand	1780	~~223 0.672 0.080 223~~ '2-2-3'	0.742	0.073
Sand	12068	~~511 0.905 0.013 511~~ '5-1-1'	0.987	0.005
Clay	407	~~151 0.416 0.121 124~~ '1-2-4'	0.604	0.149
Clay loam	78	~~115 0.202 0.132 412~~ '4-1-2'	0.276	0.081
Loam	104	~~241 0.349 0.097 313~~ '3-1-3'	0.235	0.185
Silty loam	135	~~142 0.204 0.115 511~~ '5-1-1'	0.412	0.207

Table 4. R^2 and RMSE $(\ln cm/h)$ values for linear regressions of IE vs ~~K_{sat} and log(K_{sat})~~ $\ln K_{sat}$ using the 15 different triplets for the `SC1` selection.

	R^2		RMSE	
triplet	mean	std	mean	std ~~mean std mean std~~
'1-1-5'	~~0.710 0.082 7.099 1.355~~ 0.833	0.056	0.240	0.039
'1-2-4'	~~0.570 0.058 5.132 1.139~~ 0.794	0.046	0.213	0.035
'1-3-3'	~~0.805 0.014 11.175 0.441~~ 0.915	0.031	0.402	0.059
'1-4-2'	~~0.729 0.042 7.621 0.462~~ 0.646	0.088	0.786	0.081
'1-5-1'	~~0.691 0.039 6.335 0.299~~ 0.651	0.055	0.789	0.046
'2-1-4'	~~0.714 0.019 4.979 0.205~~ 0.748	0.028	0.252	0.015
'2-2-3'	~~0.885 0.011 5.255 0.250~~ 0.807	0.015	0.485	0.017
'2-3-2'	~~0.868 0.024 3.905 0.338~~ 0.769	0.019	0.548	0.020
'2-4-1'	~~0.815 0.027 4.080 0.275~~ 0.739	0.028	0.568	0.026
'3-1-3'	~~0.837 0.026 4.441 0.340~~ 0.939	0.007	0.191	0.011
'3-2-2'	~~0.590 0.024 6.236 0.223~~ 0.799	0.009	0.429	0.011
'3-3-1'	~~0.423 0.024 7.219 0.191~~ 0.720	0.009	0.592	0.013
'4-1-2'	~~0.852 0.007 10.030 0.359~~ 0.986	0.002	0.184	0.013
'4-2-1'	~~0.852 0.007 10.006 0.359~~ 0.977	0.002	0.255	0.011
'5-1-1'	~~0.773 0.015 7.591 0.232~~ 0.927	0.011	0.426	0.030
max ~~/min 0.885 3.905~~	0.986		0.184	min
average	~~0.741 6.740~~ 0.817		0.424	

Table 5. R^2 and RMSE $(\ln cm/h)$ values for linear regressions of IE vs K_{sat} and log(K_{sat}) $\ln K_{sat}$ using the 15 different triplets for the `SC2` class.

	R^2		RMSE	
triplet	mean	std	mean	std mean std mean std
‘1-1-5’	0.204 0.095 3.226 1.162 0.623	0.092	0.634	0.103
‘1-2-4’	0.045 0.051 3.259 0.890 0.476	0.094	0.556	0.105
‘1-3-3’	0.053 0.049 3.675 0.957 0.105	0.074	0.770	0.121
‘1-4-2’	0.014 0.028 7.732 3.877 0.033	0.043	0.822	0.125
‘1-5-1’	0.074 0.095 1.414 0.292 0.268	0.173	0.633	0.129
‘2-1-4’	0.094 0.098 2.198 0.717 0.462	0.116	0.478	0.082
‘2-2-3’	0.052 0.056 3.117 0.963 0.085	0.072	0.728	0.117
‘2-3-2’	0.036 0.056 4.678 2.237 0.025	0.032	0.741	0.120
‘2-4-1’	0.038 0.052 1.523 0.261 0.156	0.091	0.760	0.087
‘3-1-3’	0.047 0.045 2.895 1.126 0.045	0.051	0.480	0.094
‘3-2-2’	0.066 0.080 5.436 3.029 0.142	0.112	0.591	0.125
‘3-3-1’	0.017 0.023 2.267 0.632 0.285	0.083	0.717	0.108
‘4-1-2’	0.093 0.113 6.656 3.951 0.331	0.188	0.465	0.135
‘4-2-1’	0.109 0.075 2.031 0.405 0.108	0.124	0.503	0.115
‘5-1-1’	0.121 0.100 2.144 0.503 0.078	0.082	0.570	0.145
max /min 0.204 1.414	0.623		0.465	min
avg	0.071 3.483 0.215		0.630	

Table 6. Comparison of parametrizing power of (IE,‘5-1-1’) against IE calculated with other triplets. The ranges of variation of IE calculated with the different triplets are compared to the ranges of variation of K_{sat} for the textural classes. The triplets are chosen to be the ones that gav the highest R^2 values at the linear regressions for K_{sat}.

Textural class	%range ‘5-1-1’ / % range ~~K_{sat}~~ $\ln K_{sat}$	% range best triplet / % range ~~K_{sat}~~ $\ln K_{sat}$
silty loam	~~8.513~~ 0.818	~~12.035~~ 1.157
sandy loam	~~0.840~~ 0.561	~~1.002~~ 0.670
sandy clay loam	~~1.022~~ 0.581	~~1.263~~ 0.718
sandy clay	~~4.359~~ 0.385	~~5.024~~ 0.444
sand	~~0.420~~ 0.528	~~0.420~~ 0.528
loamy sand	~~1.320~~ 0.426	~~2.708~~ 0.875
loam	~~2.634~~ 0.274	~~9.158~~ 0.953
clay loam	~~1.573~~ 0.124	~~12.879~~ 1.011
clay	~~1.477~~ 0.816	~~1.444~~ 0.797