Another eye into the world of the Budyko hypothesis

In a wide sweeping survey of the hydrology landscape, Sivapalan's discussion on the Budyko hypothesis or curve in evapotranspiration has piqued my curiosity (Sivapalan, 2017, lines 659-660, 911-968). This prompts the submission of this personal observation of a resemblance or similarity between it and the Bakhmeteff function describing an overland flow hydrograph.

Figures 1 and 2 show the Budyko curve (e.g., Zhang et al., 2004, Figure 1; especially Padrón et al., 2017, Figure 2) and the Bakhmeteff function (Ding, 2011, Figure 3), respectively. These are a simple expression of scientific facts from two seemingly dissimilar worlds. But their similarity is stunning; hinting at certain first principles at work.

The Budyko curve partitions the precipitation (P) into evapotranspiration (ET) explicitly and runoff (Q) implicitly and on an annual timescale, but the overland flow model does neither. (In a dimensionless S-curve hydrograph shown in Figure 2, the Bakhmeteff function on the X-axis is a time parameter, $F(v, N) = \int_{v=0}^{v} \frac{dv}{1-v^N}$.)

But based on their apparent similarity, the Bakhmeteff function, on a storm event timescale, maybe made to imitate the Budyko curve and on an annual timescale too, thereby entering an unchartered waters between the two worlds.

Since Q and ET are complementary to each other if, and only if, the storage carryover effect is negligible, i.e., $\frac{\Delta S}{\Delta t} = P - Q - ET \cong 0$, the similarity may not be a coincidence after all.

From this perspective, a new $\frac{Q}{P}$ $vs.$ ϕ curve in Figure 1 will be merely a mirror image of the Budyko curve.

In Figure 2, the dummy variable v on the Y-axis is actually a normalized runoff ratio, $v = (\frac{Q}{P})^{\frac{1}{N}}$, in which N is a storage exponent in $Q \sim S^N$. Equating the respective $\frac{Q}{P}$ term in Figures 1 and 2 yields a parametric relation below:

$$\boxed{v^N = (1 + \phi^{\omega})^{\frac{1}{\omega}} - \phi,} \tag{1}$$

The distinctly different Bakhmeteff function $F(v, N)$ thus represents a fresh eye from the variable instantaneous unit hydrograph (vIUH) model peering into the world of the Budyko hypothesis.

References

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Figure 1. The Budyko curve.

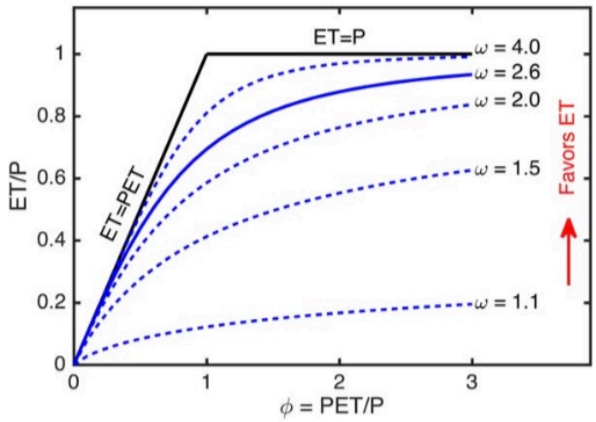

Figure 2. The original Budyko curve representing ET/P as a function of ϕ = PET/P (continuous line). Solutions to Fu's equation for different values of ω are shown as dashed lines. Note that higher values of ω result in higher ET/P, and thus favor ET over R.

Source: Padrón et al. (2017)

Figure 2. The Bakhmeteff function and its S-curve hydrograph for overland flow, $N = 1.67$ being a typical storage exponent.

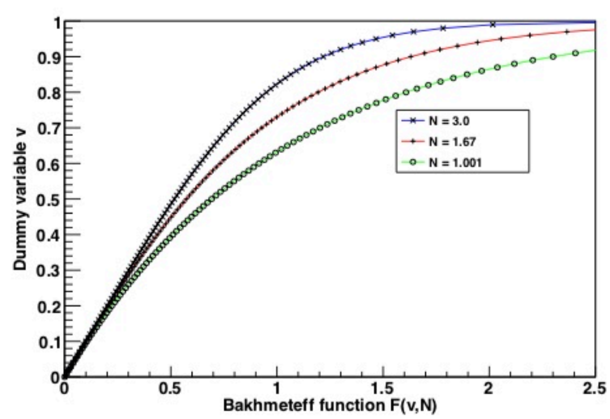

Fig. 3. The Bakhmeteff varied-flow function for three different degrees of watershed nonlinearity, N. $N = 1.001$ for nearly linear, 1.67 for moderately nonlinear, and 3.0 for highly nonlinear watersheds.

Source: Ding (2011)