Responses to Comment of Referee #2

The manuscript presents a 3D coupled (semi-)analytical model that simulates flow in both the unsaturated and saturated zones with localized recharge on the ground surface. The authors used a simplified model for flow through the unsaturated zone based on the linearized Richards´ equation. The resulting system of linear partial differential equation is then solved using the Laplace transform to eliminate the time derivative and the double Fourier transform which convert the original system of PDEs to a system of ordinary differential equation. They obtained the expressions of the heads in the unsaturated and saturated zones in terms of infinite double integrals. The final solution is presented in the Laplace domain and then numerical inversion of Laplace transform is required to get the solution in the time space.

The manuscript overall is well written and clear but it could be improved. The technique used is standard (Laplace transform coupled with cosine Fourier transform) for such kind of coupled problems, but I think the work addressed is very interesting and the topic, in my opinion, is appropriate for HESS. The consideration of the unsaturated zone in the modeling of recharge is important and to the best of my knowledge this subject has never been addressed analytically. Although coupled unsaturated/saturated flow model have been already addressed analytically in the framework of pumping tests (see for instance, Mathias and Bulter (2006), Mishra and Neuman (2010, 2011), Tartakovsky and Neuman (2007)), the mathematical model presented here is different (3D cartesian). In think this work can be considered as new contribution.

Response: Thanks for the comment.

I have however two comments that merits to be mentioned and discussed in a revised version of the manuscript:

1. The authors should mention the above cited works and discuss how they differ from their present work.

Response: Thanks for the suggestion. We added the following sentences in the revised manuscript to address the differences between our work and theirs:

"Such a coupled flow model has been proposed to investigate pumping drawdown problems by several articles (e.g., Mathias and Bulter, 2006; Tartakovsky and Neuman, 2007; Mishra and Neuman, 2010; Mishra and Neuman, 2011). They treated an extraction well as a line sink in the aquifer while we consider the localized recharge as a plane source to the aquifer. The coupled flow model in their studies is 2D written in cylindrical coordinates while that in ours is 3D expressed in Cartesian coordinates. In addition, their solutions are obtained by the Hankel transform, but ours is based on the Fourier cosine transform. The present work aims to investigate the spatiotemporal distribution of the hydraulic head due to localized recharge from the ground surface."

2. In section 3.4, the authors compare the proposed analytical solution to a finite element numerical solution obtained from the NDsolve function of Mathematica. They used the linearized system of equations (1)-(11) in the numerical solver. This is good to show the correctness of the analytical solution. However, It would be interesting to perform a comparison between a numerical solution based on the original "nonlinear" Richards´ equation to investigate the effect of linearization on the head distribution in the unsaturated and saturated zones. This may affects also the general results of the manuscripts. I think, it would not need too much efforts to include the nonlinear Richards´ equation in the NDsolve function of Mathematica.

The original nonlinear Richards´ equation writes as follows (Kroszynski and Dagan, 1975)

$$K_x \frac{\partial}{\partial x}\left(K(\phi)\frac{\partial \phi}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(K(\phi)\frac{\partial \phi}{\partial y}\right) + K_z \frac{\partial}{\partial z}\left(K(\phi)\frac{\partial \phi}{\partial z}\right) = C(\phi)\frac{\partial \phi}{\partial t}$$

with $K(\phi) = e^{\kappa(\phi - \phi_a)}$ and $C(\phi) = S_y \kappa e^{\kappa(\phi - \phi_a)}$

Response: Thanks for the suggestion. In recent days, we try very hard to solve the nonlinear Richards' equation (NRE) in our unsaturated-saturated coupled flow model using the NDSolve function of Mathematica. The associated computation consumes tremendous computing time in each computer run. The outputs, however, fail to yield reasonable results although we had tried a variety of measures to solve the NRE using fine grids, small time step, and different built-in options of the NDSolve function. A great deal of study has examined the accuracy of the linearized Richards' equation (LRE) (e.g., Kroszynski and Dagan, 1975; Tartakovsky and Neuman, 2007; Mishra and Neuman, 2010; Liang et al., 2017). Kroszynski and Dagan (1975) and Mishra and Neuman (2010), for example, achieved good agreement on aquifer drawdown estimated from an analytical solution based on the LRE and a numerical solution based on the NRE. Liang et al. (2017) also achieved agreement on the hydraulic head predicted by analytical and numerical solutions based on the LRE and the NRE, respectively. Tartakovsky and Neuman (2007) revealed that aquifer drawdown calculated by an analytical solution based on the LRE agrees well with that obtained in a field pumping test. We, therefore, add the following sentences in lines 2-5, page 11 of the revised manuscript:

"On the other hand, numerous attempts have been made by scholars to examine the accuracy and/or applicability of the linearized Richards' equation for Eqs. (2) – (4) (e.g., Kroszynski and Dagan, 1975; Tartakovsky and Neuman, 2007; Mishra and Neuman, 2010; Liang et al., 2017). Results show that the linearization of Richards' equation causes insignificant deviation on model prediction."

References

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