Responses to Comment of Referee #1

The paper presents a three-dimensional semi-analytical solution that simulates flow in an unconfined aquifer as well as groundwater recharge.

Analytical solutions for solving the Richards equation are limited in the literature because, as these authors underline, the analytical solutions don't exist in most cases. However, this semi-analytical solution is identical to the one proposed by Tartakovsky and Neuman (2007), Mishra and Neuman (2010), Mishra and Neuman (2011), the authors could have cited their work in the introduction lines 25-26 (page 2), although, Mishra's solution was applied to pumping test. Authors should highlight the differences between their analytical solution and those proposed by Mishra et al., if there is any differences.

Response: Thanks for the comment. The difference between our solution and theirs is that we consider groundwater recharge problems for a localized recharge from the ground surface as a plane source to the aquifer while they study pumping drawdown problems for an extraction well treated as a line sink in the aquifer. In addition, the governing equations (GEs) describing flow in both saturated and unsaturated zones in our study are three-dimensional expressed in Cartesian coordinates while their flow equations for saturated and unsaturated zones are two-dimensional written in cylindrical coordinates. In addition, our solution was derived by applying the Fourier cosine transform to the GEs, but theirs was developed based on the Hankel transform. Therefore, both solutions are completely different in mathematical forms. We added the following sentences in the revised manuscript: "Such a coupled flow model has been applied to study drawdown behaviors induced by well pumping (e.g., Tartakovsky and Neuman, 2007; Mishra and Neuman, 2010; Mishra and Neuman, 2011). This paper investigates spatiotemporal distribution of the hydraulic head affected by localized recharge from the ground surface."

It is certainly common in hydrogeological modeling to consider recharge as an input to hydrodynamics models of aquifers. However, several studies have been dedicated to calculate recharge in the literature, these models are both empirical or conceptual (Sophocleous et Perkins 2000; Facchi et al. 2004; Markstrom et al. 2008) and physical solving the Richards equation (Twarakavi et al. 2008; Thoms et al. 2006; Shen et Phanikumar 2010; Kuznetsov et al. 2012; Zhu et al. 2012). On contrary to what the authors stated in lines 9-10 (page 1), to be clear, they may have to add in the case of an analytical solution. Also, it is well known that the consideration of the unsaturated zone in the modeling of the recharge is important, unlike pumping. The recharge reflects the amount of water that comes from the precipitation and reaches the water table, this amount of water flow through the entire unsaturated zone. While in the case of pumping, water is directly extract from the saturated zone and several models neglect the contribution of the unsaturated zone located above. Hence the effects of the unsaturated zone in the case of pumping are discussed in the literature. I don't believe that studying the effect of the unsaturated zone in case of recharge will be something new, neither their analytical solution, since this one was already applied to pumping test. Also, the effects of Gardner parameters on the unsaturated zone flow have been discussed in (Mishra and

Neuman 2011).

Response: The sentence in the abstract of the original manuscript (lines 9-10, page 1) "Up to now, little attention has been given to the effect of unsaturated flow on the hydraulic head within the aquifer due to recharge." is changed as: "Little attention has been given to the development of analytical solutions to a coupled saturated-unsaturated flow due to a localized recharge up to now."

Although many studies have investigated unsaturated flow for groundwater recharge, our work has the following three novelties:

1. Our semi-analytical solution is indeed a new one that has not been seen in the literature. We develop the solution because it can serve as a preliminary design tool for the development of water resources management or groundwater remediation plan or a primary mean for testing and benchmarking numerical codes.
2. Analyses of quantitative results are presented based on the present solution and absolutely not found in the literature. The results demonstrate that the present solution is capable of exploring the insight into how the unsaturated flow affects the recharge efficiency and head distributions in the saturated zone. Please refer to the next response for details.
3. Sensitivity analysis assesses the response of the hydraulic head in the unsaturated zone to the change in each of the hydraulic parameters, especially the parameter associated with unsaturated flow. Please refer to the fourth conclusion in section 4 of the original manuscript.

The paper is not well written, the English must be significantly improved. Mathematical equations aren't well written and test cases (and results) are not well described. Moreover, the first three conclusions drawn are not original. I don't recommend publication of this article.

Response: This manuscript will be edited by a colleague who is good at English writing. The first three conclusions given below illustrate quantitative results which are new findings and absolutely not seen elsewhere. The first conclusion quantifies the validity of neglecting the effect of unsaturated flow on the hydraulic head in the underlying aquifer. Existing analytical solutions neglecting unsaturated flow give accurate predictions only when the quantitative conditions (i.e., $ab \geq 10$ and $b/B \leq 0.1$) are satisfied (e.g., Chang and Yeh, 2007; Illas et al., 2008; Bansal and Teloglou, 2013). Otherwise, significant deviations may happen to their predictions. The second and third ones propose a quantitative condition (i.e., $\bar{b} < 0.05$ or $\alpha > 20$) that causes almost all amount of localized recharge reaches the aquifer.

1. The effect of unsaturated flow on the hydraulic head in the aquifer is ignorable when the product of the unsaturated exponent (a) and initial unsaturated thickness (b) is greater than 10 (i.e., $ab \geq 10$) and the unsaturated thickness is less than 10 % of the initial aquifer thickness (B) (i.e., $b/B \leq 0.1$). Otherwise, the effect should be considered to avoid large deviations in calculating the head in the aquifer. Existing models considering only saturated flow can predict accurate results only when these two inequalities are

satisfied.

2. The recharge efficiency initially equals zero, increases with time, and finally approaches a constant value (below or equal to unity) depending on the values of α (= aB) and $\bar{b}$ (= b/B).
3. The ultimate recharge efficiency approaches unity when $\bar{b} < 0.05$ or $\alpha > 20$ but less than 90 % when $\bar{b} > 0.1$ and $\alpha < 10$. In other words, the surface source supplies more recharge water to the aquifer if the unsaturated zone has a large α and/or a small $\bar{b}$.

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