

1 **A discrete wavelet spectrum approach to identifying non-monotonic trend**
2 **pattern of hydroclimate data**

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23 **Abstract:** Hydroclimate system is changing non-monotonically and identifying its trend
24 pattern is a great challenge. Building on the discrete wavelet transform theory, we develop a
25 discrete wavelet spectrum (DWS) approach for identifying non-monotonic trend patterns in
26 hydroclimate time series and evaluating their statistical significance. After validating the
27 DWS approach using two typical synthetic time series, we examined the temperature and
28 potential evaporation over China from 1961-2013, and found that the DWS approach detected
29 both the “warming” and the “warming hiatus” in temperature, and the reversed changes in
30 potential evaporation. Further, the identified trend patterns showed stable significance when
31 the time series was longer than 30 years or so (i.e., the widely defined “climate” timescale).
32 Our results suggest that non-monotonic trend patterns of hydroclimate time series and their
33 significance should be carefully identified, and the DWS approach proposed has the potential
34 for wide use in hydrological and climate sciences.

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36 **Key words:** trend identification; discrete wavelet spectrum; decadal variability; statistical
37 significance; Mann-Kendall test

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45 **1. Introduction**

46 Climate and hydrological system are exhibiting great variability (Allen and Ingram, 2002;
47 Trenberth et al., 2014). Quantitatively, identifying human-induced climate change signals in
48 the usually changing hydroclimate system is of great socioeconomic significance
49 (Diffenbaugh et al., 2008; IPCC, 2013), and remains a big challenge to both scientific and
50 social communities. The simplest way to identify changes in the hydroclimate system would
51 be to fit a monotonic (e.g., linear) trend at long time scales, at which a significance level
52 would be assigned by a statistical test. Among the methods used for detection of trends, the
53 Mann-Kendall non-parametric test is most widely used and has been successfully applied in
54 climate change and its impact studies, when the time series is almost monotonic as required
55 (Burn and Hag Elnur, 2002; Yue et al., 2002). However, due to its nonlinear and
56 nonstationary nature, the hydroclimate system is changing and developing in a more
57 complicated way rather than a monotonic trend way (Cohn and McMahon, 2005; Milly et al.,
58 2008). For example, a debate on the recent change of global air temperature is receiving
59 enormous public and scientific attention that the global air temperature increased during
60 1980-1998 passing most statistical significance tests and then stabilized afterwards till now,
61 widely called “global warming hiatus” (Kosaka and Xie, 2013; Roberts et al., 2015; Medhaug
62 et al., 2017). Another known example is “evaporation paradox” (Brutsaert and Parlange, 1998;
63 Roderick and Farquhar, 2002) that potential evaporation has worldwide declined from the
64 1960s, again passing most statistical significance tests, but then reversed after the 1990s. In
65 practice, for the hydroclimate time series, the non-monotonicity is more the rule rather than
66 the exception (Dixon et al., 2006; Adam and Lettenmaier, 2008; Gong et al., 2010). Therefore,

67 identifying the non-monotonic trend pattern hidden in those hydroclimate time series and
68 assessing its statistical significance presents a significant research task for understanding
69 hydroclimatic variability.

70 Among those methods presently used in time series analysis, the wavelet method has the
71 superior capability of handling the nonstationary characteristics of time series (Percival and
72 Walden, 2000; Labat, 2005), so it may be more suitable for identifying non-monotonic trend
73 patterns in hydroclimate time series. In a seminal work, Torrence and Compo (1998) placed
74 the continuous wavelet transform in the framework of statistical analysis by formulating a
75 significance test. Since then, the continuous wavelet method has become more applicable and
76 rapidly developed to estimate the significance of variability in climate and hydrological
77 studies. However, in the continuous wavelet results of time series, a known technical issue is
78 the “data redundancy” (Gaucherel, 2002; Nourani et al., 2014), which is the redundant
79 information across timescales leading to more uncertainty.

80 On the contrary, the other type of wavelet transform, i.e., the discrete wavelet method,
81 has the potential to overcome that problem of data redundancy, in that those wavelets used for
82 discrete wavelet transform must meet the orthogonal properties. Therefore, the discrete
83 wavelet method can be more effective to identify the non-monotonic trend pattern in time
84 series (Almasri et al., 2008; de Artigas et al., 2006; Kallache et al., 2005; Partal and Kucuk,
85 2006; Nalley et al., 2012). The discrete wavelet-aided identification of trend is usually
86 influenced by some factors, such as choice of wavelet and decomposition level; moreover, the
87 uncertainty evaluation of results should also be carefully considered. To overcome these
88 problems, Sang et al. (2013) discussed the definition of trend, and further proposed a discrete

89 wavelet energy function-based method for the identification of trend by comparing the
90 difference of wavelet results between hydrological data and noise. The method used proper
91 confidence interval to assess the statistical significance of the identified trend, in which the
92 key equation for quantifying trend's significance is based on the concept of quadratic sum.
93 However, the practice of quadratic sum disobeys the common practice of computing variance
94 in spectral analysis, and sometimes cannot reasonably assess the significance of
95 non-monotonic trend, because it neglects the big influence of trend's mean value. For instance,
96 for those trends with small variation but big mean value, the quadratic sums are big values,
97 based on which the statistical significance of trends would inevitably be over-assessed.
98 Therefore, the evaluation of statistical significance of a non-monotonic trend in a time series
99 should be based on its own variability but not other factors.

100 By combining the advantages of the discrete wavelet method and successful practice in
101 the spectral analysis methods, this study aims at developing a practical but reliable discrete
102 wavelet spectrum approach for identifying non-monotonic trend patterns in hydroclimate time
103 series and quantifying their statistical significance, and further improving the understanding
104 of non-monotonic trends by investigating their variation with data length increase. To do that,
105 Section 2 presents the details of the newly developed approach building on the wavelet theory
106 and spectrum analysis. In Section 3, we use both synthetic time series and annual time series
107 of air temperature and potential evaporation over China as examples to investigate the
108 applicability of the approach, which is followed by the discussion and conclusion in the final
109 section.

110 **2. A discrete wavelet spectrum approach**

111 Here we develop an approach, termed as “discrete wavelet spectrum approach,” for
 112 identifying non-monotonic trend pattern in hydroclimate time series, in which the discrete
 113 wavelet transform (DWT) is used first to separate the trend pattern, and its statistical
 114 significance is then evaluated by using the discrete wavelet spectrum, whose confidence
 115 interval is quantified and described through the Monte-Carlo test.

116 Following the wavelet analysis theory (Percival and Walden, 2000), the discrete wavelet
 117 transform of a time series $f(t)$ can be expressed as:

$$118 \quad W_f(j, k) = \int_{-\infty}^{+\infty} f(t) \psi_{j,k}^*(t) dt \quad \text{with} \quad \psi_{j,k}(t) = a_0^{-j/2} \psi(a_0^{-j}t - b_0k) \quad (1)$$

119 where t is a time order, and $\psi^*(t)$ is the complex conjugate of mother wavelet $\psi(t)$; a_0 and b_0 are
 120 constants, and integer k is a time translation factor; $W_f(j, k)$ is the discrete wavelet coefficient
 121 under the decomposition level j (i.e., time scale a_0^j). In practice, the dyadic DWT is used
 122 widely by assigning $a_0=2$ and $b_0=1$:

$$123 \quad W_f(j, k) = \int_{-\infty}^{+\infty} f(t) \psi_{j,k}^*(t) dt \quad \text{with} \quad \psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k) \quad (2)$$

124 The highest decomposition level M can be calculated as $\log_2(L)$ (Foufoula-Georgiou and
 125 Kumar, 2014), where L is the length of series $f(t)$. The sub-signal $f_j(t)$ in the original series $f(t)$
 126 under each level j ($j = 1, 2, \dots, M$) can be reconstructed as:

$$127 \quad f_j(t) = \sum_k W_f(j, k) \psi^*(2^{-j}t - k) \quad (3)$$

128 where the sub-signal $f_j(t)$ at the highest decomposition level (when $j = M$) defines the
 129 non-monotonic trend pattern of the series $f(t)$, as generally understood. However, it should be
 130 noted that a meaningful trend closely depends on the temporal scale concerned. If the
 131 variability of series $f(t)$ on certain smaller time scale K ($K < L$) is concerned, the proper

132 decomposition level can be determined as $\log_2(K)$; then, the sum of those sub-signals at the
133 time scales bigger K can be the non-monotonic trend pattern identified.

134 Sang (2012) discussed the influence of wavelet and decomposition level choice and
135 noise type on the discrete wavelet decomposition of time series, and further proposed some
136 methods to solve them. By doing Monte-Carlo experiments, he found that those seven
137 wavelet families used for DWT can be divided into three types, and recommended the first
138 type, by which wavelet energy functions of various types of noise are stable and thus have
139 little influence on the wavelet decomposition of time series. Specifically, one chooses an
140 appropriate wavelet, according to the relationship of statistical characteristics among the
141 original series, de-noised series and removed noise, chooses a proper decomposition level by
142 analyzing the difference between energy function of the analyzed series and that of noise, and
143 then identifies the deterministic components (including trend) by conducting significance
144 testing of DWT. These methods are built on the composition and variability of hydroclimate
145 time series at different time scales. They were used here to accurately identify the
146 non-monotonic trend pattern in a time series and assess its statistical significance.

147 Further, to establish the discrete wavelet spectrum (DWS) of time series, we need to
148 specify a spectrum value $E(j)$ for each sub-signal $f_j(t)$ (in Eq. 3), based on which we can
149 quantitatively evaluate its importance and statistical significance. Here we define $E(j)$ at the
150 j th level by taking the variance of $f_j(t)$ following the general practice in conventional spectral
151 analysis methods (Fourier transform, maximum entropy spectral analysis, *etc.*):

$$152 \quad E(j) = \text{var}(f_j(t)) \quad (4)$$

153 It can accurately quantify the intensity of variation of sub-signals (including trend) by
154 eliminating the influence of their mean value, which is different from the quadratic sum-based
155 method proposed in Sang et al. (2013). For hydroclimate time series, both stochastic and
156 deterministic components generally have different characteristics from purely noise
157 components (Sang et al., 2012; Rajaram et al., 2015). Due to the grid of dyadic DWT (Partal
158 and Cigizoglu, 2008), discrete wavelet spectra $E_r(j)$ of various noise types strictly follow an
159 exponentially decreasing rule with base 2 (Sang 2012):

$$160 \quad E_r(j) = 2^{-j} \quad (5)$$

161 The discrete wavelet spectra of deterministic components and that of noise are different.
162 Hence, we define the DWS of noise as the “reference discrete wavelet spectrum (RDWS)”,
163 based on which we evaluate the statistical significance of the non-monotonic trend pattern of
164 a time series.

165 To be specific, we design a technical flowchart to show how we develop the DWS
166 approach for identifying the non-monotonic trend pattern of time series, and also for
167 evaluating the statistical significance of that trend pattern (see the detail in Figure 1):

- 168 (1) For the series $f(t)$ normalized with length L , we analyze it using the DWT in Eq. (2)
169 and (3), and calculate its discrete wavelet spectrum by Eq. (4);
- 170 (2) For the comparison purpose, we then use the Monte-Carlo method to generate the
171 normalized noise data N with the same length as the series $f(t)$, and determine its
172 RDWS by Eq. (4). Considering that discrete wavelet spectra of various types of noise
173 just consistently follow Eq. (5), here we generate the noise data following the standard
174 normal distribution;

175 (3) We repeat the above step for 5000 times, and calculate the mean value and variance of
176 the spectrum values (in Eq. 4) of the normalized noise data N at each decomposition
177 level j , based on which we can estimate an appropriate confidence interval of RDWS at
178 the concerned confidence level. In this study we mainly considered the 95%
179 confidence level;

180 (4) In comparing the discrete wavelet spectrum of the series $f(t)$ and the confidence
181 interval generated by that of the noise (i.e., the RDWS), we can easily identify the
182 deterministic components under the highest level as the non-monotonic trend pattern of
183 the series, and determine whether it is significant. Specifically, if the spectrum value of
184 the analyzed series' sub-signal under the highest level is above the confidence interval
185 of RDWS, it is thought that the non-monotonic trend pattern is statistically significant;
186 otherwise, if the spectrum value of the sub-signal under the highest level is in the
187 confidence interval of RDWS, it is not statistically significant;

188 (5) If a smaller time scale K is concerned, we can use the decomposition level $\log_2(K)$,
189 instead of M , and then repeat the steps (1-4) to identify the non-monotonic trend
190 pattern at that time scale.

191 In the following section, we mainly investigate the applicability of the DWS approach
192 for identifying non-monotonic trend and its significance, and further investigate the variation
193 of non-monotonic trend with data length increase to improve our understanding of trend.

194 < **Figure 1** >

195 **3. Results**

196 **3.1 Synthetic series analysis**

197 To test and verify the applicability of the developed discrete wavelet spectrum (DWS)
198 approach for identifying non-monotonic trend pattern of a time series, we consider the general
199 hydrological situations and use two synthetic data, generated with known signals and noise a
200 priori. For investigating the variation of non-monotonic trend with data length increase, we
201 set the length of the two series as 200, and the noise in them follows a standard normal
202 distribution. The first synthetic series S1 consists of an exponentially increasing line and a
203 periodic curve (the periodicity is 200) with some noise content (Figure 2, left panel); and the
204 second synthetic series S2 is generated by including a hemi-sine curve, a periodic curve (the
205 periodicity is 50) and some noise content (Figure 2, right panel). Using the MK test, series S1
206 shows a significant increase but the trend of series S2 is not significant.

207 In the DWS approach (Figure 1), we concern the time scale as data length, and use the
208 Daubechies (db8) wavelet to decompose series S1 into seven (i.e., $\lfloor \log_2 200 \rfloor$) sub-signals
209 using Eq. (2) and Eq. (3). Then, we take the sub-signals under the seventh level as the defined
210 non-monotonic trend pattern. As shown in Figure 2 (left panel), the identified non-monotonic
211 trend pattern in series S1 is similar to the true trend pattern. However, the linear fitting curve
212 (a monotonic curve) could not capture the detail of the trend pattern. The same approach
213 applies to series S2 in Figure 2 (right panel) and the conclusion is not changed. Moreover, for
214 series S2 with large variation at long time scales, the linear fitting curve or other monotonic
215 curves may not be physically meaningful.

216 < **Figure 2** >

217 We compute the discrete wavelet spectra of the two synthetic series using Eq. (4), and
218 use the reference discrete wavelet spectrum with 95% confidence interval to evaluate the

219 statistical significance of their trend patterns. That is, if the red point at certain data length is
220 above the 95% confidence bar, described by the blue line in Figure 3, it is thought that the
221 trend pattern is significant at 95% confidence level. Using our DWS approach, the trend
222 pattern of series S1, which is quasi-monotonic, is found significant (Figure 3a) as in the MK
223 test (Figure 3c), but the non-monotonic series S2 shows a significant trend pattern (Figure 3b),
224 which is different from the MK test (Figure 3d).

225 In Figure 3, we also present the significance of the identified trend patterns of the two
226 series using both our DWS approach and the MK test and we change the length of the series
227 to investigate the stability of the statistical significance. Generally, it would have more
228 uncertainty when evaluating the statistical significance of trend pattern with a shorter length,
229 corresponding to a bigger 95% confidence interval. Using our DWS approach, the 95%
230 confidence interval (i.e., the height of blue bars in Figure 3) for evaluating the statistical
231 significance of trend pattern generally decreases with the increase of data length, as expected.
232 However, in the MK test, the significance is always determined by the constant thresholds of
233 ± 1.96 , regardless of the data length.

234 In the DWS results in Figure 3, the significance levels of the trend patterns do not
235 consistently decrease with data length, but show some fluctuation, as the proportions of
236 different components (including trend) in the original series vary with data length.
237 Furthermore, one would expect that if the trend pattern of a series at a certain length is
238 identified statistically significant, the trend pattern would extend with the increase of data
239 length, thus its significance may be more stable with a larger length of data considered. Using
240 our DWS approach, the trend pattern of series S1 is significant when the data length is larger

241 than 55 (Figure 3a), being similar to the result of the MK test (Figure 3c); the trend pattern of
242 the series S2 is statistically significant when the data length is larger than 75 (Figure 3b).
243 However, using the MK test, the monotonic trend pattern of series S2 is significant only when
244 the data length is between 40 and 185 (Figure 3d). In summary, the significance of trend
245 pattern identified by our DWS approach is more stable than that detected by the MK test,
246 demonstrating the advantage of the DWS approach in dealing with non-monotonic
247 hydroclimate time series.

248 < **Figure 3**>

249 **3.2 Observed data analysis**

250 We use the annual time series of mean air temperature (denoted as TEM) and potential
251 evaporation (denoted as PET) over China to further verify the applicability of our developed
252 DWS approach for identifying non-monotonic trend pattern of a time series. The two series
253 were obtained from the hydroclimate data measured at 740 meteorological stations over China,
254 with the same measurement years from 1961 to 2013. The data have been quality-checked to
255 ensure their reliability for scientific researches. The PET series was calculated from the
256 Penman-Monteith approach (Chen et al., 2005).

257 Given the general nonstationary nature of observed hydroclimate time series, linear
258 trends or more generally monotonic curves could not capture the trend pattern with large
259 decadal variations and therefore are not particularly physically meaningful. In Figure 4 (left
260 panel), we present the annual TEM time series visually showing nonstationary characteristics
261 and non-monotonic variation. The TEM series decreases till the 1980s with fluctuations and
262 then sharply rises till the 2000s, followed by a decreasing tendency. The large fluctuation of

263 the mean air temperature after late 1990s is the known phenomenon of the “global warming
264 hiatus” (Roberts et al., 2015). The linear fitting curve obviously missed out the more
265 complicated trend pattern of the observed temperature time series. Using our DWS approach,
266 we decompose the TEM series into five (i.e., $\langle \log_2 53 \rangle$) sub-signals using Eq. (2) and Eq. (3),
267 and take the sub-signals under the fifth level as the trend pattern, which realistically presents
268 the nonstationary variability of temperature over China (Figure 4, left panel).

269 We also apply this DWS approach to the annual PET time series. In the time series of
270 PET (Figure 4, right panel), there was a decreasing trend for the period from 1961 to the
271 1990s, which is the known “evaporation paradox” leading to controversial interpretations
272 continuing over the last decade on hydrological cycles (Brutsaert and Parlange, 1998;
273 Roderick and Farquhar, 2002). That decreasing trend was then followed by an abrupt increase
274 in around the 1990s, almost the same time when solar radiation was observed to be reversing
275 its trend, widely termed as “global dimming to brightening” (Wild, 2009). Surprisingly, after
276 the mid-2000s, PET starts to decrease again (Figure 4, right panel). Sometimes, one would
277 propose to fit linear curves for separate periods. Again, linear curves could not capture the
278 overall trend pattern of the PET series. Using the same DWS approach, we identify
279 non-monotonic trend pattern of the PET series (Figure 4, right panel), which captures the two
280 turning points of the changing trends in the 1990s and the 2000s.

281 **< Figure 4 >**

282 The changes of trends in terms of magnitudes and signs for different periods lead to the
283 difficulty in assessing and interpreting the significance of trends. For example, the PET time
284 series shows a significant decrease using the MK test ($-3.76 < -1.96$) during 1961-1992

285 (Figure 5d). At that moment before the reversed trend reported, the significant decrease could
286 be literally interpreted as that PET has significantly declined and might be declining in the
287 future. However, the PET time series reversed after the 1990s and again in the 2000s, coming
288 with an insignificant overall trend for the whole period of 1961-2013. For the more or less
289 monotonic time series of the TEM series (1961-2013), the MK test detects a significant
290 increase ($6.00 > 1.96$) (Figure 5c), which leads to the surprise when air temperature was
291 reported to have stopped increasing after late the 1990s. In summary, it becomes vital to
292 develop an approach for testing the significance of trend pattern, which is suitable for
293 non-monotonic time series.

294 In this study, building on the discrete wavelet transform theory, we propose an
295 operational approach, i.e., the DWS, for evaluating the significance of non-monotonic trend
296 pattern in the TEM (Figure 5a) and PET (Figure 5b) series. For comparison purpose, we also
297 conduct the significance test for the two time series using the MK test (Figure 5c and 5d).
298 Similar to Figure 3, we change the data length to investigate the stability of statistical
299 significance (Figure 5). Again, the result indicates that the 95% confidence interval for
300 evaluating the statistical significance generally decreases with the data length, which is
301 different from the constants ± 1.96 adopted in the MK test. The significance test using our
302 DWS approach appears to be more stable with the data length than the MK test (Figure 5).
303 Using our DWS approach the trend pattern in the TEM series becomes significant when the
304 data length is 30 and the significance is more stable when it is greater than 35 (Figure 5a). For
305 the case of the PET series, the trend pattern becomes statistically significant when the data
306 length is larger than 25 (Figure 5b). The findings here have important implications for

307 non-monotonic hydroclimate time series analysis, in that the timescale of defining *climate* and
308 *climate change* by the World Meteorological Organization is usually 30 years (Arguez and
309 Vose, 2011) and in hydrological practice it is between 25-30 years.

310 For the whole time series investigated here, whose length is larger than 30 years, we are
311 able to examine the significance using the developed DWS approach. Combining the trend
312 pattern in Figure 4 (left panel) and the significance test in Figure 5a, we can confirm that the
313 trend pattern of the TEM time series from 1961-2013 identified in this study is significant at
314 the 95% confidence interval. Similarly, the trend pattern in PET is also significant (Figure 4
315 right panel and Figure 5b). The significance test results suggest that the three main stages of
316 the series (red lines, Figure 4) are detectable as the overall trend pattern from the variability of
317 the series and are vital to understanding how the temperature and the PET series are changing.
318 In particular, the reversed changes in PET and its significance can be revealed by our DWS
319 approach, which can provide more useful and physically meaningful information. Our results
320 suggest that the non-monotonic trend pattern of hydroclimate time series and its significance
321 should be carefully identified and evaluated.

322 < **Figure 5** >

323 **4. Summary and Conclusion**

324 Climate and hydrological system are changing non-monotonically. Identification of
325 linear (or monotonic) trends in hydroclimate time series, as a common practice, cannot
326 capture the detail of the trend pattern in the time series at long time scales, and then can lead
327 to misinterpreting climatic and hydrological changes. Therefore, revealing the trend pattern of
328 the time series and assessing its significance from the usually varying hydroclimate system

329 remains a great challenge. To that end, we develop the discrete wavelet spectrum (DWS)
330 approach for identifying the non-monotonic trend in hydroclimate time series, in which the
331 discrete wavelet transform is used first to separate the trend pattern, and its statistical
332 significance is then evaluated by using the discrete wavelet spectrum (Figure 1). Using two
333 typical synthetic time series, we examine the developed DWS approach, and find that it can
334 precisely identify non-monotonic trend pattern in the synthetic time series (Figure 2) and has
335 the advantage in significance testing (Figure 3).

336 Using our DWS approach, we identify the trend pattern in the annual time series of
337 average temperature and potential evaporation over China from 1961-2013 (Figure 4). The
338 identified non-monotonic trend patterns precisely describe how temperature and PET are
339 changing. Of particularly interest here is that the DWS approach can help detect both the
340 “warming” and the “warming hiatus” in the temperature time series, and reveal the reversed
341 changes and the latest decrease in the PET time series. The DWS approach can provide other
342 aspects of information on the trend pattern in the time series, i.e., the significance test. Results
343 show that the trend pattern becomes more significant and the significance test becomes more
344 stable when the time series is longer than a certain period like 30 years or so, the widely
345 defined “climate” time scale (Figure 5). Using the DWS approach, in both time series of mean
346 air temperature and potential evaporation, the identified trend patterns are found significant
347 (Figure 5).

348 In summary, our results suggest that the non-monotonic trend pattern of hydroclimate
349 time series and its statistical significance should be carefully identified and evaluated, and the

350 DWS approach developed in this study has the potential for wide use in hydrological and
351 climate sciences.

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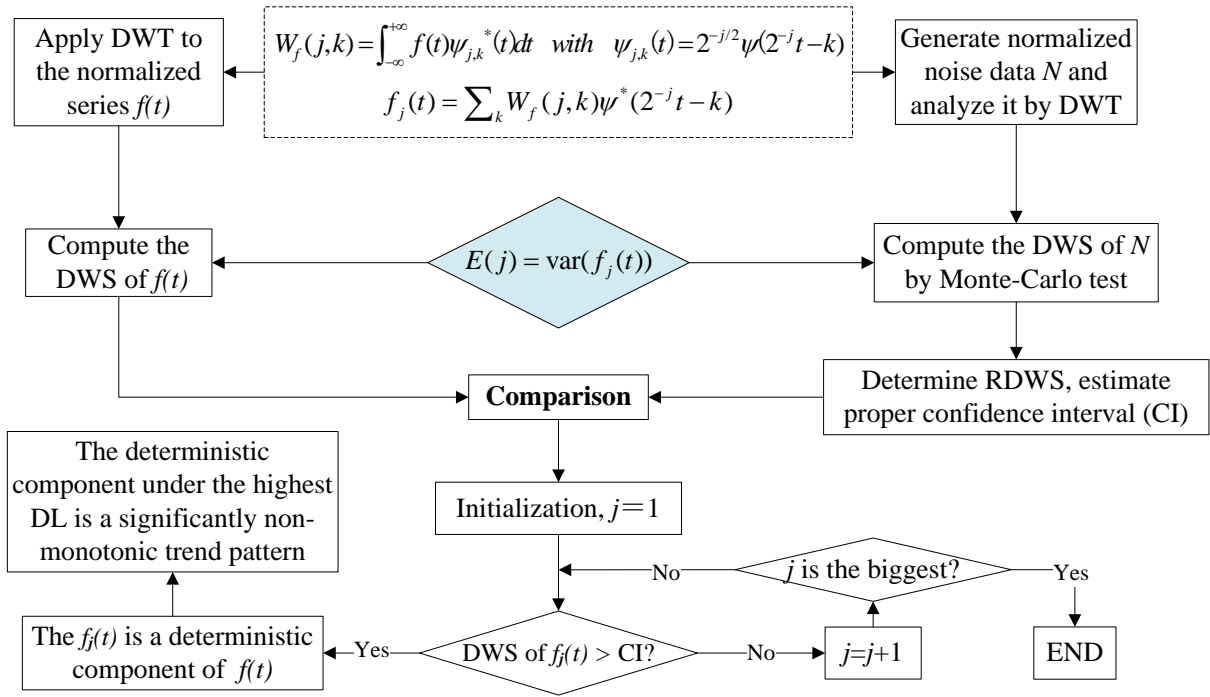
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459 **Figures**

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464 **Figure 1.** Technical flowchart for identification of the non-monotonic trend pattern in a time

465 series using the discrete wavelet spectrum approach developed. In the figure, “DWT” is

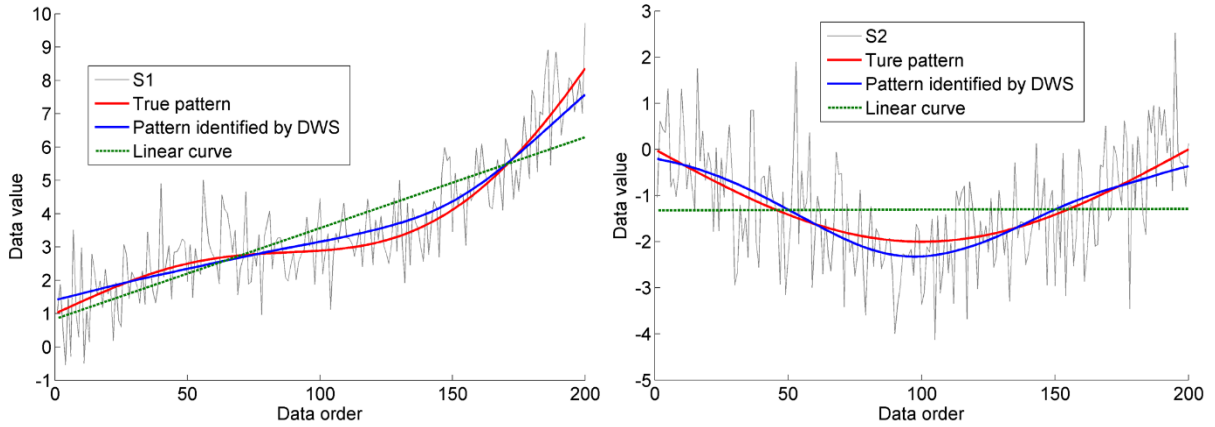
466 the discrete wavelet transform, “DWS” is the discrete wavelet spectrum, “RDWS” is the

467 reference discrete wavelet spectrum, “DL” is the decomposition level, and “CI” is the

468 confidence interval.

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472 **Figure 2.** Non-monotonic trend patterns in the synthetic series S1 and S2 identified by the

473 discrete wavelet spectrum (DWS) approach, and the linear trends in the two series.

474 Synthetic series S1 is generated as: $S1=1.112^{0.1t}+0.8 \times \sin(0.01\pi t)+\alpha$; and synthetic series

475 S2 is generated as: $S2=\sin(0.04\pi t)+2 \times \sin(\pi+0.005\pi t)+\alpha$, where α is a random process

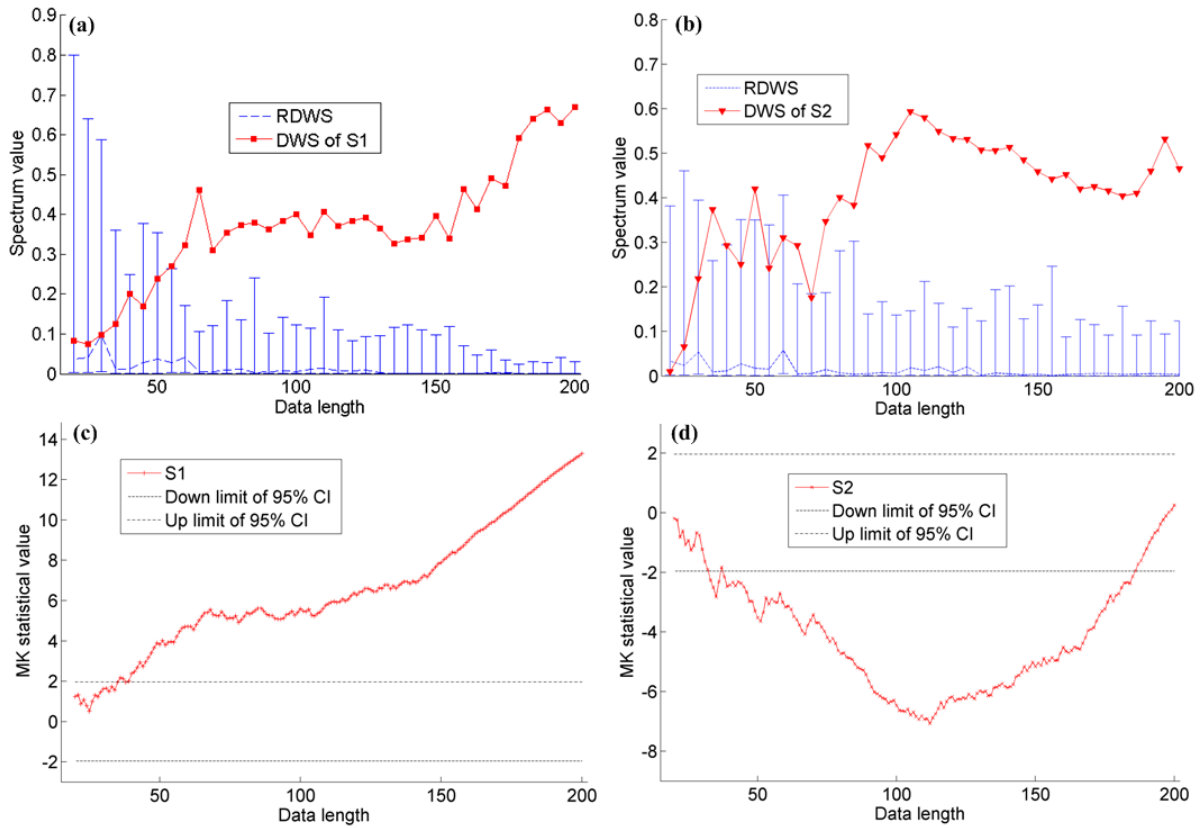
476 following the standard normal distribution.

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Figure 3. Evaluation of statistical significance of non-monotonic trend patterns in the

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synthetic series S1 (a) and S2 (b) with different data length by the discrete wavelet

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spectrum (DWS) approach, and the results by the Mann-Kendall (MK) test (c and d). In

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figure a and b, the blue line is the reference discrete wavelet spectrum (RDWS) with 95%

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confidence interval under each data length; if the red point at certain data length is above

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the blue bar, it is thought that the trend pattern is significant at 95% confidence level. In

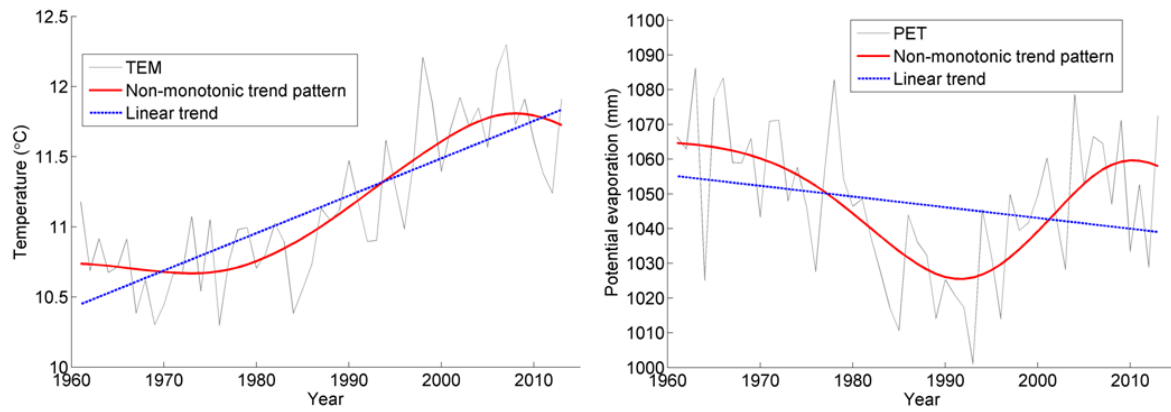
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figure c and d, the two black dash lines indicate 95% confidence interval (CI) with the

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thresholds of ± 1.96 in the MK test.

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492 **Figure 4.** Non-monotonic trend patterns in the annual time series of the mean air temperature

493 (TEM) and the potential evaporation (PET) over China from 1961-2013 identified by the

494 discrete wavelet spectrum (DWS) approach, and the linear trends in the two series.

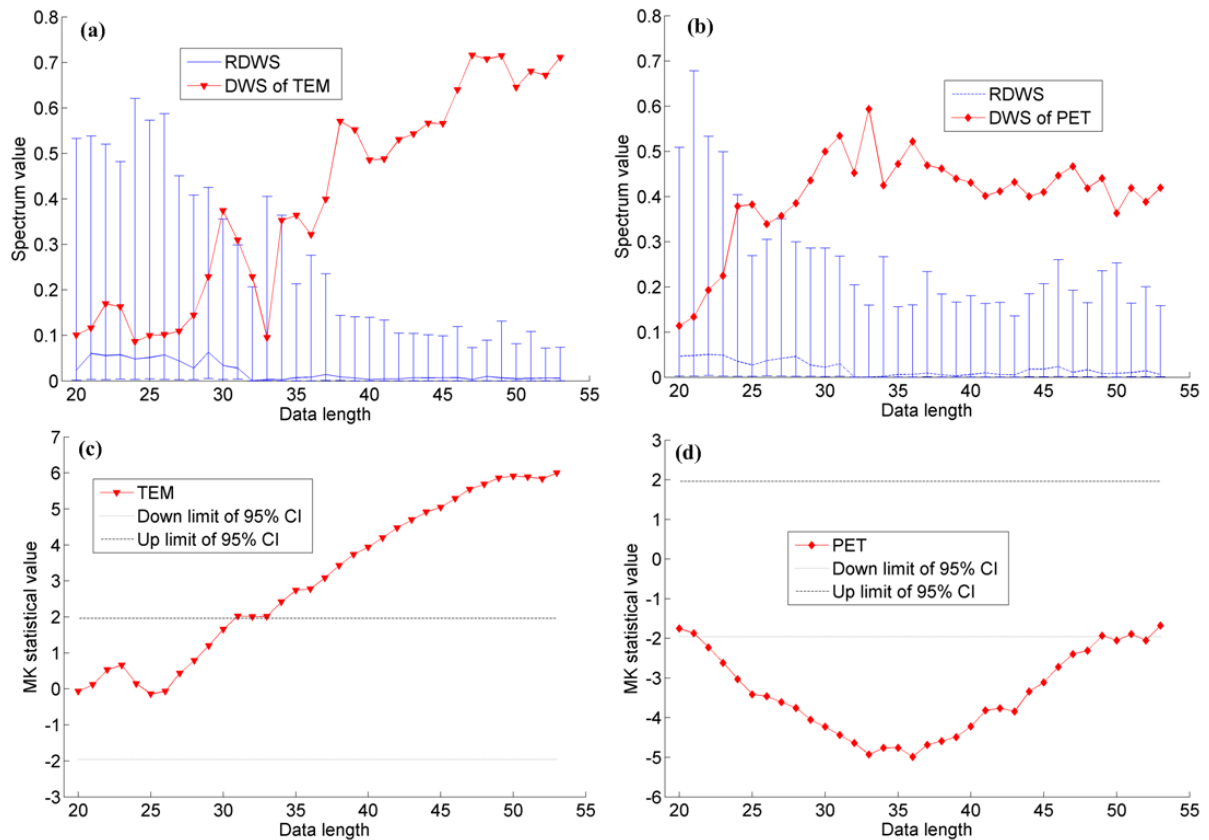
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501 **Figure 5.** Evaluation of statistical significance of non-monotonic trend patterns in the annual

502 time series of the mean air temperature (TEM, a) and the potential evaporation (PET, b)

503 over China with different data length by the discrete wavelet spectrum (DWS) approach,

504 and the results by the Mann-Kendall (MK) test (c and d). In figure a and b, The blue line

505 is the reference discrete wavelet spectrum (RDWS) with 95% confidence interval under

506 each data length; and in figure c and d, the two black dash lines indicate 95% confidence

507 interval (CI) with the thresholds of ± 1.96 in the MK test.